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The Check Problem of Food Thermal Processes: a Mathematical Solution

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Abstract

To calculate the sterilizing value *U*, and hence, the microbial lethality *F* in thermal processes of the canned food, starting from the knowledge of heating time *B*, a mathematical modeling was carried out. Therefore it's useful to verify the desired microbial destruction (check problem) and it was obtained by reversing the mathematical approach carried out in a previous work [23] for the design problem, namely to calculate the retort heating time *B*, starting from a desired lethality *F* and, hence from the f_h/U parameter. A comparison between the predicted f_h/U, related to the lethality *F* calculated with the mathematical model of the present work and the desired Stumbo's values of f_h/U, provided the following statistical indices: a mean relative error MRE=1.18±2.11%, a mean absolute error MAE=1.61±11.7 and a determination coefficient R^2=0.991, better than ANN models. The mathematical procedure, quickly usable also with a spreadsheet, replaces the 57 Stumbo's tables and 18512 data sets in the Ball formula method.

Keywords: Mathematical modeling, Thermal process check, Canned food, Food engineering

1. Introduction

Among the various methods for lengthening the food shelf life, the canned food is the best compromise between food safety, nutritional value and costs [1 and 2].
The two mathematical laws introduced by Bigelow [3], describing the destruction of a microbial population and the alteration of the constituents (enzymes, proteins

and vitamins) by using the couple temperature-time, are insufficient for modeling and subsequent calculating the thermal processes of canned food. In fact it's necessary to complete the two Bigelow laws by the mathematical one, describing the heat penetration in the mass of canned food.

The same Bigelow [3], proposed a graphical method (known as general method), by combining the two laws on microbial destruction and the experimental heat penetration curve, to determine the optimum heating time (*B*) of the canned food to obtain a given sterilization.

The general method was improved during more steps by various authors [4, 5, 6, and 7], achieving an excellent result [8].

As an alternative to general method, Ball [9] proposed a method (known as formula method) by combining the two Bigelow laws and some equations of heat penetration. The same Ball [10] improved his method, but a great expansion to cover the whole range of thermal death parameters was made by Stumbo [11]: 57 tables of Stumbo vs. 1 table of Ball.

A comparison of the Ball-Stumbo formula method with other formula methods later developed [12 and 13], was conducted by Smith and Tung [14]. They established the most accurate results by using the first.

Unfortunately the consultation of 57 Stumbo's tables with 18,513 datasets poses serious problems on its computerization. In last years, a third method was also proposed by using computational thermo-fluid dynamics. These CFD methods proved to be a valuable tool to ensure food safety and nutritional quality [15, 16, 17, 18]. Nevertheless for this third numerical approach, a high computing power and long calculation times are necessary [19]. Besides, many input data like the heat transfer coefficient of the heating and cooling medium, thermal diffusivity of the food product, can shape and dimensions, processing conditions and a good practice in the use of the CFD (right choice of mesh, etc.) are required.

As an alternative, in recent years, ANN method (artificial neural networks) was proposed [19, 20 and 21]. It consists in a high number of algebraic equations solved by an information processing system (black box) that learns from 18,513 Stumbo's datasets.

In a previous work [22], the 18,513 Stumbo's datasets, were also transformed into a mathematical model, based on ten equations for the computerized solution of check problem, that is the verification of desired microbial *lethality F*.

Instead for the design problem, i.e. the calculation of the *g* value, necessary to compute directly the thermal process time *B* with Ball's formula, known a priori the *process lethality F*, a mathematical approach, which consisted of only three equations, was proposed [23].

Considering the need to simplify and to improve the previous mathematical model of ten equations [22], proposed for the solution of the check food thermal process problem and considering the new type of mathematical solution found for the design problem [23], the overall objective of this paper was to develop a new mathematical procedure, that solves the check problem i.e. to furnish more quickly and precisely the microbial *lethality F* for a given heating time (*B*) of the canned food.

2. The mathematical problem

For a given constant temperature (i.e. 121.1°C) and for a given number of decimal reductions *n* of viable microorganisms, the heating time that needed for the thermal death of microorganisms is known as *F* (first Bigelow law):

$$F = n \cdot D_{121.1} \tag{1}$$

where $D_{121.1}$ is the decimal reduction time at reference temperature 121.1°C, experimentally known as function of the target microorganism and where *F* is known as *process lethality*.

During food thermal processes (sterilization, cooking, pasteurisation etc.), the temperature inside the canned food is not constant. It slowly increases over the heating time and then slowly decreases over the subsequent cooling time (Fig. 1).

The alteration in temperature *T* vs. time *t* in the coldest point of the canned food causes a modification in the decimal reduction time, now called D_T instead $D_{121.1}$. Accordingly the *process lethality F* becomes [23]:

$$F = n \cdot D_{121.1} = \int_0^t 10^{\frac{T-121.1}{z}} dt = \int_0^B 10^{\frac{T-121.1}{z}} dt \tag{2}$$

Equation (2) summarizes the two Bigelow's laws. For its integration up to total heating time *B*, it needs the relationship between the coldest point temperature *T* and the time *t* (also known as temperature-time history or heat penetration curves).

Excluding a possible initial lag period, the temperature-time equation considered by Ball [9] was:

$$T = T_R - (T_R - T_0) \cdot J_{ch} \cdot e^{-\frac{2.3 \cdot t}{f_h}} \tag{3}$$

where T_R (°C) is the retort temperature, T_0 (°C) is the initial food temperature, J_{ch} is the heating rate lag factor at the can center (coldest point), and f_h (min) is the heating rate index.

Ball and Olson [10] and Stumbo [11], used also the equation (3) to obtain the relationship among $g = (T_R - T_g)$ (°C), that is the difference between the retort temperature T_R and coldest point temperature T_g(°C) at the end of the heating process (figure 1) and the total heating time B:

$$B = f_h \cdot \log\left[\frac{J_{ch}(T_R - T_0)}{g}\right] \tag{4}$$

If the retort temperature T_R is not equal to the conventional 121.1°C, preferably a higher value for improving exergetic efficiency [24], the time required to obtain a given *F* value, is known as the *sterilizing value U*:

$$U = F \cdot 10^{\frac{121.1 - T_R}{z}} \tag{5}$$

In the design problem, it is required the prediction of process time *B* to obtain a given *lethality* (*F* or *U*) [25]. On the contrary, in the check problem it needs the verification of lethality (*F* or *U*) for a given heating time *B*.
In any case a preliminary experimental assessment of the parameters of the heating and cooling curves (f_h, J_{ch}, f_c, J_{cc}), is needed.

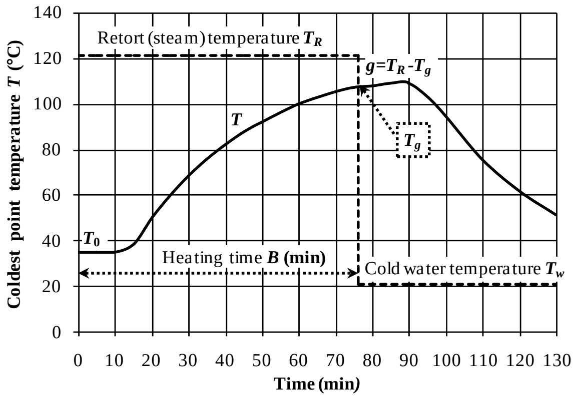

FIGURE 1 Heat penetration curve or temperature-time history of the coldest point of the canned food during the entire thermal process from a first heating period to a second cooling period

In the *heating phase*, with the retort temperature considered constant, combining equations (3) and (2) and then by integrating, Ball and Olson obtained [10]:

$$F_h = -\frac{f_h}{2.3} \cdot 10^{\frac{T_R - 121.1}{z}} \left[\mathrm{Ei}\left(\frac{-2.3 \cdot g}{z} \right) \right] \tag{6}$$

where Ei is the Exponential integral function and F_h is the *lethality* during the heating phase. Equation (6) is valid under Ball condition: $z \le 15°C$.
Recalling equation (5), the *sterilizing value* U_h becomes:

$$U_h = -\frac{f_h}{2.3} \cdot \mathrm{Ei}\left(\frac{-2.3 \cdot g}{z} \right) \tag{7}$$

During the subsequent *cooling phase* (fig. 1), Ball represented the temperature-time history with a hyperbola and an equation similar to the function (3). Both equations are valid under the Ball conditions: $z \le 15°C$ and J_{cc} =1.41 (J_{cc} is cooling lag factor).
Then, integrating equation (2), Ball obtained the equation for *cooling process lethality* F_c. It was more complicated than that of the *heating process lethality* F_h and it is indicated briefly as follows:

$$F_c = -\frac{f_c}{2.3} \cdot 10^{\frac{T_R - 121.1}{z}} \cdot F_c(g, z, T_w) \tag{8}$$

where $F_c(g, z, T_w)$ represents the influence of the *g* value, z value and the cold water temperature T_w, used in the retort during cooling.
Considering T_w constant and equal to 21.1°C (70°F) and the cooling rate index $f_c = f_h$ [11], the *sterilizing value* U_c, during the *cooling phase,* is then:

$$U_c = -\frac{f_h}{2.3} \cdot F_c(g, z) \tag{9}$$

The sum of U_h and U_c, is the *sterilizing value U* of the whole thermal process:

$$U = U_h + U_c = -\frac{f_h}{2.3}\left[\text{Ei}\left(\frac{-2.3 \cdot g}{z}\right) + F_c(g, z)\right] \tag{10}$$

For each value of $z \le 15°\text{C}$, equation (10) can be summarized:

$$\frac{f_h}{U} = f(g) \tag{11}$$

Because of *f*(*g*) contains the Ei function, that is tabulated, Ball obtained a table that furnished *g* value, in correspondence of each $\frac{f_h}{U}$ values between 0.3 and 1000, and each *z* values between 3.3 and 15°C.
With *z* value outside these limits and the cooling lag factor J_{cc} different to 1.41, the values of f_h/U are different.
Starting from this problem, Stumbo [11] enlarged the initial Ball table to 57 tables of $f_h/U{:}g$, valid for a wide range of *z* values (5.5°-111.1°C; 10°-200°F) and cooling lag factor J_{cc} values (0.4-2), keeping valid the hypothesis that the cooling rate index $f_c = f_h$.
Therefore, Stumbo expanded the applicability of Ball's formula method for any kind of thermal process (*z* value) and for any canned food (J_{cc} value). Unfortunately, the method is unsuitable for its computerization. Rather, Stumbo has worsened this problem because the Ball's table has turned into 57 Stumbo's tables representing the function:

$$\frac{f_h}{U} = f(g, z, J_{cc}) \tag{12}$$

3. Proposal of a solution method

The indirect problem, that is the check problem, consists in the attainment of lethality *F* for a given heating time *B* of the canned food.
By reversing the Ball's formula (4), *g* value, that is the difference between the retort temperature T_R and coldest point temperature T_g at the end of the heating process, is obtained:

$$g = J_{ch}\left(T_R - T_0\right)\cdot\exp\left(\frac{-2.3B}{f_h}\right) \tag{13}$$

In a previous work [23], about the thermal processes design, an equation correlating g value with the thermo-physical and geometrical parameters of canned food (z, J_{cc} and f_h) and the sterilizing value U, was obtained:

$$g = \frac{z}{2.3}\cdot\exp\left(-\gamma - \frac{2.3\cdot U}{f_h}\right)\cdot H\cdot K \tag{14}$$

where γ was Euler's constant (γ=0.5772…); H was the product of two polynomials, understood as the first correction factor, $H = \left(au^3 + bu^2 + cu + d\right)\cdot\left(Ay^3 + By^2 + Cy + D\right)$; K was a second polynomial correction factor $K = \left[1 + \left(J_{cc} - 0.4\right)\cdot\left(pu^2 + qu + r\right)\cdot\left(Pz^2 + Qz + R\right)\right]$; $u = \ln\left(f_h/U\right)$, $y = \ln(z)$.
Now, if we consider J_{cc}=0.4, then K=1 and if we reverse the equation (14), we obtain:

$$U = \frac{f_h}{2.3}\cdot\left[\ln\left(\frac{z\cdot G}{2.3\cdot g}\right) - \gamma\right] = \frac{f_h}{2.3}\ln\left(\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}}\right) \tag{15}$$

Where G is also a correction factor and it coincides with the product of two polynomials; the second polynomial is a function of $y = \ln(z)$, like the previous H, but the first one is a function of $v = \ln(1+g)$, instead of $u = \ln\left(f_h/U\right)$ used in H:

$$G = \left(av^3 + bv^2 + cv + d\right)\cdot\left(Ay^3 + By^2 + Cy + D\right) \tag{16}$$

The equation (16), as a double polynomial, is a result of a careful analysis of the data in Stumbo's tables and as a consequence of a trial-and-error approach.
By developing the product of two polynomials, the products of the coefficients *a, b, c, d, A, B, C* and *D* were obtained through a multiple analysis [26] using Stumbo's datasets relating f_h/U, g and z, with J_{cc}=0.4, $f_h/U \geq 0.3$ and $10° \leq z \leq 111°C$. The results is shown in table 1 and the regression analysis was characterized by a R^2=0.9998.
For a better fitting of Stumbo's datasets a second regression was carried out. In this case using always the values of J_{cc}=0.4 and only the values of f_h/U, g and z that satisfied the following inequality:

$$\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}} \leq 1.055 \tag{17}$$

This because of, when $\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}} \to 1$ and hence $\ln\left(\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}}\right) \to 0$, any tiny error of the first regression represented by equation (16), can easily produce, by equation (15), negative values of sterilizing value *U*. So, a second polynomial *G'* was arranged:

$$G' = \left(a'v^3 + b'v^2 + c'v + d'\right)\cdot\left(A'y^3 + B'y^2 + C'y + D'\right) \quad (18)$$

Similarly to the equation (16), the products of the coefficients *a'*, *b'*, *c'*, *d'*, *A'*, *B'*, *C'* and *D'*, obtained by multiple regression with R^2=0.99999, are shown in table 1. Therefore the second polynomial *G'* must substitute the first one *G*, only when $\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}} \leq 1.055$. To solve this problem, a combination of *G* and *G'* by using the hyperbolic function tanh, was studied and here proposed:

$$G = \tau^{+}\cdot G + \tau^{-}\cdot G' \quad (19)$$

Where:

$$\tau^{+} = \frac{1}{2} + \frac{1}{2}\tanh\left[1000\left(\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}} - 1.055\right)\right] \quad (20)$$

$$\tau^{-} = \frac{1}{2} + \frac{1}{2}\tanh\left[-1000\left(\frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}} - 1.055\right)\right] \quad (21)$$

The functions τ^{+} and τ^{-} vs. $x = \frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}}$ is shown in figure 2.

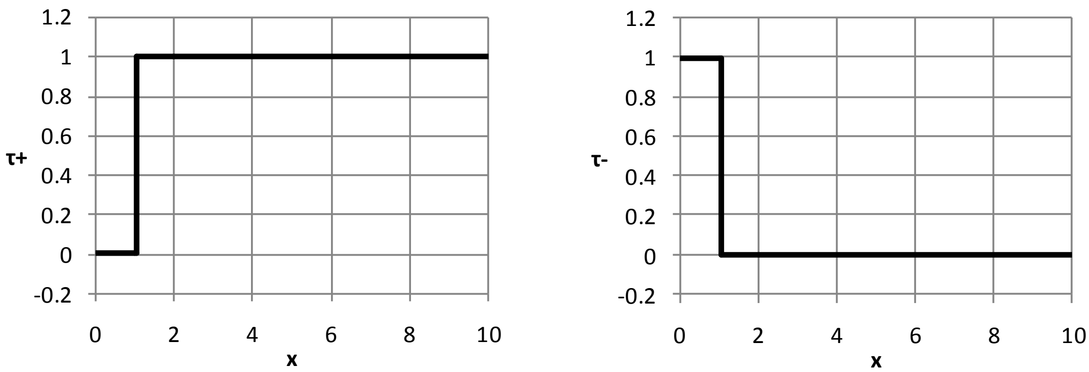

FIGURE 2 Functions τ^{+} and τ^{-} vs. $x = \frac{z\cdot G}{2.3\cdot g\cdot e^{\gamma}}$

Recall that equation (15) is valid under condition J_{cc}=0.4. Looking at the Stumbo's datasets, for $f_h/U \geq 0.3$ and $10° \leq z \leq 111°C$, it was observed that the relationship $g : J_{cc}$ is continuous and monotonically increasing. Therefore, to extend the use of equation (15) for J_{cc}>0.4, it's necessary to modify the g value, obtained from equation (13), before introducing it in equation (15). For this reason a function $g(J_{cc} \geq 0.4) \rightarrow g(J_{cc} = 0.4)$ was developed, starting from Stumbo's datasets. For greater convenience $g(J_{cc} \geq 0.4)$ was indicated with symbol g, like in equation (13), and $g(J_{cc} = 0.4)$ with symbol g_m (g modified). In fact, after a careful analysis of Stumbo's datasets, it was observed that $g = g_m + \Delta g \cdot 5 \cdot (J_{cc} - 0.399)$, where the increment Δg was depending only of g_m and z through polynomials:

$$g = g_m + \left(pg_m^2 + qg_m\right) \cdot \left(Pz^3 + Qz^2 Rz + S\right) \cdot 5 \cdot \left(J_{cc} - 0.399\right) \tag{22}$$

By developing the product of two polynomials, the previous equation symbolically becomes:

$$g = \alpha \cdot g_m^2 + \beta \cdot g_m \tag{23}$$

Where:

$$\alpha = \left(pPz^3 + pQz^2 + pRz + pS\right) \cdot 5 \cdot \left(J_{cc} - 0.399\right) \tag{24}$$

$$\beta = \left(qPz^3 + qQz^2 + qRz + qS\right) \cdot 5 \cdot \left(J_{cc} - 0.399\right) + 1 \tag{25}$$

Solving equation (23), g_m was obtained:

$$g_m = \frac{-\beta + \sqrt{\beta^2 + 4\alpha \cdot g}}{2\alpha} \tag{26}$$

Therefore, it needs to pay attention in equation (15), that becomes:

$$U = \frac{f_h}{2.3} \cdot \left[\ln\left(\frac{z \cdot G}{2.3 \cdot g_m} \right) - \gamma \right] = \frac{f_h}{2.3} \cdot \ln\left(\frac{z \cdot G}{2.3 \cdot g_m \cdot e^{\gamma}} \right) \tag{27}$$

Where G in equation (16), (17), (18), (19), (20) and (21) must be correlated to $v = \ln(1 + g_m)$ instead of $v = \ln(1 + g)$.

TABLE 1 Coefficients of polynomials *G and G'*

a·A	-0.03117	*a'·A'*	-0.06892
a·B	0.40441	*a'·B'*	0.31714
a·C	-1.81641	*a'·C'*	0
a·D	2.83278	*a'·D'*	0
b·A	0.03365	*b'·A'*	-0.05119

TABLE 1 (Continued): Coefficients of polynomials *G and G'*

$b{\cdot}B$	-0.49036	$b'{\cdot}B'$	-0.57530
$b{\cdot}C$	2.48951	$b'{\cdot}C'$	0
$b{\cdot}D$	-4.32509	$b'{\cdot}D'$	0
$c{\cdot}A$	0.00394	$c'{\cdot}A'$	0.60785
$c{\cdot}B$	0.12946	$c'{\cdot}B'$	0
$c{\cdot}C$	-1.41272	$c'{\cdot}C'$	0
$c{\cdot}D$	3.41010	$c'{\cdot}D'$	0
$d{\cdot}A$	-0.05496	$d'{\cdot}A'$	-0.61067
$d{\cdot}B$	0.48405	$d'{\cdot}B'$	0
$d{\cdot}C$	-1.45712	$d'{\cdot}C'$	0
$d{\cdot}D$	2.32634	$d'{\cdot}D'$	3.39600

TABLE 2 Coefficients of polynomial Δg of equation (22)

$p{\cdot}P$	$-3.47449 \cdot 10^{-08}$
$p{\cdot}Q$	$6.42525 \cdot 10^{-07}$
$p{\cdot}R$	$2.14817 \cdot 10^{-05}$
$p{\cdot}S$	$2.51153 \cdot 10^{-04}$
$q{\cdot}P$	$4.19566 \cdot 10^{-08}$
$q{\cdot}Q$	$2.15151 \cdot 10^{-05}$
$q{\cdot}R$	$2.86640 \cdot 10^{-03}$
$q{\cdot}S$	$5.17551 \cdot 10^{-02}$

4. Results and discussion

The calculated f_h/U values, obtained by $\frac{f_h}{U} = \frac{2.3}{\ln\left(\frac{z \cdot G}{2.3 \cdot g_m \cdot e^{\gamma}}\right)}$, deduced from equation (27), and by using the other equations (16), (18), (19), (20), (21), (24), (25) and (26) proposed in present mathematical model, were compared to the desired f_h/U values obtained from Stumbo's tables (fig. 3).

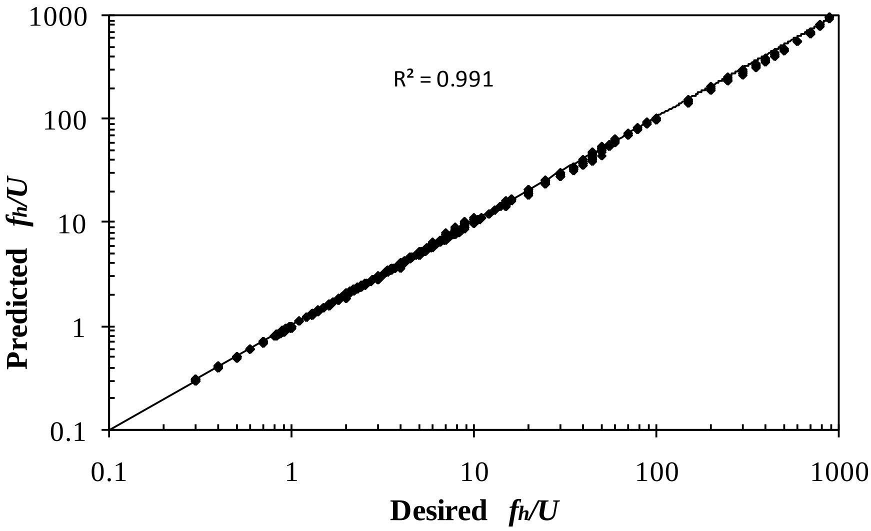

FIGURE 3 - Predicted f_h/U values using the mathematical model versus desired Stumbo's values of f_h/U.

Table 3 also shows the statistical indices: mean relative error *MRE*, mean absolute error *MAE* and determination coefficient R^2.

The results of this work are better with respect to the previous mathematical model [22] and to the ANN models of Sablani & Shayya [20] and Mittal & Zhang [21].

TABLE 3 Comparison of *MRE* and *MAE* obtained as difference between predicted f_h/U by various authors and desired f_h/U from Stumbo's tables

	f_h/U				
	MRE	*S.D.*	*MAE*	*S.D.*	R^2 in Fig. 2
Sablani & Shayya [20]	2.42	3.90	3.21	20.76	0.979
Friso[22]	2.46	3.38	3.38	20.49	0.982
Mittal & Zhang [21]	1.41	3.40	2.43	15.97	n.d.
This work	1.18	2.11	1.61	11.27	0.991

MRE = Mean Relative Error, *MAE* = Mean Absolute Error, *S.D.* = Standard Deviations

5. Conclusions

Among the various methods for food thermal calculations, the Ball's formula method is still interesting for the cannery industry. In its complete version, it is associated with 18,512 Stumbo's datasets which are to be consulted both for the check problem as well as for the design problem.
In this work, for a faster solution of the microbial *lethality* check, it was proposed a mathematical model, made of nine equations to be sequentially solved.
In this way it's possible to eliminate the consultation of 57 Stumbo's tables both manual as well as through the computerized storage and interpolation of 18,512 Stumbo's datasets.

The process *lethality F* and, hence, the *sterilizing values U* and, finally, the f_h/U values, which could be called number of Ball, calculated by applying the mathematical approach of this work, were closer to the Stumbo f_h/U values, compared to those obtained by a previous mathematical model [22] and by ANN models [20 and 21].

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