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Bayesian Tail Risk Interdependence Using Quantile Regression

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Abstract. Recent financial disasters emphasised the need to investigate the consequences associated with the tail co-movements among institutions; episodes of contagion are frequently observed and increase the probability of large losses affecting market participants' risk capital. Commonly used risk management tools fail to account for potential spillover effects among institutions because they only provide individual risk assessment. We contribute to the analysis of the interdependence effects of extreme events, providing an estimation tool for evaluating the co-movement Value-at-Risk. In particular, our approach relies on a Bayesian quantile regression framework. We propose a Markov chain Monte Carlo algorithm, exploiting the representation of the Asymmetric Laplace distribution as a location-scale mixture of Normals. Moreover, since risk measures are usually evaluated on time series data and returns typically change over time, we extend the model to account for the dynamics of the tail behaviour. We apply our model to a sample of U.S. companies belonging to different sectors of the Standard and Poor's Composite Index and we provide an evaluation of the marginal contribution to the overall risk of each individual institution.

Keywords: Bayesian quantile regression, time-varying conditional quantile, risk measures, state space models.

1 Introduction

In recent years particular attention has been devoted to measuring and quantifying the level of financial risk within a firm or investment portfolio. The Value-at-Risk (VaR) has become one of the most diffuse risk measurement tools. It measures the maximum loss in value of a portfolio over a predetermined time period for a given confidence level. In fact, in the current banking regulation framework, the VaR has become an important risk capital evaluation tool where different institutions are considered as independent entities. Unfortunately, such a risk measure fails to consider the institution as part of a system which might itself experience instability and thus spread new sources of systemic risk. For a comprehensive and up to date overview of VaR and related risk measures see, for example, Jorion (2007) and McNeil et al. (2005). Recent financial disasters emphasised the need for a thorough investigation of the co-movement among institutions in order to evaluate their tail interdependence relationships. Especially during periods of financial distress, episodes of contagion among institutions are not rare and thus need to be taken into account in order to analyse the overall level of health of a financial system: company specific risk can not be appropriately assessed in isolation, without accounting

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for potential spillover effects to and from other firms. For this reason different risk measures have been proposed in the literature that analyse the tail-risk interdependence, (see Acharya et al., 2012, 2010; Adams et al., 2010; Brownlees and Engle, 2012; Billio et al., 2012). Recently, Adrian and Brunnermeier (2011) introduced the so called Conditional Value-at-Risk (CoVaR), which is defined as the overall VaR of an institution, conditional on another institution being in distress. There are many possible ways to infer on these risk measures; in particular, the most common approaches to estimate VaR are the variance–covariance methodology, historical and Monte Carlo simulations. For an overview of alternative parametric and nonparametric methodologies and processes to generate VaR estimates see Jorion (2007) and Lee and Su (2012). See also Chao et al. (2012) and Taylor (2008) for recent developments. Bernardi (2013) and Bernardi et al. (2012) propose to estimate VaR and related risk measures by fitting asymmetric mixture models to the unconditional distribution of returns. Moreover, Adrian and Brunnermeier (2011) use a quantile regression approach to estimate the CoVaR in a frequentist framework; Girardi and Ergün (2013) propose a multivariate Generalized ARCH model to estimate it; Bernardi et al. (2013) and Bernardi and Petrella (2014) consider the class of multivariate hidden Markov models; and Castro and Ferrari (2014) propose a CoVaR-based hypothesis testing procedure to rank systemically important institutions.

In this paper, we measure tail risk interdependence using the CoVaR framework of Adrian and Brunnermeier (2011) and since this measure is a quantile of a conditional distribution calculated at a given quantile of its conditioning distribution, we address the estimation problem using a quantile regression approach. Quantile regression has been popular as a simple, robust and distribution free modeling tool since the seminal work of Koenker and Basset (1978) and Koenker (2005). It provides a way to model the conditional quantiles of a response variable with respect to some covariates, in order to have a more complete picture of the entire conditional distribution than traditional linear regression. In fact, sometimes problem-specific features, such as skewness, fat-tails, outliers, truncated and censored data, and heteroskedasticity, can shadow the nature of the dependence between the variable of interest and the covariates so that the conditional mean may not be enough to understand the nature of that dependence. In particular, not only is the quantile regression approach appropriate when the underlying model is nonlinear or the innovation terms are non-Gaussian, but also when modeling the tail behaviour of the underlying distribution is the primary interest. There are a number of papers on quantile regression utilising both frequentist and Bayesian frameworks dealing with parametric and nonparametric approaches. For a detailed review and references, see for example, Lum and Gelfand (2012) and Koenker (2005).

In quantile regression, the quantile of order τ of a dependent variable Y is expressed as a function of covariates $\mathbf{X}$, say $q^{\tau}(\mathbf{X})$. In literature different representations have been proposed to specify the quantile function $q^{\tau}(\mathbf{x})$; the most common specification is the linear one adopted hereafter:

$$q^{\tau}(\mathbf{x}) = \mathbf{x}^{\mathsf{T}}\boldsymbol{\theta}, \tag{1}$$

where $\mathbf{x}^{\mathsf{T}}$ denotes the transpose of $\mathbf{x}$.

The problem of estimating $q^\tau(\mathbf{x})$ through quantile regression has been considered both from the frequentist and Bayesian points of view. In the former case, Koenker and Basset (1978) show that the quantile estimation problem is solved by the following minimisation problem:

$$\text{argmin}_{q^\tau} \sum_{t=1}^{T} \rho_\tau \left(y_t - q^\tau(\mathbf{x}_t)\right), \tag{2}$$

where $(y_t, \mathbf{x}_t)$ for $t = 1, \ldots, T$ are observations from $(Y, \mathbf{X})$ and $\rho_\tau(y) = y(\tau - \mathbb{1}(y < 0))$ is the quantile loss function, where $\mathbb{1}(\cdot)$ is the indicator function. The Bayesian quantile regression approach (see Yu and Moyeed, 2001; Kottas and Gelfand, 2001; Kottas and Krnjajic, 2009; Sriram et al., 2013) instead considers the distribution of $Y \mid \mathbf{x}$ as belonging to the Asymmetric Laplace distribution family, denoted by $\text{ALD}(\tau, q^\tau(\mathbf{x}), \sigma)$, with positive σ, whose density function is given by:

$$\text{ald}(y \mid q^\tau(\mathbf{x}), \sigma) = \frac{\tau(1-\tau)}{\sigma} \exp\left\{-\frac{\rho_\tau(y - q^\tau(\mathbf{x}))}{\sigma}\right\} \mathbb{1}_{(-\infty,\infty)}(y). \tag{3}$$

A useful feature of the $\text{ALD}(\tau, q^\tau(\mathbf{x}), \sigma)$ distribution is that the regression function $q^\tau(\mathbf{x})$ corresponds exactly to the theoretical τ-th quantile of $Y \mid \mathbf{x}$.

Quantile regression methods have been extensively considered in the literature as an approach for evaluating the VaR (see, among others, Huang, 2012; Schaumburg, 2010; Chernozhukov and Du, 2008; Kuester et al., 2006; Taylor, 2008; Gerlach et al., 2011; Chen et al., 2012a,b,c); recently Adrian and Brunnermeier (2011), Chao et al. (2012), Fan et al. (2013), Hautsch et al. (2014), Chao et al. (2012) and Castro and Ferrari (2014) considered the same approach to also calculate the CoVaR.

In this paper, we propose the Bayesian approach to cast the CoVaR within a quantile regression framework and we show how to model and estimate it as a quantile of the conditional distribution of an institution, k, given a particular quantile of another institution, j. Bayesian methods are very useful and flexible tools for combining data with prior information in order to provide the entire posterior distribution for the parameters of interest. It also allows for parameter uncertainty to be taken into account when making predictions. In the context of the present paper, since the quantities of interest are risk measures, understanding about the whole distribution becomes more relevant due to the interpretation of the VaR and CoVaR as financial losses. In the Bayesian quantile regression framework, the inference on the unknown parameters is made analytically tractable because it relies on the exact likelihood function for the quantiles of interest, see equation (3). Moreover, by post–processing the Markov chain Monte Carlo (MCMC) output we are able to make inference on the CoVaR function as well as to calculate its posterior credible sets which is useful to assess the statistical accuracy of our estimates.

In the second part of the paper, we extend the proposed model to account for the dynamics of the tail behaviour, since risk measures are usually evaluated on time series data and returns typically change over time. We use time-varying quantiles to link the future tail behaviour of a time series to its past movements which is important in a risk management context. In particular, based on the ideas of De Rossi and Harvey (2009), we propose a dynamic model to capture the evolution of VaR and CoVaR. In order to provide a flexible solution for quantile modelisation whilst retaining a parsimonious representation, the time evolution of the process should be carefully selected. Hence, throughout the paper, we model the dynamics of the quantile functions' parameters as local linear trends; this is a suitable compromise between the degree of smoothness of the resulting quantiles and the ability of the model to capture changes over time. Time-varying quantiles represent a valid alternative to conditional quantile autoregression proposed in different contexts by Engle and Manganelli (2004), Gerlach et al. (2011), Gourieroux and Jasiak (2008) and Koenker and Xiao (2006).

To implement the dynamic Bayesian inference, we cast VaR and CoVaR models in state space representation and we run a Gibbs sampler algorithm using the Exponential-Gaussian mixture representation of Asymmetric Laplace distributions (see e.g. Kotz et al., 2001). This approach allows us to obtain a conditionallly Gaussian state space representation which permits an efficient numerical solution to the inferential problem. In order to make posterior inference, we use the maximum a posteriori summarising criterion and we prove that it leads to estimated quantiles having good sample properties according to De Rossi and Harvey (2009) results.

There are several applications of CoVaR which are interesting in both economics and finance. In this paper we analyse different U.S. companies belonging to several sectors of the Standard and Poor's Composite Index (S&P500) in order to evaluate the marginal contribution to the overall systemic risk of a single institution belonging to it. The empirical results show that the proposed models provide realistic and informative characterisation of extreme tail co-movements. Moreover, our findings suggest that the dynamic model we propose is more appropriate when dealing with financial time series data.

The paper is organised as follows: Section 2 contains a brief definition of Value-at-Risk and Conditional Value-at-Risk measures; Section 3 builds the time invariant Bayesian model and provides details on how to make inference using MCMC algorithms; Section 4 contains an extension of the previous framework to the time-varying case, representing marginal and conditional quantiles as functions of latent processes; Section 5 details the prior hyperparameters used in the following Section 6 which applies the proposed models to real data; Section 7 concludes.

2 VaR and CoVaR representations

Let $(Y_1, \ldots, Y_d)$ be a d-dimensional $(d > 1)$ random vector where each Y_j is expressed through some covariates $\mathbf{X} = (X_1, X_2, \ldots, X_M)$, $(M \geq 1)$. Keeping in mind that, for any $j \in \{1, \ldots, d\}$, Y_j, the variable of interest of institution j depends on some covariates $\mathbf{X}$ and that for any $k \in \{1, \ldots, d\}$, $k \neq j$, the behaviour of the variable Y_k, related to either institution k or the whole system, depends on covariates $\mathbf{X}$ as well as on the behaviour of the variable of institution j, Y_j. Without loss of generality, thereafter, we fix $\tau \in (0, 1)$ and suppose that we are interested in institutions j and k for $j \neq k$ and $(j, k) \in \{1, \ldots, d\} \times \{1, \ldots, d\}$.

Let us recall that the Value-at-Risk, $\mathrm{VaR}_j^{\mathbf{x},\tau}$ of institution j is the τ-th level conditional quantile of the random variable $Y_j \mid \mathbf{X} = \mathbf{x}$, i.e.

$$\mathbb{P}\left(Y_j \le \mathrm{VaR}_j^{\mathbf{x},\tau} \mid \mathbf{X} = \mathbf{x}\right) = \tau.$$

The Conditional Value-at-Risk $\left(\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}\right)$ is the Value-at-Risk of institution k conditional on $Y_j = \mathrm{VaR}_j^{\mathbf{x},\tau}$ at the level τ, i.e., $\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}$ satisfies the following equation

$$\mathbb{P}\left(Y_k \le \mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau} \mid \mathbf{X} = \mathbf{x}, Y_j = \mathrm{VaR}_j^{\mathbf{x},\tau}\right) = \tau. \tag{4}$$

Note that the CoVaR corresponds to the τ-th quantile of the conditional distribution of $Y_k \mid \{\mathbf{X} = \mathbf{x}, Y_j = \mathrm{VaR}_j^{\mathbf{x},\tau}\}$. Assuming the linear representation (1) of the quantiles of interest, we can write:

$$\mathrm{VaR}_j^{\mathbf{x},\tau} = \theta_{j,0}^{\tau} + \theta_{j,1}^{\tau} x_1 + \theta_{j,2}^{\tau} x_2 + \ldots + \theta_{j,M}^{\tau} x_M \tag{5}$$

$$\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau} = \theta_{k,0}^{\tau} + \theta_{k,1}^{\tau} x_1 + \theta_{k,2}^{\tau} x_2 + \ldots + \theta_{k,M}^{\tau} x_M + \beta^{\tau} \mathrm{VaR}_j^{\mathbf{x},\tau}, \tag{6}$$

where $\theta_{l,m}^{\tau}$ and β are unknown parameters with $l \in \{j,k\}$ and $m = 0,\ldots,M$. For simplicity we consider the same τ for both VaR and CoVaR and for ease of reading we drop the τ index from all parameters.

3 Time invariant quantile model

The use of Bayesian inference in a quantile regression context is now standard practice. In what follows, we adopt the approach used in Yu and Moyeed (2001) where data come from an Asymmetric Laplace distribution which is a convenient tool to deal with quantile regression problems in a Bayesian framework. Suppose that we observe $(\mathbf{y},\mathbf{x}) = (\mathbf{y}_t, \mathbf{x}_t)_{t=1}^T = (y_{j,t}, y_{k,t}, \mathbf{x}_t)_{t=1}^T$, T independent realizations of $(Y_j, Y_k, \mathbf{X})$. To estimate $\mathrm{VaR}_j^{\mathbf{x},\tau}$ and $\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}$ we consider the following equations:

$$y_{j,t} = \mathbf{x}_t^{\mathsf{T}} \boldsymbol{\theta}_j + \epsilon_{j,t} \tag{7}$$

$$y_{k,t} = \mathbf{x}_t^{\mathsf{T}} \boldsymbol{\theta}_k + \beta y_{j,t} + \epsilon_{k,t}, \tag{8}$$

for $t = 1,2,\ldots,T$, where β, $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}_k$ are unknown parameters of dimension 1, $(M+1)$, and $(M+1)$ respectively, and the first component of $\mathbf{x}_t$ is equal to 1, including a constant term in the regression function. Here, for any $t \in \{1,\ldots,T\}$, $\epsilon_{j,t}$ and $\epsilon_{k,t}$ are independent random variables distributed according to $\mathrm{ALD}(\tau, 0, \sigma_j)$ and $\mathrm{ALD}(\tau, 0, \sigma_k)$ respectively, with positive σ_j and σ_k. Due to the property of Asymmetric Laplace distributions, the functions $\mathbf{x}^{\mathsf{T}}\boldsymbol{\theta}_j$ and $\mathbf{x}^{\mathsf{T}}\boldsymbol{\theta}_k + \beta y_j$ correspond to the τ-th quantiles of $Y_j \mid \mathbf{X} = \mathbf{x}$ and $Y_k \mid \{\mathbf{X} = \mathbf{x}, Y_j = y_j\}$, respectively.

For a Bayesian modeling, we need to specify the prior distribution for the vector of the unknown parameter $\boldsymbol{\gamma} = (\boldsymbol{\theta}, \beta, \sigma_j, \sigma_k)$. We assume the following priors independent on the value of τ:

$$\pi(\boldsymbol{\gamma}) = \pi(\boldsymbol{\theta})\, \pi(\beta)\, \pi(\sigma_j)\, \pi(\sigma_k), \tag{9}$$

with $\boldsymbol{\theta} = (\boldsymbol{\theta}_j, \boldsymbol{\theta}_k)^\intercal \sim \mathcal{N}_{(2M+2)}\left(\boldsymbol{\theta}^0, \Sigma^0\right)$, $\beta \sim \mathcal{N}\left(\beta^0, \sigma_\beta^2\right)$, $\sigma_j \sim \mathcal{IG}\left(a_j^0, b_j^0\right)$ and $\sigma_k \sim \mathcal{IG}\left(a_k^0, b_k^0\right)$, and where $\Sigma^0 = \text{diag}\left(\Sigma_j^0, \Sigma_k^0\right)$, $\boldsymbol{\theta}^0 = \left(\boldsymbol{\theta}_j^0, \boldsymbol{\theta}_k^0\right)^\intercal$, β^0, $\sigma_\beta^2 > 0$, $a_j^0 > 0$, $b_j^0 > 0$, $a_k^0 > 0$ and $b_k^0 > 0$ are given hyperparameters, with Σ_j^0 and Σ_k^0 positive definite square matrices of dimension $M+1$. Notations $\mathcal{N}$ and $\mathcal{IG}$ refer to Gaussian and Inverse Gamma distributions, respectively. This choice is quite standard in the literature on quantile regression, having the advantage of giving closed form solution for the full conditional distributions. An alternative solution is proposed by Yu and Moyeed (2001) and Tokdar and Kadane (2012) where they assume improper priors for the quantile regression and scale parameters. In our model we opt for a proper prior structure to be coherent with that imposed on the time-varying framework, where the posterior is not guaranteed to be proper under improper priors. More details about the priors' hyperparameters elicitation procedure are given in Section 5.

As discussed in Yu and Moyeed (2001), due to the complexity of the likelihood function, the resulting posterior density for the regression parameters $\boldsymbol{\theta}$ and β does not admit a closed form representation for the full conditional distributions, and needs to be sampled by using MCMC-based algorithms. Following Kozumi and Kobayashi (2011), we instead adopt the well-known representation (see e.g. Kotz et al., 2001 and Park and Casella, 2008) of $\epsilon \sim \text{ALD}(\tau, 0, \sigma)$ as a location-scale mixture of Gaussian distributions:

$$\epsilon = \lambda\omega + \delta\sqrt{\sigma\omega}z, \tag{10}$$

where $\omega \sim \mathcal{E}\text{xp}\left(\sigma^{-1}\right)$ and $z \sim \mathcal{N}(0,1)$ are independent random variables and $\mathcal{E}\text{xp}(\cdot)$ denotes the Exponential distribution. Moreover, the parameters λ and δ^2 are fixed equal to

$$\lambda = \frac{1-2\tau}{\tau(1-\tau)}, \qquad \delta^2 = \frac{2}{\tau(1-\tau)}, \tag{11}$$

in order to ensure that the $\tau-$th quantile of ϵ is equal to zero. The previous representation (10) allows us to use a Gibbs sampler algorithm detailed in the next subsection. Exploiting this augmented data structure, the model defined by equations (7) and (8) admits, conditionally on w, the following Gaussian representation:

$$\begin{aligned} y_{j,t} &= \mathbf{x}_t^\intercal\boldsymbol{\theta}_j + \lambda\omega_{j,t} + \delta\sqrt{\sigma_j\omega_{j,t}}z_{j,t} & (12)\\ y_{k,t} &= \mathbf{x}_t^\intercal\boldsymbol{\theta}_k + \beta y_{j,t} + \lambda\omega_{k,t} + \delta\sqrt{\sigma_k\omega_{k,t}}z_{k,t}, & (13)\end{aligned}$$

for $t = 1, 2, \ldots, T$, where $z_{j,t}$, $z_{k,t}$ are independent and $\omega_{j,t}$, $\omega_{k,t}$ are independently drawn from $\mathcal{E}\text{xp}\left(\sigma_j^{-1}\right)$ and $\mathcal{E}\text{xp}\left(\sigma_k^{-1}\right)$, respectively. From equations (12) and (13), the distribution of $\mathbf{Y}$ conditional on the parameters vector $\boldsymbol{\gamma}$, the observed exogenous variables $\mathbf{x}$ and the augmented variables $\boldsymbol{\omega} = (\omega_{j,t}, \omega_{k,t})_{t=1}^T$, becomes

$$f(\mathbf{y} \mid \boldsymbol{\omega}, \mathbf{x}, \boldsymbol{\gamma}) = \prod_{t=1}^T \mathcal{N}\left(y_{j,t} \mid \omega_{j,t}, \mathbf{x}_t, \boldsymbol{\theta}_j, \sigma_j\right) \prod_{t=1}^T \mathcal{N}\left(y_{k,t} \mid \omega_{k,t}, y_{j,t}, \mathbf{x}_t, \beta, \boldsymbol{\theta}_k, \sigma_k\right).$$

3.1 Computations

Due to the Gaussian representation shown above, we are able to implement a partially collapsed Gibbs sampler algorithm based on data augmentation (see Liu, 1994 and

Van Dyk and Park, 2008). The key idea of the complete collapsed Gibbs sampler is to avoid simulations from the full conditional distributions of all of the model parameters $(\boldsymbol{\theta}_j, \boldsymbol{\theta}_k, \beta, \sigma_j, \sigma_k)$ by analytically marginalising them out. This approach has several advantages with respect to a systematic sampling because it reduces the computational time and increases the convergence rate of the sampler. In our model, this complete collapsed approach is not possible since the predictive distribution of the augmented variables $(\omega_{j,t}, \omega_{k,t})$ does not have a closed form expression. Instead, given the observations, it is possible to integrate out the variables $(\omega_{j,t}, \omega_{k,t})$ from the full conditionals of the scale parameters (σ_j, σ_k). We implement a partially collapsed Gibbs sampler that is an iterative simulation procedure from the following full conditional distributions:

1. The full conditional distributions of the scale parameters σ_j and σ_k are sampled by integrating out the augmented latent factors $(\omega_{j,t}, \omega_{k,t})_{t=1}^T$, becoming:

$$\pi\left(\sigma_l \mid \mathbf{y}_l, \mathbf{x}, \boldsymbol{\theta}_l\right) \propto \mathcal{IG}\left(\widetilde{a}_l, \widetilde{b}_l\right), \quad \mathbf{y}_l = \left(y_{l,t}\right)_{t=1}^T, \forall l \in \{j,k\}$$

where

$$\begin{array}{ll} \widetilde{a}_j = a_j^0 + T, & \widetilde{b}_j = b_j^0 + \sum_{t=1}^T \rho_\tau\left(y_{j,t} - \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_j\right), \\ \widetilde{a}_k^0 = a_k^0 + T, & \widetilde{b}_k = b_k^0 + \sum_{t=1}^T \rho_\tau\left(y_{k,t} - \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_k - \beta y_{j,t}\right). \end{array} \tag{14}$$

2. $\pi\left(\omega_{j,t}^{-1} \mid y_{j,t}, \mathbf{x}_t, \boldsymbol{\theta}_j, \sigma_j\right) \propto \mathcal{IN}\left(\psi_{j,t}, \phi_j\right)$, $\forall t = 1, \ldots, T$, i.e., an Inverse Gaussian with parameters

$$\psi_{j,t} = \sqrt{\frac{\lambda^2 + 2\delta^2}{\left(y_{j,t} - \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_j\right)^2}}, \qquad \phi_j = \frac{\lambda^2 + 2\delta^2}{\delta^2\sigma_j}.$$

3. $\pi\left(\omega_{k,t}^{-1} \mid \mathbf{y}_t, \mathbf{x}_t, \boldsymbol{\theta}_k, \beta, \sigma_k\right) \propto \mathcal{IN}\left(\psi_{k,t}, \phi_k\right)$, $\forall t = 1, \ldots, T$, with parameters

$$\psi_{k,t} = \sqrt{\frac{\lambda^2 + 2\delta^2}{\left(y_{k,t} - \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_k - \beta y_{j,t}\right)^2}}, \qquad \phi_k = \frac{\lambda^2 + 2\delta^2}{\delta^2\sigma_k}.$$

4. $\pi\left(\boldsymbol{\theta}_j \mid \mathbf{y}_j, \mathbf{x}, \boldsymbol{\omega}_j, \sigma_j\right) \propto \mathcal{N}_{M+1}\left(\widetilde{\boldsymbol{\theta}}_j, \widetilde{\Sigma}_j\right)$, where $\boldsymbol{\omega}_j = \left(\omega_{j,t}\right)_{t=1}^T$, with

$$\begin{array}{rcl} \widetilde{\boldsymbol{\theta}}_j & = & \boldsymbol{\theta}_j^0 + \mathrm{K}_j\left(\mathbf{y}_j - \mathbf{x}^\mathsf{T}\boldsymbol{\theta}_j^0 - \lambda\boldsymbol{\omega}_j\right) \\ \widetilde{\Sigma}_j & = & \left(\mathbb{I}_{M+1} - \mathrm{K}_j\mathbf{x}\right)\Sigma_j^0 \\ \mathrm{K}_j & = & \Sigma_j^0\mathbf{x}^\mathsf{T}\left(\mathrm{W}_j + \mathbf{x}\Sigma_j^0\mathbf{x}^\mathsf{T}\right)^{-1} \\ \mathrm{W}_j & = & \mathrm{diag}\left(\left(\omega_{j,t} \times \delta^2 \times \sigma_j\right)_{t=1}^T\right) \end{array}$$

and $\mathbb{I}_{M+1}$ denotes the identity matrix of size $(M+1)$.

5. $\pi\left(\left(\boldsymbol{\theta}_k, \beta\right)^\mathsf{T} \mid \mathbf{y}, \mathbf{x}, \boldsymbol{\omega}_k, \sigma_k\right) \propto \mathcal{N}_{M+2}\left(\left(\widetilde{\boldsymbol{\theta}}_k, \widetilde{\beta}\right), \widetilde{\Sigma}_k\right)$, where $\boldsymbol{\omega}_k = \left(\omega_{k,t}\right)_{t=1}^T$ with

$$\left(\widetilde{\boldsymbol{\theta}}_k, \widetilde{\beta}\right)^\mathsf{T} = \left(\boldsymbol{\theta}_k^0, \beta^0\right)^\mathsf{T} + \mathrm{K}_k\left(\mathbf{y}_k - \left(\mathbf{x}, \mathbf{y}_j\right)^\mathsf{T}\left(\boldsymbol{\theta}_k^0, \beta^0\right)^\mathsf{T} - \lambda\boldsymbol{\omega}_k\right)$$

$$\begin{aligned}
\widetilde{\Sigma}_k &= \left(\mathbb{I}_{M+2} - \mathrm{K}_k\left(\mathbf{x}, \mathbf{y}_j\right)\begin{pmatrix} \Sigma_k^0 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix}\right) \\
\mathrm{K}_\mathrm{k} &= \begin{pmatrix} \Sigma_k^0 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix}\left(\mathbf{x}, \mathbf{y}_j\right)^\intercal \left(\mathrm{W}_k + \left(\mathbf{x}, \mathbf{y}_j\right)\begin{pmatrix} \Sigma_k^0 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix}\left(\mathbf{x}, \mathbf{y}_j\right)^\intercal\right)^{-1} \\
\mathrm{W}_k &= \mathrm{diag}\left(\left(\omega_{k,t} \times \delta^2 \times \sigma_k\right)_{t=1}^T\right).
\end{aligned}$$

Updating the parameters in this order ensures that the posterior distribution is the stationary distribution of the generated Markov chain. By combining steps 1 and 2 sample draws are produced from $\pi\left(\sigma_j, \sigma_k, \boldsymbol{\omega}_j, \boldsymbol{\omega}_k \mid \boldsymbol{\theta}_k, \boldsymbol{\theta}_j, \beta, \mathbf{y}, \mathbf{x}\right)$, i.e. the conditional posterior distribution and the partially collapsed Gibbs sampler is a blocked version of the ordinary Gibbs sampler, see Van Dyk and Park (2008) and Park and Van Dyk (2009). To initialise the Gibbs sampling algorithm we simulate a random draw from the joint prior distribution of the parameters defined in equation (9), and conditionally on that, we simulate the initial values of the augmented variables $\left(\omega_{j,t}, \omega_{k,t}\right)_{t=1}^T$ from their exponential distributions.

3.2 VaR and CoVaR posterior estimation

From a Bayesian point of view, simulations retrieved from the posterior distribution can be summarised in several ways. Lin and Chang (2012) use the maximisation of the posterior density to make inference for the quantile regression parameters, and show that this is equivalent to the minimisation problem (2) in the frequentist context. This leads us to consider the Maximum a Posteriori (MaP) criterion as an estimate of all the posterior parameters in equations (7) and (8), assuming Asymmetric Laplace distributions for the error terms and diffuse priors on the regressor parameters. For all the MaP parameters involved in the marginal and conditional quantiles, that is $\left(\boldsymbol{\theta}_j^{\mathrm{MaP}}, \boldsymbol{\theta}_k^{\mathrm{MaP}}, \beta^{\mathrm{MaP}}\right)$, the estimators of $\mathrm{VaR}_j^{\mathbf{x},\tau}$ and the $\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}$ are then derived from equations (5) and (6) as follows:

$$\begin{aligned}
\left(\mathrm{VaR}_j^{\mathbf{x},\tau}\right)^{\mathrm{MaP}} &= \mathbf{x}^\intercal \boldsymbol{\theta}_j^{\mathrm{MaP}} \\
\left(\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}\right)^{\mathrm{MaP}} &= \mathbf{x}^\intercal \boldsymbol{\theta}_k^{\mathrm{MaP}} + \beta^{\mathrm{MaP}}\left(\mathrm{VaR}_j^{\mathbf{x},\tau}\right)^{\mathrm{MaP}}.
\end{aligned}$$

Credible sets at a given confidence level for both $\mathrm{VaR}_{k|j}^{\mathbf{x},\tau}$ and $\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau}$ estimates can be calculated by marginalising out the scale parameters $\left(\sigma_j, \sigma_k\right)$ and the latent variables $\left(\boldsymbol{\omega}_j, \boldsymbol{\omega}_k\right)$, using the sample draws of the MCMC algorithm. Monte Carlo estimates of the marginal posterior densities of the quantile functions are given by

$$\begin{aligned}
\pi\left(\mathrm{VaR}_j^{\mathbf{x},\tau} \mid \mathbf{y}_j\right) &= \frac{1}{G}\sum_{g=1}^G \pi\left(\mathbf{x}^\intercal \boldsymbol{\theta}_j \mid \sigma_j^{(g)}, \boldsymbol{\omega}_j^{(g)}, \mathbf{y}_j\right), \\
\pi\left(\mathrm{CoVaR}_{k|j}^{\mathbf{x},\tau} \mid \mathbf{y}_k\right) &= \frac{1}{G}\sum_{g=1}^G \pi\left(\mathbf{x}^\intercal \boldsymbol{\theta}_k + \beta\left(\mathrm{VaR}_j^{\mathbf{x},\tau}\right)^{(g)} \mid \sigma_k^{(g)}, \boldsymbol{\omega}_k^{(g)}, \mathbf{y}_k\right),
\end{aligned}$$

where G denotes the number of post burn-in iterations. The 95% High Posterior Credible intervals $\text{HPD}_{95\%}$ for the τ-th quantile can be obtained from the samples $\left\{\mathbf{x}^\mathsf{T}\boldsymbol{\theta}_j \mid \sigma_j^{(g)}, \boldsymbol{\omega}_j^{(g)}, \mathbf{y}_j\right\}_{g=1}^G$ and $\left\{\mathbf{x}^\mathsf{T}\boldsymbol{\theta}_k + \beta\left(\text{VaR}_j^{\mathbf{x},\tau}\right)^{(g)} \mid \sigma_k^{(g)}, \boldsymbol{\omega}_k^{(g)}, \mathbf{y}_k\right\}_{g=1}^G$.

4 Time-varying quantile model

As mentioned before, VaR and CoVaR are respectively unconditional and conditional quantiles, given current information, of future portfolio values. It is typically the case that returns change over time and for this reason it can be interesting to build suitable models for time-varying VaR and CoVaR. In particular, when modeling time-varying quantiles, it is important to link future tail behaviours of time series to past movements, to account for risk management arguments. Recently, the topic of time varying quantiles has received increased attention and different econometric models have been proposed: the most well-known are the Conditional Autoregressive Value-at-Risk (CAViaR) model of Engle and Manganelli (2004), the Quantile Autoregressive (QAR) model of Koenker and Xiao (2006), and the Dynamic Additive Quantile (DAQ) model of Gourieroux and Jasiak (2008). Most of these include an autoregressive structure in their modeling, which is intuitively attractive, as series of financial returns tend to exhibit time-varying conditional moments, fat tails and volatility clustering. More recently, Gerlach et al. (2011) and Chen et al. (2012a,b), deal with the problem of estimating the conditional dynamic VaR using a Bayesian CAViaR approach. The resulting conditional quantile for the variable of interest is directly modeled as a smooth function of the observed past returns.

In this paper we propose a different approach for introducing dynamics in the quantiles, modeling both the VaR and CoVaR as a function of latent variables having their own time dependence. The introduction of latent states having a dynamic evolution allows the future behaviour of the modeled quantiles to depend upon their past movements in a flexible way. In particular we estimate, from a Bayesian point of view, the required quantiles simultaneously and we allow the quantiles to depend on exogenous variables. In doing so we are consistent with the result of De Rossi and Harvey (2009) and Kurose and Omori (2012) who modeled the unconditional quantile curve using smoothing spline interpolation. More precisely, we model the observed vector at each point in time $(y_{j,t}, y_{k,t})$, as a function of independent latent processes $(\mu_{j,t}, \mu_{k,t})$ and the regressor terms in the following way:

$$y_{j,t} = \mu_{j,t} + \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_j + \epsilon_{j,t} \tag{15}$$

$$y_{k,t} = \mu_{k,t} + \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_k + \beta_t y_{j,t} + \epsilon_{k,t}, \tag{16}$$

$\forall t \in 1, \ldots, T$, where $\epsilon_{j,t} \sim \text{ALD}(\tau, 0, \sigma_j)$, $\epsilon_{k,t} \sim \text{ALD}(\tau, 0, \sigma_k)$ are independent random variables. The intercept terms $\mu_{l,t}$ with $l \in \{j, k\}$ are introduced to account for time dependence in the quantile functions. In fact, we propose the following smooth time-varying dynamics for $\mu_{l,t}$ and $l \in \{j, k\}$:

$$\mu_{l,t+1} = \mu_{l,t} + \mu^*_{l,t} + \eta_{l,t} \tag{17}$$

$$\mu^*_{l,t+1} = \mu^*_{l,t} + \eta^*_{l,t}, \tag{18}$$

where $\left(\mu_{l,1}, \mu_{l,1}^*\right)^\mathsf{T} \sim \mathcal{N}_2\left(0, \kappa \mathbb{I}_2\right)$ with $\kappa > 0$ being sufficiently large, $(\eta_{l,t}, \eta_{l,t}^*)^\mathsf{T} \sim \mathcal{N}_2(0, S_l)$ and $S_l = s_l^2 V = s_l^2 \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$, with $s_l > 0$ allowing for a certain degree of smoothness of the quantile process. The V hyperparameter is considered as fixed in order to control for the quantiles' smoothness. In particular, to get a local linear trend specification for the unobservable processes, we chose V to be non-diagonal. Concerning the specific choice of the V entries, following Harvey (1989), Durbin and Koopman (2012), De Rossi and Harvey (2009) and Kurose and Omori (2012) on related literature on smoothing splines, we allowed them to be fixed to pre-specified values. Since one of our main focuses is to analyse the dynamic co-movement of two institutions, we also allow the parameter β_t to change over time. To reflect different impacts between institutions we consider the following evolution for β_t:

$$\beta_{t+1} = \beta_t + \beta_t^* + \eta_{\beta,t} \tag{19}$$
$$\beta_{t+1}^* = \beta_t^* + \eta_{\beta,t}^*, \tag{20}$$

$(\beta_1, \beta_1^*)^\mathsf{T} \sim \mathcal{N}_2\left(0, \kappa \mathbb{I}_2\right)$ and $(\eta_{\beta,t}, \eta_{\beta,t}^*)^\mathsf{T} \sim \mathcal{N}_2(0, s_\beta^2 V)$ where V and κ is defined as before. Throughout the paper we assume that $\forall l \in \{j, k, \beta\}$, $(\eta_{l,t}, \eta_{l,t}^*)$ is independent of $(\epsilon_{j,t}, \epsilon_{k,t})$ (here we use β as an index since there is no ambiguity). Since we are interested in modelling how different economic phases may influence the relationship between VaR and CoVaR, we allow only the conditional quantile loading parameter β to change over time while retaining the covariate's parameters $(\boldsymbol{\theta}_j, \boldsymbol{\theta}_k)$ as fixed. In fact, we retain that the dynamics of the time series covariates are informative enough for the evolution of the economic and the specific characteristics, in accordance with a parsimony criterion.

In order to estimate the model parameters we rewrite equations (15)–(20) using a state space representation so that $\forall t \in \{1, \ldots, T\}$,

$$\mathbf{y}_t = Z_t \boldsymbol{\xi}_t + \mathbf{x}_t^\mathsf{T} \boldsymbol{\theta} + \boldsymbol{\epsilon}_t \tag{21}$$
$$\boldsymbol{\xi}_{t+1} = A \boldsymbol{\xi}_t + \boldsymbol{\eta}_t \tag{22}$$
$$\boldsymbol{\xi}_1 \sim \mathcal{N}_6\left(0, \kappa \mathbb{I}_6\right), \tag{23}$$

where

- $\boldsymbol{\epsilon}_t = (\epsilon_{j,t}, \epsilon_{k,t})$ is the vector of independent ALDs as defined in equations (7)–(8),
- $Z_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & y_{j,t} & 0 \end{pmatrix}$ is the time-varying matrix of loading factors,
- $\boldsymbol{\xi}_t = \left(\mu_{j,t}, \mu_{j,t}^*, \mu_{k,t}, \mu_{k,t}^*, \beta_t, \beta_t^*\right)^\mathsf{T}$ is the vector of latent states whose dynamic is given by the transition matrix A, with $A = \mathbb{I}_3 \otimes B$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and $\otimes$ denotes the Kronecker product,
- $\boldsymbol{\theta} = (\boldsymbol{\theta}_j, \boldsymbol{\theta}_k)$ is a $(M \times 2)$ matrix of time invariant coefficients,

- $\boldsymbol{\eta}_t = \left(\eta_{j,t}, \eta^*_{j,t}, \eta_{k,t}, \eta^*_{k,t}, \eta_{\beta,t}, \eta^*_{\beta,t}\right)^{\mathsf{T}}$ is the time-varying error vector distributed according to $\mathcal{N}_6(0, \Omega)$, where $\Omega = \text{diag}\left(s_j^2, s_k^2, s_\beta^2\right) \otimes V$.

The Bayesian model specification requires the selection of a prior distribution for all the fixed parameters $(\boldsymbol{\theta}, \sigma_j, \sigma_k)$, the variance of the latent factors $\left(s_j^2, s_k^2, s_\beta^2\right)$ and the initial distribution for the vector of first states $\boldsymbol{\xi}_1$. Concerning the regression parameters $\boldsymbol{\theta}$ and the nuisance parameters (σ_j, σ_k), which are common to the time invariant model, we impose the same structure of conjugate proper priors defined in Section 3. The scale of the unobserved components has an independent Inverse Gamma distribution, i.e. $s_l^2 \sim \mathcal{IG}\left(r_l^0, v_l^0\right)$, with positive r_l^0 and v_l^0 $\forall l \in \{j, k, \beta\}$. This choice is motivated by the need to have closed forms for the full conditional distributions of those parameters. In addition, we assume that the vector of first states $\boldsymbol{\xi}_1$ is distributed as in equation (23), where the Gaussian distribution is assumed to preserve the conditional Gaussianity of the state space model defined in (21)–(22). The motivation for the specific choice of the parameter κ as well as other prior hyperparameters is explained in Section 5.

The linear state space model introduced in (21)–(23) for modeling time-varying conditional quantiles is non-Gaussian because of the assumption made on the innovation terms. So in those circumstances optimal filtering techniques used to analytically marginalise out the latent states based on the Kalman filter recursions can not be applied (see Durbin and Koopman, 2012). Considering the (10) representation of the innovation terms in (21) it is easy to recognise that the non-Gaussian state space model admits a conditionally Gaussian representation. More specifically equations (21) and (22) become:

$$\mathbf{y}_t = \mathbf{c}_t + Z_t\boldsymbol{\xi}_t + \mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta} + G_t\boldsymbol{\nu}_t, \quad \boldsymbol{\nu}_t \sim \mathcal{N}_2(0, \mathbb{I}_2) \tag{24}$$
$$\boldsymbol{\xi}_{t+1} = A\boldsymbol{\xi}_t + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim \mathcal{N}(0, \Omega) \tag{25}$$
$$\boldsymbol{\xi}_1 \sim \mathcal{N}_6(0, \kappa\mathbb{I}_6), \tag{26}$$

$\forall t \in 1, \ldots, T$, where the time-varying vector $\boldsymbol{c}_t$, and matrix G_t are respectively $\boldsymbol{c}_t = (\lambda\omega_{j,t}, \lambda\omega_{k,t})^{\mathsf{T}}$ and $G_t = \begin{pmatrix} \delta\sqrt{\sigma_j\omega_{j,t}} & 0 \\ 0 & \delta\sqrt{\sigma_k\omega_{k,t}} \end{pmatrix}$; $\omega_{j,t}$ and $\omega_{k,t}$ are independent with $\omega_{l,t} \sim \mathcal{E}\text{xp}\left(\sigma_l^{-1}\right)$ for $l \in (j, k)$ and $\sigma_l > 0$; λ and δ are defined in equation (11).

The complete-data likelihood of the unobservable components $\boldsymbol{\omega} = (\omega_{j,t}, \omega_{k,t})_{t=1}^T$ and $(\boldsymbol{\xi}_t)_{t=1}^T$ and all parameters $\boldsymbol{\gamma} = \left(\boldsymbol{\theta}, s_j^2, s_k^2, s_\beta^2, \sigma_j, \sigma_k\right)$ can be factorised as follows:

$$\begin{aligned}
&\mathcal{L}\left((\boldsymbol{\xi}_t)_{t=1}^T, \boldsymbol{\omega}, \boldsymbol{\gamma} \mid \mathbf{y}, \mathbf{x}\right) \\
&\propto \prod_{t=1}^T f\left(y_{j,t} \mid \boldsymbol{\xi}_t, \omega_{j,t}, \sigma_j, \mathbf{x}_t\right) \prod_{t=1}^T f\left(y_{k,t} \mid y_{j,t}, \boldsymbol{\xi}_t, \omega_{k,t}, \sigma_k, \mathbf{x}_t\right) \\
&\times \prod_{t=1}^T f\left(\omega_{j,t} \mid \sigma_j\right) \prod_{t=1}^T f\left(\omega_{k,t} \mid \sigma_k\right) f\left(\boldsymbol{\xi}_1\right) \prod_{t=1}^{T-1} f\left(\boldsymbol{\xi}_{t+1} \mid \boldsymbol{\xi}_t, s_j^2, s_k^2, s_\beta^2\right) (\sigma_j\sigma_k)^{-\frac{T}{2}}
\end{aligned}$$

$$
\times \exp\left\{-\frac{1}{2}\sum_{t=1}^{T}\left(\mathbf{y}_t-\mathbf{c}_t-Z_t\boldsymbol{\xi}_t-\mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}\right)^{\mathsf{T}}\left(G_tG_t^{\mathsf{T}}\right)^{-1}\left(\mathbf{y}_t-\mathbf{c}_t-Z_t\boldsymbol{\xi}_t-\mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}\right)\right\}
$$

$$
\times\prod_{t=1}^{T}\left(\omega_{j,t}\omega_{k,t}\right)^{-\frac{1}{2}}\left(\sigma_j\right)^{-T}\exp\left\{-\frac{\sum_{t=1}^{T}\omega_{j,t}}{\sigma_j}\right\}\left(\sigma_k\right)^{-T}\exp\left\{-\frac{\sum_{t=1}^{T}\omega_{k,t}}{\sigma_k}\right\}
$$

$$
\times \exp\left\{-\frac{1}{2\kappa}\boldsymbol{\xi}_1^{\mathsf{T}}\boldsymbol{\xi}_1\right\}\exp\left\{-\frac{1}{2}\sum_{t=1}^{T-1}\left(\boldsymbol{\xi}_{t+1}-A\boldsymbol{\xi}_t\right)^{\mathsf{T}}\Omega^{-1}\left(\boldsymbol{\xi}_{t+1}-A\boldsymbol{\xi}_t\right)\right\}. \tag{27}
$$

4.1 Computations

Using the complete-data likelihood in (27) and the prior distributions stated in sections 3 and 4 we are able to write the joint posterior distribution of the parameters and the unobservable components. The form of the posterior allows us to sample from the complete conditional distributions and to use the Gibbs sampler algorithm, as shown below.

After choosing a set of initial values for the parameter vector $\boldsymbol{\gamma}^{(0)}$, simulations from the posterior distribution at the i-th iteration of $\boldsymbol{\gamma}^{(i)}$, $\{\boldsymbol{\xi}_t, t=1,2,\ldots,T\}^{(i)}$ and $\{\omega_{l,t}, l\in j,k, t=1,2,\ldots,T\}^{(i)}$ for $i=1,2,\ldots,$ are obtained by the following Gibbs sampling scheme:

1. For $l\in\{j,k,\beta\}$, generate s_l^2 from $\pi\left(s_l^2\mid\mathbf{y}_l,\boldsymbol{\xi}_t^l\right)\propto\mathcal{IG}\left(\widetilde{r}_l,\widetilde{v}_l\right)$ with parameters

$$
\widetilde{r}_l=r_l^0+(T-1),\qquad \widetilde{v}_l=v_l^0+\sum_{t=1}^{T-1}\left(\boldsymbol{\xi}_{t+1}^l-B\boldsymbol{\xi}_t^l\right)^{\mathsf{T}}V^{-1}\left(\boldsymbol{\xi}_{t+1}^l-B\boldsymbol{\xi}_t^l\right)
$$

   where $\boldsymbol{\xi}_t^l$ denotes the vector consisting of the elements $\left(\mu_{j,t},\mu_{j,t}^*\right)$ for $l=j$, $\left(\mu_{k,t},\mu_{k,t}^*\right)$ for $l=k$, and (β_t,β_t^*) for $l=\beta$.

2. For $l=j,k$, generate σ_l from $\pi\left(\sigma_l\mid\mathbf{y}_l,\boldsymbol{\xi}_t^l,\boldsymbol{\theta}_l\right)\propto\mathcal{IG}\left(\widetilde{a}_l,\widetilde{b}_l\right)$ with parameters

$$
\widetilde{a}_l=a_l^0+T,\qquad \widetilde{b}_l=b_l^0+\sum_{t=1}^{T}\rho_\tau\left(y_{l,t}-z_{l,t}\boldsymbol{\xi}_t-\mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}_l\right),
$$

   where $z_{l,t}$ denotes the first row of the matrix Z_t for $l=j$ or the second one for $l=k$.

3. For all $t\in\{1,2,\ldots,T\}$, generate $\omega_{j,t}^{-1}$ from $\pi\left(\omega_{j,t}^{-1}\mid y_{j,t},\mathbf{x}_t,\boldsymbol{\theta}_j,\sigma_j,\mu_{j,t}\right)$ distributed according with $\mathcal{IN}\left(\psi_{j,t},\phi_j\right)$, with parameters

$$
\psi_{j,t}=\sqrt{\frac{\lambda^2+2\delta^2}{\left(y_{j,t}-z_{j,t}\mu_{j,t}-\mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}_j\right)^2}},\qquad \phi_j=\frac{\lambda^2+2\delta^2}{\delta^2\sigma_j}
$$

and generate $\omega_{k,t}^{-1}$ from $\pi\left(\omega_{k,t}^{-1} \mid \mathbf{y}_t, \mathbf{x}_t, \boldsymbol{\theta}_k, \beta_t, \sigma_k, \mu_{k,t}\right) \propto \mathcal{IN}\left(\psi_{k,t}, \phi_k\right)$, with parameters

$$\psi_{k,t} = \sqrt{\frac{\lambda^2 + 2\delta^2}{\left(y_{k,t} - \beta_t y_{j,t} - z_{k,t}\mu_{k,t} - \mathbf{x}_t^\mathsf{T}\boldsymbol{\theta}_k\right)^2}}, \qquad \phi_k = \frac{\lambda^2 + 2\delta^2}{\delta^2 \sigma_k}.$$

4. For $l = j, k$, denote $\mathbf{y}_l = (y_{l,t})_{t=1}^T$, $\mathbf{z}_l \boldsymbol{\mu}_l = (z_{l,t} \times \mu_{j,t})_{t=1}^T$, $\boldsymbol{\beta}\mathbf{y}_j = (\beta_t \times y_{j,t})_{t=1}^T$. Then, generate $\boldsymbol{\theta}_l$ from $\pi\left(\boldsymbol{\theta}_l \mid \mathbf{y}, \mathbf{x}, \boldsymbol{\omega}_l, \sigma_l, \boldsymbol{\mu}_l\right) \propto \mathcal{N}_M\left(\widetilde{\boldsymbol{\theta}}_l, \widetilde{\Sigma}_l\right)$ with parameters

$$\begin{aligned}
\widetilde{\boldsymbol{\theta}}_j &= \boldsymbol{\theta}_j^0 + \mathrm{K}_j\left(\mathbf{y}_j - z_j\boldsymbol{\mu}_j - \mathbf{x}^\mathsf{T}\boldsymbol{\theta}_j^0 - \lambda\boldsymbol{\omega}_j\right)\\
\widetilde{\Sigma}_j &= \left(\mathbb{I}_M - \mathrm{K}_j\mathbf{x}\right)\Sigma_j^0\\
K_j &= \Sigma_j^0\mathbf{x}^\mathsf{T}\left(\mathrm{W}_j + \mathbf{x}\Sigma_j^0\mathbf{x}^\mathsf{T}\right)^{-1}\\
\mathrm{W}_j &= \mathrm{diag}\left(\left(\omega_{j,t} \times \delta^2 \times \sigma_j\right)_{t=1}^T\right)
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{\boldsymbol{\theta}}_k &= \boldsymbol{\theta}_k^0 + \mathrm{K}_k\left(\mathbf{y}_k - z_k\boldsymbol{\mu}_k - \boldsymbol{\beta}\mathbf{y}_j - \mathbf{x}^\mathsf{T}\boldsymbol{\theta}_k^0 - \lambda\boldsymbol{\omega}_k\right)\\
\widetilde{\Sigma}_k &= \left(\mathbb{I}_M - K_k\mathbf{x}\right)\Sigma_k^0\\
\mathrm{K}_k &= \Sigma_k^0\mathbf{x}^\mathsf{T}\left(\mathrm{W}_k + \mathbf{x}\Sigma_k^0\mathbf{x}^\mathsf{T}\right)^{-1}\\
\mathrm{W}_k &= \mathrm{diag}\left(\left(\omega_{k,t} \times \delta^2 \times \sigma_k\right)_{t=1}^T\right).
\end{aligned}$$

5. For all $t \in \{1, 2, \ldots, T\}$, generate $\pi\left(\boldsymbol{\xi}_t, \beta_t \mid \mathbf{y}, \mathbf{x}, \boldsymbol{\theta}_j, \boldsymbol{\theta}_k, \sigma_j, \sigma_k, \boldsymbol{\omega}_j, \boldsymbol{\omega}_k, s_j^2, s_k^2, s_\beta^2\right)$. Since it is conditional on the augmented latent states $(\omega_{j,t}, \omega_{k,t})_{t=1}^\mathsf{T}$ the state space model defined in equations (24)–(26), is linear and Gaussian, the latent dynamics can be marginalised out by running the Kalman filter-smoothing algorithm. We draw $(\boldsymbol{\xi}_t, \beta_t)_{t=1}^T$ jointly using the multi-move simulation smoother of Durbin and Koopman (2002). This entails running a Kalman filter forward with the state equation defined as in (26). As in Johannes and Polson (2009), equation (16) for $y_{k,t}$ is a measurement equation with time-varying coefficients, because $y_{j,t}$ is known and represents a time-varying factor loading. Once the Kalman filter is run forward, we run the Kalman smoother backward in order to get the moments of the joint full conditional distribution of the latent states (27). Finally, we simulate a sample path by drawing from this joint distribution. For a similar simulation algorithm based on forward-filtering backward-smoothing see also Carter and Kohn (1994, 1996) and Frühwirth-Schnatter (1994).

In line with Van Dyk and Park (2008) and Park and Van Dyk (2009), the ordering of the full conditional simulation ensures that the posterior distribution is the stationary distribution of the generated Markov chain. This is because the combination of steps

2 and 3 above essentially ensures draws from the conditional posterior distribution $\pi\left(\sigma_j, \sigma_k, \boldsymbol{\omega}_j, \boldsymbol{\omega}_k \mid \{\beta_t, \boldsymbol{\xi}_t\}_{t=1}^{T}, \boldsymbol{\theta}_k, \boldsymbol{\theta}_j, \mathbf{y}, \mathbf{x}, s_j^2, s_k^2, s_\beta^2\right)$ and the partially collapsed Gibbs sampler is a blocked version of the ordinary Gibbs sampler. As in the time invariant case, the Gibbs sampler algorithm is initialised by simulating a random draw from the joint prior distribution of the parameters.

4.2 Maximum a Posteriori

As in the time invariant case, once we retrieve simulations from the posterior distribution, we use the maximum a posteriori summarising criterion in order to make posterior inference. In what follows we prove that using this criterion, the estimated quantiles have good sample properties according to Proposition 3 of De Rossi and Harvey (2009), i.e. a generalisation of the "fundamental property", where the sample quantile has the appropriate number of observations above and below.

Proposition 4.1. *For the state space model defined in equations (15)–(20) with prior distributions specified in Sections 3 and 4, κ large enough and a diffuse prior on $\boldsymbol{\theta}$, the MaP quantile estimates $\mu_{j,t}^{\text{MaP}} + \mathbf{x}_t^\top \boldsymbol{\theta}_j^{\text{MaP}}$ and $\mu_{k,t}^{\text{MaP}} + \mathbf{x}_t^\top \boldsymbol{\theta}_k^{\text{MaP}} + y_{j,t}\beta_t^{\text{MaP}}$ satisfy*

$$\begin{aligned} \sum_{t \notin C} (x_{m,t} + 1)\, \chi_\tau \left(y_{j,t} - \left(\mu_{j,t}^{\text{MaP}} + \mathbf{x}_t^\top \boldsymbol{\theta}_j^{\text{MaP}} \right) \right) &= 0 \\ \sum_{t \notin C} (y_{j,t} + x_{m,t} + 1)\, \chi_\tau \left(y_{k,t} - \left(\mu_{k,t}^{\text{MaP}} + \mathbf{x}_t^\top \boldsymbol{\theta}_k^{\text{MaP}} + y_{j,t}\beta_t^{\text{MaP}} \right) \right) &= 0, \end{aligned}$$

$\forall m \in \{1, \ldots, M\}$, where $C \subset \{1, \ldots, T\}$ is the set of all points such that the MaP quantile estimate coincides with observations and

$$\chi_\tau : \ z \to \begin{cases} \tau - 1 & \text{if} \quad z < 0 \\ \tau & \text{if} \quad z > 0. \end{cases} \tag{28}$$

Proof of Proposition 4.1. *For $t \in \{1, \ldots, T\}$, $\boldsymbol{\xi}_t^j = \left(\mu_{j,t}, \mu_{j,t}^*\right)^\top$. Define $\boldsymbol{\xi}_t^{k,\beta} = \left(\mu_{k,t}, \mu_{k,t}^*, \beta_t, \beta_t^*\right)^\top$, $\boldsymbol{\xi}^j = \left(\boldsymbol{\xi}_t^j\right)_{t=1}^{T}$ and $\boldsymbol{\xi}^{k,\beta} = \left(\boldsymbol{\xi}_t^{k,\beta}\right)_{t=1}^{T}$. From equations (15)–(20), let us write the complete-posterior distribution $p\left(\boldsymbol{\theta}, \boldsymbol{\xi}^j, \boldsymbol{\xi}^{k,\beta}, s_j^2, s_k^2, s_\beta^2, \sigma_j, \sigma_k \mid (\mathbf{y}_t)_{t=1}^{T}\right)$ as proportional to the product of two parts $Post_{marg}$ and $Post_{cond}$ where*

$$\begin{aligned} Post_{marg} \quad = \quad & \prod_{t=1}^{T} \text{ald}\left(y_{j,t} \mid \mu_{j,t} + \mathbf{x}_t^\top \boldsymbol{\theta}_j, \sigma_j\right) s_j^{-1} \exp\left\{ -\frac{1}{2\kappa} \left(\boldsymbol{\xi}_1^j\right)^\top \boldsymbol{\xi}_1^j \right\} \\ & \times \exp\left\{ -\frac{12}{2s_j^2} \sum_{t=1}^{T-1} \left(\boldsymbol{\xi}_{t+1}^j - B\boldsymbol{\xi}_t^j\right)^T V^{-1} \left(\boldsymbol{\xi}_{t+1}^j - B\boldsymbol{\xi}_t^j\right) \right\} \\ & \times \exp(-\frac{1}{2}(\boldsymbol{\theta}_{\boldsymbol{j}} - \boldsymbol{\theta}_j^0)^\top (\Sigma_j^0)^{-1} (\boldsymbol{\theta}_{\boldsymbol{j}} - \boldsymbol{\theta}_j^0)) \times \mathcal{IG}(r_j^0, v_j^0) \times \mathcal{IG}(a_j^0, b_j^0) \end{aligned}$$

$$
\begin{aligned}
Post_{cond} \quad = \quad & \prod_{t=1}^{T} \mathrm{ald}\left(y_{k,t} \mid \mu_{k,t} + \mathbf{x}_t^{\mathsf{T}} \boldsymbol{\theta}_k + \beta_t y_{j,t}, \sigma_k\right) \exp\left\{-\frac{1}{2\kappa}\left(\boldsymbol{\xi}_1^{k,\beta}\right)^{\mathsf{T}} \boldsymbol{\xi}_1^{k,\beta}\right\} (s_k s_\beta)^{-1} \\
& \times \exp\left\{-\frac{12}{2} \sum_{t=1}^{T-1} \left(\Delta\boldsymbol{\xi}_{t+1}^{k,\beta}\right)^{\mathsf{T}} \left(\left[\begin{array}{cc} s_k^2 & 0 \\ 0 & s_\beta^2 \end{array}\right] \otimes V\right)^{-1} \Delta\boldsymbol{\xi}_{t+1}^{k,\beta}\right\} \\
& \times \exp\left(-\frac{1}{2}\left(\boldsymbol{\theta}_k - \boldsymbol{\theta}_k^0\right)^{\mathsf{T}} \left(\Sigma_k^0\right)^{-1} \left(\boldsymbol{\theta}_k - \boldsymbol{\theta}_k^0\right)\right) \\
& \times \mathcal{IG}\left(r_k^0, v_k^0\right) \times \mathcal{IG}\left(a_k^0, b_k^0\right) \times \mathcal{IG}\left(r_\beta^0, v_\beta^0\right),
\end{aligned}
$$

where $\Delta\boldsymbol{\xi}_{t+1}^{k,\beta} = \boldsymbol{\xi}_{t+1}^{k,\beta} - (\mathbb{I}_2 \otimes B)\, \boldsymbol{\xi}_t^{k,\beta}$.

The MaP of $p\big(\boldsymbol{\theta}, \boldsymbol{\xi}^j, \boldsymbol{\xi}^{k,\beta}, s_j^2, s_k^2, s_\beta^2, \sigma_j, \sigma_k \mid (\mathbf{y}_t)_{t=1}^T\big)$ *in* $\big(\boldsymbol{\theta}, (\mu_{j,t})_{t=1}^T, (\mu_{k,t})_{t=1}^T, (\beta_t)_{t=1}^T\big)$ *is obtained by maximizing separately* $Post_{marg}$ *and* $Post_{cond}$ *with respect to* $\big(\boldsymbol{\theta}_j, (\mu_{j,t})_{t=1}^T\big)$ *and* $\big(\boldsymbol{\theta}_k, (\mu_{k,t})_{t=1}^T, (\beta_t)_{t=1}^T\big)$*, respectively. Note also that the check function* $\rho_\tau(\cdot)$ *of the Asymmetric Laplace distribution is derivable everywhere except at zero and its derivative corresponds to the function* $\chi_\tau(\cdot)$ *defined in equation (28).*

Differentiating $\log(Post_{marg})$ $\forall t = \{1, 2, \ldots, T\} \setminus \{C\}$*, we obtain:*

$$
\begin{aligned}
\frac{\partial \log(Post_{marg})}{\partial \mu_{j,1}} \quad = \quad & \frac{-\mu_{j,1} - \mu_{j,1}^*}{\kappa} + \frac{1}{\sigma_j}\chi_\tau\left(y_{j,1} - \mu_{j,1} - \mathbf{x}_1^{\mathsf{T}}\boldsymbol{\theta}_j\right) \\
& + \frac{6}{s_j^2}\left\{2\left(\mu_{j,2} - \mu_{j,1} - \mu_{j,1}^*\right) - \left(\mu_{j,2}^* - \mu_{j,1}^*\right)\right\} \\
\frac{\partial \log(Post_{marg})}{\partial \mu_{j,t}} \quad = \quad & \frac{1}{\sigma_j}\chi_\tau\left(y_{j,t} - \mu_{j,t} - \mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}_j\right) - \frac{12}{s_j^2}\left(\mu_{j,t} - \mu_{j,t-1} - \mu_{j,t-1}^*\right) \\
& + \frac{12}{s_j^2}\left(\mu_{j,t+1} - \mu_{j,t} - \mu_{j,t}^*\right) - \frac{6}{s_j^2}\left(\mu_{j,t+1}^* - 2\mu_{j,t}^* + \mu_{j,t-1}^*\right)
\end{aligned}
$$

$\forall t \in \{2, \ldots, T-1\}$*, and*

$$
\begin{aligned}
\frac{\partial \log(Post_{marg})}{\partial \mu_{j,T}} \quad = \quad & \frac{1}{\sigma_j}\chi_\tau\left(y_{j,T} - \mu_{j,T} - \mathbf{x}_T^{\mathsf{T}}\boldsymbol{\theta}_j\right) \\
& + \frac{6}{s_j^2}\left\{-2\left(\mu_{j,T} - \mu_{j,T-1} - \mu_{j,T-1}^*\right) + \left(\mu_{j,T}^* - \mu_{j,T-1}^*\right)\right\} \\
\frac{\partial \log(Post_{marg})}{\partial \boldsymbol{\theta}_j} \quad = \quad & \frac{1}{\sigma_j}\sum_{t=1}^{T}\mathbf{x}_t\chi_\tau\left(y_{j,t} - \mu_{j,t} - \mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}_j\right) + \left(\Sigma_j^0\right)^{-1}\left(\boldsymbol{\theta}_j - \boldsymbol{\theta}_j^0\right),
\end{aligned}
\tag{29}
$$

where we use $S_j^{-1} = \frac{12}{s_j^2}V^{-1} = \frac{12}{s_j^2}\left(\begin{array}{cc} 1 & -1/2 \\ -1/2 & 1/3 \end{array}\right)$. *It turns out that*

$$
\sum_t \frac{\partial \log(Post_{marg})}{\partial \mu_{j,t}} = \frac{-\mu_{j,1} - \mu_{j,1}^*}{\kappa} + \frac{1}{\sigma_j}\sum_t \chi_\tau\left(y_{j,t} - \mu_{j,t} - \mathbf{x}_t^{\mathsf{T}}\boldsymbol{\theta}_j\right),
$$

which, combined with equation (29) and choosing κ sufficiently large enough and a diffuse prior on $\boldsymbol{\theta}_j$, implies that the maximiser of $Post_{marg}$ satisfies the following equation

$$\sum_{t \notin C} (x_{m,t}+1)\, \chi_\tau(y_{j,t} - \mu_{j,t} - \mathbf{x}_t^\intercal \boldsymbol{\theta}_j) = 0, \quad \forall m \in \{1, 2, \dots, M\}.$$

The derivatives of $\log(Post_{cond})$ *with respect to* $\left(\boldsymbol{\theta}_k, (\mu_{k,t})_{t=1}^T\right)$ *can be obtained in the same way as those for* $\log(Post_{marg})$*. Deriving* $\log(Post_{cond})$ *with respect to* $(\beta_t)_{t=1}^T$*,* $\forall t = \{1, 2, \dots, T\} \setminus \{C\}$*, leads to:*

$$\begin{aligned}
\frac{\partial \log(Post_{cond})}{\partial \beta_1} &= \frac{-\beta_1 - \beta_1^*}{\kappa} + \frac{y_{j,1}}{\sigma_k} \chi_\tau \left(y_{k,1} - \mu_{k,1} - \mathbf{x}_1^\intercal \boldsymbol{\theta}_k - y_{j,1}\beta_1\right) \\
&\quad + \frac{6}{s_\beta^2} \left\{2(\beta_2 - \beta_1 - \beta_1^*) - (\beta_2^* - \beta_1^*)\right\} \\
\frac{\partial \log(Post_{cond})}{\partial \beta_t} &= \frac{y_{j,t}}{\sigma_k} \chi_\tau \left(y_{k,t} - \mu_{k,t} - \mathbf{x}_t^\intercal \boldsymbol{\theta}_k - y_{j,t}\beta_t\right) - \frac{12}{s_\beta^2}\left(\beta_t - \beta_{t-1} - \beta_{t-1}^*\right) \\
&\quad + \frac{6}{s_\beta^2} \left\{2(\beta_{t+1} - \beta_t - \beta_t^*) - \left(\beta_{t+1}^* - 2\beta_t^* + \beta_{t-1}^*\right)\right\},
\end{aligned}$$

$\forall t \in \{2, \dots, T-1\}$*, and*

$$\begin{aligned}
\frac{\partial \log(Post_{cond})}{\partial \beta_T} &= \frac{y_{j,T}}{\sigma_k} \chi_\tau \left(y_{k,T} - \mu_{k,T} - \mathbf{x}_T^\intercal \boldsymbol{\theta}_k - y_{j,T}\beta_t\right) \\
&\quad + \frac{6}{s_\beta^2} \left\{-2\left(\beta_T - \beta_{T-1} - \beta_{T-1}^*\right) + \left(\beta_T^* - \beta_{T-1}^*\right)\right\}.
\end{aligned}$$

It turns out that

$$\sum_t \frac{\partial \log(Post_{cond})}{\partial \beta_t} = \frac{-\beta_1 - \beta_1^*}{\kappa} + \frac{1}{\sigma_k} \sum_t y_{j,t} \chi_\tau \left(y_{k,t} - \mu_{k,t} - \mathbf{x}_t^\intercal \boldsymbol{\theta}_k - y_{j,t}\beta_t\right).$$

By choosing a sufficiently large κ and a diffuse prior on $\boldsymbol{\theta}_k$ the maximiser of $Post_{cond}$ satisfies the following equation

$$\sum_{t \notin C} (y_{j,t} + x_{m,t} + 1)\, \chi_\tau \left(y_{k,t} - \mu_{k,t} - \mathbf{x}_t^\intercal \boldsymbol{\theta}_k - y_{j,t}\beta_t\right) = 0,$$

$\forall m \in \{1, 2, \dots, M\}$*, which concludes the proof of Proposition 4.1.*

Corollary 4.1. *The MaP estimate of the quantiles* $\left(q_t^\tau(\mathbf{x}_t)\right)^{\mathrm{MaP}} = \mu_{j,t}^{\mathrm{MaP}} + \mathbf{x}_t^\intercal \boldsymbol{\theta}_j^{\mathrm{MaP}}$ *and* $\left(q_t^\tau(\mathbf{x}_t, y_{j,t})\right)^{\mathrm{MaP}} = \mu_{k,t}^{\mathrm{MaP}} + \mathbf{x}_t^\intercal \boldsymbol{\theta}_k^{\mathrm{MaP}} + y_{j,t}\beta_t^{\mathrm{MaP}}$ *satisfies a generalisation of the fundamental property of sample time-varying quantiles, that is:*

$$\sum_{t \in A} h_t \le (1-\tau) \sum_{t=1}^T h_t - \sum_{t \in C^-} h_t \quad \text{and} \quad \sum_{t \in B} h_t \le \tau \sum_{t=1}^T h_t - \sum_{t \in C^-} h_t, \tag{30}$$

where

$$h_t = \begin{cases} x_{m,t} + 1 & \text{for the VaR} \\ y_{j,t} + x_{m,t} + 1 & \text{for the CoVaR,} \end{cases} \tag{31}$$

and $A \cup B \cup C = \{1, \ldots, T\}$. Here, A and B denote the set of indices such that observations are respectively (strictly) above and (strictly) below the MaP quantile estimates, and $C = C^+ \cup C^- = \{t \in C : h_t \geq 0\} \cup \{t \in C : h_t < 0\}$ is the set of indices such that the observations coincide with the quantile estimates.

Proof of Corollary 4.1. *Following the proof of Proposition 3 in De Rossi and Harvey (2009), using Proposition 4.1 and the following inequalities*

$$\begin{aligned} -\tau \sum_{t \in C^+} h_t + (1-\tau) \sum_{t \in C^-} h_t &< \sum_{t \notin C} h_t \chi_\tau \left(y_{j,t} - \left(q_t^\tau\left(\mathbf{x}_t\right)\right)^{\text{MaP}} \right) \\ \sum_{t \notin C} h_t \chi_\tau \left(y_{j,t} - \left(q_t^\tau\left(\mathbf{x}_t\right)\right)^{\text{MaP}} \right) &< (1-\tau) \sum_{t \in C^+} h_t - \tau \sum_{t \in C^-} h_t, \end{aligned} \tag{32}$$

where $h_t = x_{m,t} + 1$, the result in Corollary 4.1 is obtained by rewriting the left-hand side term in (32) as follows:

$$\sum_{t \notin C} h_t \chi_\tau \left(y_{j,t} - \left(q_t^\tau\left(\mathbf{x}_t\right)\right)^{\text{MaP}} \right) = \tau \sum_{t \in A} h_t + (\tau - 1) \sum_{t \in B} h_t. \tag{33}$$

The same occurs for the CoVaR.

Remark 4.1. *If $h_t > 0 \quad \forall t = 1, 2, \ldots, T$, or if the distribution of $(Y_{j,t}, Y_{k,t})$ is continuous, then inequalities (30) coincide with the ones stated in Proposition 3 of De Rossi and Harvey (2009). When $h_t \equiv h \quad \forall t$, then (30) corresponds to the fundamental property of sample time-varying quantiles.*

5 Specification of the prior hyperparameters

In this section we outline the elicitation of the hyperparameters for the prior distributions specified in Sections 3 and 4. Concerning the regression parameters $\boldsymbol{\theta}$, which are common to both time invariant and time-varying models, we typically set $\boldsymbol{\theta}^0 = \mathbf{0}$ and the variance-covariance matrices $\Sigma_j^0 = \Sigma_k^0 = 100\mathbb{1}_d$, where $d = 2M + 2$ in the first case and $d = 2M$ in the latter case. This choice is quite standard in the Bayesian quantile regression literature when one wants to be as non informative as possible retaining a proper distribution. For the same reason we impose the same mean and variance hyperparameters for the loading factor β, i.e. $\beta^0 = 0$ and $\sigma_\beta^2 = 100$. Regarding the nuisance parameters (σ_j, σ_k), we choose $a_j^0 = b_j^0 = a_k^0 = b_k^0 = 0.0001$ which corresponds to proper Inverse Gamma distributions with infinite second moments. The same prior hyperparameters are adopted even in the time-varying case. This choice is motivated by the lack of knowledge of the range of the data and it is particularly appropriate when dealing with financial data characterised by large kurtosis.

In the time-varying framework we have the additional parameters related to the latent states dynamics. For those parameters we fix the location and shape of the Inverse Gamma prior to $r_l^0 = 2$ and $v_l^0 = 0.0001$ for $l \in \{j, k, \beta\}$, which corresponds to a prior mean equal to 0.001 and variance equal to 0.0001. Since these parameters are related to the smoothness of the quantile estimates, choosing those values allows us to reduce their variability through time. Concerning the initial values of the latent states $\boldsymbol{\xi}_1$, we employ a diffuse initialisation of the Kalman filter, by allowing the first state to be Normally distributed with zero mean and variance-covariance matrix proportional to $\kappa = 10^6$, as suggested by Durbin and Koopman (2012).

6 Empirical application

Throughout this section we apply the methodology previously discussed to real data. In particular we separately analyse the time-invariant specification of CoVaR proposed in Section 3 and the time-varying version considered in Section 4. Our aim is to study the tail co-movements between an individual institution j and the whole system k it belongs to. The financial data we utilise are taken from the Standard and Poor's Composite Index (k) for the U.S. market, where different sectors (j) are included. For both the institutions and for the whole system, we consider microeconomics and macroeconomics variables, in order to account for individual information and for global economic conditions respectively. The analysis is based on weekly observations; however, as microeconomic variables are not observable at the weekly frequency, we build a smoothing state space model to fill the missing values. The aim of the empirical application is to show how CoVaR provides interesting insights into the tail risk interdependence. In addition, we show the relevance of introducing dynamics in the extreme quantiles in order to effectively capture the contribution of individual institutions to the evolution of systemic risk. Approaching VaR and CoVaR estimation in a Bayesian framework allows us to calculate their credible sets which are necessary to assess the accuracy of estimates.

6.1 The data

Our empirical analysis is based on publicly traded U.S. companies belonging to different sectors of the Standard and Poor's Composite Index (S&P500) listed in Table 1. The sectors considered are: Financials, Consumer Goods, Energy, Industrials, Technologies and Utilities. Financials consists of banks, diversified financial services and consumer financial services. Consumer Goods consists of the food and beverage industry, primary food industry and producers of personal and household goods. The Energy sector consists of companies producing or supplying energy and it includes companies involved in the exploration and development of oil or gas reserves, oil and gas drilling, or integrated power firms. Industrials consist of industries such as construction and heavy equipment, as well as industrial goods and services that include containers, packing and industrial transport. Technologies are related to the research, development and/or distribution of technologically based goods and services, while Utilities consists of the provision of gas and electricity.

Daily equity price data are converted to weekly log-returns (in percentage points) for the sample period from January 2, 2004 to December 28, 2012, covering the recent global financial crisis. Table 1 provides summary statistics for the weekly returns. Except for some companies, the mean return during the estimation period is positive. Consumer Goods and Energy have the highest average return, while banks and financial services have the lowest. Focusing on the sample correlation with the market index return, the

Name	Ticker	Sector	Mean	Min	Max	Std. Dev.	Corr.	1% Str. Lev.
CITIGROUP INC.	C	Financial	-0.490	-92.632	78.797	9.756	0.673	-26.267
BANK OF AMERICA CORP.	BAC	Financial	- 0.212	-59.287	60.672	7.990	0.705	-27.231
COMERICA INC.	CMA	Financial	-0.072	-31.744	33.104	5.945	0.718	-19.794
JPMORGAN CHASE & CO.	JPM	Financial	0.086	-41.684	39.938	5.826	0.711	-12.785
KEYCORP	KEY	Financial	-0.210	-61.427	40.976	7.428	0.694	-24.231
GOLDMAN SACHS GROUP INC.	GS	Financial	0.071	-36.564	39.320	5.641	0.719	-16.850
MORGAN STANLEY	MS	Financial	-0.170	-90.465	69.931	8.342	0.699	-19.706
MOODY'S CORP.	MCO	Financial	0.125	-27.561	28.300	5.615	0.688	-20.719
AMERICAN EXPRESS CO.	AXP	Financial	0.091	-28.779	24.360	5.104	0.772	-16.108
MCDONALD'S CORP.	MCD	Consumer	0.320	-12.130	11.878	2.606	0.554	-4.978
NIKE INC.	NKE	Consumer	0.260	-18.462	18.723	3.746	0.655	-11.596
CHEVRON CORP.	CVX	Energy	0.254	-31.674	15.467	3.585	0.745	-8.275
EXXON MOBIL CORP.	XOM	Energy	0.197	-22.301	8.717	3.081	0.696	-7.123
BOEING CO.	BA	Industrial	0.164	-25.294	16.034	4.258	0.727	-12.468
GENERAL ELECTRIC CO.	GE	Industrial	-0.023	-18.680	30.940	4.311	0.715	-15.578
INTEL CORP.	INTC	Technology	-0.052	-17.038	16.935	4.117	0.671	-12.533
ORACLE	ORCL	Technology	0.203	-15.518	12.135	3.762	0.632	-9.986
AMEREN CORP.	AEE	Utilities	0.015	-29.528	9.485	3.118	0.682	-8.172
PUBLIC SERVICE ENT.	PEG	Utilities	0.143	-26.492	10.568	3.318	0.553	-8.191
STANDARD AND POOR 500	S&P500	Index	0.052	-20.083	11.355	2.637	1.000	-7.258

Table 1: Summary statistics of the company's returns and market index (S&P500) returns (in percentage). The sixth column, denoted by "Corr", is the correlation coefficient with the market returns while the last column, denoted by "1% Str. Lev." is the 1% empirical quantile of the returns distribution.

correlation involving the Financials and Industrials is the largest on average. The correlation with the market index return varies substantially across sectors, ranging from 0.553, Public Service Enterprise Inc. (PEG) to 0.772 American Express Co. (AXP). Interestingly, Nike Inc. (NKE) and Ameren Corp. (AEE), which belong to bellwether sectors like consumer and utilities, show a surprisingly high correlation level compared to that of McDonald's Corp. (MCD) and Public Service Enterprise Inc. (PEG) which belong to the same sectors, respectively. A possible explanation for this empirical evidence is that the correlation among financial stocks increases dramatically during times of turbulence and this was particularly evident in late 2008 as the global financial crisis intensified. Finally, the last column of Table 1 provides the sample 1% stress level of each institution's return evaluated over the entire time period. By comparing these values with the number of standard deviations away from their mean, we can see that asset return distributions do not appear highly skewed. We study the risk interdependence through the CoVaR tool due to the characteristics of the data summarised in Table 1.

To control for the general economic conditions we use observations of the following macroeconomic regressors as suggested by Adrian and Brunnermeier (2011) and Chao et al. (2012):

(I) the VIX index (VIX), measuring the model-free implied stock market volatility as evaluated by the Chicago Board Options Exchange (CBOE);

(II) a short term liquidity spread (LIQSPR), computed as the difference between the 3-month collateral repo rate and the 3-month Treasury Bill rate;

(III) the weekly change in the three-month Treasury Bill rate (3MTB);

(IV) the change in the slope of the yield curve (TERMSPR), measured by the difference of the 10-year Treasury rate and the 3-month Treasury Bill rate;

(V) the change in the credit spread (CREDSPR) between 10-year BAA rated bonds and the 10-year Treasury rate;

(VI) the weekly return of the Dow Jones U.S. Real Estate Index (DJUSRE).

Historical data for the volatility index (VIX) can be downloaded from the Chicago Board Options Exchange's website, while the remaining variables are from the Federal Reserve Board H.15 database. Data are available on a daily frequency and subsequently converted to a weekly frequency.

To capture the individual firms' characteristics, we include observations from the following microeconomic regressors:

(VII) leverage (LEV), calculated as the value of total assets divided by total equity (both measured in book values);

(VIII) the market to book value (MK2BK), defined as the ratio of the market value to the book value of total equity;

(IX) the size (SIZE), defined by the logarithmic transformation of the market value of total assets;

(X) the maturity mismatch (MM), calculated as short term debt net of cash divided by the total liabilities.

Microeconomic variables are downloaded from the Bloomberg database and are available only on a quarterly basis. Since our analysis builds on weekly frequencies we choose to impute missing observations by smoothing spline interpolation. Details on the procedure are given in Appendix 1.

6.2 Time-invariant risk beta

In what follows we provide the Bayesian empirical analysis for the time-invariant CoVaR model stated in Section 3. In order to implement the inference we specify the hyper-parameters values for each prior distribution defined therein. We ran the MCMC algorithm illustrated in Section 3 for 200,000 times, with a burn-in phase of 100,000 iterations. Tables 3–4, 5–6 and 7–8 in the appendix report the estimated systemic risk β and the exogenous parameters as well as the $\text{HPD}_{95\%}$ credible sets, for $\tau = (0.01, 0.025, 0.05)$, for all the considered institutions. To check the MCMC convergence we also calculate Geweke's convergence diagnostics (see Geweke, 1992, 2005) which are not reported here to save space but suggest that the convergence has been achieved.

For all reported institutions, β's parameters are positive and significantly different from zero. Note that a positive β indicates that a decrease in $\text{VaR}_j^{\mathbf{x},\tau}$ (expressed as a larger negative value) yields a greater negative $\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$, i.e. a higher risk of system losses. Moreover, by comparing $\text{HPD}_{95\%}$ for the β parameters, it is also evident that the extent of the contribution to systemic risk is significantly different across institutions belonging to different sectors. Hence the result highlights the evidence for a sector-specific effect of individual losses to the overall systemic risk. We also observe that on average the systemic β has a lower value for institutions belonging to the Financials sector and is higher for institutions belonging to Consumer Goods and Energy sectors. These results provide evidence that sectors have different sensitivity to risk exposure. It is worth noting that the β parameter sometimes displays a huge variation even within the same sector; as it is the case for the Industrial sector, where the estimated β coefficient for GE is significantly different from the one for BA whose credible sets are not overlapping. Furthermore, we observe as expected that the order of β's parameter estimates does not correspond to the order of the empirical correlation in Table 1. This is particularly evident for the consumer sector, where MCD has an estimated β larger than that of NKE but displays a lower correlation. This result is in line with what we expected as the correlation coefficient does not provide enough information about the relationship between extreme events. Finally, we compare the βs for the different values of τ considered and we observe that, on average, higher values of the parameter tend to be associated with smaller values of the confidence level τ, meaning that the co-movement between asset and market is stronger for extreme returns.

Considering the influence of macroeconomic variables, from Tables 3–4, 5–6 and 7–8 in the appendix we observe some remarkable differences among assets, in particular:

- Except for the volatility (VIX) and the U.S. real estate (DJUSRE) indices, the impact of the remaining variables changes in magnitude and significance as we move from one asset to another. This heterogeneous behaviour seems to be transversal with respect to sectors, at least in some cases such as liquidity spread (LIQSPRD). As expected, the estimated parameters for the VIX index are always significantly negative while the opposite occurs for the DJUSRE Index. This is true for both the VaR and CoVaR regressions and for all the considered levels of τ.

- Some macroeconomic variables, such as the change in three-month Treasury Bill rate (3MTB) or the term spread (TERMSPR) have different impacts on the CoVaR and VaR. In the first case the 3MTB displays positive or non significant coefficients, while in the case of TERMSPR we find negative coefficients for some sectors (Financial and Utilities). This means that an increase in the spread between 10-year Treasury Bond rates and three-month Treasury Bill rates (CREDSPR) produces a decrease of the CoVaR while reducing individual risks in the case of the Financial and Utilities sectors. Thus large traded firms, such as the ones typically operating in the mentioned sectors, benefit from an increase in the interest rate spread because it increases the opportunity cost of different financing strategies. As expected, the change in credit spread has a negative impact on the firms' level of risk.

- For different values of the confidence level τ the macroeconomic variables exert a different impact on the marginal and conditional quantiles, becoming in general less significant as the τ-level increases.

For the microeconomics exogenous regressors, we note that:

- The leverage regressor (LEV) displays a different impact on the VaR and the CoVaR respectively. The VaR is greatly enhanced in highly leveraged companies, in fact the significant coefficients are negative signed. In contrast, the CoVaR regression produces mixed results for the impact of the leverage that is always positive for American Express Co. (AXP), JP Morgan Chase & Co. (JPM) and Exxon Mobile Corp. (XOM) and always negative for Goldman and Sachs Group Inc. (GS), Chevron Corp. (CVX) and Public Enterprise Service icn. (PEG). Moreover as the τ level decreases to 0.01 a larger number of institutions belonging to the Financials display significantly negative coefficients.

- The impact of the market capitalisation (SIZE) on the VaR is nearly exclusively significantly positive while for the CoVaR regression its sign varies across institutions belonging to the same sector. This evidence suggests that large institutions are more risky if considered in isolation. Moreover, the extent to which large companies contribute to the overall risk is not clear, and depends on the "degree of connection" among institutions and on diversification of their portfolios.

- The maturity mismatch coefficient (MM) is nearly always negative for the VaR regression in the Financials while it is positive for all the remaining sectors. In this instance, the CoVaR regression shows a different influence of the MM regressor; that is positive and significant for Financials (except for AXP). This signals the existence of positive dependence between financial imbalances and systemic riskiness at least for financials firms.

- The market-to-book ratio (MK2BK), again, shows an opposite impact on the VaR and the CoVaR: significantly negative for the non financial sectors in the VaR regression, and positive, when significant, for the CoVaR regression.

To have a complete picture of the contributions from individual and systemic risk we plot the estimated VaR and CoVaR for some of the assets listed in Table 1 in Figure 1. Looking at individual risk assessment, it is clear that the VaR profiles are relatively similar across institutions, displaying strong negative downside effects upon the occurrence of the recent financial crises of 2008 and 2010 and the sovereign debt crisis of 2012. However, the analysis of the time series evolution of the marginal contribution to the systemic risk, measured by CoVaR, reveals different behaviors for the considered assets. In particular, Citigroup (C), which belongs to the Financials, seems to contribute more to the overall risk than other assets do. Inspecting Figure 1 and Tables 3–4, 5–6 and 7–8 in the appendix, we note that institutions which have low β coefficients provide major contribution to the 2008 financial crisis. On the contrary, MCD, which belongs to the Consumer Goods sector, has a large estimated β, and as seen in the CoVaR plot in Figure 1 its contribution to the overall systemic risk is much lower than that of financial institutions.

For the selected companies, Figure 2 plots the CoVaR for two different confidence levels $\tau = 0.025$ and $\tau = 0.1$. This figure highlights the different impact of the Global

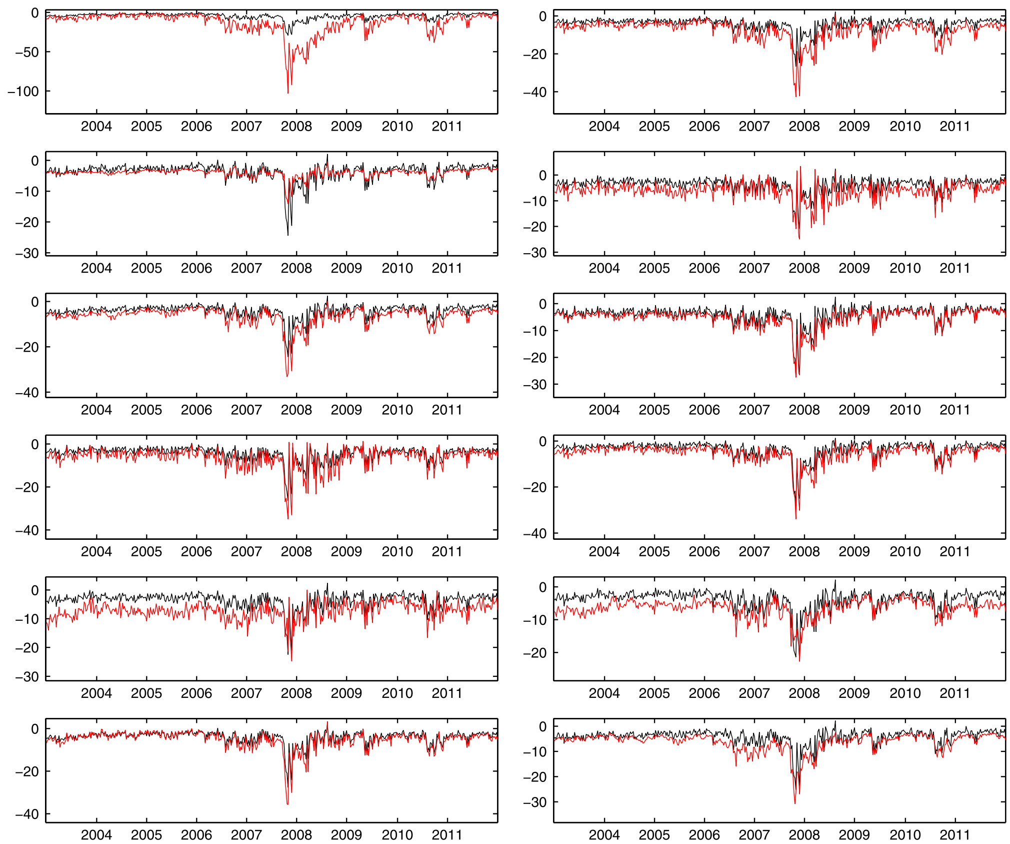

Figure 1: Time series plot of the $\text{VaR}_j^{\mathbf{x},\tau}$ (red line) and $\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ (gray line) at the confidence level $\tau = 0.025$, obtained by fitting the model time invariant model defined in equations (7)–(8) for the following assets: top panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); fourth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); bottom panel (utilities): AEE (left), PEG (right).

Financial crisis among sectors. For the Financials, for example, we note that the difference between $\text{CoVaR}_j^{\mathbf{x},0.025}$ and $\text{CoVaR}_j^{\mathbf{x},0.1}$ is much larger than for assets belonging to other sectors, implying that the Financials had a huge impact on the extreme systemic risk during the 2008 crisis, as witnessed by the extremely large losses.

6.3 Time-varying risk beta

We estimate time-varying systemic risk betas according to the dynamic model defined in equations (15)–(20) using the same exogenous variables described in Section 6.1. Tables 9–10, 11–12 and 13–14 in the appendix list the posterior estimates of the exogenous

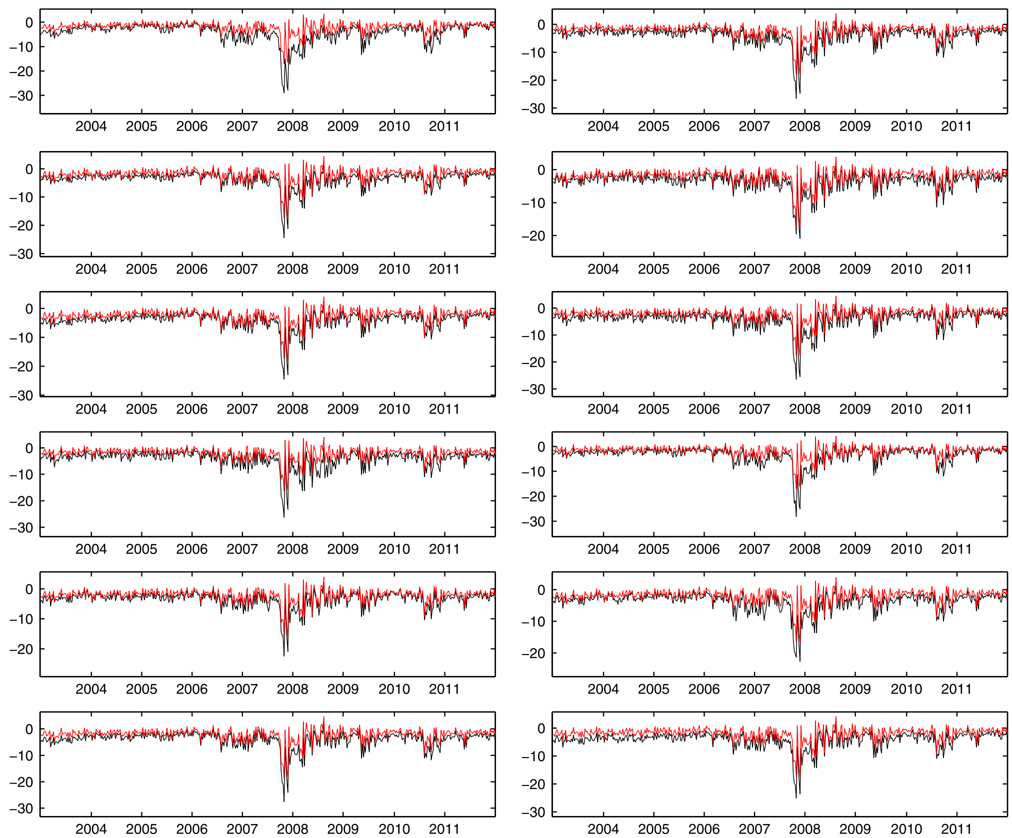

Figure 2: Time series plot of the $\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ at the confidence levels $\tau = 0.025$ (gray line) and $\tau = 0.1$ (red line), obtained by fitting the time invariant model defined in equations (7)–(8) for the following assets: top panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); forth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); bottom panel (utilities): AEE (left), PEG (right).

regressor parameters $\boldsymbol{\theta}$ for both $\text{VaR}_j^{\tau,\mathbf{x}}$ and $\text{CoVaR}_{k|j}^{\tau,\mathbf{x}}$ regressions, at the confidence level $\tau = 0.01$, $\tau = 0.025$ and $\tau = 0.05$, respectively. We observe that all the macroeconomic variables related to the term structure have a positive impact on both VaR and CoVaR or are non significant, except for a few cases. This means that an upward shift in the term structure of interest rate provides a marginal positive contribution to individual and systemic risks. For the remaining macroeconomic variables, the VIX index always has a negative impact, while the rate of change of the DJUSRE index always has a positive effect on both VaR and CoVaR. Interestingly, American Express Co. (AXP) is the only financial institution displaying a negative coefficient for the variable credit spread (CREDSPR) in the individual VaR regression. This essentially means that an

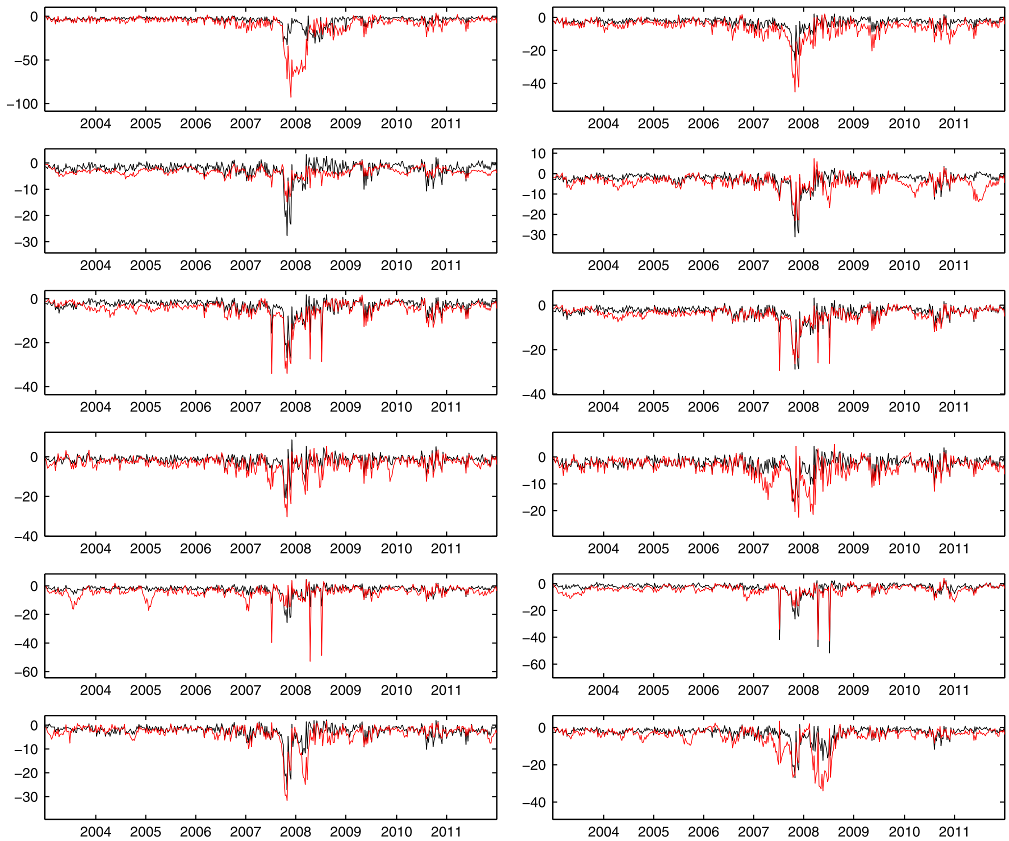

Figure 3: Time series plot of the $\text{VaR}_j^{\mathbf{x},\tau}$ (red line) and $\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ (gray line) at the confidence level $\tau = 0.025$, obtained by fitting the time-varying model defined in equations (15)–(20) for the following assets: top panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); forth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); bottom panel (utilities): AEE (left), PEG (right).

increase in the credit spread increases the individual riskiness of that institution. This result seems to be coherent with American Express's institutional activity, since it is a global financial services institution whose main offerings are charge and credit cards. Concerning the microeconomic variables, SIZE is the only variable which shows a clear positive effect for all the considered institutions in the case of the VaR regression. The effect of the SIZE variable on CoVaR changes according to the τ-level, moving from a predominance of positive effects for higher τs to negative effect for lower τs. The MM is non significant for almost all the reported institutions for both VaR and CoVaR regression, while the MK2BK and the LEV variables reveal heterogeneous sector-specific impacts on the risk measures. In this respect the analysis is not exhaustive and the

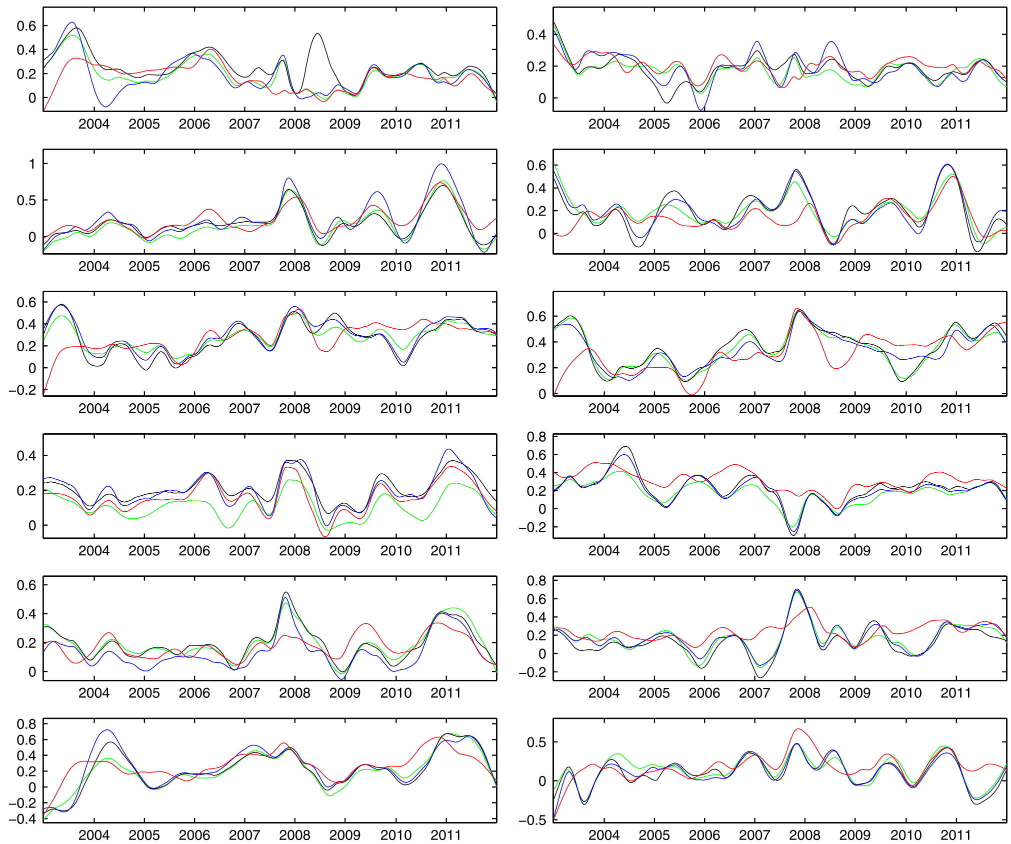

Figure 4: Time series plot of the dynamic β_t for all confidence level $\tau = 0.01$, (green line), $\tau = 0.025$, (dark line), $\tau = 0.05$, (blue line) and $\tau = 0.5$ (red line), obtained by fitting the time-varying model defined in equations (15)–(20) for the following assets, top panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); forth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); bottom panel (utilities): AEE (left), PEG (right).

identification of sector-specific risk factors deserves further investigation using a greater number of institutions.

Figure 3, which is the dynamic counterpart of Figure 1, shows that the dynamic CoVaR risk measure suddenly adapts to capture extreme negative losses especially during the 2008 financial crisis. Comparing this evidence with that shown in Figure 1, it is clear that the dynamic model provides a better characterisation of extreme tail co-movements when dealing with time series data.

Turning our attention to the time-varying β's results, Figure 4 plots the evolution of the MaP estimates for all confidence levels of τ. As expected the evolution of the

β's for $\tau = 0.01$ (green line), $\tau = 0.025$ (black line) and $\tau = 0.05$ (blue line), is almost identical for all the reported institutions. Moreover, the behaviour of the systemic risk β displays huge cross-sectional heterogeneity. For example, the systemic risk β of the energy institutions (third panel in Figure 4) increases over time, showing their highest values during the financial crisis at the end of 2008 and 2011. Conversely, during the same period, we observe a large drop down for the β_t of General Electric (GE) (fourth panel in Figure 4). The time series behaviour of the systemic risk β reveals a different impact of the crisis periods on the overall marginal risk contribution of each institution.

6.4 Quantile backtesting

In order to evaluate the predictive ability of the quantiles' models (7)–(8) and (15)–(16) we perform out–of–sample backtesting procedures. We consider observations from January 2, 2004 to February, 21 2014 comprising 530 weekly log–returns. The full data period is divided into a learning sample: January 2, 2004 to September 25, 2009; and a forecasting sample: October 2, 2009 to the end of the sample period. The out–of–sample forecasting period comprises 230 observations. For each weekly return $(y_{j,T+n}, y_{k,T+n})$, $n = 1, 2, \ldots, 230$ in the forecast sample, parameters are estimated by employing an estimation window of 300 observations till time $T + n - 1$, then the forecasts for the next week's τ–level quantiles are generated using $25,000$ post burn–in MCMC draws. Bayesian quantile forecasting procedures are described in Clarke and Clarke (2012). To assess the backtesting performances we calculate the actual over expected number of violations (A/E), the expected loss given a violation (AD), and the well known conditional and unconditional coverage tests of Kupiec (1995) and Christoffersen (1998), and the Dynamic Quantile (DQ) test of Engle and Manganelli (2004), for $\tau = 0.05$ and for all the considered institutions. Although these procedures are not Bayesian, they are commonly used tools to assess the validity of the quantile models. The two tables summarising all the results for both invariant and time-varying models are available as supplementary material. In both cases we reach satisfactory results in terms of A/E and AD violations. For all considered institutions, the number of violations is in line with their expected values, except in a few instances. Moreover, neither tests of conditional and unconditional coverage reject the null hypothesis that violations series are martingale difference sequences. Concerning the AD series, results highlight the goodness models performance in terms of loss magnitudes. Comparing the static and dynamic backtesting results, however, we can state that the latter is preferable in terms of maximum losses.

6.5 Measuring marginal contribution to systemic risk

In their paper Adrian and Brunnermeier (2011) introduced the $\Delta\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ as a measure of the marginal contribution to system risk, defined as

$$\Delta\text{CoVaR}_{k|j}^{\mathbf{x},\tau} = \text{CoVaR}_{k|Y_j=\text{VaR}_j^{\mathbf{x},\tau}}^{\mathbf{x},\tau} - \text{CoVaR}_{k|Y_j=\text{VaR}_j^{\mathbf{x},0.5}}^{\mathbf{x},\tau}$$

where $\text{CoVaR}_{k|Y_j=\text{VaR}_j^{\mathbf{x},\tau}}^{\mathbf{x},\tau} = \text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ and $\text{CoVaR}_{k|Y_j=\text{VaR}_{k|j}^{\mathbf{x},0.5}}^{\mathbf{x},\tau}$ satisfies equation (4) with $\text{VaR}_j^{\mathbf{x},0.5}$ instead of $\text{VaR}_j^{\mathbf{x},\tau}$. To illustrate the behaviour of $\Delta\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ in our ap-

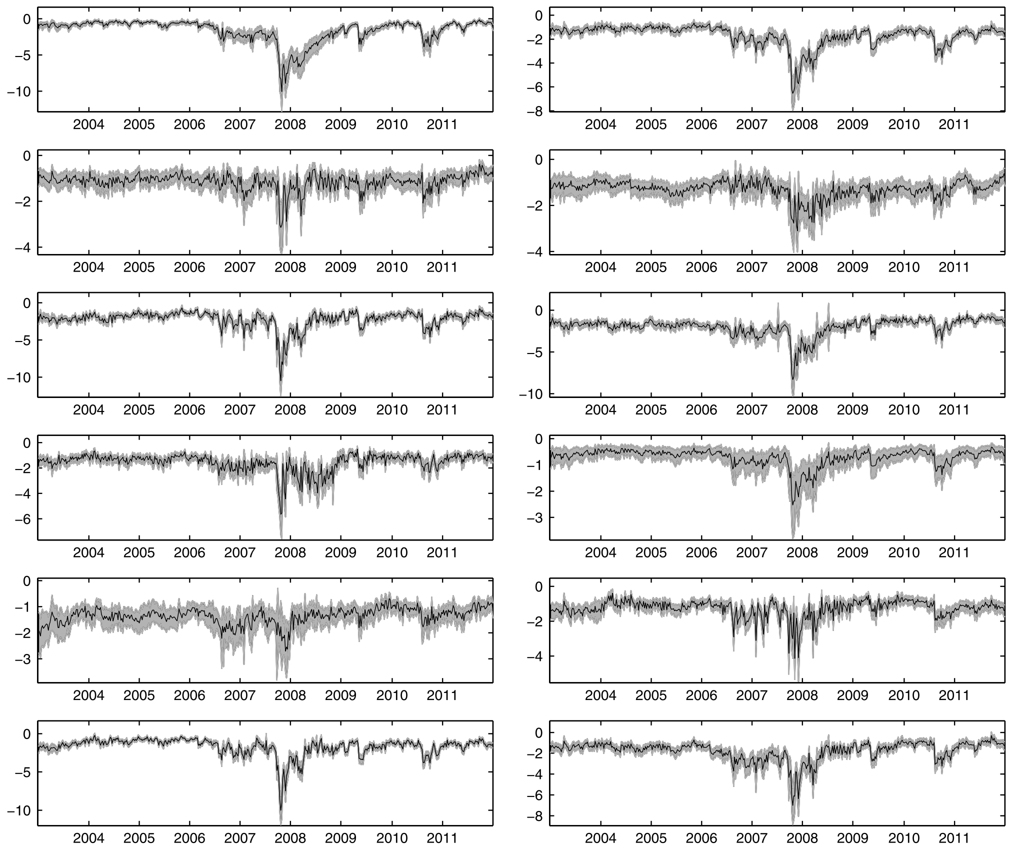

Figure 5: Time series plot of the static $\Delta\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ at the confidence level $\tau = 0.025$ along with $\text{HPD}_{95\%}$ credible sets, obtained by fitting the model defined in equations (8) for the following assets, first panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); forth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); last panel (utilities): AEE (left), PEG (right).

plication we plot the time-invariant version in Figure 5, and its dynamic version in Figure 6. As expected, for all the considered companies, the marginal contribution to systemic risk increases during market turbulences, showing their lowest values during the financial crisis of 2008. Moreover, some important differences among companies belonging to different sectors are evident: in particular, the Financials sector, (first panel of Figures 5–6), the Energy sector (third panel of Figures 5–6) and the Utilities sector (bottom panel of Figures 5–6), display the largest drop in value. In contrast, the Technology sector displays the lowest variations of the ΔCoVaR measure during the 2008 recession. The grey areas correspond to the $\text{HPD}_{95\%}$ associated to the ΔCoVaR contributions providing information about the size of the risk contribution for the given

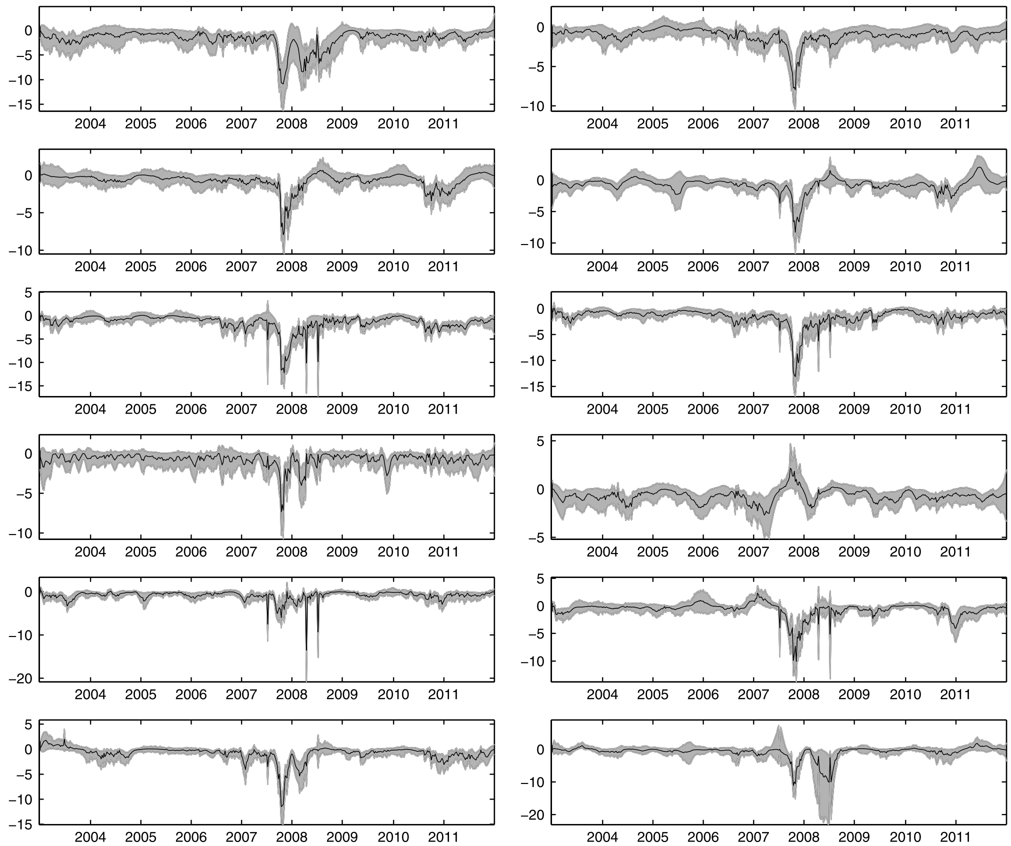

Figure 6: Time series plot of the dynamic $\Delta\text{CoVaR}_{k|j}^{\mathbf{x},\tau}$ at the confidence level $\tau = 0.025$ along with $\text{HPD}_{95\%}$ credible sets, obtained by fitting the time-varying model defined in equations (15)–(20) for the following assets, top panel (financial): C (left), GS (right); second panel (consumer): MCD (left) and NKE (right); third panel (energy): CVX (left), XOM (right); forth panel (industrial): BA (left), GE (right); fifth panel (technology): INTC (left), ORCL (right); bottom panel (utilities): AEE (left), PEG (right).

confidence level. We note a huge cross-sectional heterogeneity on the credible sets behaviour, with some sectors such as the Consumer Goods, Industrials and Technology being characterised by large uncertainty of the ΔCoVaR estimates.

Comparing Figures 5 and 6, it is evident that the dynamic ΔCoVaR estimates are smoother than the corresponding time-invariant ones, which is a clear and useful signal for policy maker purposes. Moreover, Figure 6 provides a clear indication of the high flexibility of the dynamic model implying a prompt reaction of the risk measure to the economic and financial downturns. These considerations argue in favour of the dynamic model when dealing with time series data.

	CoVaR		ΔCoVaR	
Name	Time invariant	Time-varying	Time invariant	Time-varying
C	1.325	1.035	0.401	0.325
BAC	1.084	0.983	0.230	0.298
CMA	1.109	1.264	0.109	0.177
JPM	1.259	0.573	0.136	0.111
KEY	1.078	-0.481	0.229	0.165
GS	1.085	1.276	0.234	0.188
MS	**1.003**	0.604	0.275	0.132
MCO	1.085	-4.139	0.130	0.052
AXP	1.110	18.352	0.171	0.205
MCD	0.933	3.500	0.062	0.192
NKE	0.842	1.251	0.095	0.222
CVX	**0.986**	2.376	0.291	0.354
XOM	1.099	1.280	0.276	0.365
BA	**1.017**	2.666	0.169	0.156
GE	1.179	21.245	0.078	-0.039
INTC	0.874	5.928	0.060	0.097
ORCL	**0.998**	3.985	0.066	0.229
AEE	1.054	1.441	0.346	0.296
PEG	**1.001**	5.123	0.205	0.271

Table 2: δ_1 estimates for the regressions in equations (34)–(35) for both the time invariant and time varying models. Bold numbers indicate that the corresponding coefficients are not significantly different from 1.

Finally, we consider the time series relationship between VaR and CoVaR or ΔCoVaR. In what is perhaps the key result of Adrian and Brunnermeier (2011), they find that the CoVaR (ΔCoVaR) of two institutions may be significantly different even if the VaR of the two institutions are similar. On this basis, they suggest that the policy maker employ the CoVaR (ΔCoVaR) risk measure, as a valid alternative to the VaR, when forming policy regarding an institution's risk. Results obtained with our modeling support their thesis. In fact, building the following simple regression model for each institution j

$$\text{CoVaR}_{k|j}^{\tau,\mathbf{x}} = \delta_0 + \delta_1 \text{VaR}_k^{\tau,\mathbf{x}} + \nu, \tag{34}$$

or

$$\Delta\text{CoVaR}_{k|j}^{\tau,\mathbf{x}} = \delta_0 + \delta_1 \text{VaR}_k^{\tau,\mathbf{x}} + \nu, \tag{35}$$

with $\mathbb{E}\left(\nu|\text{VaR}_k^{\tau,\mathbf{x}}\right) = 0$, where k is the S&P500 Index we test $H_0 : \delta_1 = 1$ to show that $\text{CoVaR}_{k|j}^{\tau,\mathbf{x}}$ ($\Delta\text{CoVaR}_{k|j}^{\tau,\mathbf{x}}$) is significantly different from the $\text{VaR}_k^{\tau,\mathbf{x}}$. Bold numbers in Table 2 indicate cases where the corresponding coefficient δ_1 is not significantly different from one. Except for a few cases the null hypothesis is rejected and this is more evident for the dynamic model than for the time invariant one. In particular, for the ΔCoVaR regression there is strong evidence that the estimated δ_1 parameter is related to the sector the institutions belong to, being higher for Financials, Energies and Utilities. Interestingly, the regression coefficients are almost zero for the Technology sector.

7 Conclusion

One of the major issues policy makers deal with during financial crisis is the evaluation of the extent to which risky tail events spread across financial institutions. In fact, during financial turmoils, the correlations among asset returns tend to rise, a phenomenon known in the economic and financial literature as contagion. From a statistical point of view the risk of contagion essentially implies that the joint probability of observing large losses increases during recessions. The common risk measures recently imposed by the public regulators, (the Basel Committee, for the bank sector) such as the VaR, fail to account for such risk spillover among institutions. The CoVaR risk measure recently introduced by Adrian and Brunnermeier (2011) overcomes this problem as it is able to account for the dependence among institutions' extreme events.

In this paper we address the problem of estimating the CoVaR in a Bayesian framework using quantile regression. We first consider a time-invariant model allowing for interactions only among contemporaneous variables. The model is subsequently extended in a time-varying framework where the constant part and the CoVaR parameter β are modeled as functions of unobserved processes having their own dynamics. In order to make posterior inference, we use the maximum a posteriori summarising criterion and we prove that it leads to estimated quantiles having good sample properties according to the results of De Rossi and Harvey (2009) and we improve the efficiency of Gibbs sampler algorithms based on data augmentation for the two models considered.

The Bayesian approach used throughout the paper allows us to infer on the entire posterior distribution of the quantities of interest and their credible sets which are important to assess the accuracy of the point estimates. Since the quantities of interest in this context are risk measures, understanding the whole distribution becomes more relevant due to the interpretation of the VaR and CoVaR as financial losses. In addition, credible sets provide upper and lower limits for the capital requirements for banks and financial institutions.

To verify the reliability of the built models we analyse weekly time series for nineteen institutions belonging to six different sectors of the Standard and Poor's 500 composite index spanning the period from 2nd January, 2004 to 31st December, 2012. We use micro and macro exogenous variables to characterise the quantile functions and to give insights into which variables have the most influences on both the VaR and the CoVaR. In order to thoroughly investigate the joint relevance of exogenous variables we are currently developing a Bayesian variable selection approach in a similar context. Nevertheless, from the empirical results, it is clear that the model and the proposed approach give an exact estimate of the marginal and conditional quantiles providing a more realistic and informative characterisation of extreme tail co-movements. In particular, the dynamic version of the model we propose outperforms the time invariant specification when the analysis is based on time series data. This is, to our knowledge, the first attempt to implement a Bayesian inference for the CoVaR.

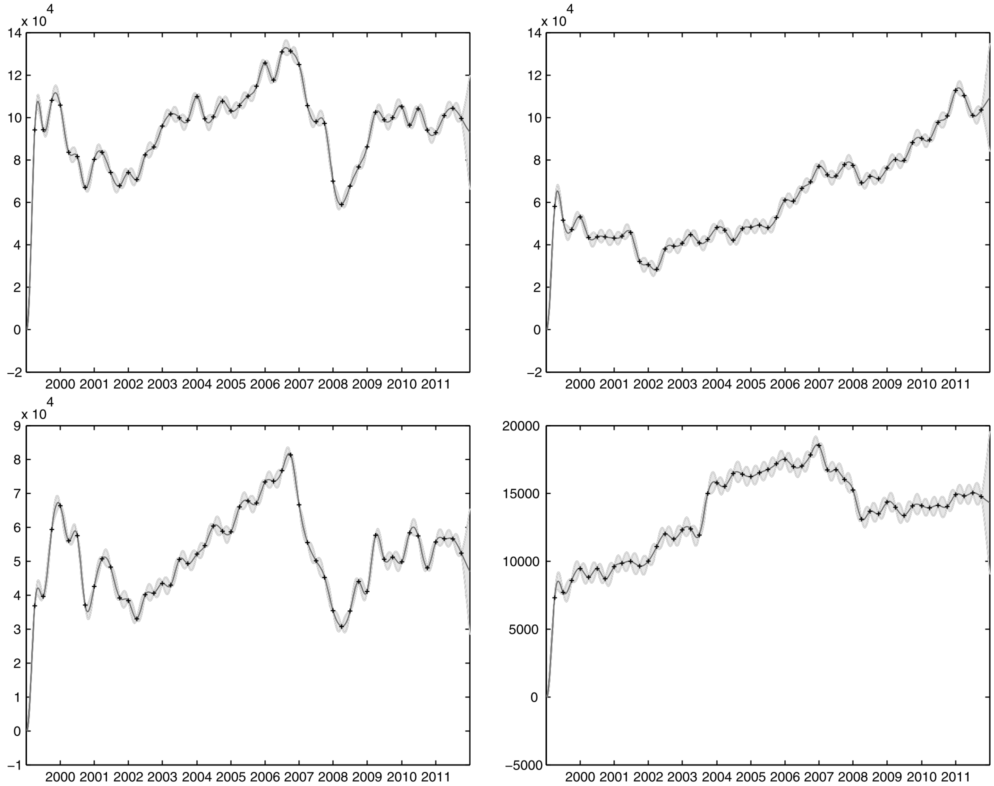

Figure 7: Missing values imputation using smoothing spline of the SIZE variable for AXP (top, left), MCD (top, right), BA (bottom, left) and AEE (bottom, right). In each plot predicted values (gray line) as well as 95% HPD credible sets (gray area) are displayed. Black cross denotes observed values.

Appendix 1: Missing values treatment

In this appendix we give details on the procedure used to impute missing observations. In particular for each variable, starting from balance sheets data available only on a quarterly basis, we implement a nonparametric smoothing cubic spline, see for example Koopman (1991) and Koopman et al. (1998). Cubic splines have been previously applied to local linear forecasts of time series by Hyndman et al. (2005) and used for handling missing data in Koopman (1991) and Koopman et al. (1998). This non-parametric technique represents a valid and flexible alternative to parametric methodologies without relying on strong assumptions about the underlying data generating process.

Supposing we have a univariate time series $y_{\tau_1}, y_{\tau_2}, \ldots, y_{\tau_T}$, not necessarily equispaced in time, and define $\delta_t = \tau_t - \tau_{t-1}$, for $t = 1, 2, \ldots, T$ as the difference between two consecutive observations, and assume the time series is not entirely observed, we approximate the series by a sufficiently smooth function $s(\tau_t)$. Following the standard approach, we choose $s(\tau_t)$ by minimizing the following penalized-least squares criterion:

$$\mathcal{L}(\lambda) = \sum_{i=1}^{T} \left[y_{\tau_t} - s(\tau_i)\right] + \lambda \sum_{i=1}^{T} \left[\Delta^m s(\tau_i)\right]^2, \tag{36}$$

with respect to $s(\tau_t)$, for a given penalization term λ. The function $s(\tau_t), \forall t = 1, 2, \dots, T$ is a polynomial spline of order $m+1$ and when $m = 2$, we have a smoothing cubic spline model.

To estimate the smoothing parameter λ and to forecast missing observations model (36) can be cast in state space form that is, for $m = 2$:

$$\begin{aligned} y(\tau_t) &= s(\tau_t) + \epsilon(\tau_t) \\ s(\tau_{t+1}) &= s(\tau_t) + \delta_t \ell(\tau_t) + \zeta_1(\tau_t) \\ \ell(\tau_{t+1}) &= \ell(\tau_t) + \zeta_2(\tau_t) \end{aligned}$$

where $\left[s(\tau_1), \ell(\tau_1)\right]^{\mathsf{T}} \sim \mathcal{N}(\mathbf{0}_2, \kappa \mathbb{I}_2)$, with κ sufficiently large enough to ensure a diffuse initialization of the latent states, the transition equation innovations vector is $\left[\zeta_1(\tau_t), \zeta_2(\tau_t)\right]^{\mathsf{T}} \sim \mathcal{N}(\mathbf{0}_2, \mathbf{S}_\zeta)$ with $\mathbf{S}_\zeta = \sigma_\zeta^2 \begin{bmatrix} \frac{1}{3}\delta_t^3 & \frac{1}{2}\delta_t^2 \\ \frac{1}{2}\delta_t^2 & \delta_t \end{bmatrix}$, the measurement innovation is $\epsilon(\tau_t) \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$ and the penalty parameter λ coincides with the signal-to-noise ratio $\lambda = \sigma_\zeta^2 / \sigma_\epsilon^2$. We fix the parameter σ_ϵ to one and estimate $\lambda = \sigma_\zeta^2$ in a Bayesian framework imposing a diffuse Inverse Gamma prior distribution, i.e. $\lambda = \sigma_\zeta^2 \sim \mathcal{IG}\left(\alpha_\lambda^0, \beta_\lambda^0\right)$. We fit the model using MCMC techniques, in particular, the Gibbs sampler with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987; Gelfand and Smith, 1990). The Gibbs sampler consists of the following two steps:

S1. Simulate the latent process $\boldsymbol{\xi}_t^{(i+1)} = \left[s(\tau_t), \ell(\tau_t)\right]^{\mathsf{T}}$, $\forall t = 1, 2, \dots, T$, using the disturbance simulation smoothing algorithm of De Jong and Shephard (1995) appropriately adjusted to handle missing observations and the diffuse initialization of the state vector (see also the augmented Kalman filter and smoother of De Jong, 1991).

S2. Simulate the $\lambda^{(i+1)}$ parameter from the complete full conditional distribution which is an Inverse Gamma distribution $\lambda^{(i+1)} \sim \mathcal{IG}\left(\widetilde{\alpha}_\lambda^{(i+1)}, \widetilde{\beta}_\lambda^{(i+1)}\right)$ with parameters:

$$\begin{aligned} \widetilde{\alpha}_\lambda^{(i+1)} &= \alpha_\lambda^0 + \frac{T-1}{2} \\ \widetilde{\beta}_\lambda^{(i+1)} &= \beta_\lambda^0 + \frac{1}{2} \sum_{t=1}^{T-1} \left(\boldsymbol{\xi}_{t+1}^{(i+1)} - T_t \boldsymbol{\xi}_t^{(i+1)}\right)^{\mathsf{T}} \boldsymbol{S}_\zeta^{-1} \left(\boldsymbol{\xi}_{t+1}^{(i+1)} - T_t \boldsymbol{\xi}_t^{(i+1)}\right), \end{aligned}$$

where $T_t = \begin{bmatrix} 1 & \delta_t \\ 0 & 1 \end{bmatrix}$, for $i = 1, 2, \dots, G$, with G equal to the number of draws. The algorithm is initialized at $i = 0$ by simulating the λ parameter from its prior distribution. The state space representation of the smoothing spline along with the Bayesian paradigm allow us to efficiently deal with the estimation of the model parameters as well as to predict the missing observations. In fact, the missing value interpolation is obtained by running the efficient recursive Kalman filter and smoothing algorithms and represents a byproduct of the Bayesian inferential procedure (see also Durbin and Koopman (2012) and Harvey (1989) for an extensive treatment of missing values using state space methods). Missing information is then estimated by averaging simulated points of $s(\tau_t)$ across Gibbs sampler draws. The advantage of the implemented smoothing technique is that it takes all available values into consideration instead of only past observations, providing more credible estimates of the missing information.

Appendix 2: Tables

VaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	-44.628	-13.362	-27.581	-24.574	-7.746	-16.918	-30.206
	(-46.623,-11.638)	(-42.309,-3.855)	(-30.366,-16.108)	(-48.497,-13.360)	(-38.721,-0.696)	(-26.912,-4.081)	(-46.345,-9.539)
VIX	-0.535	-1.193	-0.284	-0.538	-0.742	-0.336	-0.901
	(-0.857,-0.445)	(-1.424,-0.872)	(-0.346,-0.267)	(-0.700,-0.476)	(-0.933,-0.609)	(-0.409,-0.319)	(-1.049,-0.819)
LIQSPR	0.1	0.066	-0.031	0.009	0.026	0.061	-0.013
	(0.033,0.142)	(0.003,0.139)	(-0.059,-0.016)	(-0.016,0.048)	(-0.025,0.087)	(0.032,0.082)	(-0.056,0.038)
3MTB	-0.174	0.067	-0.096	0.113	-0.376	0.173	-0.133
	(-0.270,0.056)	(-0.086,0.221)	(-0.126,-0.057)	(-0.032,0.191)	(-0.443,-0.016)	(0.086,0.244)	(-0.221,0.006)
TERMSPR	0.041	0.071	0.059	0.071	-0.289	0.101	-0.186
	(0.010,0.174)	(-0.026,0.147)	(0.043,0.090)	(-0.007,0.112)	(-0.339,-0.111)	(0.026,0.153)	(-0.266,-0.028)
CREDSPR	-0.146	0.381	-0.13	-0.066	-0.094	0.021	-0.551
	(-0.197,0.020)	(0.179,0.428)	(-0.151,-0.078)	(-0.172,-0.019)	(-0.171,0.036)	(-0.089,0.104)	(-0.592,-0.433)
DJUSRE	0.998	0.383	0.42	0.641	0.508	0.586	0.695
	(0.580,1.110)	(0.069,0.552)	(0.230,0.460)	(0.416,0.742)	(0.271,0.776)	(0.438,0.727)	(0.548,0.830)
LEV	-2.283	-0.445	-0.187	0.027	-1.647	-0.439	0.052
	(-2.646,-1.669)	(-2.140,1.005)	(-0.421,0.196)	(-0.139,0.822)	(-1.717,-0.310)	(-0.568,-0.115)	(-0.251,0.347)
MK2BK	0.649	2.511	1.201	0.582	0.713	4.463	2.66
	(-1.892,1.894)	(-0.533,4.771)	(0.113,2.027)	(-1.982,2.834)	(-4.232,2.301)	(1.900,4.923)	(-1.869,6.012)
SIZE	6.679	3.1	2.917	1.906	3.05	1.597	3.875
	(3.726,6.629)	(2.078,6.008)	(1.649,3.092)	(1.019,3.670)	(1.715,6.095)	(0.669,2.508)	(1.839,5.640)
MM	-25.5	-52.111	-42.737	14.609	2.068	-2.704	-25.432
	(-30.261,7.397)	(-65.770,-36.803)	(-67.752,-32.009)	(-9.267,20.620)	(-12.345,16.225)	(-13.493,2.910)	(-38.203,-10.582)
σ_j	0.16	0.166	0.089	0.108	0.182	0.096	0.151
	(0.147,0.176)	(0.143,0.173)	(0.081,0.097)	(0.095,0.114)	(0.156,0.187)	(0.085,0.103)	(0.133,0.160)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	9.007	-30.075	-30.055	0.455	-27.4	1.174
	(-2.914,29.766)	(-52.988,-18.805)	(-42.302,-4.708)	(-22.700,12.720)	(-28.084,9.970)	(-23.484,12.143)
VIX	0.061	-0.299	-0.039	-0.015	-0.315	-0.222
	(-0.034,0.103)	(-0.284,-0.181)	(-0.071,-0.025)	(-0.061,0.012)	(-0.350,-0.302)	(-0.310,-0.196)
LIQSPR	0.123	-0.06	-0.003	0.035	-0.013	0.061
	(0.040,0.142)	(-0.080,-0.047)	(-0.023,0.008)	(0.022,0.059)	(-0.034,0.002)	(0.023,0.079)
3MTB	-0.257	-0.271	0.03	0.164	0.037	0.095
	(-0.334,-0.057)	(-0.270,-0.198)	(-0.002,0.095)	(0.096,0.198)	(-0.012,0.097)	(0.037,0.138)
TERMSPR	-0.287	-0.084	-0.028	0.116	-0.02	0.002
	(-0.322,-0.089)	(-0.089,-0.042)	(-0.043,-0.000)	(0.060,0.136)	(-0.043,-0.001)	(-0.032,0.041)
CREDSPR	-0.1	-0.195	-0.073	-0.002	-0.116	0.087
	(-0.168,0.070)	(-0.209,-0.149)	(-0.094,-0.025)	(-0.057,0.035)	(-0.156,-0.078)	(0.041,0.127)
DJUSRE	1.332	0.372	0.084	0.692	0.333	0.541
	(1.051,1.381)	(0.312,0.428)	(0.065,0.139)	(0.610,0.817)	(0.193,0.361)	(0.435,0.637)
LEV	0.134	-0.723	4.567	-4.709	-0.924	-7.992
	(0.109,0.201)	(-1.344,-0.657)	(-0.508,6.736)	(-11.706,2.860)	(-5.548,1.234)	(-10.352,9.795)
MK2BK	0.001	0.153	-0.807	1.905	1.082	-2.907
	(-0.001,0.001)	(-0.151,0.868)	(-1.146,-0.010)	(1.072,3.732)	(-0.967,1.491)	(-3.723,-2.104)
SIZE	-3.284	3.413	1.784	-0.876	2.412	1.807
	(-5.401,-1.910)	(2.443,5.541)	(-0.139,2.972)	(-1.712,0.538)	(-0.022,2.477)	(-0.882,3.192)
MM	-21.133	12.664	-5.104	-2.177	16.138	10.614
	(-25.495,-18.562)	(5.438,24.334)	(-9.962,2.912)	(-6.483,1.551)	(10.043,22.250)	(-2.527,10.777)
σ_j	0.145	0.083	0.059	0.104	0.08	0.073
	(0.132,0.158)	(0.073,0.087)	(0.052,0.063)	(0.090,0.108)	(0.074,0.088)	(0.065,0.078)

VaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-9.948	-23.374	-34.916	-36.633	2.812	-30.87
	(-32.756,-2.359)	(-46.142,-8.340)	(-37.384,-1.083)	(-52.563,-19.746)	(-7.549,13.665)	(-36.585,-2.333)
VIX	-0.188	-0.197	-0.048	-0.173	-0.396	-0.294
	(-0.228,-0.112)	(-0.251,-0.144)	(-0.127,-0.043)	(-0.275,-0.164)	(-0.444,-0.376)	(-0.317,-0.262)
LIQSPR	0.007	-0.119	0.055	-0.05	-0.045	-0.142
	(-0.001,0.034)	(-0.177,-0.056)	(0.015,0.068)	(-0.076,-0.029)	(-0.063,-0.031)	(-0.164,-0.090)
3MTB	0.026	-0.072	0.149	0.158	-0.054	-0.009
	(-0.025,0.041)	(-0.086,0.020)	(0.089,0.200)	(0.023,0.196)	(-0.103,-0.041)	(-0.047,0.040)
TERMSPR	0.041	-0.001	0.116	0.108	-0.024	-0.067
	(0.009,0.061)	(-0.014,0.056)	(0.082,0.151)	(0.024,0.120)	(-0.069,-0.013)	(-0.086,-0.031)
CREDSPR	-0.069	-0.073	0.086	-0.015	-0.15	-0.044
	(-0.198,-0.044)	(-0.079,-0.005)	(0.044,0.143)	(-0.127,0.007)	(-0.200,-0.127)	(-0.082,0.004)
DJUSRE	0.851	0.645	0.553	-0.024	0.51	0.174
	(0.768,0.964)	(0.467,0.705)	(0.355,0.588)	(-0.150,0.085)	(0.408,0.535)	(0.100,0.281)
LEV	0.025	-1.288	8.828	12.738	-6.526	0.366
	(0.021,0.031)	(-2.522,-0.543)	(1.262,12.417)	(8.731,21.628)	(-9.410,-3.876)	(-0.757,0.601)
MK2BK	-0.003	-0.47	-6.14	0.161	-3.163	-0.706
	(-0.010,0.015)	(-2.384,0.173)	(-7.466,-5.944)	(-0.306,0.539)	(-4.499,-2.599)	(-1.058,1.128)
SIZE	0.716	2.613	2.526	0.876	2.511	3.169
	(0.057,2.828)	(1.390,4.818)	(-0.062,3.222)	(-1.148,2.056)	(1.753,3.235)	(0.265,3.738)
MM	18.439	-4.285	-7.173	2.152	7.183	34.012
	(17.263,40.139)	(-7.426,9.228)	(-12.137,-5.689)	(-0.158,4.083)	(4.891,19.742)	(7.889,38.060)
σ_j	0.091	0.081	0.106	0.094	0.076	0.085
	(0.079,0.095)	(0.071,0.086)	(0.097,0.116)	(0.083,0.100)	(0.066,0.079)	(0.076,0.091)

Table 3: VaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.01$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	22.686	-8.613	-2.556	0.439	-12.936	1.15	-15.852
	(13.746,28.843)	(-5.061,15.474)	(-5.260,3.271)	(-15.336,5.558)	(-17.946,5.696)	(-6.157,3.317)	(-19.356,-8.646)
VIX	-0.264	-0.177	-0.2	-0.293	-0.102	-0.112	-0.068
	(-0.290,-0.247)	(-0.202,-0.146)	(-0.243,-0.177)	(-0.304,-0.276)	(-0.180,-0.102)	(-0.133,-0.085)	(-0.082,-0.051)
LIQSPR	-0.008	-0.01	0.012	0.007	-0.004	-0.011	0.008
	(-0.017,0.004)	(-0.024,0.008)	(-0.009,0.032)	(-0.006,0.024)	(-0.015,0.004)	(-0.021,0.004)	(-0.004,0.013)
3MTB	0.075	0.115	0.094	0.046	0.091	0.055	0.088
	(0.044,0.097)	(0.056,0.123)	(0.046,0.110)	(0.040,0.108)	(0.058,0.132)	(0.015,0.089)	(0.057,0.110)
TERMSPR	0.002	0.03	0.044	0.035	0.055	0.033	0.056
	(-0.011,0.013)	(0.019,0.039)	(0.030,0.057)	(0.023,0.047)	(0.047,0.070)	(0.008,0.041)	(0.044,0.059)
CREDSPR	0.04	0.022	0.024	0.03	0.01	0.022	0.047
	(0.025,0.056)	(-0.016,0.027)	(0.010,0.048)	(0.035,0.072)	(0.001,0.032)	(0.004,0.049)	(0.041,0.062)
DJUSRE	0.162	0.308	0.235	0.307	0.245	0.296	0.321
	(0.148,0.239)	(0.233,0.345)	(0.210,0.304)	(0.300,0.390)	(0.196,0.265)	(0.252,0.357)	(0.302,0.346)
LEV	-0.005	-0.102	-0.096	0.381	-0.391	-0.038	-0.054
	(-0.035,0.031)	(-0.199,0.157)	(-0.275,0.123)	(0.239,0.408)	(-0.384,-0.068)	(-0.093,-0.010)	(-0.090,-0.040)
MK2BK	-0.901	-1.384	-1.64	-3.438	-1.069	-0.045	-0.939
	(-1.335,-0.414)	(-1.459,-0.412)	(-2.046,-1.097)	(-5.040,-3.190)	(-2.024,-0.294)	(-0.629,0.322)	(-1.447,-0.344)
SIZE	-2.031	0.981	0.757	0.009	1.961	0.004	1.487
	(-2.501,-1.229)	(-1.276,0.607)	(0.037,1.119)	(-0.411,1.354)	(-0.212,2.467)	(-0.211,0.591)	(0.809,1.792)
MM	19.698	2.619	-8.594	6.362	6.168	-1.458	3.025
	(12.129,23.386)	(-0.896,8.075)	(-21.921,-0.429)	(3.488,20.180)	(-3.064,10.506)	(-3.610,4.881)	(1.496,5.958)
β	0.162	0.102	0.155	0.146	0.14	0.265	0.118
	(0.114,0.173)	(0.086,0.129)	(0.118,0.180)	(0.115,0.209)	(0.099,0.153)	(0.239,0.291)	(0.116,0.145)
σ_k	0.032	0.036	0.033	0.035	0.035	0.032	0.029
	(0.031,0.038)	(0.032,0.038)	(0.032,0.038)	(0.032,0.039)	(0.031,0.037)	(0.030,0.036)	(0.027,0.032)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	19.58	8.419	13.405	-12.615	-1.496	-17.332
	(11.940,21.390)	(8.225,27.859)	(-17.500,17.692)	(-25.305,4.103)	(-6.785,26.106)	(-21.370,-3.897)
VIX	-0.256	-0.235	-0.189	-0.174	-0.101	-0.17
	(-0.281,-0.237)	(-0.265,-0.223)	(-0.195,-0.143)	(-0.216,-0.170)	(-0.140,-0.092)	(-0.184,-0.129)
LIQSPR	0.024	0.018	0.002	-0.018	0.001	0.005
	(0.001,0.020)	(-0.004,0.016)	(-0.006,0.020)	(-0.031,-0.008)	(-0.016,0.004)	(0.000,0.024)
3MTB	0.044	0.076	-0.024	0.094	0.018	0.014
	(0.017,0.053)	(0.061,0.101)	(-0.028,0.025)	(0.030,0.106)	(-0.022,0.034)	(-0.004,0.049)
TERMSPR	0.02	0.054	-0.001	0.044	0.035	0.006
	(0.009,0.035)	(0.041,0.063)	(-0.022,0.012)	(0.017,0.055)	(0.023,0.051)	(-0.006,0.027)
CREDSPR	-0.024	0.024	-0.095	-0.009	-0.041	-0.025
	(-0.035,0.003)	(0.026,0.065)	(-0.095,-0.045)	(-0.026,0.004)	(-0.050,-0.012)	(-0.037,0.002)
DJUSRE	0.2	0.181	0.231	0.297	0.228	0.297
	(0.162,0.228)	(0.118,0.192)	(0.236,0.351)	(0.257,0.330)	(0.218,0.276)	(0.276,0.366)
LEV	-0.013	0.585	-5.605	1.704	-2.015	10.978
	(-0.017,-0.007)	(0.109,0.590)	(-8.487,-2.422)	(-2.616,7.181)	(-5.341,-1.096)	(5.837,11.966)
MK2BK	0	0.146	0.755	0.079	-0.083	0.052
	(-0.000,0.000)	(-0.229,0.096)	(-0.122,0.965)	(-0.749,0.335)	(-0.524,0.581)	(-0.556,0.418)
SIZE	-1.852	-1.273	-0.324	0.899	0.397	-0.347
	(-1.996,-1.009)	(-2.566,-1.042)	(-0.570,2.472)	(-0.193,1.765)	(-1.418,0.794)	(-0.574,-0.147)
MM	1.669	-15.891	5.462	-2.861	-2.533	-2.555
	(1.130,2.738)	(-15.223,-4.542)	(4.728,9.597)	(-5.027,-0.386)	(-4.565,1.213)	(-3.409,2.701)
β	0.127	0.196	0.24	0.276	0.292	0.287
	(0.099,0.173)	(0.198,0.274)	(0.205,0.359)	(0.169,0.295)	(0.255,0.312)	(0.237,0.364)
σ_k	0.031	0.034	0.037	0.039	0.028	0.03
	(0.030,0.036)	(0.030,0.036)	(0.035,0.041)	(0.033,0.040)	(0.027,0.033)	(0.028,0.033)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-0.757	43.748	-34.985	2.623	-10.804	-10.487
	(-5.088,8.764)	(29.376,52.771)	(-43.813,-20.351)	(-5.429,8.097)	(-9.430,8.813)	(-14.445,-4.795)
VIX	-0.132	-0.332	-0.186	-0.18	-0.132	-0.088
	(-0.158,-0.126)	(-0.338,-0.310)	(-0.203,-0.175)	(-0.187,-0.154)	(-0.166,-0.125)	(-0.110,-0.069)
LIQSPR	-0.001	0.044	0.006	-0.003	0.006	0.017
	(-0.012,0.024)	(0.021,0.054)	(-0.011,0.011)	(-0.008,0.010)	(0.003,0.013)	(0.004,0.021)
3MTB	-0.025	0.029	0.024	0.014	0.053	0.055
	(-0.041,0.001)	(-0.010,0.052)	(-0.012,0.035)	(0.014,0.071)	(0.034,0.057)	(0.026,0.089)
TERMSPR	-0.031	0.018	0.013	-0.012	0.059	0.047
	(-0.039,-0.012)	(-0.009,0.025)	(-0.009,0.025)	(-0.027,0.019)	(0.044,0.064)	(0.027,0.061)
CREDSPR	-0.044	0.01	-0.07	-0.059	0.007	-0.045
	(-0.059,-0.030)	(-0.009,0.020)	(-0.079,-0.053)	(-0.079,-0.026)	(-0.002,0.020)	(-0.067,-0.011)
DJUSRE	0.301	0.247	0.251	0.252	0.185	0.269
	(0.247,0.316)	(0.205,0.275)	(0.236,0.298)	(0.225,0.337)	(0.148,0.195)	(0.257,0.355)
LEV	0.006	0.686	-5.23	-0.181	2.377	-0.359
	(0.002,0.010)	(0.362,0.876)	(-6.660,-2.059)	(-1.925,0.642)	(-0.038,2.603)	(-0.686,-0.141)
MK2BK	0.003	-0.375	-0.967	-0.288	-1.677	0.274
	(-0.003,0.004)	(-0.868,0.180)	(-1.165,-0.347)	(-0.520,0.021)	(-2.432,-1.565)	(-0.197,0.472)
SIZE	-0.043	-3.514	4.011	-0.032	0.611	1.049
	(-0.899,0.383)	(-4.290,-2.267)	(2.432,4.765)	(-0.470,0.676)	(-0.880,0.625)	(0.522,1.480)
MM	-10.568	-1.581	2.25	-0.926	7.636	-6.775
	(-12.839,-2.524)	(-5.204,2.112)	(1.285,2.936)	(-1.342,-0.149)	(6.418,10.077)	(-11.854,6.084)
β	0.234	0.137	0.201	0.21	0.373	0.308
	(0.200,0.257)	(0.102,0.197)	(0.160,0.223)	(0.169,0.258)	(0.324,0.397)	(0.257,0.350)
σ_k	0.037	0.035	0.031	0.035	0.032	0.036
	(0.033,0.039)	(0.031,0.038)	(0.030,0.036)	(0.033,0.039)	(0.028,0.034)	(0.033,0.039)

Table 4: CoVaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.01$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

VaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	-23.453	-23.73	-24.575	-3.706	-25.803	-13.518	-40.232
	(-41.309,-5.136)	(-42.958,-11.088)	(-28.871,-15.193)	(-28.752,2.264)	(-40.714,-3.834)	(-22.909,-0.779)	(-58.293,-21.802)
VIX	-0.881	-0.766	-0.3	-0.371	-0.905	-0.398	-0.603
	(-1.295,-0.747)	(-0.855,-0.552)	(-0.350,-0.246)	(-0.459,-0.364)	(-0.975,-0.717)	(-0.436,-0.350)	(-0.778,-0.466)
LIQSPR	0.014	0.097	-0.034	0	0.022	0.042	-0.018
	(-0.076,0.041)	(0.003,0.088)	(-0.058,-0.012)	(-0.014,0.028)	(-0.029,0.078)	(0.000,0.057)	(-0.064,0.043)
3MTB	-0.007	0.092	-0.084	-0.025	-0.159	0.116	0.09
	(-0.207,0.071)	(-0.117,0.141)	(-0.107,-0.019)	(-0.049,0.054)	(-0.235,0.020)	(0.054,0.134)	(-0.081,0.101)
TERMSPR	0.11	0.093	0.083	0.014	-0.137	0.053	-0.027
	(0.005,0.179)	(-0.017,0.145)	(0.059,0.106)	(0.003,0.062)	(-0.233,-0.050)	(0.015,0.075)	(-0.117,0.008)
CREDSPR	-0.003	0.072	-0.13	-0.038	0.033	-0.018	-0.213
	(-0.130,0.110)	(-0.136,0.118)	(-0.153,-0.061)	(-0.071,0.002)	(-0.043,0.119)	(-0.133,-0.023)	(-0.411,-0.161)
DJUSRE	0.776	0.566	0.368	0.67	0.576	0.492	0.788
	(0.319,0.917)	(0.456,0.737)	(0.298,0.506)	(0.496,0.689)	(0.277,0.788)	(0.340,0.510)	(0.517,0.917)
LEV	-1.046	0.093	-0.147	-0.081	-0.463	-0.281	-0.025
	(-1.268,-0.095)	(-1.027,1.295)	(-0.505,0.176)	(-0.186,0.336)	(-1.042,0.007)	(-0.312,-0.053)	(-0.193,0.126)
MK2BK	-1.61	0.477	0.773	1.191	-4.699	1.86	-3.526
	(-3.923,0.055)	(-0.730,2.861)	(0.524,2.384)	(-0.152,2.452)	(-6.073,-1.249)	(-0.144,2.376)	(-5.719,0.044)
SIZE	4.276	3.033	2.689	0.215	5.003	1.701	4.328
	(2.574,5.648)	(1.881,4.307)	(1.570,3.023)	(-0.199,2.292)	(2.287,6.438)	(0.518,2.311)	(2.398,6.036)
MM	-16.018	-40.628	-65.857	8.852	-5.523	-5.775	4.814
	(-33.629,1.883)	(-47.632,-24.672)	(-58.769,-23.794)	(-9.480,11.641)	(-14.424,16.149)	(-13.134,3.581)	(-10.206,19.449)
σ_j	0.388	0.317	0.212	0.225	0.306	0.195	0.324
	(0.342,0.410)	(0.290,0.348)	(0.191,0.229)	(0.197,0.235)	(0.301,0.362)	(0.193,0.232)	(0.300,0.362)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	0.151	-30.096	3.999	-0.26	-18.94	-23.383
	(-13.902,17.419)	(-44.705,-13.426)	(-32.696,1.486)	(-24.156,12.067)	(-23.875,13.031)	(-22.646,8.844)
VIX	-0.329	-0.332	-0.092	-0.104	-0.314	-0.237
	(-0.399,-0.231)	(-0.346,-0.228)	(-0.104,-0.064)	(-0.106,-0.020)	(-0.381,-0.273)	(-0.321,-0.244)
LIQSPR	0.025	-0.048	0.017	0.008	-0.01	0.037
	(0.003,0.063)	(-0.080,-0.009)	(0.005,0.034)	(-0.007,0.049)	(-0.037,0.019)	(0.010,0.061)
3MTB	0.025	-0.11	0.015	0.067	0.017	0.029
	(0.006,0.159)	(-0.240,-0.079)	(-0.031,0.034)	(0.038,0.144)	(-0.041,0.094)	(-0.021,0.088)
TERMSPR	-0.011	-0.036	-0.004	0.057	0.006	0.005
	(-0.023,0.112)	(-0.083,-0.004)	(-0.025,0.009)	(-0.006,0.109)	(-0.050,0.017)	(-0.017,0.038)
CREDSPR	0.203	-0.097	-0.075	0.056	-0.109	-0.024
	(0.073,0.257)	(-0.159,-0.068)	(-0.088,-0.044)	(-0.010,0.057)	(-0.169,-0.047)	(-0.032,0.054)
DJUSRE	0.785	0.348	0.112	0.592	0.217	0.287
	(0.484,0.914)	(0.318,0.460)	(0.077,0.165)	(0.527,0.794)	(0.145,0.333)	(0.248,0.436)
LEV	-0.008	-0.546	0.288	-4.157	-1.443	-0.671
	(-0.057,0.030)	(-1.165,-0.205)	(-0.057,6.935)	(-10.445,3.260)	(-8.269,-0.155)	(-0.363,14.448)
MK2BK	0	0.072	0.482	1.518	-0.177	-1.013
	(-0.000,0.001)	(-0.285,0.601)	(-0.671,0.389)	(1.177,3.623)	(-1.937,1.344)	(-2.299,-0.483)
SIZE	-0.493	3.392	-0.859	-0.54	2.127	2.284
	(-2.255,0.992)	(1.910,4.684)	(-0.885,1.978)	(-1.869,0.903)	(0.146,2.636)	(-1.739,1.377)
MM	-9.419	5.392	-2.905	-5.08	22.866	13.566
	(-11.613,-5.467)	(-0.754,16.497)	(-7.443,0.710)	(-7.222,0.039)	(6.004,22.529)	(-1.730,12.212)
σ_j	0.305	0.179	0.122	0.216	0.197	0.165
	(0.279,0.334)	(0.167,0.200)	(0.117,0.140)	(0.186,0.223)	(0.172,0.206)	(0.145,0.174)

VaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-30.23	-30.936	-9.147	-31.432	7.56	-19.406
	(-38.347,-14.434)	(-54.967,-19.101)	(-35.447,2.443)	(-46.134,-18.303)	(-5.195,17.662)	(-32.558,-0.159)
VIX	-0.164	-0.321	-0.168	-0.166	-0.267	-0.301
	(-0.189,-0.112)	(-0.326,-0.234)	(-0.178,-0.094)	(-0.220,-0.153)	(-0.318,-0.238)	(-0.337,-0.262)
LIQSPR	-0.03	-0.002	0.006	-0.041	0.016	-0.054
	(-0.039,0.007)	(-0.031,0.019)	(-0.081,0.028)	(-0.068,-0.026)	(-0.026,0.022)	(-0.094,-0.009)
3MTB	0.014	-0.01	0.208	0.167	-0.046	-0.036
	(-0.034,0.056)	(-0.054,0.034)	(0.058,0.242)	(0.109,0.200)	(-0.091,-0.026)	(-0.062,0.064)
TERMSPR	0.036	0.033	0.175	0.081	-0.044	-0.042
	(-0.035,0.051)	(0.002,0.050)	(0.105,0.208)	(0.054,0.124)	(-0.095,-0.029)	(-0.066,0.003)
CREDSPR	-0.101	0.011	0.123	0.022	-0.167	-0.092
	(-0.150,-0.044)	(-0.045,0.040)	(0.036,0.152)	(-0.033,0.074)	(-0.198,-0.126)	(-0.116,-0.042)
DJUSRE	0.757	0.488	0.423	0.095	0.41	0.106
	(0.676,0.838)	(0.311,0.517)	(0.225,0.486)	(-0.031,0.200)	(0.327,0.455)	(0.022,0.193)
LEV	0.027	0.69	2.208	4.951	-7.226	0.121
	(0.021,0.036)	(-0.447,0.625)	(-7.393,7.797)	(4.051,10.956)	(-10.479,-3.950)	(-0.697,0.468)
MK2BK	-0.005	-1.675	-3.986	-0.007	-3.435	-1.473
	(-0.011,0.005)	(-2.534,-0.481)	(-4.901,-0.982)	(-0.267,0.386)	(-4.128,-1.743)	(-2.744,-0.237)
SIZE	2.705	2.632	1.016	1.745	2.03	2.348
	(1.232,3.415)	(1.774,4.884)	(-0.200,3.634)	(0.168,2.671)	(1.544,2.673)	(0.482,3.732)
MM	16.945	-6.272	-6.329	0.686	3.878	-1.645
	(14.240,36.859)	(-11.209,-2.179)	(-9.374,2.354)	(-1.218,2.378)	(-0.720,17.106)	(-12.453,19.996)
σ_j	0.187	0.157	0.242	0.176	0.173	0.179
	(0.175,0.210)	(0.148,0.178)	(0.220,0.263)	(0.180,0.216)	(0.147,0.175)	(0.172,0.205)

Table 5: VaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.025$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	21.132	-0.745	12.509	-8.071	0.218	0.027	-7.597
	(7.972,27.108)	(-0.198,14.951)	(-3.539,10.333)	(-15.303,0.607)	(-10.848,8.853)	(-6.376,2.947)	(-17.399,-5.490)
VIX	-0.252	-0.188	-0.253	-0.26	-0.195	-0.118	-0.1
	(-0.276,-0.211)	(-0.204,-0.160)	(-0.245,-0.176)	(-0.303,-0.258)	(-0.213,-0.154)	(-0.146,-0.109)	(-0.105,-0.044)
LIQSPR	-0.008	0.001	0.01	0.014	0.012	0.004	0.013
	(-0.013,0.008)	(-0.010,0.018)	(0.008,0.036)	(0.000,0.022)	(0.004,0.018)	(-0.000,0.015)	(-0.009,0.011)
3MTB	0.069	0.023	0.044	0.046	0.036	0.014	0.063
	(0.021,0.080)	(-0.008,0.050)	(0.028,0.081)	(0.000,0.092)	(0.020,0.058)	(-0.017,0.024)	(0.043,0.103)
TERMSPR	0.003	0.015	0.042	0.031	0.029	0.014	0.037
	(-0.012,0.012)	(-0.001,0.026)	(0.026,0.050)	(0.017,0.044)	(0.024,0.053)	(0.002,0.021)	(0.032,0.053)
CREDSPR	0.002	-0.009	0.022	-0.005	0.002	-0.013	0.033
	(-0.022,0.031)	(-0.044,0.004)	(-0.025,0.032)	(-0.025,0.038)	(-0.012,0.030)	(-0.030,0.011)	(0.038,0.063)
DJUSRE	0.151	0.234	0.209	0.236	0.242	0.284	0.316
	(0.137,0.223)	(0.188,0.274)	(0.191,0.277)	(0.209,0.311)	(0.193,0.258)	(0.229,0.309)	(0.302,0.348)
LEV	-0.019	0.086	-0.023	0.327	-0.093	-0.072	-0.027
	(-0.065,0.019)	(-0.140,0.245)	(-0.383,0.028)	(0.212,0.428)	(-0.204,0.039)	(-0.121,-0.055)	(-0.091,-0.012)
MK2BK	-0.971	-1.415	-1.176	-3.896	-1.429	-0.073	-0.881
	(-1.533,-0.521)	(-1.486,-0.471)	(-2.168,-1.074)	(-4.775,-3.528)	(-2.010,-0.568)	(-0.214,0.620)	(-1.207,-0.284)
SIZE	-1.853	0.221	-0.946	0.687	0.394	0.228	0.779
	(-2.333,-0.685)	(-1.141,0.128)	(-0.618,1.069)	(0.042,1.281)	(-0.635,1.624)	(-0.033,0.844)	(0.559,1.649)
MM	18.739	0.459	-6.417	12.946	9.352	-2.369	1.805
	(11.500,21.613)	(-1.346,6.252)	(-17.247,2.584)	(9.701,19.381)	(-2.890,11.002)	(-6.649,1.025)	(-0.543,5.387)
β	0.138	0.116	0.147	0.14	0.107	0.243	0.112
	(0.103,0.158)	(0.103,0.138)	(0.113,0.168)	(0.086,0.169)	(0.097,0.142)	(0.207,0.256)	(0.114,0.148)
σ_k	0.076	0.076	0.072	0.081	0.076	0.076	0.065
	(0.071,0.085)	(0.068,0.082)	(0.071,0.085)	(0.070,0.084)	(0.067,0.080)	(0.063,0.076)	(0.059,0.071)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	16.835	25.524	-9.039	-10.414	9.677	-12.747
	(13.627,22.211)	(6.678,26.090)	(-25.983,7.463)	(-15.093,8.644)	(3.317,23.778)	(-18.034,-7.015)
VIX	-0.24	-0.243	-0.149	-0.128	-0.099	-0.152
	(-0.269,-0.228)	(-0.248,-0.201)	(-0.195,-0.139)	(-0.141,-0.091)	(-0.107,-0.065)	(-0.165,-0.105)
LIQSPR	0.022	0.005	0.01	-0.01	0.012	0.009
	(0.008,0.027)	(0.000,0.016)	(-0.001,0.021)	(-0.023,-0.004)	(0.005,0.016)	(-0.002,0.014)
3MTB	0.039	0.089	0.007	0.031	0.027	0.026
	(0.017,0.056)	(0.050,0.106)	(-0.023,0.032)	(0.001,0.064)	(0.006,0.045)	(0.006,0.045)
TERMSPR	0.024	0.047	0.007	0.037	0.023	0.02
	(0.012,0.037)	(0.030,0.053)	(-0.008,0.017)	(0.016,0.044)	(0.010,0.038)	(0.004,0.029)
CREDSPR	-0.019	0.054	-0.044	-0.002	-0.04	-0.006
	(-0.034,-0.001)	(0.024,0.063)	(-0.085,-0.039)	(-0.027,0.022)	(-0.052,-0.014)	(-0.034,0.003)
DJUSRE	0.219	0.136	0.301	0.286	0.237	0.318
	(0.177,0.235)	(0.127,0.210)	(0.228,0.303)	(0.235,0.326)	(0.211,0.282)	(0.277,0.348)
LEV	-0.015	0.257	-2.304	1.907	-3.26	8.835
	(-0.016,-0.006)	(0.114,0.436)	(-4.171,0.565)	(-3.872,3.391)	(-5.988,-2.268)	(5.627,10.242)
MK2BK	0	0.07	0.226	-0.067	0.154	-0.001
	(-0.000,0.000)	(-0.031,0.249)	(-0.280,0.705)	(-0.293,0.537)	(-0.324,0.737)	(-0.404,0.450)
SIZE	-1.584	-2.402	1.246	0.722	-0.337	-0.342
	(-2.133,-1.225)	(-2.499,-0.800)	(-0.217,2.637)	(-0.602,1.033)	(-1.233,0.102)	(-0.400,-0.042)
MM	0.893	-8.13	5.641	-1.648	0.145	-0.555
	(0.716,1.954)	(-12.256,-6.441)	(1.580,6.941)	(-4.143,-1.265)	(-4.200,1.809)	(-3.485,2.361)
β	0.153	0.262	0.327	0.252	0.297	0.34
	(0.120,0.181)	(0.218,0.287)	(0.187,0.324)	(0.162,0.244)	(0.294,0.354)	(0.310,0.421)
σ_k	0.073	0.071	0.08	0.079	0.066	0.059
	(0.067,0.081)	(0.065,0.078)	(0.074,0.088)	(0.073,0.087)	(0.058,0.070)	(0.059,0.071)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-2.66	38.01	-8.476	-2.013	8.342	-12.301
	(-13.725,3.097)	(22.465,48.669)	(-24.689,-1.164)	(-7.721,2.699)	(-9.701,5.974)	(-14.436,-2.596)
VIX	-0.122	-0.325	-0.129	-0.159	-0.153	-0.108
	(-0.145,-0.097)	(-0.333,-0.283)	(-0.159,-0.114)	(-0.173,-0.129)	(-0.185,-0.134)	(-0.139,-0.087)
LIQSPR	0.013	0.038	-0.003	0.012	0.013	0.018
	(-0.010,0.020)	(0.018,0.049)	(-0.014,0.007)	(-0.000,0.014)	(0.007,0.018)	(0.008,0.023)
3MTB	-0.004	0.026	-0.006	0.027	0.033	0.048
	(-0.052,0.019)	(-0.005,0.068)	(-0.019,0.031)	(0.023,0.062)	(0.022,0.049)	(0.015,0.076)
TERMSPR	-0.007	0.009	0.015	-0.006	0.037	0.042
	(-0.028,0.000)	(-0.005,0.027)	(0.003,0.030)	(-0.007,0.020)	(0.030,0.056)	(0.026,0.059)
CREDSPR	-0.046	0.008	-0.054	-0.071	0.009	-0.057
	(-0.084,-0.015)	(-0.031,0.025)	(-0.067,-0.033)	(-0.096,-0.056)	(-0.012,0.018)	(-0.067,-0.030)
DJUSRE	0.287	0.228	0.235	0.273	0.18	0.271
	(0.217,0.351)	(0.184,0.284)	(0.222,0.275)	(0.223,0.294)	(0.157,0.212)	(0.236,0.310)
LEV	0.009	0.685	-1.409	-1.496	0.861	-0.399
	(0.004,0.014)	(0.272,0.865)	(-4.305,-0.949)	(-2.520,-0.357)	(-0.137,3.041)	(-0.636,-0.180)
MK2BK	0.002	-0.59	-0.23	-0.258	-1.54	-0.104
	(-0.002,0.004)	(-0.975,0.025)	(-0.762,-0.044)	(-0.315,0.004)	(-2.621,-1.617)	(-0.533,0.282)
SIZE	0.211	-3.012	1.014	0.558	-0.961	1.439
	(-0.289,1.266)	(-3.923,-1.705)	(0.428,2.745)	(0.077,1.005)	(-0.574,0.623)	(0.437,1.684)
MM	-0.504	-0.796	0.47	-0.492	8.684	-11.421
	(-3.486,8.583)	(-5.450,2.629)	(0.202,1.696)	(-1.003,0.082)	(4.172,9.289)	(-14.582,-3.803)
β	0.219	0.153	0.194	0.179	0.353	0.272
	(0.193,0.290)	(0.086,0.193)	(0.164,0.216)	(0.157,0.233)	(0.277,0.370)	(0.220,0.315)
σ_k	0.085	0.075	0.073	0.074	0.065	0.081
	(0.074,0.088)	(0.072,0.086)	(0.066,0.078)	(0.070,0.084)	(0.062,0.074)	(0.070,0.084)

Table 6: CoVaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.025$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

VaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	-30.567	-22.619	-15.032	-29.325	-22.208	-3.429	-42.321
	(-40.095,-5.584)	(-43.424,-9.687)	(-23.739,-8.861)	(-35.705,-2.935)	(-42.447,-5.758)	(-15.338,7.775)	(-51.416,-15.868)
VIX	-0.571	-0.416	-0.397	-0.339	-0.434	-0.313	-0.275
	(-0.853,-0.424)	(-0.776,-0.446)	(-0.409,-0.222)	(-0.464,-0.313)	(-0.646,-0.366)	(-0.297,-0.169)	(-0.443,-0.233)
LIQSPR	0.038	0.005	-0.048	0.008	0.006	0.035	-0.032
	(-0.020,0.058)	(0.015,0.090)	(-0.056,-0.013)	(-0.017,0.027)	(-0.042,0.025)	(-0.002,0.050)	(-0.079,0.007)
3MTB	-0.051	-0.051	-0.065	-0.011	-0.104	0.105	0.132
	(-0.137,0.058)	(-0.161,0.048)	(-0.110,-0.011)	(-0.079,0.030)	(-0.134,0.045)	(0.062,0.191)	(0.036,0.191)
TERMSPR	0.071	0.058	0.081	0.008	-0.075	0.042	0.028
	(0.022,0.141)	(-0.078,0.062)	(0.017,0.098)	(-0.027,0.054)	(-0.081,0.016)	(0.021,0.112)	(-0.028,0.090)
CREDSPR	-0.088	-0.073	-0.022	-0.011	-0.09	-0.082	-0.158
	(-0.189,0.004)	(-0.134,0.070)	(-0.202,-0.056)	(-0.083,0.038)	(-0.066,0.113)	(-0.162,-0.003)	(-0.254,-0.084)
DJUSRE	0.531	0.45	0.376	0.819	0.52	0.419	0.533
	(0.350,0.733)	(0.346,0.651)	(0.334,0.655)	(0.570,0.839)	(0.331,0.702)	(0.342,0.570)	(0.449,0.820)
LEV	-0.513	-0.426	0.565	0.025	-0.972	-0.22	-0.122
	(-0.812,-0.108)	(-0.239,1.553)	(-0.275,0.730)	(-0.197,0.474)	(-1.278,-0.258)	(-0.285,-0.069)	(-0.224,0.039)
MK2BK	-1.548	2.418	-0.225	1.114	-0.654	2.424	-4.885
	(-2.669,0.452)	(-1.796,1.546)	(-0.570,1.529)	(-1.574,2.181)	(-3.448,0.637)	(0.778,3.299)	(-5.854,-0.994)
SIZE	3.796	2.604	1.262	2.546	3.932	0.811	3.45
	(1.785,4.808)	(0.987,3.809)	(0.633,2.376)	(0.322,2.952)	(2.099,6.253)	(-0.492,1.409)	(1.129,4.616)
MM	-7.724	-19.824	-36.894	-6.028	12.85	-12.917	29.372
	(-30.441,1.276)	(-33.085,-10.879)	(-47.880,-13.141)	(-12.368,12.512)	(-7.523,22.752)	(-15.137,3.840)	(9.365,32.212)
σ_j	0.653	0.521	0.415	0.349	0.525	0.382	0.528
	(0.591,0.708)	(0.489,0.588)	(0.359,0.431)	(0.343,0.411)	(0.484,0.580)	(0.350,0.420)	(0.489,0.588)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	20.671	-21.941	-19.8	-7.908	2.736	-5.371
	(-1.387,21.765)	(-43.386,-12.236)	(-28.864,5.137)	(-19.865,15.642)	(-24.191,13.474)	(-11.947,15.798)
VIX	-0.359	-0.334	-0.084	-0.128	-0.311	-0.203
	(-0.396,-0.264)	(-0.352,-0.233)	(-0.109,-0.070)	(-0.158,-0.089)	(-0.304,-0.186)	(-0.256,-0.120)
LIQSPR	0.038	0.008	0.014	0	-0.006	0.046
	(0.001,0.040)	(-0.011,0.038)	(0.007,0.031)	(-0.024,0.040)	(-0.041,0.006)	(0.020,0.068)
3MTB	0.102	-0.05	-0.002	0.084	0.013	0.013
	(0.052,0.139)	(-0.107,0.019)	(-0.027,0.023)	(0.017,0.123)	(0.018,0.135)	(-0.024,0.108)
TERMSPR	0.087	0.007	0.001	0.056	-0.01	-0.014
	(0.047,0.120)	(-0.040,0.024)	(-0.023,0.012)	(0.015,0.097)	(-0.010,0.078)	(-0.021,0.056)
CREDSPR	0.087	-0.062	-0.057	0.014	-0.051	-0.055
	(0.003,0.114)	(-0.118,-0.024)	(-0.080,-0.035)	(-0.043,0.059)	(-0.090,0.033)	(-0.084,-0.008)
DJUSRE	0.449	0.493	0.168	0.384	0.246	0.249
	(0.348,0.638)	(0.487,0.655)	(0.104,0.202)	(0.330,0.570)	(0.146,0.326)	(0.150,0.337)
LEV	0.009	-0.02	3.318	-2.905	-4.431	4.91
	(-0.021,0.031)	(-0.401,0.347)	(-0.643,5.836)	(-10.835,2.790)	(-7.671,1.642)	(-2.016,9.679)
MK2BK	0	-0.224	-0.375	0.124	-1.514	-1.044
	(-0.000,0.000)	(-0.543,0.171)	(-0.536,0.491)	(-0.816,1.518)	(-3.069,0.710)	(-2.365,-0.464)
SIZE	-2.326	2.254	1.105	0.925	0.984	-0.049
	(-2.402,-0.003)	(1.369,4.078)	(-1.053,1.796)	(-0.812,1.816)	(-0.030,2.663)	(-1.449,0.795)
MM	-5.023	-5.437	-2.498	-1.23	12.417	9.032
	(-7.120,-3.242)	(-12.994,2.934)	(-7.179,-0.608)	(-3.759,3.553)	(6.140,22.127)	(1.043,11.944)
σ_j	0.515	0.337	0.232	0.35	0.357	0.305
	(0.440,0.528)	(0.290,0.347)	(0.212,0.254)	(0.311,0.373)	(0.306,0.367)	(0.258,0.310)

VaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-24.596	-40.958	-17.548	8.218	-5.352	-10.077
	(-36.554,-12.861)	(-49.954,-15.272)	(-27.708,6.410)	(-34.212,-3.006)	(-12.458,14.775)	(-20.598,1.331)
VIX	-0.09	-0.284	-0.128	-0.191	-0.244	-0.144
	(-0.157,-0.077)	(-0.331,-0.219)	(-0.169,-0.087)	(-0.238,-0.172)	(-0.292,-0.161)	(-0.229,-0.096)
LIQSPR	-0.023	0.002	-0.005	-0.048	0.004	0.010
	(-0.053,-0.002)	(-0.027,0.024)	(-0.005,0.035)	(-0.045,0.005)	(-0.016,0.033)	(-0.038,0.032)
3MTB	0.004	0.041	0.126	0.159	-0.073	0.021
	(-0.044,0.047)	(-0.023,0.059)	(0.080,0.157)	(0.070,0.165)	(-0.094,-0.014)	(-0.014,0.077)
TERMSPR	-0.012	0.078	0.121	0.108	-0.084	-0.023
	(-0.033,0.034)	(0.017,0.080)	(0.067,0.133)	(0.039,0.108)	(-0.121,-0.053)	(-0.062,0.029)
CREDSPR	-0.037	0.044	0.068	0.012	-0.151	-0.024
	(-0.163,0.002)	(-0.001,0.080)	(0.001,0.102)	(-0.007,0.095)	(-0.185,-0.101)	(-0.087,0.020)
DJUSRE	0.601	0.436	0.33	0.072	0.295	0.223
	(0.562,0.722)	(0.334,0.523)	(0.231,0.419)	(0.056,0.266)	(0.295,0.450)	(0.123,0.324)
LEV	0.029	0.691	-5.424	4.029	-3.167	-0.068
	(0.006,0.038)	(-0.807,0.915)	(-12.964,-1.582)	(2.517,9.632)	(-8.613,-0.274)	(-0.672,0.450)
MK2BK	-0.001	-2.181	-0.742	-0.564	-2.288	-2.250
	(-0.007,0.008)	(-2.650,-0.889)	(-2.200,-0.673)	(-0.616,0.019)	(-4.353,-0.753)	(-3.168,-0.808)
SIZE	2.138	3.495	2.136	-1.19	2.053	1.365
	(1.066,3.274)	(1.610,4.523)	(0.181,3.605)	(-0.554,1.967)	(0.995,2.441)	(0.197,2.503)
MM	19.721	-4.243	1.605	-0.012	-0.605	3.511
	(4.277,27.544)	(-10.316,2.410)	(-1.659,4.307)	(-1.844,1.166)	(-9.528,7.868)	(-9.541,17.568)
σ_j	0.353	0.279	0.409	0.348	0.257	0.320
	(0.313,0.375)	(0.259,0.312)	(0.351,0.421)	(0.320,0.384)	(0.254,0.304)	(0.304,0.365)

Table 7: VaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.05$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
CONST	19.181	5.401	3.905	-0.906	3.785	-5.954	1.791
	(6.034,20.256)	(-1.032,9.738)	(-2.861,10.222)	(-11.724,2.908)	(-6.847,12.108)	(-8.639,1.038)	(-10.801,4.484)
VIX	-0.167	-0.168	-0.126	-0.164	-0.167	-0.115	-0.096
	(-0.181,-0.117)	(-0.179,-0.103)	(-0.161,-0.093)	(-0.196,-0.148)	(-0.189,-0.130)	(-0.149,-0.106)	(-0.099,-0.051)
LIQSPR	0.006	0.007	0.025	0.012	0.003	0.002	0.009
	(-0.002,0.013)	(-0.006,0.013)	(0.006,0.027)	(-0.000,0.014)	(0.001,0.016)	(-0.000,0.017)	(0.001,0.016)
3MTB	0.05	0.03	0.058	0.035	0.037	0.005	0.044
	(0.015,0.059)	(0.011,0.059)	(0.032,0.081)	(0.022,0.064)	(0.022,0.060)	(-0.021,0.016)	(0.021,0.063)
TERMSPR	0.026	0.026	0.041	0.025	0.038	0.003	0.021
	(0.010,0.034)	(-0.000,0.035)	(0.020,0.054)	(0.013,0.038)	(0.020,0.051)	(-0.006,0.018)	(0.011,0.033)
CREDSPR	0.029	-0.015	0.006	0.006	0.013	-0.004	0.009
	(-0.008,0.034)	(-0.030,0.013)	(-0.030,0.019)	(-0.027,0.030)	(-0.033,0.036)	(-0.033,0.004)	(-0.010,0.039)
DJUSRE	0.255	0.227	0.284	0.335	0.22	0.256	0.316
	(0.222,0.311)	(0.223,0.318)	(0.243,0.332)	(0.258,0.337)	(0.188,0.268)	(0.233,0.298)	(0.304,0.364)
LEV	0.006	-0.027	-0.077	0.064	-0.045	-0.051	-0.017
	(-0.046,0.037)	(-0.121,0.249)	(-0.240,0.028)	(0.032,0.237)	(-0.222,0.021)	(-0.114,-0.036)	(-0.054,0.003)
MK2BK	-0.283	-0.843	-1.236	-3.108	-0.82	0.135	-0.361
	(-0.922,-0.118)	(-1.172,-0.318)	(-1.708,-0.632)	(-3.712,-2.428)	(-1.543,-0.184)	(-0.200,0.541)	(-0.913,-0.075)
SIZE	-1.7	-0.245	-0.15	0.185	-0.125	0.783	-0.075
	(-1.734,-0.546)	(-0.753,0.146)	(-0.809,0.708)	(-0.151,0.998)	(-0.998,1.093)	(0.190,1.054)	(-0.381,1.071)
MM	10.06	-0.078	-10.758	12.859	3.783	-3.935	-0.132
	(3.613,13.537)	(-2.005,3.748)	(-16.063,3.034)	(7.380,17.038)	(-3.388,8.257)	(-7.353,0.088)	(-1.542,3.406)
β	0.102	0.124	0.101	0.123	0.136	0.222	0.143
	(0.082,0.124)	(0.093,0.131)	(0.098,0.166)	(0.102,0.161)	(0.108,0.159)	(0.184,0.239)	(0.118,0.151)
σ_k	0.13	0.124	0.136	0.133	0.128	0.12	0.116
	(0.122,0.146)	(0.121,0.145)	(0.124,0.148)	(0.120,0.144)	(0.120,0.144)	(0.113,0.135)	(0.106,0.127)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
CONST	16.287	24.691	-2.188	-18.412	12.451	-11.071
	(13.990,24.617)	(7.882,30.063)	(-19.552,9.145)	(-20.668,2.366)	(6.583,21.842)	(-17.948,-7.645)
VIX	-0.212	-0.229	-0.105	-0.106	-0.075	-0.103
	(-0.249,-0.164)	(-0.248,-0.177)	(-0.128,-0.081)	(-0.116,-0.080)	(-0.087,-0.055)	(-0.133,-0.093)
LIQSPR	0.017	0.012	0.002	-0.005	0.007	0.013
	(0.008,0.024)	(0.002,0.020)	(-0.006,0.013)	(-0.017,-0.001)	(0.003,0.012)	(-0.002,0.012)
3MTB	0.035	0.081	0.023	0.028	0.022	0.019
	(0.029,0.073)	(0.030,0.083)	(0.014,0.056)	(0.012,0.056)	(0.015,0.043)	(-0.002,0.033)
TERMSPR	0.026	0.029	0.021	0.03	0.026	0.012
	(0.028,0.058)	(0.016,0.043)	(0.011,0.043)	(0.020,0.046)	(0.014,0.035)	(0.002,0.028)
CREDSPR	-0.033	0.045	-0.026	0	-0.018	-0.033
	(-0.030,0.020)	(0.016,0.061)	(-0.040,0.010)	(-0.025,0.025)	(-0.036,-0.004)	(-0.048,-0.004)
DJUSRE	0.208	0.219	0.338	0.33	0.301	0.324
	(0.190,0.280)	(0.138,0.246)	(0.292,0.385)	(0.271,0.347)	(0.242,0.324)	(0.271,0.346)
LEV	-0.011	0.218	-1.148	3.143	-3.68	6.78
	(-0.011,0.003)	(0.083,0.441)	(-3.442,1.094)	(-3.116,3.717)	(-5.408,-2.476)	(5.551,9.462)
MK2BK	0	0.069	0.377	-0.055	-0.064	0.169
	(-0.000,0.000)	(-0.003,0.297)	(-0.111,0.742)	(-0.355,0.407)	(-0.449,0.510)	(-0.385,0.216)
SIZE	-1.54	-2.301	0.281	1.294	-0.479	-0.216
	(-2.384,-1.355)	(-2.841,-0.889)	(-0.616,1.799)	(-0.013,1.608)	(-1.094,-0.101)	(-0.355,0.126)
MM	1.05	-8.901	-0.244	-2.492	-0.534	1.287
	(0.174,1.486)	(-12.656,-6.095)	(-2.352,2.512)	(-3.112,-0.722)	(-2.232,2.380)	(-3.180,0.446)
β	0.153	0.245	0.282	0.169	0.347	0.386
	(0.105,0.181)	(0.199,0.292)	(0.189,0.304)	(0.147,0.217)	(0.316,0.388)	(0.308,0.398)
σ_k	0.144	0.133	0.132	0.14	0.11	0.11
	(0.123,0.147)	(0.118,0.141)	(0.125,0.150)	(0.120,0.144)	(0.100,0.120)	(0.103,0.124)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
CONST	-0.134	18.708	-4.144	-7.104	-2.633	-10.080
	(-12.840,1.177)	(4.842,36.457)	(-15.879,5.770)	(-11.422,0.227)	(-9.876,7.335)	(-11.788,3.139)
VIX	-0.078	-0.193	-0.083	-0.092	-0.13	-0.120
	(-0.104,-0.057)	(-0.226,-0.146)	(-0.108,-0.069)	(-0.107,-0.072)	(-0.171,-0.129)	(-0.127,-0.090)
LIQSPR	0.018	0.025	0.001	0.004	0.017	0.012
	(0.000,0.016)	(0.014,0.034)	(-0.005,0.008)	(-0.005,0.010)	(0.009,0.022)	(0.004,0.017)
3MTB	0.002	0.022	0.049	0.035	0.036	0.038
	(-0.011,0.037)	(0.002,0.052)	(0.011,0.040)	(0.017,0.060)	(0.009,0.042)	(0.022,0.064)
TERMSPR	0.005	0.019	0.027	0.018	0.035	0.047
	(0.001,0.033)	(0.004,0.036)	(0.014,0.036)	(0.004,0.029)	(0.012,0.038)	(0.032,0.059)
CREDSPR	-0.013	-0.019	-0.029	-0.048	-0.017	-0.030
	(-0.045,0.019)	(-0.034,0.015)	(-0.047,-0.005)	(-0.060,-0.009)	(-0.037,0.019)	(-0.046,-0.007)
DJUSRE	0.273	0.27	0.309	0.312	0.184	0.268
	(0.259,0.354)	(0.221,0.310)	(0.254,0.330)	(0.277,0.353)	(0.167,0.244)	(0.249,0.318)
LEV	0.006	0.258	-2.954	0.064	0.535	-0.263
	(-0.000,0.009)	(-0.006,0.540)	(-4.081,0.188)	(-2.276,0.479)	(-0.317,3.537)	(-0.671,-0.104)
MK2BK	0.002	-0.689	-0.239	-0.069	-2.004	-0.249
	(-0.003,0.003)	(-1.158,0.062)	(-0.592,0.002)	(-0.223,0.057)	(-2.553,-1.282)	(-0.507,0.270)
SIZE	0.004	-1.381	0.776	0.621	0.41	1.264
	(-0.109,1.182)	(-2.895,-0.288)	(-0.267,1.791)	(0.196,1.090)	(-0.751,0.546)	(-0.100,1.419)
MM	7.242	0.405	0.888	-0.532	5.599	-10.513
	(1.734,11.166)	(-3.810,3.196)	(-0.032,1.554)	(-0.726,0.290)	(1.983,8.818)	(-15.061,-0.298)
β	0.23	0.097	0.174	0.154	0.328	0.208
	(0.171,0.244)	(0.084,0.170)	(0.170,0.225)	(0.147,0.211)	(0.275,0.356)	(0.156,0.268)
σ_k	0.133	0.142	0.119	0.135	0.119	0.140
	(0.126,0.151)	(0.126,0.152)	(0.111,0.132)	(0.120,0.143)	(0.113,0.135)	(0.124,0.148)

Table 8: CoVaR parameter estimates obtained by fitting the time invariant model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.05$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

VaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.542	-0.395	-0.352	-0.244	-0.895	-0.442	-1.107
	(-0.635,-0.190)	(-0.549,-0.210)	(-0.504,-0.230)	(-0.350,-0.102)	(-1.068,-0.626)	(-0.457,-0.305)	(-1.157,-0.834)
LIQSPR	0.033	0.031	0.013	0.009	0.116	0.023	0.054
	(-0.070,0.049)	(-0.041,0.034)	(-0.036,0.020)	(-0.036,0.036)	(0.023,0.120)	(-0.021,0.044)	(0.015,0.129)
3MTB	0.022	-0.026	-0.127	-0.01	-0.084	0.077	-0.033
	(0.009,0.175)	(-0.044,0.083)	(-0.114,0.015)	(-0.035,0.093)	(-0.122,0.014)	(0.056,0.149)	(-0.040,0.118)
TERMSPR	0.038	0.05	0.036	0.021	0	0.056	-0.016
	(0.035,0.157)	(0.014,0.116)	(0.028,0.121)	(0.010,0.089)	(-0.045,0.042)	(0.050,0.123)	(-0.026,0.101)
CREDSPR	-0.055	-0.059	-0.066	-0.136	0.084	-0.004	0.01
	(-0.159,0.080)	(-0.150,0.043)	(-0.111,0.082)	(-0.201,0.068)	(-0.051,0.140)	(-0.080,0.058)	(-0.106,0.099)
DJUSRE	0.691	0.51	0.425	0.763	0.534	0.43	0.624
	(0.573,0.935)	(0.455,0.738)	(0.338,0.561)	(0.668,0.922)	(0.426,0.669)	(0.402,0.622)	(0.498,0.789)
LEV	-6.222	-0.932	-9.06	-1.689	-11.082	-3.494	-3.703
	(-7.631,-6.158)	(-2.288,0.748)	(-10.882,-8.630)	(-2.540,-0.609)	(-15.097,-10.828)	(-3.842,-3.099)	(-4.297,-3.054)
MK2BK	2.858	5.98	-3.522	-0.909	-1.671	-0.655	1.203
	(-1.770,7.826)	(2.343,14.281)	(-3.216,5.159)	(-5.159,3.915)	(-3.514,10.233)	(-2.024,6.441)	(-0.253,9.801)
SIZE	-0.819	-7.374	1.52	-6.291	4.387	-2.844	-0.644
	(-1.339,1.021)	(-9.217,-6.558)	(0.396,1.812)	(-7.623,-5.942)	(3.600,8.900)	(-4.517,-2.205)	(-3.440,0.139)
MM	0.916	-1.953	1.3	0.798	1.823	2.615	0.701
	(-7.183,5.579)	(-7.432,4.338)	(-6.468,5.498)	(-6.424,5.701)	(-5.505,6.409)	(-6.451,5.809)	(-6.442,5.383)
σ_j	0.109	0.098	0.079	0.078	0.083	0.076	0.096
	(0.091,0.112)	(0.084,0.102)	(0.067,0.082)	(0.065,0.080)	(0.067,0.085)	(0.062,0.076)	(0.080,0.100)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-1.277	-0.287	-0.228	-0.347	-0.572	-0.431
	(-1.803,-1.078)	(-0.420,-0.261)	(-0.241,-0.144)	(-0.377,-0.177)	(-0.603,-0.476)	(-0.467,-0.359)
LIQSPR	0.055	0.012	0.055	0.034	0.044	0.039
	(-0.102,0.086)	(-0.046,0.021)	(0.032,0.061)	(-0.016,0.051)	(-0.027,0.048)	(-0.011,0.043)
3MTB	-0.01	-0.045	-0.016	0.026	0.044	0.057
	(-0.010,0.167)	(-0.100,0.020)	(-0.039,0.032)	(0.015,0.114)	(-0.021,0.087)	(0.030,0.129)
TERMSPR	-0.02	0.015	-0.014	0.035	0.02	0.037
	(-0.009,0.118)	(-0.004,0.054)	(-0.029,0.025)	(0.011,0.095)	(-0.013,0.051)	(0.028,0.078)
CREDSPR	0.05	-0.104	0.001	0.058	0.072	0.111
	(0.038,0.309)	(-0.149,-0.009)	(-0.076,0.056)	(0.001,0.132)	(-0.038,0.070)	(0.054,0.160)
DJUSRE	0.389	0.522	0.09	0.291	0.16	0.212
	(0.272,0.553)	(0.420,0.609)	(0.021,0.150)	(0.283,0.486)	(0.078,0.246)	(0.177,0.331)
LEV	11.432	-5.374	-8.644	-10.635	-10.713	-9.716
	(11.409,11.631)	(-7.605,-4.242)	(-9.840,0.900)	(-10.913,-0.472)	(-13.221,-1.394)	(-14.139,-6.290)
MK2BK	0	-0.823	-1.162	-5.508	-8.117	-8.893
	(-0.000,0.001)	(-1.506,3.971)	(-1.960,0.490)	(-6.296,-0.357)	(-10.543,-1.494)	(-11.345,-6.177)
SIZE	-13.189	-1.48	-3.293	-1.419	-1.456	-0.539
	(-13.846,-12.719)	(-2.672,-1.389)	(-5.757,-3.163)	(-4.751,-1.780)	(-3.140,-1.442)	(-0.871,-0.214)
MM	1.904	7.453	-6.742	0.32	-3.427	0.04
	(-7.567,3.219)	(-4.944,7.394)	(-10.809,0.278)	(-3.915,6.975)	(-4.433,7.604)	(-4.717,6.863)
σ_j	0.081	0.062	0.052	0.058	0.061	0.049
	(0.064,0.082)	(0.053,0.065)	(0.043,0.053)	(0.048,0.061)	(0.052,0.065)	(0.042,0.051)

VaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.492	-0.158	-0.256	-0.344	-0.423	-0.445
	(-0.534,-0.330)	(-0.214,-0.052)	(-0.302,-0.143)	(-0.381,-0.191)	(-0.472,-0.264)	(-0.509,-0.378)
LIQSPR	0.033	-0.035	0.081	0.039	0.041	0.018
	(-0.058,0.025)	(-0.104,-0.031)	(0.030,0.088)	(-0.034,0.039)	(0.001,0.054)	(-0.034,0.038)
3MTB	0.045	0.019	0.092	-0.002	-0.03	0.025
	(0.018,0.117)	(0.011,0.111)	(0.052,0.148)	(-0.001,0.118)	(-0.033,0.040)	(-0.008,0.077)
TERMSPR	0.028	0.052	0.078	0.006	-0.026	-0.008
	(0.028,0.098)	(0.036,0.101)	(0.066,0.133)	(-0.001,0.082)	(-0.040,0.013)	(-0.042,0.026)
CREDSPR	-0.041	-0.027	0.102	0	-0.043	0.042
	(-0.100,0.091)	(-0.051,0.048)	(0.048,0.148)	(-0.060,0.094)	(-0.092,0.014)	(-0.007,0.108)
DJUSRE	0.417	0.558	0.153	0.155	0.296	0.226
	(0.368,0.547)	(0.468,0.622)	(0.111,0.303)	(0.165,0.372)	(0.240,0.380)	(0.168,0.310)
LEV	-0.486	-1.956	-11.981	-8.537	-11.568	-0.655
	(-0.635,-0.465)	(-6.971,-0.917)	(-11.725,-0.477)	(-15.913,-6.510)	(-18.736,-6.351)	(-1.239,0.858)
MK2BK	-0.011	2.664	-3.362	-1.504	-4.758	-4.970
	(-0.011,0.006)	(1.554,8.157)	(-6.393,0.319)	(-1.621,0.619)	(-5.942,5.346)	(-7.268,-2.350)
SIZE	-6.303	-6.011	-2.273	-1.58	-4.479	-7.040
	(-6.691,-6.310)	(-6.763,-4.244)	(-4.385,-2.177)	(-2.685,-1.140)	(-7.295,-2.731)	(-8.113,-6.680)
MM	-1.883	2.247	-3.302	-0.252	0.928	3.580
	(-5.849,5.671)	(-6.318,6.360)	(-5.752,5.989)	(-2.707,2.464)	(-5.988,5.313)	(-5.652,6.566)
σ_j	0.057	0.051	0.066	0.065	0.047	0.057
	(0.046,0.057)	(0.043,0.053)	(0.055,0.068)	(0.053,0.069)	(0.039,0.048)	(0.047,0.058)

Table 9: VaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.01$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.147	-0.144	-0.153	-0.32	-0.18	-0.175	-0.071
	(-0.241,-0.107)	(-0.323,-0.132)	(-0.274,-0.149)	(-0.471,-0.308)	(-0.256,-0.167)	(-0.281,-0.152)	(-0.181,-0.069)
LIQSPR	-0.002	0.005	0.016	0.037	-0.005	-0.007	0.017
	(-0.020,0.010)	(-0.017,0.024)	(-0.011,0.016)	(0.004,0.047)	(-0.023,0.000)	(-0.027,0.001)	(-0.017,0.013)
3MTB	0.023	0.037	0.05	0.025	0.038	0.019	0.021
	(0.009,0.057)	(0.017,0.063)	(0.033,0.073)	(0.017,0.058)	(0.034,0.067)	(0.018,0.056)	(0.012,0.053)
TERMSPR	0.007	0.022	0.021	0.004	0.033	0.021	0.013
	(0.001,0.036)	(0.011,0.047)	(0.018,0.048)	(0.008,0.045)	(0.032,0.061)	(0.020,0.049)	(0.010,0.046)
CREDSPR	0.021	0.038	0.023	0.069	0.054	0.053	0.01
	(-0.010,0.050)	(0.017,0.093)	(0.002,0.056)	(0.047,0.116)	(0.031,0.086)	(0.027,0.084)	(-0.005,0.048)
DJUSRE	0.265	0.311	0.286	0.26	0.318	0.275	0.327
	(0.224,0.332)	(0.256,0.354)	(0.240,0.329)	(0.198,0.309)	(0.266,0.354)	(0.228,0.319)	(0.286,0.370)
LEV	0.272	-0.006	0.43	-2.897	-2.693	1.342	1.634
	(0.211,0.501)	(-4.745,0.733)	(0.196,0.792)	(-5.199,-2.204)	(-3.387,-2.348)	(0.881,1.487)	(1.091,1.807)
MK2BK	24.223	22.535	35.034	16.241	36.495	24.587	30.898
	(23.781,25.946)	(21.860,25.367)	(34.752,36.653)	(15.626,25.870)	(35.586,36.701)	(23.106,30.548)	(27.986,34.991)
SIZE	2.716	3.116	0.035	7.265	4.163	1.892	0.152
	(2.333,2.742)	(2.457,7.559)	(-0.443,0.201)	(6.527,9.092)	(3.824,5.109)	(1.278,1.834)	(-0.151,0.777)
MM	0.705	-0.609	0.735	-3.314	-2.992	-6.275	2.546
	(-2.519,8.826)	(-4.575,9.650)	(-6.295,5.446)	(-6.350,6.034)	(-6.171,6.292)	(-1.648,9.617)	(3.284,14.718)
σ_k	0.023	0.022	0.02	0.025	0.021	0.022	0.022
	(0.020,0.025)	(0.020,0.026)	(0.018,0.022)	(0.022,0.028)	(0.019,0.023)	(0.019,0.024)	(0.019,0.024)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-0.392	-0.188	-0.189	-0.208	-0.074	-0.055
	(-0.706,-0.401)	(-0.252,-0.158)	(-0.248,-0.169)	(-0.355,-0.199)	(-0.204,-0.067)	(-0.098,-0.026)
LIQSPR	0.028	0.011	0.017	-0.004	0	-0.005
	(0.002,0.036)	(-0.012,0.014)	(-0.008,0.018)	(-0.024,0.003)	(-0.016,0.006)	(-0.021,-0.001)
3MTB	0.013	0.021	0.043	0.018	0.038	0.027
	(0.011,0.057)	(0.014,0.055)	(0.032,0.086)	(0.007,0.053)	(0.031,0.062)	(0.031,0.067)
TERMSPR	0.014	0.015	0.039	0.023	0.03	0.03
	(0.018,0.064)	(0.012,0.044)	(0.036,0.074)	(0.019,0.051)	(0.025,0.054)	(0.028,0.062)
CREDSPR	0.059	0.019	0.037	0.037	0.034	0.011
	(0.055,0.118)	(0.001,0.056)	(0.027,0.079)	(0.014,0.065)	(0.021,0.079)	(0.007,0.068)
DJUSRE	0.279	0.27	0.298	0.325	0.298	0.329
	(0.217,0.319)	(0.241,0.329)	(0.274,0.341)	(0.237,0.334)	(0.252,0.324)	(0.296,0.363)
LEV	2.3	0.429	-0.529	1.18	9.039	12.578
	(2.290,2.327)	(-0.026,0.814)	(-2.709,4.961)	(-7.046,7.137)	(2.710,15.592)	(1.613,15.804)
MK2BK	0	10.526	-8.54	11.762	12.079	0.073
	(-0.000,0.000)	(9.417,10.817)	(-10.374,-7.839)	(10.893,16.560)	(9.641,12.891)	(-0.796,2.762)
SIZE	4.289	0.707	7.625	0.246	0.118	1.633
	(4.330,4.643)	(0.349,1.296)	(6.772,8.292)	(-0.761,0.466)	(-0.437,1.218)	(1.070,2.886)
MM	-20.368	-0.764	0.702	2.658	-2.435	5.064
	(-26.990,-16.699)	(-7.180,3.715)	(-5.433,5.198)	(-2.902,6.393)	(-5.688,5.050)	(-9.562,5.699)
σ_k	0.019	0.02	0.024	0.021	0.021	0.024
	(0.018,0.023)	(0.018,0.023)	(0.021,0.026)	(0.019,0.023)	(0.018,0.022)	(0.021,0.025)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.465	-0.28	-0.219	-0.119	-0.139	-0.127
	(-0.899,-0.423)	(-0.345,-0.245)	(-0.333,-0.222)	(-0.190,-0.093)	(-0.215,-0.115)	(-0.207,-0.108)
LIQSPR	0.015	0.031	0.016	0.004	0.004	0.008
	(-0.005,0.042)	(-0.002,0.041)	(-0.007,0.015)	(-0.030,-0.003)	(-0.012,0.008)	(-0.006,0.011)
3MTB	0.002	0.004	0.018	0.011	0.056	0.017
	(-0.011,0.044)	(-0.006,0.038)	(0.011,0.052)	(0.016,0.060)	(0.044,0.077)	(0.018,0.064)
TERMSPR	0.012	0.008	0.014	0.016	0.042	0.032
	(0.002,0.059)	(-0.002,0.033)	(0.012,0.044)	(0.022,0.055)	(0.038,0.063)	(0.029,0.059)
CREDSPR	0.044	0.02	0.019	0.005	0.044	0.033
	(0.019,0.114)	(-0.008,0.052)	(-0.000,0.052)	(-0.004,0.040)	(0.017,0.072)	(0.020,0.076)
DJUSRE	0.249	0.312	0.292	0.315	0.308	0.354
	(0.189,0.323)	(0.278,0.373)	(0.249,0.322)	(0.252,0.324)	(0.278,0.361)	(0.281,0.368)
LEV	2.071	1.011	3.926	-0.8	4.88	8.731
	(2.070,2.084)	(-2.282,2.142)	(-2.538,5.525)	(-2.197,1.867)	(3.748,6.666)	(8.329,11.382)
MK2BK	0.006	16.977	11.732	2.185	20.842	-0.221
	(0.001,0.009)	(16.480,18.312)	(11.720,13.832)	(2.018,2.951)	(19.713,23.402)	(-0.852,1.288)
SIZE	4.186	1.383	0.585	2.767	0.118	0.101
	(4.106,4.845)	(0.859,2.995)	(0.124,1.497)	(2.110,2.864)	(-0.542,0.284)	(-1.094,0.295)
MM	1.125	-4.474	-6.021	1.405	-2.916	-3.057
	(-6.725,5.965)	(-6.694,2.795)	(-6.011,0.542)	(-0.014,2.914)	(-4.480,6.182)	(-5.900,4.319)
σ_k	0.018	0.023	0.021	0.024	0.023	0.027
	(0.018,0.024)	(0.020,0.025)	(0.019,0.023)	(0.021,0.026)	(0.020,0.024)	(0.023,0.028)

Table 10: CoVaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.01$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

VaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.372	-0.402	-0.366	-0.143	-0.606	-0.275	-0.715
	(-0.602,-0.120)	(-0.464,-0.084)	(-0.469,-0.314)	(-0.256,-0.025)	(-0.824,-0.460)	(-0.400,-0.220)	(-1.019,-0.668)
LIQSPR	-0.01	0.051	-0.008	0.027	0.094	0.004	0.065
	(-0.046,0.051)	(-0.021,0.062)	(-0.031,0.018)	(0.006,0.069)	(0.045,0.136)	(-0.048,0.039)	(0.037,0.140)
3MTB	0.126	0.017	-0.038	0.088	-0.054	0.038	-0.002
	(0.036,0.186)	(-0.054,0.074)	(-0.097,0.006)	(-0.015,0.088)	(-0.118,-0.012)	(0.031,0.136)	(-0.087,0.060)
TERMSPR	0.143	0.066	0.113	0.107	0.004	0.05	0.018
	(0.060,0.194)	(-0.011,0.099)	(0.062,0.124)	(0.044,0.116)	(-0.048,0.027)	(0.039,0.134)	(-0.046,0.056)
CREDSPR	0.042	0.135	-0.006	0.121	0.067	0.014	-0.068
	(-0.029,0.169)	(-0.078,0.126)	(-0.074,0.058)	(0.024,0.150)	(-0.036,0.114)	(-0.087,0.063)	(-0.130,0.050)
DJUSRE	0.821	0.443	0.326	0.621	0.574	0.543	0.632
	(0.588,0.980)	(0.387,0.690)	(0.309,0.539)	(0.541,0.830)	(0.396,0.671)	(0.380,0.637)	(0.524,0.797)
LEV	4.01	-2.095	-2.126	-5.057	-9.054	3.715	0.784
	(3.060,5.498)	(-2.776,0.234)	(-2.110,-0.036)	(-8.468,-5.457)	(-11.707,-5.887)	(3.426,4.314)	(0.496,1.740)
MK2BK	-10.536	-3.206	-2.164	-6.315	-2.946	-9.493	-0.836
	(-20.510,-6.433)	(-10.438,-1.068)	(-5.478,0.780)	(-11.379,-1.873)	(-7.522,-0.918)	(-12.312,-2.865)	(-6.877,3.179)
SIZE	6.666	7.57	2.617	25.876	20.569	5.981	4.095
	(6.514,9.782)	(6.104,8.709)	(0.795,2.780)	(26.485,30.494)	(17.537,24.030)	(4.867,6.426)	(2.716,4.767)
MM	2.854	3.491	2.974	-2.903	-3.318	-3.859	1.419
	(-6.838,5.443)	(-5.925,6.244)	(-7.669,4.549)	(-6.271,5.793)	(-6.506,5.477)	(-7.390,4.397)	(-6.704,5.186)
σ_j	0.241	0.214	0.186	0.145	0.162	0.176	0.188
	(0.203,0.249)	(0.197,0.239)	(0.171,0.211)	(0.133,0.165)	(0.144,0.190)	(0.143,0.179)	(0.181,0.222)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-0.034	-0.211	-0.178	-0.257	-0.401	-0.308
	(-0.018,0.900)	(-0.283,-0.130)	(-0.206,-0.114)	(-0.298,-0.156)	(-0.514,-0.350)	(-0.418,-0.282)
LIQSPR	-0.012	-0.013	0.045	0.027	0.038	0.017
	(-0.026,0.124)	(-0.036,0.024)	(0.030,0.060)	(-0.017,0.049)	(-0.009,0.063)	(-0.019,0.067)
3MTB	0.104	-0.039	-0.02	0.09	0.062	0.021
	(0.024,0.215)	(-0.070,0.017)	(-0.030,0.023)	(0.010,0.095)	(-0.007,0.108)	(-0.019,0.109)
TERMSPR	0.084	0.029	-0.015	0.063	0.026	0.028
	(0.054,0.179)	(-0.001,0.059)	(-0.029,0.018)	(0.009,0.084)	(-0.004,0.060)	(-0.009,0.070)
CREDSPR	0.073	-0.107	-0.009	0.079	0.047	0.051
	(0.045,0.230)	(-0.168,-0.079)	(-0.027,0.068)	(0.019,0.122)	(0.000,0.108)	(0.006,0.144)
DJUSRE	0.581	0.585	0.13	0.468	0.221	0.326
	(0.497,0.863)	(0.485,0.672)	(0.061,0.205)	(0.337,0.518)	(0.122,0.276)	(0.171,0.344)
LEV	13.564	4.993	-3.784	-3.996	-8.426	1.008
	(13.316,13.575)	(3.357,5.100)	(-5.398,6.451)	(-9.360,1.539)	(-8.127,3.678)	(-4.034,8.182)
MK2BK	0	0.19	4.503	2.413	-5.572	1.46
	(0.000,0.001)	(0.914,3.565)	(1.988,5.062)	(-1.859,3.453)	(-8.929,-1.863)	(-3.095,2.299)
SIZE	30.641	11.211	8.86	3.074	14.158	5.035
	(29.507,30.644)	(10.611,12.669)	(7.376,9.753)	(2.399,4.800)	(12.581,14.745)	(3.936,6.351)
MM	2.211	-2.196	-0.94	0.266	-4.235	3.713
	(-2.740,12.412)	(-8.834,2.961)	(-9.186,0.908)	(-7.130,4.815)	(-7.558,3.962)	(-7.566,4.130)
σ_j	0.095	0.126	0.092	0.128	0.135	0.115
	(0.085,0.115)	(0.114,0.139)	(0.090,0.112)	(0.108,0.135)	(0.122,0.149)	(0.104,0.133)

VaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.411	-0.082	-0.074	-0.226	-0.131	-0.164
	(-0.511,-0.262)	(-0.164,-0.010)	(-0.086,0.069)	(-0.317,-0.167)	(-0.331,-0.081)	(-0.179,0.016)
LIQSPR	0.005	-0.052	0.047	0.047	0.024	0.015
	(-0.048,0.026)	(-0.068,-0.005)	(0.002,0.082)	(-0.024,0.059)	(0.008,0.078)	(-0.043,0.077)
3MTB	0.072	0.028	0.076	0.1	-0.03	0.043
	(0.016,0.102)	(0.008,0.081)	(0.015,0.105)	(0.028,0.119)	(-0.043,0.035)	(-0.006,0.091)
TERMSPR	0.064	0.062	0.065	0.068	-0.026	0.021
	(0.028,0.090)	(0.034,0.079)	(0.032,0.090)	(0.013,0.082)	(-0.043,0.015)	(-0.021,0.043)
CREDSPR	0.02	-0.011	0.129	0.107	-0.164	-0.067
	(-0.073,0.068)	(-0.050,0.035)	(0.063,0.163)	(0.024,0.146)	(-0.154,-0.033)	(-0.117,-0.002)
DJUSRE	0.431	0.512	0.317	0.216	0.295	0.337
	(0.354,0.527)	(0.406,0.571)	(0.210,0.374)	(0.136,0.329)	(0.229,0.399)	(0.278,0.476)
LEV	0.872	1.752	10.882	3.213	29.995	7.278
	(0.875,1.003)	(-2.053,2.712)	(-0.518,13.613)	(-2.475,9.482)	(27.499,33.212)	(0.963,8.038)
MK2BK	0.008	-1.991	-35.73	-11.746	7.257	21.233
	(0.003,0.011)	(-4.704,-0.329)	(-45.045,-35.591)	(-11.470,-5.786)	(6.441,15.177)	(15.872,23.850)
SIZE	7.428	6.276	34.56	15.592	2.974	13.221
	(7.432,7.850)	(5.967,8.552)	(34.792,38.321)	(12.874,16.147)	(1.877,3.235)	(13.295,17.800)
MM	-1.985	1.846	8.831	-5.146	-1.11	2.157
	(-5.881,6.317)	(-4.292,6.978)	(2.176,12.974)	(-5.219,3.953)	(-6.208,5.558)	(-5.796,6.345)
σ_j	0.121	0.115	0.114	0.121	0.094	0.117
	(0.093,0.115)	(0.093,0.115)	(0.107,0.133)	(0.114,0.141)	(0.086,0.107)	(0.121,0.153)

Table 11: VaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.025$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.157	-0.066	-0.137	-0.186	0.023	-0.141	-0.01
	(-0.332,-0.163)	(-0.115,-0.045)	(-0.194,-0.106)	(-0.258,-0.180)	(-0.080,0.030)	(-0.193,-0.118)	(0.033,0.179)
LIQSPR	0	-0.002	-0.006	0.004	0.005	-0.021	0.005
	(-0.032,0.013)	(-0.008,0.016)	(-0.018,0.010)	(0.004,0.028)	(-0.020,0.015)	(-0.039,-0.009)	(-0.032,0.008)
3MTB	0.031	0.063	0.04	0.04	0.076	0.025	0.013
	(-0.003,0.052)	(0.027,0.068)	(0.035,0.073)	(0.020,0.056)	(0.044,0.088)	(0.002,0.042)	(0.004,0.061)
TERMSPR	0.017	0.024	0.025	0.023	0.065	0.026	0.023
	(0.009,0.049)	(0.009,0.037)	(0.017,0.045)	(0.010,0.036)	(0.040,0.071)	(0.012,0.040)	(0.017,0.058)
CREDSPR	0.027	0.005	0.022	-0.007	0.034	0.026	0.025
	(-0.026,0.054)	(-0.030,0.025)	(-0.003,0.045)	(-0.021,0.024)	(0.016,0.077)	(0.002,0.054)	(-0.042,0.028)
DJUSRE	0.221	0.276	0.301	0.302	0.361	0.285	0.365
	(0.145,0.279)	(0.263,0.336)	(0.234,0.332)	(0.252,0.329)	(0.259,0.371)	(0.238,0.313)	(0.306,0.423)
LEV	7.634	0.618	4.47	0.063	10.988	2.324	4.355
	(7.664,10.884)	(0.383,1.344)	(4.459,5.943)	(-0.680,0.487)	(11.007,16.783)	(2.167,4.758)	(3.473,4.345)
MK2BK	14.821	-0.346	-31.865	-3.014	-25.14	0.628	-16.423
	(14.334,26.435)	(-2.572,0.101)	(-42.140,-31.434)	(-6.063,-0.162)	(-43.667,-25.572)	(-22.032,2.998)	(-28.846,-15.475)
SIZE	-1.482	0.537	-0.628	-0.36	8.4	0.132	7.96
	(-6.462,-1.366)	(0.080,1.116)	(-1.121,-0.179)	(-0.777,-0.085)	(3.858,8.642)	(-1.006,0.259)	(7.821,11.076)
MM	6.625	1.549	7.691	-2.843	-1.273	1.333	0.624
	(-3.881,8.264)	(-4.259,5.645)	(-4.822,7.239)	(-5.071,5.153)	(-7.136,5.921)	(-8.886,3.295)	(-5.515,7.515)
σ_k	0.068	0.054	0.042	0.057	0.055	0.051	0.073
	(0.061,0.079)	(0.049,0.060)	(0.039,0.048)	(0.050,0.062)	(0.049,0.061)	(0.047,0.059)	(0.069,0.087)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-0.604	-0.402	-0.217	-0.143	-0.073	-0.104
	(-0.969,-0.556)	(-0.447,-0.330)	(-0.285,-0.209)	(-0.234,-0.113)	(-0.122,-0.059)	(-0.104,-0.064)
LIQSPR	0.005	-0.012	-0.004	-0.013	-0.01	-0.009
	(0.004,0.041)	(-0.018,0.013)	(-0.010,0.017)	(-0.026,-0.002)	(-0.016,0.002)	(-0.015,-0.003)
3MTB	0.022	0.036	0.051	0.034	0.04	0.04
	(-0.007,0.045)	(0.011,0.052)	(0.030,0.071)	(0.005,0.039)	(0.020,0.052)	(0.028,0.052)
TERMSPR	0.003	0.027	0.044	0.035	0.034	0.037
	(0.002,0.045)	(0.008,0.037)	(0.033,0.065)	(0.020,0.047)	(0.016,0.044)	(0.029,0.049)
CREDSPR	0.023	0.048	0.038	0.034	0.028	0.03
	(0.019,0.100)	(0.015,0.065)	(0.017,0.061)	(0.010,0.056)	(0.008,0.052)	(0.005,0.041)
DJUSRE	0.251	0.264	0.279	0.332	0.316	0.33
	(0.163,0.282)	(0.181,0.272)	(0.275,0.347)	(0.255,0.332)	(0.271,0.336)	(0.288,0.343)
LEV	5.113	-1.802	5.172	7.65	15.472	-13.124
	(5.065,5.148)	(-4.170,-1.854)	(-2.759,5.557)	(1.598,14.057)	(11.412,17.700)	(-15.960,-12.372)
MK2BK	0	19.025	8.285	-1.378	-0.858	-1.336
	(-0.000,0.000)	(18.810,23.131)	(8.536,12.628)	(-6.127,-0.223)	(-2.193,0.426)	(-1.808,-0.253)
SIZE	-13.493	-15.499	-0.241	3.798	3.458	-0.169
	(-13.792,-13.138)	(-15.773,-14.937)	(-1.371,0.580)	(2.302,6.045)	(3.002,4.214)	(-0.347,-0.037)
MM	-13.168	3.921	-0.965	0.029	1.628	1.932
	(-24.580,-12.211)	(-1.947,8.920)	(-5.739,5.604)	(-7.708,4.510)	(-6.276,4.218)	(-3.530,6.667)
σ_k	0.035	0.042	0.049	0.056	0.05	0.044
	(0.033,0.044)	(0.037,0.047)	(0.043,0.053)	(0.048,0.062)	(0.045,0.055)	(0.043,0.052)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.321	-0.297	-0.185	-0.067	-0.165	-0.150
	(-0.428,-0.297)	(-0.298,-0.216)	(-0.269,-0.186)	(-0.087,-0.014)	(-0.188,-0.111)	(-0.156,-0.025)
LIQSPR	0.015	0.012	0.002	-0.014	-0.003	0.017
	(-0.027,0.020)	(-0.008,0.021)	(-0.008,0.010)	(-0.024,-0.001)	(-0.014,0.007)	(-0.012,0.025)
3MTB	0.042	0.015	0.037	-0.006	0.058	0.044
	(0.016,0.063)	(0.006,0.044)	(0.015,0.048)	(0.000,0.045)	(0.050,0.078)	(0.021,0.068)
TERMSPR	0.04	0.025	0.018	0.016	0.053	0.036
	(0.020,0.063)	(0.009,0.036)	(0.010,0.033)	(0.014,0.042)	(0.042,0.065)	(0.027,0.062)
CREDSPR	0.042	0.056	0.008	0.008	0.043	0.033
	(0.042,0.108)	(0.007,0.058)	(-0.005,0.036)	(-0.001,0.046)	(0.022,0.064)	(-0.025,0.047)
DJUSRE	0.251	0.329	0.274	0.23	0.3	0.339
	(0.186,0.303)	(0.270,0.358)	(0.254,0.309)	(0.230,0.316)	(0.256,0.333)	(0.298,0.426)
LEV	-3.985	14.072	2.518	-0.079	8.039	-6.957
	(-3.990,-3.962)	(13.437,17.983)	(-7.174,3.374)	(0.497,5.535)	(7.772,14.210)	(-12.426,-6.557)
MK2BK	-0.003	-17.047	0.554	-0.257	-6.849	0.253
	(-0.006,-0.001)	(-22.498,-16.637)	(0.544,6.286)	(-2.604,0.236)	(-21.980,-7.119)	(-6.298,0.724)
SIZE	-4.621	0.181	0.776	16.263	-0.116	0.330
	(-4.685,-4.427)	(-1.315,0.517)	(0.063,1.322)	(15.511,16.903)	(-0.570,0.313)	(0.058,3.843)
MM	-1.984	0.522	-6.696	0.007	-1.442	-2.947
	(-4.162,7.792)	(-7.930,2.710)	(-14.254,-3.877)	(-2.214,0.916)	(-7.059,4.074)	(-6.749,4.022)
σ_k	0.033	0.05	0.045	0.06	0.051	0.057
	(0.029,0.038)	(0.043,0.054)	(0.042,0.052)	(0.058,0.071)	(0.044,0.055)	(0.051,0.064)

Table 12: CoVaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.025$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

VaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.289	-0.449	-0.328	-0.255	-0.575	-0.195	-0.415
	(-0.582,-0.172)	(-0.608,-0.328)	(-0.461,-0.280)	(-0.311,-0.104)	(-0.810,-0.459)	(-0.249,-0.114)	(-0.579,-0.292)
LIQSPR	-0.005	0.033	0.023	0.027	0.091	0.012	-0.002
	(-0.053,0.053)	(-0.026,0.052)	(-0.021,0.032)	(-0.008,0.053)	(0.038,0.151)	(-0.007,0.047)	(-0.075,0.009)
3MTB	0.098	-0.011	-0.093	0.011	-0.065	0.098	0.055
	(0.042,0.189)	(-0.082,0.058)	(-0.102,0.019)	(-0.037,0.069)	(-0.121,0.002)	(0.076,0.184)	(0.050,0.190)
TERMSPR	0.107	0.03	0.053	0.092	-0.001	0.051	-0.011
	(0.066,0.186)	(-0.027,0.078)	(0.034,0.114)	(0.015,0.104)	(-0.048,0.028)	(0.024,0.103)	(-0.025,0.083)
CREDSPR	0.066	0.038	0.002	-0.006	0.045	-0.116	-0.167
	(-0.047,0.169)	(-0.106,0.077)	(-0.065,0.100)	(-0.048,0.081)	(-0.039,0.121)	(-0.117,0.013)	(-0.153,0.003)
DJUSRE	0.621	0.642	0.461	0.602	0.385	0.569	0.875
	(0.548,0.922)	(0.432,0.698)	(0.346,0.590)	(0.562,0.835)	(0.384,0.690)	(0.403,0.609)	(0.549,0.829)
LEV	-4.748	-2.016	7.434	-2.195	-6.049	-0.18	-2.172
	(-4.609,-1.123)	(-2.480,0.960)	(5.754,8.236)	(-4.715,-1.883)	(-6.803,1.339)	(-1.131,0.000)	(-4.018,-1.410)
MK2BK	-1.813	3.965	-5.967	-5.682	-3.652	2.577	-2.421
	(-6.966,-1.801)	(-5.516,2.773)	(-7.438,-0.040)	(-7.437,1.429)	(-9.469,0.754)	(0.075,4.741)	(-5.909,2.442)
SIZE	10.337	4.014	1.037	11.342	21.974	1.99	3.975
	(6.938,10.738)	(1.450,4.901)	(0.589,2.404)	(10.561,14.765)	(14.293,23.179)	(1.376,5.079)	(2.931,6.892)
MM	2.416	-2.6	2.491	-2.095	-5.669	-4.732	-1.288
	(-6.762,5.264)	(-5.379,6.361)	(-7.012,5.229)	(-5.415,6.715)	(-6.873,5.029)	(-7.044,4.453)	(-7.058,5.047)
σ_j	0.398	0.423	0.33	0.349	0.353	0.344	0.49
	(0.397,0.486)	(0.424,0.512)	(0.320,0.393)	(0.294,0.362)	(0.279,0.389)	(0.335,0.403)	(0.432,0.527)

VaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-0.214	-0.267	-0.157	-0.036	-0.51	-0.23
	(-0.390,-0.124)	(-0.312,-0.177)	(-0.228,-0.114)	(-0.351,-0.095)	(-0.575,-0.413)	(-0.335,-0.197)
LIQSPR	0.039	-0.026	0.045	-0.015	0.014	0.012
	(-0.017,0.049)	(-0.053,0.024)	(0.026,0.056)	(-0.022,0.051)	(-0.020,0.058)	(-0.022,0.028)
3MTB	0.078	0.002	-0.026	0.086	0.009	-0.021
	(0.009,0.133)	(-0.103,0.023)	(-0.032,0.032)	(0.011,0.097)	(-0.000,0.120)	(-0.044,0.047)
TERMSPR	0.026	0.025	-0.011	0.063	0.023	-0.026
	(-0.023,0.084)	(-0.014,0.050)	(-0.026,0.026)	(0.005,0.084)	(-0.006,0.071)	(-0.030,0.028)
CREDSPR	0.037	-0.07	-0.026	0.023	0.041	-0.009
	(-0.065,0.147)	(-0.164,-0.048)	(-0.054,0.061)	(-0.003,0.127)	(-0.015,0.097)	(-0.040,0.049)
DJUSRE	0.624	0.493	0.092	0.478	0.183	0.276
	(0.409,0.676)	(0.423,0.614)	(0.065,0.208)	(0.324,0.533)	(0.110,0.277)	(0.174,0.335)
LEV	0.12	-4.647	-3.147	-1.379	-2.262	-1.543
	(-0.112,1.184)	(-8.058,-4.422)	(-7.432,4.933)	(-10.645,1.433)	(-6.651,4.404)	(-3.639,7.919)
MK2BK	0	-0.919	-1.065	-8.302	-5.97	-2.912
	(-0.000,0.000)	(-3.836,0.050)	(-2.061,1.416)	(-9.582,0.143)	(-7.866,-0.416)	(-4.142,-0.508)
SIZE	9.592	1.528	0.113	-0.279	4.189	7.332
	(9.239,11.702)	(1.399,3.866)	(-2.178,0.983)	(-3.704,1.133)	(2.326,4.373)	(5.671,7.854)
MM	-1.058	-3.205	-5.392	1.594	-0.998	-1.434
	(-4.568,5.630)	(-8.965,3.998)	(-9.433,-0.051)	(-4.613,8.208)	(-6.219,5.500)	(-6.526,4.561)
σ_j	0.294	0.326	0.193	0.25	0.242	0.264
	(0.253,0.345)	(0.255,0.320)	(0.192,0.234)	(0.205,0.271)	(0.239,0.298)	(0.230,0.277)

VaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.45	0.02	-0.09	-0.261	-0.144	-0.195
	(-0.506,-0.315)	(-0.140,0.016)	(-0.246,-0.063)	(-0.340,-0.168)	(-0.248,-0.097)	(-0.276,-0.093)
LIQSPR	-0.031	-0.051	0.052	0.019	0.002	-0.018
	(-0.071,0.020)	(-0.084,-0.014)	(0.033,0.098)	(-0.022,0.064)	(0.005,0.065)	(-0.036,0.033)
3MTB	0.082	0.051	0.118	0.083	-0.015	-0.004
	(0.027,0.131)	(0.015,0.089)	(0.065,0.152)	(0.033,0.123)	(-0.066,0.015)	(-0.034,0.065)
TERMSPR	0.094	0.058	0.118	0.069	-0.056	-0.043
	(0.040,0.114)	(0.039,0.089)	(0.061,0.131)	(0.020,0.085)	(-0.091,-0.026)	(-0.059,0.022)
CREDSPR	0.107	-0.024	0.141	0.079	-0.203	-0.037
	(-0.031,0.120)	(-0.040,0.038)	(0.055,0.156)	(0.023,0.142)	(-0.231,-0.135)	(-0.083,0.029)
DJUSRE	0.346	0.591	0.244	0.176	0.358	0.329
	(0.324,0.520)	(0.426,0.590)	(0.144,0.350)	(0.130,0.329)	(0.323,0.482)	(0.223,0.398)
LEV	-1.657	-1.076	-4.507	0.57	22.493	9.380
	(-1.753,-1.567)	(-2.138,4.223)	(-7.159,4.734)	(-2.984,8.990)	(21.227,33.193)	(5.406,9.456)
MK2BK	-0.005	-5.311	-3.947	-6.556	6.416	7.269
	(-0.009,0.007)	(-9.451,-5.360)	(-8.723,-2.496)	(-13.354,-5.472)	(1.652,10.367)	(5.484,14.125)
SIZE	1.06	6.71	2.545	12.098	3.623	4.409
	(0.427,1.090)	(5.197,7.616)	(1.133,3.660)	(11.082,15.720)	(2.771,4.066)	(3.580,6.515)
MM	-4.732	-3.612	4.854	-0.169	0.299	-1.372
	(-6.274,6.346)	(-6.257,5.845)	(-5.056,6.705)	(-4.141,3.884)	(-7.265,4.380)	(-5.711,6.326)
σ_j	0.202	0.191	0.28	0.257	0.263	0.263
	(0.176,0.220)	(0.182,0.226)	(0.250,0.315)	(0.220,0.273)	(0.242,0.291)	(0.219,0.274)

Table 13: VaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.025$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

CoVaR	C	BAC	CMA	JPM	KEY	GS	MS
VIX	-0.094 (-0.166,-0.055)	-0.121 (-0.243,-0.099)	-0.129 (-0.165,-0.086)	-0.279 (-0.238,-0.128)	-0.157 (-0.228,-0.127)	-0.124 (-0.162,-0.100)	-0.333 (-0.419,-0.310)
LIQSPR	-0.005 (-0.029,0.003)	0.017 (-0.014,0.020)	0.011 (-0.013,0.017)	-0.006 (-0.029,0.007)	-0.009 (-0.014,0.012)	0 (-0.016,0.001)	-0.006 (-0.022,0.006)
3MTB	0.036 (0.012,0.061)	0.034 (0.018,0.065)	0.038 (0.034,0.072)	0.043 (0.030,0.075)	0.04 (0.038,0.073)	0.038 (0.018,0.050)	0.046 (0.019,0.056)
TERMSPR	0.021 (0.002,0.035)	0.011 (0.007,0.040)	0.03 (0.017,0.045)	0.022 (0.008,0.040)	0.035 (0.027,0.051)	0.031 (0.016,0.038)	0.03 (0.012,0.037)
CREDSPR	-0.002 (-0.013,0.052)	-0.011 (-0.037,0.030)	0.007 (-0.008,0.037)	0.066 (0.020,0.084)	0.017 (0.005,0.057)	0.03 (0.008,0.044)	0.036 (0.003,0.050)
DJUSRE	0.293 (0.246,0.341)	0.319 (0.271,0.373)	0.347 (0.275,0.358)	0.239 (0.222,0.319)	0.375 (0.271,0.377)	0.292 (0.239,0.308)	0.292 (0.269,0.342)
LEV	2.63 (2.268,2.707)	-6.06 (-7.307,-5.267)	4.65 (4.248,6.207)	3.058 (2.416,3.528)	-3.064 (-3.571,-2.549)	0.268 (0.287,0.856)	-0.064 (-0.121,0.227)
MK2BK	0.3 (-1.643,0.835)	-8.786 (-11.358,-6.699)	-3.066 (-14.052,-0.820)	-1.627 (-4.903,3.550)	-8.476 (-14.151,-7.705)	1.076 (-0.043,2.216)	25.007 (20.889,34.748)
SIZE	3.978 (4.127,4.899)	-4.92 (-5.530,-3.787)	-0.179 (-0.803,0.159)	11.516 (11.033,11.849)	-4.149 (-4.625,-2.734)	-0.405 (-1.208,-0.310)	-15.947 (-17.190,-15.678)
MM	8.429 (-2.087,9.599)	-0.878 (-8.888,3.136)	0.429 (-5.980,5.878)	-0.759 (-4.267,7.510)	-3.787 (-8.054,4.351)	4.763 (-5.082,5.185)	6.95 (0.364,11.800)
σ_k	0.123 (0.103,0.126)	0.121 (0.101,0.129)	0.094 (0.078,0.103)	0.1 (0.097,0.122)	0.089 (0.075,0.096)	0.112 (0.094,0.113)	0.094 (0.083,0.102)

CoVaR	MCO	AXP	MCD	NKE	CVX	XOM
VIX	-0.172 (-0.309,-0.122)	-0.188 (-0.292,-0.186)	-0.161 (-0.193,-0.119)	-0.11 (-0.182,-0.107)	-0.08 (-0.120,-0.067)	-0.106 (-0.147,-0.081)
LIQSPR	0.023 (0.007,0.050)	-0.004 (-0.009,0.018)	0.006 (-0.005,0.015)	-0.008 (-0.015,0.002)	0.002 (-0.011,0.002)	-0.009 (-0.020,-0.004)
3MTB	0.04 (0.019,0.066)	0.031 (0.022,0.057)	0.063 (0.028,0.072)	0.021 (0.010,0.046)	0.019 (0.025,0.053)	0.033 (0.028,0.054)
TERMSPR	0.035 (0.017,0.053)	0.022 (0.010,0.042)	0.045 (0.028,0.056)	0.021 (0.015,0.041)	0.012 (0.020,0.044)	0.033 (0.028,0.052)
CREDSPR	0.035 (0.030,0.103)	0.037 (0.018,0.066)	0.009 (-0.010,0.040)	-0.021 (-0.025,0.021)	0.021 (0.009,0.046)	0.02 (0.001,0.041)
DJUSRE	0.366 (0.287,0.396)	0.301 (0.236,0.310)	0.29 (0.249,0.324)	0.326 (0.294,0.368)	0.294 (0.255,0.314)	0.283 (0.278,0.341)
LEV	0.86 (0.782,1.070)	-0.366 (-0.706,1.757)	8.045 (-0.980,9.195)	-22.402 (-24.988,-18.350)	7.352 (1.225,10.041)	-35.741 (-38.175,-34.204)
MK2BK	0 (-0.000,0.000)	5.322 (4.433,6.322)	-0.279 (-1.372,-0.065)	-0.787 (-1.913,-0.332)	-1.459 (-2.508,0.178)	-6.012 (-6.691,-4.370)
SIZE	26.698 (26.549,27.138)	-5.3 (-6.802,-4.744)	0.296 (0.129,2.058)	-1.783 (-2.431,-1.403)	-0.274 (-0.793,0.752)	-0.263 (-0.460,-0.138)
MM	-0.906 (-3.319,3.090)	2.56 (-2.423,10.332)	-1.681 (-7.046,0.119)	3.131 (-0.371,5.001)	1.197 (-3.896,4.845)	3.288 (-2.685,8.078)
σ_k	0.1 (0.084,0.106)	0.08 (0.071,0.090)	0.106 (0.095,0.115)	0.107 (0.097,0.117)	0.092 (0.081,0.098)	0.091 (0.077,0.094)

CoVaR	BA	GE	INTC	ORCL	AEE	PEG
VIX	-0.314 (-0.393,-0.300)	-0.296 (-0.319,-0.217)	-0.143 (-0.207,-0.107)	-0.132 (-0.140,-0.063)	-0.169 (-0.206,-0.132)	-0.089 (-0.136,-0.063)
LIQSPR	0.003 (-0.019,0.018)	0.008 (-0.006,0.025)	0.007 (-0.008,0.016)	-0.007 (-0.020,-0.002)	0.002 (-0.011,0.005)	0.001 (-0.009,0.007)
3MTB	0.037 (0.011,0.054)	-0.001 (0.002,0.042)	0.009 (0.001,0.043)	0.018 (0.002,0.040)	0.059 (0.046,0.074)	0.031 (0.021,0.063)
TERMSPR	0.039 (0.024,0.056)	0.006 (0.006,0.034)	0.024 (0.016,0.045)	0.03 (0.014,0.038)	0.048 (0.040,0.062)	0.038 (0.032,0.060)
CREDSPR	0.075 (0.034,0.093)	0.015 (0.000,0.050)	-0.012 (-0.002,0.060)	0.025 (-0.014,0.028)	0.064 (0.027,0.073)	0.008 (-0.006,0.044)
DJUSRE	0.286 (0.219,0.300)	0.307 (0.271,0.353)	0.295 (0.260,0.347)	0.268 (0.246,0.319)	0.274 (0.246,0.318)	0.350 (0.323,0.406)
LEV	-1.119 (-1.154,-1.111)	9.08 (8.050,10.669)	10.716 (6.473,18.929)	-0.779 (-1.657,3.050)	-11.4 (-13.780,-6.341)	9.890 (8.222,11.550)
MK2BK	-0.003 (-0.005,0.000)	-6.857 (-12.815,-6.164)	-4.2 (-18.357,-2.606)	-0.396 (-0.807,0.152)	-4.331 (-4.284,1.753)	-0.780 (-1.347,0.237)
SIZE	-1.691 (-1.796,-1.624)	-0.4 (-0.922,0.768)	13.69 (12.511,17.506)	7.174 (6.381,7.386)	-3.359 (-5.721,-2.889)	1.080 (0.501,1.572)
MM	5.928 (-7.234,5.686)	-1.908 (-7.929,2.842)	6.849 (1.663,17.154)	-0.743 (-1.229,0.620)	3.734 (-3.442,6.773)	-1.384 (-6.165,3.729)
σ_k	0.068 (0.054,0.070)	0.091 (0.084,0.107)	0.114 (0.092,0.118)	0.1 (0.096,0.117)	0.094 (0.088,0.108)	0.107 (0.100,0.121)

Table 14: CoVaR parameter estimates obtained by fitting the time-varying model to each of the 19 assets vs S&P500 and all the exogenous variables, for the confidence levels $\tau = 0.05$. For each regressor the first row reports parameter estimates by Maximum a Posteriori, while the second row reports the 95% High Posterior Density (HPD) credible sets.

Supplementary Material

Supplementary Materials for "Bayesian tail risk interdependence using quantile regression" (DOI: 10.1214/14-BA911SUPP; .pdf).

References

Acharya, V., Engle, R., and Richardson, M. (2012). "Capital shortfall: a new approach to ranking and regulating systemic risks." *American Economic Review*, 102: 59–64. 554

Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. (2010). "Measuring Systemic Risk." In Acharya, V., Cooley, T., Richardson, M., and Walter, I. (eds.), *Regulating Wall Street: The Dodd–Frank Act and the New Architecture of Global Finance*. John Wiley & Sons. 554

Adams, Z., Füss, R., and Gropp, R. (2010). "Modeling spillover effects among financial institutions: a state-dependent sensitivity Value-at-Risk (SDSVaR) approach." *Working paper*, European Business School. 554

Adrian, T. and Brunnermeier, M. (2011). "CoVaR." *Working paper*, NBER(17454). 554, 555, 571, 579, 582, 583

Bernardi, M. (2013). "Risk measures for skew normal mixtures." *Statistics & Probability Letters*, 83: 1819–1824. 554

Bernardi, M., Maruotti, A., and Petrella, L. (2012). "Skew mixture models for loss distributions: a Bayesian approach." *Insurance: Mathematics and Economics*, 51: 617–623. 554

— (2013). "Multivariate Markov-switching models and tail risk interdependence." *arXiv.1312.6407*. 554

Bernardi, M. and Petrella, L. (2014). "Interconnected risk contributions: a heavy-tail approach to analyse US financial sectors." *arXiv.1401.6408*. 554

Billio, M., Getmansky, M., Lo, A., and Pellizon, L. (2012). "Econometric measures of connectedness and systemic risk in the finance and insurance sectors." *Journal of Financial Econometrics*, 101: 535–559. 554

Brownlees, C. and Engle, R. (2012). "Volatility, correlation and tails for systemic risk measurement." *Working paper*, European Business School. 554

Carter, C. and Kohn, R. (1994). "On Gibbs sampling for state space models." *Biometrika*, 81: 541–553. 565

— (1996). "Markov chain Monte Carlo in conditionally Gaussian state space models." *Biometrika*, 83: 589–601. 565

Castro, C. and Ferrari, S. (2014). "Measuring and testing for the systemically important financial institutions." *Journal of Empirical Finance*, 25: 1–14. 554, 555

Chao, S.-K., Härdle, W., and Chang, W. (2012). "Quantile Regression in Risk Calibration." In Lee, C.-F. (ed.), *Handbook for Financial Econometrics and Statistics*. Springer Verlag. 554, 555, 571

Chen, C., Gerlach, R., Hwang, B., and McAleer, M. (2012a). "Forecasting Value-at-Risk using nonlinear regression quantiles and the intra-day range." *International Journal of Forecasting*, 28: 557–574. 555, 561

Chen, C., Gerlach, R., Lin, E., and Lee, W. (2012b). "Bayesian forecasting for financial risk management, pre and post the global financial crisis." *Journal of Forecasting*, 31: 661–687. 555, 561

Chen, C., Lin, S., and Yu, P. (2012c). "Smooth transition quantile capital asset pricing models with heteroscedasticity." *Computational Economics*, 40: 19–48. 555

Chernozhukov, V. and Du, S. (2008). "Extremal Quantiles and Value-at-Risk." In Durlauf, S. and Blume, L. (eds.), *The New Palgrave Dictionary of Economics*, 1–72. Palgrave Macmillan. 555

Christoffersen, P. (1998). "Evaluating interval forecasts." *Journal of Empirical Finance*, 39: 841–862. 579

Clarke, B. and Clarke, J. (2012). "Prediction in several conventional contexts." *Statistical survey*, 6: 1–73. 579

De Jong, P. (1991). "The diffuse Kalman filter." *Annals of Statistics*, 19: 1073–1083. 585

De Jong, P. and Shephard, N. (1995). "The simulation smoother for time series models." *Biometrika*, 82: 339–350. 585

De Rossi, G. and Harvey, A. (2009). "Quantiles, expectiles and splines." *Journal of Econometrics*, 152: 179–185. 556, 561, 562, 566, 569, 583

Durbin, J. and Koopman, S. (2002). "A simple and efficient simulation smoother for state space time series analysis." *Biometrika*, 89: 603–615. 565

Durbin, J. and Koopman, S. J. (2012). *Time Series Analysis by State Space Methods*. Oxford: Oxford University Press, 2nd edition. 562, 563, 570, 586

Engle, R. and Manganelli, S. (2004). "CAViaR: Conditional autoregressive value at risk by regression quantiles." *Journal of Business & Economic Statistics*, 22: 367–381. 556, 561, 579

Fan, Y., Härdle, W., Wang, W., and Zhu, L. (2013). "Composite quantile regression for the single-index model." *Sonderforschungsbereich 649: Ökonomisches Risiko 10*, (SFB 649 Papers). 555

Frühwirth-Schnatter, S. (1994). "Data augmentation and dynamic linear models." *Journal of Time Series Analysis*, 15: 183–202. 565

Gelfand, A. and Smith, F. (1990). "Sampling-based approaches to calculating marginal densities." *Journal of the American Statistical Association*, 85: 398–409. 585

Geman, S. and Geman, D. (1984). "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images." *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6: 721–741. 585

Gerlach, R., Chen, C., and Chan, N. (2011). "Bayesian time-varying quantile forecasting for Value-at-Risk in financial markets." *Journal of Business & Economic Statistics*, 29: 481–492. 555, 556, 561

Geweke, J. (1992). "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments." In Bernardo, J. M., Berger, J., Dawid, A. P., and Smith, A. F. M. (eds.), *Bayesian Statistics*, 169–193. Oxford University Press, 4th edition. 572

— (2005). *Contemporary Bayesian Econometrics and Statistics*. Hoboken: Wiley Series in Probability and Statistics, Wiley. 572

Girardi, G. and Ergün, A. (2013). "Systemic risk measurement: Multivariate GARCH estimation of CoVaR." *Journal of Banking & Finance*, 37: 3169–3180. 554

Gourieroux, C. and Jasiak, J. (2008). "Dynamic quantile models." *Journal of Econometrics*, 147: 198–205. 556, 561

Harvey, A. (1989). *Forecasting, structural time series models and the Kalman filter*. Cambridge: Cambridge University Press. 562, 586

Hautsch, N., Schaumburg, J., and Schienle, M. (2014). "Financial Network Systemic Risk Contributions." *Review of Finance*. 555

Huang, A. (2012). "Value at risk estimation by quantile regression and kernel estimator." *Review of Quantitative Finance and Accounting*, 41: 225–251. 555

Hyndman, R., King, M., Pitrun, I., and Billah, B. (2005). "Local linear forecasts using cubic smoothing splines." *Australian & New Zealand Journal of Statistics*, 41: 87–99. 584

Johannes, M. and Polson, N. (2009). "MCMC methods for Financial Econometrics." In Ait-Sahalia, Y. and Hansen, L. (eds.), *Handbook of Financial Econometrics*, 1–72. North Holland. 565

Jorion, P. (2007). *Value-at-Risk: The new benchmark for managing financial risk*. Chicago: McGraw-Hill. 553, 554

Koenker, B. (2005). *Quantile Regression*. Cambridge: Cambridge University Press. 554

Koenker, B. and Basset, G. (1978). "Regression Quantiles." *Econometrica*, 46: 33–50. 554, 555

Koenker, B. and Xiao, Z. (2006). "Quantile autoregression." *Journal of the American Statistical Association*, 101: 980–990. 556, 561

Koopman, S. (1991). "Efficient smoothing algorithms for time series models." *Doctoral Thesis*. 584

Koopman, S., Shephard, N., and Doornik, J. (1998). "Statistical algorithms for models in state space using Ssf Pack 2.2." *Econometrics Journal*, 1: 1–55. 584

Kottas, A. and Gelfand, A. (2001). "Bayesian semiparametric median regression modeling." *Journal of the American Statistical Association*, 96: 1458–1468. 555

Kottas, A. and Krnjajic, M. (2009). "Bayesian semiparametric modelling in quantile regression." *Scandinavian Journal of Statistics*, 36: 297–319. 555

Kotz, S., Kozubowski, T., and Podgórski, K. (2001). *The Laplace distribution and generalizations. A revisit with Applications to Communications, Economics, Engineering, and Finance*. Birkhäuser. 556, 558

Kozumi, H. and Kobayashi, G. (2011). "Gibbs Sampling Methods for Bayesian Quantile Regression." *Journal of Statistical Computation and Simulation*, 81: 1565–1578. 558

Kuester, K., Mittnik, S., and Paolella, M. (2006). "Value-at-Risk Prediction: A Comparison of Alternative Strategies." *Journal of Financial Econometrics*, 4: 53–89. 555

Kupiec, P. (1995). "Techniques for verifying the accuracy of risk measurement models." *Journal of Derivatives*, 3: 73–84. 579

Kurose, Y. and Omori, Y. (2012). "Bayesian analysis of time-varying quantiles using a smoothing spline." *Journal of the Japan Statistical Society*, 42: 23–46. 561, 562

Lee, C. and Su, J. (2012). "Alternative statistical distributions for estimating value-at-risk: theory and evidence." *Review of Quantitative Finance and Accounting*, 39: 309–331. 554

Lin, N. and Chang, C. (2012). "Comment on Article by K. Lum and A.E. Gelfand, "Spatial Quantile Multiple Regression Using the Asymmetric Laplace Process"." *Bayesian Analysis*, 7: 263–270. 560

Liu, J. (1994). "The Collapsed Gibbs Sampler in Bayesian Computations with Applications to a Gene Regulation Problem." *Journal of the American Statistical Association*, 89: 958–966. 558

Lum, K. and Gelfand, A. (2012). "Spatial Quantile Multiple Regression Using the Asymmetric Laplace Process." *Bayesian Analysis*, 7: 235–258. 554

McNeil, A., Frey, R., and Embrechts, P. (2005). *Quantitative risk management: Concepts, Techniques, Tools*. Princeton: Princeton Series in Finance. 553

Park, T. and Casella, G. (2008). "The Bayesian Lasso." *Journal of the American Statistical Association*, 103: 681–686. 558

Park, T. and Van Dyk, D. (2009). "Partially Collapsed Gibbs Samplers: Illustrations and Applications." *Journal of Computational and Graphical Statistics*, 18: 283–305. 560, 565

Schaumburg, J. (2010). "Predicting extreme VaR: nonparametric quantile regression with refinements from extreme value theory." *Sonderforschungsbereich 649: Ökonomisches Risiko 9*, (SFB 649 Papers). 555

Sriram, K., Ramamoorthi, R., and Ghosh, P. (2013). "Posterior consistency of Bayesian quantile regression based on the misspecified Asymmetric Laplace density." *Bayesian Analysis*, 8: 479–504. 555

Tanner, M. and Wong, W. (1987). "The calculation of posterior distributions by data augmentation." *Journal of the American Statistical Association*, 82: 528–550. 585

Taylor, J. (2008). "Simultaneous linear quantile regression: a semiparametric Bayesian approach." *Bayesian Analysis*, 7: 51–72. 554, 555

Tokdar, S. and Kadane, J. (2012). "Using exponentially weighted quantile regression to estimate value at risk and expected shortfall." *Journal of Financial Econometrics*, 6: 382–406. 558

Van Dyk, D. and Park, T. (2008). "Partially collapsed Gibbs samplers: Theory and Methods." *Journal of the American Statistical Association*, 110: 790–796. 559, 560, 565

Yu, K. and Moyeed, R. (2001). "Bayesian quantile regression." *Statistics & Probability Letters*, 54: 437–447. 555, 557, 558

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