UPDATING AND DOWNDATING THE LINEAR DECODER FOR THE UPLINK OF MULTIUSER MASSIVE MIMO SYSTEMS

THESIS

ELECTRICAL ENGINEERING

CONCENTRATION ELECTRICAL ELECTRONIC CONTROL SYSTEM

Declared qualified to obtain
a Master degree in Engineering

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BRAWIJAYA UNIVERSITY
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STATEMENT OF THESIS ORIGINALITY

I declare that, to the best of my knowledge, this Thesis does not contain scientific article published by other person to gain academic degree in a University or higher institution, as well as ideas or opinions that have been written or published by other person, except the ones written as quotations in this Thesis and mentioned in the sources/Thesis References.

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CURRICULUM VITAE

The author's name is Soraya Norma Mustika. I was born in Malang, August 9, 1992. I had been living in Jl.Cikaso 16 Malang, Jawa Timur, Indonesia. In 1998-2004, the first formal education he took was in SDN Rampal Celaket V. In 2004-2007, the second formal education I took was National Junior High School 20 in Malang. In 2007-2010, the third formal education I took was Kolese Santo Yusup Malang. I began to study in Electrical Engineering, Faculty of Engineering, Brawijaya University in 2010 and graduated in 2014. In 2015, I continued my study in Double Degree Program provided by Brawijaya University and Sun Yat Sen University.

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Author

PREFACE

In the name of *Allah Subhanahu wa ta'ala*, The Most Beneficent and The Most Merciful. Peace be upon *Muhammad Shallallahu 'alaihi wasallam*. ALLAH the great one gives me blessings and strength.Finally, the thesis book "Updating and Downdating the Linear Decoder for The Uplink of Multiuser Massive MIMO Systems" is finished.At last, I hope this book gives merits especially engineering students. In general, this book gives benefits for all community.

Kaohsiung, July 24, 2018

Author

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Deteksi Zero forcing dan MMSE umumnya sering digunakan untuk memperkirakan atau menghitung berapa approximation channel di massive MIMO. Di penelitian syang telah dilakukan sebelumnya banyak menggunakan salah satu detector saja dalam menghitung channel. Di penelitian ini, algoritma yang dibuat dapat dipergunakan untuk kedua tipe detector ini zero forcing atau MMSE ketika salah satu user dating atau pergi dari channel. Penelitian ini berdasarkan algoritma Singular Value Decomposition (SVD) dari matrik channel yang dikembangkan dengan gabungan dari Gram Schmid saat updating (salah satu user datang/masuk channel) dan menggunakan Given Rotation saat downdating. (salah satu user meninggalkan channel) untuk menjaga bentuk matriks dalam SVD.

Kita menggunakan Given rotation untuk membuat matrix menjadi bidiagonal dan Golub Kahan untuk menghilangkan matriks diagonalnya. Hasil penelitian ini mengindikasikan bahwa penelitian ini menghasilkan hasil performansi yang lebih baik dari skela yang lain dengan kompleksitas yang lebih rendah.

Kata kunci : *Massive MIMO, SVD, Zero Forcing, MMSE, Given Rotation, Golub Kahan*

SUMMARY

Soraya Norma Mustika, Teknik Elektro, Fakultas Teknik, Universitas Brawijaya, Juli 2017, *Updating and Downdating The Linear Decoder for The Uplink of Multiuser Massive MIMO Systems*, Dosen Pembimbing: Rini Nur Hasanah dan Wijono

Zero forcing and MMSE detector are both commonly used detectors in approximation channel of massive MIMO. In previous research, there are often developments in one of the detectors. In this research developed an algorithm that can be used in zero forcing or MMSE when one user is updated or one user is downdate. This research is based on Singular Value Decomposition (SVD) of channel matrix which is the development of Gram Schmid procedure on updating and using Given Rotation on downdating.

To keep them form SVD, we used Given rotation to make matrix converge to bidiagonal and Golub Kahan to decreasing off diagonal matrix. The results of this research indicate that this research produces better performance than other scheme with lower complexity.

Keyword: Massive MIMO**,** SVD, Zero Forcing, MMSE, Given Rotation, Golub Kahan

CHAPTER 1
INTRODUCTION

1.1 Significance

One of subtantial in 5G telecommunication technologies is Massive Multiple Input Multiple Output (MIMO), which will give data rates which is high and efficiency of enegy over conventional MIMO system. In the uplink transmission in this system, computational complexity increasing exponentially along more antenna which is used. The main complexity focused in inverse of huge matrix dimension. Because of this reason many researcher found way to approximate MMSE or ZF detection [1]-[6].

1.2 Related works

In many previous research [1],[2],and [6] is just can using MMSE estimation and still calculation many multiplication although the complexity reduced from exact MMSE estimation. In [1] author try to approximate MMSE equalization using Neumann Series Approximation but many error occurred. Whereas, when approximate using gauss seidel method [2] and newton iteration method [6] result are still have error but less than neumann series approximation .

1.3 Contribution

In this research, we want to find alternative way to estimate received data which can used for MMSE or Zero Forcing estimation. This research want to estimate signal without calculate inverse whole the equalization matrix when one user join or leave the base station. After make SVD decomposition of channel, we using algorithm updated and removing cell in [4]. After updating or removing cell completed, to make sure matrix remain to be SVD decomposition, we using some algorithm. First algorithm is using Householder matrix [7] to make result updating or removing cell to be bidiagonal form. Second algorithm is using Given rotation and Golub Kahan method [10] to focused on main diagonal of diagonal matrix in SVD Decomposition.

1.4 Estimation Result

Because this approximation wont calculate whole matrix inverse, so the complexity less than other method. Apparently, when using Golub Kahan method, result will have some error because in the algorithm have some subtracted equation although, the subtracted value is very small.

1.5 Outline structure

Chapter 2 describe more detail about related works, some of the previous research as Neumann series method[1], Gauss Seidel method [2] and newton iteration [6]. Chapter 3 give explanation about system model along with our proposed scheme. In this chapter we describe each method which is we used as householder, given rotation and Golub Kahan. After that we show, when we used this method when updated or downdate.

The last chapter, we give simulation BER and SNR comparison of related work with proposed scheme and CDF graph to show how many iteration used for specific error. Moreover, we show table to compare complexity of related works and proposed algorithm.

CHAPTER 2
STUDY LITERACY

This chapter give summary explanation about related works which eventually will used for comparison validity result of proposed method. There are five journal related to this research. First journal is Large scale MIMO Detection for 3GPP LTE: Algorithm and FPGA Implementation, its journal is the basis of large MIMO detection. Second journal is Low Complexity Soft Output Signal Detection Based on Gauss Seidel Method for Uplink Multiuser Large scale MIMO System which give alternative to approximate transmitted signal using Gauss Seidel method. For the third related works, A Near-Optimal Detection Scheme Based on Joint Steepest Descent and Jacobi Method for Uplink Massive MIMO System, author make approach based on both joint steepest descent and Jacobi method. Next journal which works related to this research is High Precision Low Complexity Matrix Inversion Based on Newton Iteration for Data Detection in the Massive MIMO purpose to find approximation transmitted data using newton iteration.The last research is source of this proposed algorithm. Robust Update Algorithms for Zero-Forcing Detection in Uplink Large-scale MIMO System giving alternative how to found transmitted data when one user is leave or join the base station.

2.1 Large scale MIMO Detection for 3GPP LTE: Algorithm and FPGA Implementation, its journal is the basis of large MIMO detection

This research concern in the complexity issue of data detection base large scale MIMO system in the uplink. Author main point in linear soft –output detection in combination with new approximate matrix version method relying on a Neumann series expansion.This research using the most common approach to linear MIMO detection is the minimum-mean square error (MMSE) equalizer,

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{y} \tag{2-1}$$

with MMSE equalization matrix denote that

$$\mathbf{W} = \left(\mathbf{H}^H \mathbf{H} + N_0 E_s^{-1} \mathbf{I}_{LU}\right)^{-1} \mathbf{H}^H \tag{2-2}$$

where $\hat{\mathbf{s}}$ is the symbol transmitted, $\mathbf{y}$ is the symbol received signal on base station antenna, $\mathbf{H}$ is channel frequency ,N_0 is noise energy, E_s is user energy, $\mathbf{I}$ is Identity matrix and the gram matrix have definition as

$$\mathbf{G} = \mathbf{H}^H\mathbf{H} \quad (2\text{-}3)$$

followed by forming the regularized Gram matrix

$$\mathbf{A} = \mathbf{G} + N_0 E_s^{-1}\mathbf{I}_U \quad (2\text{-}4)$$

$\mathbf{A}$ is regularized Gram matrix, actually have matched filter(MF) output as

$$\mathbf{y}^{\mathrm{MF}}=\mathbf{H}^H\boldsymbol{y} \quad (2\text{-}5)$$

where, $\mathbf{y}^{\mathrm{MF}}$ is the matched-filter (MF) output. From (2-3),(2-4),(2-5) it can define as

$$\hat{s} = \mathbf{A}^{-1}\mathbf{y}^{\mathrm{MF}} \quad (2\text{-}6)$$

For MMSE, the computation inverse $\mathbf{A}^{-1}$ is major computational complexity in large scale MIMO system. Especially for this large scale MIMO system,where the number of receive antennas is larger than the number of single-antenna users,., for U□ B, where

U = number of antenna users

B = number of antenna in the Base Station

the gram matrix $\mathbf{G}$ and consentquently $\mathbf{A}^{-1}$ become diagonally dominant. Additionally, for i.i.d Gaussian channel matrices $\mathbf{H}$ in the large antenna limit denote that $\mathbf{G} \to \mathbf{I}$. Based on this, the author want to derive a low-complexity approximation of the inverse with let

$$\mathbf{A} \approx \mathbf{D} , \quad (2\text{-}7)$$

where $\mathbf{D}$ is the main diagonal of $\mathbf{A}$ can be approximate by $\mathbf{D}^{-1}$ which sure have much lower complexity than the exact inverse. But, if used simple approximation would cause high error. To get at an accurate approximation of the inverse at low computational complexity , this research propose to use a Neumann series expansion. The inverse $\mathbf{A}^{-1}$ with the following Neumann series expansion :

$$\mathbf{A}^{-1} = \sum_{n=0}^{\infty}\left(\mathbf{X}^{-1}(\mathbf{X} - \mathbf{A})\right)^n \mathbf{X}^{-1} \quad (2\text{-}8)$$

After that, decomposing $\mathbf{A}$ the regulized Gram matrix such that $\mathbf{A} = \mathbf{D} + \mathbf{E}$ where $\mathbf{D}$ is the main diagonal of $\mathbf{A}$ and the $\mathbf{E}$ is the hollow, so author rewrite the regulized Gram matrix in the Neumann series expansion as

$$\mathbf{A}^{-1} = \sum_{n=0}^{\infty}\left(\mathbf{D}^{-1}(\mathbf{D} - \mathbf{A})\right)^n \mathbf{D}^{-1} \quad (2\text{-}9)$$

where they substitute X by $\mathbf{D}_w$. Concretely, computed a *K*-term approximation is needed as

$$\widetilde{\mathbf{A}}_K^{-1} = \sum_{n=0}^{K-1}\left(\mathbf{D}^{-1}(\mathbf{D}-\mathbf{A}\ \ )\right)^n \mathbf{D}^{-1} \quad (2\text{-}10)$$

which can be computed at low computational complexity for approximation consisting of only a few Neumann series term,i.e for small values of *K*. In this research result, for $K \geq 4$,computing exact inverse can be lower complexitythan proposed algorithm.

2.2 Low-Complexity Soft-Output Signal Detection Based onGauss–Seidel Method

This paper background is because author realizing that the advantage of Large Scale MIMO in practice faces some problem, one of which is the practical signal detection algorithm in the uplink because increases number transmit of antenna make complexity exponentially increases too which is make high computational complexity in large scale MIMO system. This research propose to use diagonal component of the MMSE filtering matrix to obtain diagonal-approximate initial solution to the Gauss Seidel method, which can accelerate the convergence rate. This research use an uplink large scale MIMO system with *N* antenna at the *BS* to simultaneously serve *K* selected single antenna UE device, where usually have $N \geq K$,.e.g., *N*=128 and *K*=16.

$$\mathbf{W} = \mathbf{G} + \delta^2 \mathbf{I}_k \quad (2\text{-}11)$$

Wis MMSE filtering matrix In uplink large-scale MIMO systems, the channel matrix **H** is column full rank and colum asymptotically orthogonal, which make sure that the MMSE filtering matrix **W**is Hermitian positive definite can be decomposed as

$$\mathbf{W} = \mathbf{D} + \mathbf{L} + \mathbf{L}^{\mathbf{H}} \quad (2\text{-}12)$$

Where

Dis diagonal component

Lis strictly lower triangular component of **W**

$\mathbf{L}^{\mathbf{H}}$ is strictly upper triangular component of **W**

then, using Gauss Seidel method to estimate the transmitted signal vector $\boldsymbol{s}$ as follows

$$\mathbf{s}^{(i)} = (\mathbf{D} + \mathbf{L})^{-1}(\bar{\mathbf{y}} - \mathbf{L}^{\mathbf{H}}\mathbf{s}^{i-1}), i = 1,2, \ldots \tag{2-13}$$

where i is the number of iteration and $\mathbf{s}^{(0)}$ denotes the initial solution as follows

$$\mathbf{s}^{(0)} = \mathbf{D}^{-1}\bar{\mathbf{y}} \tag{2-14}$$

because approximation error of $\mathbf{s}^{(0)}$should be small, it is show that the proposed diagonal-approximate initial solution will be closer to the final MMSE estimate $\hat{\mathbf{s}}$ compared with the traditional zero-vector initial solution. In this research proved that the proposed GS based algorithm have better performance in simulation result of BER performance against the signal-to-noise ratio (SNR) against proposed Neumann-base Algorithm

2.3 A Near-Optimal Detection Scheme Based on Joint Steepest Descent and Jacobi Method for Uplink Massive MIMO Systems

This research consider the uplink massive MIMO system with B antennas at at base station and U single antenna users($U \leq B$,e.g. U=16,B=128) .

$$\mathbf{b} = \mathbf{H}^{\mathbf{H}}\mathbf{y} \tag{2-15}$$

from (2-4), (2-6), (2-14) can define the approximate $\hat{\mathbf{x}}$ by

$$\hat{\mathbf{x}} = \mathbf{A}^{-1}\mathbf{b} \tag{2-16}$$

And author the MMSE algorithm is converted into solving the linear equation as

$$\mathbf{Ax=b} \tag{2-17}$$

The proposed joint algorithm, consist of three step:

1. First step,*Initialization* Set up the model as (2-15)

2. Second step, *performing steepest descent algorithm once.* This step to obtain an efficient searching direction for following Jacobi iteration to get a fast convergence rate.
3. Third step,,*Employing (K-1) time Jacobi iteration.* Jacobi iteration has low complexity while it is able to converges fast exact solution .it is because **A** is diagonnaly dominant.

The initial estimation of **x**, employed on the proposed algorithm in first step. Because the diagonally dominant property of A, the diagonal-apprimating canused for define initial estimation as

$$\mathbf{x^0} = \mathbf{D^{-1}b} \tag{2-18}$$

where, **D**is diagonal component of **A**.

In this research used the Jacobi iteration for replacing exact computing as

$$\mathbf{x}^{(N)} = \mathbf{D}^{-1}\left[(\mathbf{D}-\mathbf{A})\mathbf{x}^{(N-1)} + \mathbf{b}\right] \tag{2-19}$$

$$\mathbf{x}^{(N)} = \mathbf{x}^{(\mathrm{N}-1)} + \mathbf{D}^{-1}\left(\mathbf{b} - \mathbf{A}\mathbf{x}^{(\mathrm{N}-1)}\right) \tag{2-20}$$

$$\mathbf{r}^{(i)} = \mathbf{b} - \mathbf{A}\mathbf{x}^{(\mathrm{i})} \tag{2-21}$$

from it can define $\mathbf{x}^{(N)}$as

$$\mathbf{x}^{(N)} = \mathbf{x}^{(\mathrm{N}-1)} + \mathbf{D^{-1}}\mathbf{r}^{(N-1)} \tag{2-22}$$

For *step 2*,author employing the steepest descent algorithm for the first time iteration to make result of Jacobi iteration direction efficient. Suppose that

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} + u\mathbf{r}^{(0)} \tag{2-23}$$

where

$$u = \frac{\mathbf{r}^{(0)H}\mathbf{r}^{(0)}}{\left(A\mathbf{r}^{(0)}\right)^H \mathbf{r}^{(0)}} \tag{2-24}$$

after that substituting(2-22)at (2-23)for the first to make convergence. Therefore, within K iteration, the proposed joint algorithm constructed.This paper consider the 64-QAM modulation scheme with have result that has good performance when K is large (e.g $K \geq 2$).

2.4 High Precision Low Complexity Matrix Inversion Based on Newton Iteration

This letter consider a multi user(MU) massive MIMO system focused on uplink data detection. For massive MIMO has special property that $\mathbf{A}$ is diagonally dominant. The inversion of $\mathbf{A}$ is the main complex computation for the system. In this paper make proposed method based on Neumann series. Polynomial Expansion Method transform the matrix inversion of $\mathbf{A}$ to L-order matrix polynomial as

$$\mathbf{A}^{-1} \approx \sum_{n=0}^{L} (\mathbf{X}(\mathbf{X}^{-1} - \mathbf{A})^n)\mathbf{X} \tag{2-25}$$

where $\mathbf{X}$is an invertible matrix close to $\mathbf{A}^{-1}$ which under condition

$$\lim_{n\to\infty} (\mathbf{I} - \mathbf{XA})^n = 0 \tag{2-26}$$

Denoted that **A=D+E**, where **D** and **E** is the main diagonal elements and the off diagonal elements of **A**. Because **A** is diagonally dominant matrix, **X** can be set as $\mathbf{D}^{-1}$, so equation (2-24)can be written as

$$\mathbf{A}^{-1} \approx \sum_{n=0}^{L} (-\mathbf{D}^{-1} - \mathbf{E})^n \mathbf{D}^{-1} \tag{2-27}$$

In this paper, author not just using Polynomial Expansion method but also Newton Iteration method. Newton iteration method is derived from Taylor series which only considers the 1-st order, and can improve the precision by the iteration.The iteration estimation as

$$\mathbf{X}_{n+1}^{iter} = \mathbf{X}_n^{iter}\left(2\mathbf{I} - \mathbf{A}\mathbf{X}_n^{iter}\right) \tag{2-28}$$

If $\mathbf{X}_0^{iter}$is the rough and original estimation of$\mathbf{A}^{-1}$, (2-27) must satisfy condition

$$\|\mathbf{I} - \mathbf{A}\mathbf{X}_0^{iter}\| < 1 \tag{2-29}$$

then (4) will converges quadratically to $\mathbf{A}^{-1}$.

This paper provide the relationship between PE and NI for matrix inversion and propose the diagonal band Newton Iteration method,which is above relation indicate that show as

$$\mathbf{X}_0^{iter} = \mathbf{X}_0^{iter} = \mathbf{D}^{-1} \quad (2\text{-}30)$$

$$\mathbf{X}_1^{iter} = \mathbf{X}_1^{pe} = \mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1} \quad (2\text{-}31)$$

$$\begin{cases} \mathbf{X}_n^{iter} = \mathbf{X}_n^{pe} & n = 0, \\ \mathbf{X}_n^{iter} = \mathbf{X}^{pe}_{\sum_{i=0}^{pe} \mathbf{2}^{i}} & n > 0 \end{cases} \quad (2\text{-}32)$$

from above PE and NI indicate that $\mathbf{X}_1^{iter}$ and $\mathbf{X}_1^{pe}$ have same precision and complexity and for more iteration, Newton Iteration method much faster than Polynomial method.

Considering the data detectionand approximate matrix inversion there, the estimation transmitted signal $\hat{\mathbf{x}}$ can denoted as

$$\hat{\mathbf{x}}_{pe} = (\mathbf{D}^{-1} - \mathbf{D}^{-1}\mathbf{E}\mathbf{D}^{-1})\mathbf{H}^{+}\mathbf{y} \quad (2\text{-}33)$$

$$\hat{\mathbf{x}}_{iter} = \mathbf{X}_1^{iter}\left(2\mathbf{H}^{+}\mathbf{y} - \mathbf{A}\mathbf{X}_1^{iter}(\mathbf{H}^{+}\mathbf{y})\right) =$$

$$\left(\mathbf{D}^{-1} - \mathbf{D}^{-1}\hat{\mathbf{E}}\mathbf{D}^{-1}\right)2\mathbf{H}^{+}\mathbf{y} - \mathbf{A}\left(\mathbf{D}^{-1} - \mathbf{D}^{-1}\hat{\mathbf{E}}\mathbf{D}^{-1}\right)\mathbf{H}^{+}\mathbf{y} \quad (2\text{-}34)$$

$\hat{\mathbf{x}}_{pe}$is the estimation transmitted signal $\hat{\mathbf{x}}$ using PE method

$\hat{\mathbf{x}}_{iter}$isthe estimation transmitted signal $\hat{\mathbf{x}}$ using NI method

From using PE method and NI method to find the estimation transmitted signal $\hat{\mathbf{x}}$,it is found that the BER of DBNI method is close to the exact inversion when user number is small,and always better than PE method for all kind of configuration.

2.5 Robust Update Algorithms for Zero Forcing Detection in Uplink Large-Scale MIMO Systems

This research proposes a Zero Forcing (ZF) detector, referred to as the S-ZF detector based on the singular value decomposition (SVD) of channel matrix. Computationally efficient algorithm proposed to update the decomposition of the channel matrix whenever a user equipment (UE) leaves or join the cell. Gram Schmidt procedure to added channel vector is performed when UE join the cell, the decomposition of the inflated channel matrix is updated. Given rotation is performed when a UE leaves the cell, the removed channel vector is ejected from the decomposition. After downdating or updating, the new channel matrix can be factored as

$$\mathbf{H} = \mathbf{U}_1\mathbf{R}\mathbf{V}^{\mathbf{H}} \qquad (2\text{-}35)$$

without recalculating the SVD, where the matrices $\mathbf{U}_1$ and $\mathbf{V}^{\mathbf{H}}$have orthonormal colums and **R** is upper triangular matrix.

Consider the uplink channel of a large scale multiuser MIMO system in which the base station is equipped with *M*antenna and serve *K* (K < M) UEs with a single antenna.

In the assumption of perfect channel state information at the base station, the ZF detection of the data vector is given by

$$\hat{\mathbf{x}}_{ZF} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y} \qquad (2\text{-}36)$$

After substituting SVD decomposition (2-34) into (2-35), the ZF detector can be defined as

$$\hat{\mathbf{x}}_{ZF} = \mathbf{V}\boldsymbol{\Sigma}_1^{-1}\mathbf{U}_1^{\mathbf{H}}\mathbf{y} \qquad (2\text{-}37)$$

With combine the (2-47) and (2-44)it will reformulated as

$$\hat{\mathbf{x}}_{ZF} = \mathbf{V}\mathbf{R}^{-1}\mathbf{U}_1^{\mathbf{H}}\mathbf{y} \qquad (2\text{-}38)$$

2.5.1 Adding UE

If inflated channel matrix define as $\breve{\mathbf{H}} = [\mathbf{H} \quad \mathbf{h}_{K+1}]$ where $\mathbf{h}_{K+1}$is the vector of the channel coefficient from the new UE to the BS.The algorithm when a UE joins cell, the channel matrix needs to be inflated by one column.

1. Initialization Given $\mathbf{H} = \mathbf{U}_1\mathbf{R}\mathbf{V}^{\mathbf{H}}$, $\mathbf{h}_{K+1}$
2. Calculate

$$\breve{\mathbf{u}} = \frac{1}{\alpha}(\mathbf{I} - \mathbf{U}_1\mathbf{U}_1^H)\mathbf{h}_{K+1}, \qquad (2\text{-}39)$$

And $\alpha = \|(\mathbf{I} - \mathbf{U}_1\mathbf{U}_1^H)\mathbf{h}_{K+1}\|_2$ (2-40)

3. Define new channel as

$$\breve{\mathbf{H}} = \underbrace{\begin{bmatrix}\mathbf{U}_1 & \breve{\mathbf{u}}\end{bmatrix}}_{\breve{\mathbf{U}}_1}\underbrace{\begin{bmatrix}\mathbf{R} & \mathbf{U}_1^H\mathbf{h}_{K+1}\\ 0 & \alpha\end{bmatrix}}_{\breve{\mathbf{R}}}\underbrace{\begin{bmatrix}\mathbf{V}^H & 0\\ 0 & 1\end{bmatrix}}_{\breve{\mathbf{V}}^H} \qquad (2\text{-}41)$$

2.5.2 Removing UE

Upon UE-*j* leaves the cell , the *j-th* column of **H**is removed from the channel matrix. If new channel matrix after deflated channel matrix define by $\widetilde{\mathbf{H}}$, so the new channel can be expressed by

$$\left[\widetilde{\mathbf{H}} \quad \mathbf{h}_j\right] = \mathbf{HP}_{j,K} = \mathbf{U}_1\mathbf{RV}_j^H \qquad (2\text{-}42)$$

Where $\mathbf{V}_j$ is result permutation matrix of $\mathbf{V}$, to make column UE which is want to removed moved tolast column of $\mathbf{V}$. If we define new channel matrix as

$$\widetilde{\mathbf{H}} = \widetilde{\mathbf{U}}\widetilde{\mathbf{R}}\widetilde{\mathbf{V}}^H \qquad (2\text{-}43)$$

The algorithm when a UE leaves cell, the channel matrix needs to be deflated by one column.

1. Initialization Given $\mathbf{H} = \mathbf{U_1RV^H}$
2. Delete last row of $\mathbf{V}$as

$$\left[\widetilde{\mathbf{H}} \quad \mathbf{h}_j\right] = \widetilde{\mathbf{U}}\widetilde{\mathbf{R}}\begin{bmatrix}\widetilde{\mathbf{V}}^H & 0_{k-1} \\ 0_{K-1}^T & 1\end{bmatrix} \qquad (2\text{-}44)$$

   Using given rotation
3. Obtain new $\mathbf{U_1}, \widetilde{\mathbf{U}}$by deleting last colum of $\mathbf{U_1}$
4. Obtain $\widetilde{\mathbf{R}}$ and $\widetilde{\mathbf{V}}$ by deleting the last row and column of $\mathbf{R}^{(K-1)}$ and $\mathbf{V}^{(K-1)}$ respectively

The simulation result of this research shown that the proposed detector outperform existing method like Neumann series, Gauss Seidel, Newton Iteration, and Steepeest Jacobi .

CHAPTER 3
RESEARCH METHODOLOGY

3.1 System Model

In the case uplink or downlink in the Massive MIMO system computational complexity tend to really high. This is happen because major calculation for MMSE or Zero Forcing lies on inverse of large matrix. We proposed algorithm that can reduce the complexity of inverse MMSE or Zero Forcing equalizer when one user come or one user go out from range of base station. An approximate USV decomposition of channel will be calculated to complete the calculation of the update or downdate signal transmitted without renumerate the new inverse.

We consider Massive MIMO applied *K*selected single antenna of user and *M*antennas at Base Station,e.g., *K*=16 and *M*=128 . The received signal by base station denoted by **y**,while user transmitted signal can be denoted by **x** which can obtained from **y** by filteringrayleigh fading channel matrix **H** that entries fulfill $CN(0,1)$ [2]. In the receiver, we add Gaussian noise **n** that fulfill $CN(0,\sigma^2)$. Then received vector in base station can be stated by:

$$3.2\mathbf{y} = \mathbf{H}\mathbf{x}+\mathbf{n} \qquad (3\text{-}1)$$

With above vector model, base station has found clearly the channel **H** .We compute the

$$\mathrm{SNR} = \mathrm{M}\frac{\sigma_x^2}{\sigma_n^2} \qquad (3\text{-}2)$$

σ_x^2 is source power

σ_n^2 is noise power

To show the error in the algorithm used, the following error equations used in this research, for Downdating algorithm error define as ,.

$$\varepsilon = \frac{Tr((\tilde{\mathbf{H}}-\mathbf{H})(\tilde{\mathbf{H}}-\mathbf{H})^H)}{Tr(\mathbf{H})(\mathbf{H})^H} \qquad (3\text{-}3)$$

meanwhile for Updating Algorithm same as downdating but change $\tilde{\mathbf{H}}$ be $\breve{\mathbf{H}}$

Where

$\tilde{\mathbf{H}}$ =Approximate channel using proposed Downdating algorithm

$\breve{\mathbf{H}}$ =Approximate channel using proposed Updating algorithm

$\mathbf{H}$ =Real Channel

3.2 Channel Equalization using MMSE

The famous equalizer for uplink or downlink is MMSE equalizer, because it has good performance but more complicated that Zero Forcing. The MMSE equalizer detection can be formulated by

$$\hat{\mathbf{x}} = (\mathbf{H}^H\mathbf{H} + \boldsymbol{\sigma}^2\mathbf{I}_K)^{-1}\mathbf{H}^H\mathbf{y} \quad (3\text{-}4)$$

where $\boldsymbol{\sigma}^2$ is user power $\mathbf{I}_K$ is identity matrix and if we have USV decomposition of new **H,**

$$\mathbf{H} = \mathbf{U\Sigma V} \quad (3\text{-}5)$$

Where $\mathbf{U} \in \mathbb{C}^{M\times K}$ and $\mathbf{V} \in \mathbb{C}^{K\times K}$ have orthonormal column and $\boldsymbol{\Sigma}$ is diagonal column. For approximate $\hat{\mathbf{x}}$, we don't have recomputed above formula and inverse massive matrix.

$$\boldsymbol{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_K), \quad (3\text{-}6)$$

$$\hat{\mathbf{x}} = \mathbf{V}\boldsymbol{\Sigma}^H(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^H + \boldsymbol{\sigma}^2\mathbf{I}_K)^{-1}\mathbf{U}^H\mathbf{y} \quad (3\text{-}7)$$

The inverse of approximate $\hat{\mathbf{x}}$ which using USV decomposition has less complexity because matrix which is inversed is diagonal matrix compare that original MMSE ,it has to calculate inverse of full rank of matrix.

3.3 Channel Equalization using Zero Forcing

Zero forcing equalization less complexity than MMSE, although for the early computing, has descent difference, but in the end the difference compare both using zero forcing equalization and MMSE have small difference. The Zero Forcing detection can be denoted as

$$\hat{\mathbf{x}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y} \quad (3\text{-}8)$$

and if we have USV decomposition of new **H,** recomputed above formula and inverse massive matrix. We can calculate new approximate $\hat{\mathbf{x}}$ as

$$\hat{\mathbf{x}} = \mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\mathbf{y} \quad (3\text{-}9)$$

The inverse of approximate $\hat{\mathbf{x}}$ which using USV decomposition has less complexity because matrix which is inversed of $\mathbf{U}$ that is diagonal matrix.

3.4 Proposed Algorithm

In this proposed algorithm, we provide some step how to calculating approximate $\hat{\mathbf{x}}$ without recalculating whole channel from beginning. In this algorithm we using some lemma which is proved has good result by Yi Wu,Huang[4]. To make upper triangular be bidiagonal, we using householder algorithm. Given rotation and Golub Kahan formula are used for construct diagonal matrix from bidiagonal matrix. Basically, updating and downdating based USV decomposition need three step which has to be done. It is three step for updating :

Step-1 :Reconsturct update for USV decomposition[4], result this step is $\hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}$

Step-2 :Using Householder algorithm for make $\hat{\mathbf{S}}$ result from step 1 to be bidiagonal matrix $\bar{\mathbf{S}}$ and get new $\bar{\mathbf{U}}\bar{\mathbf{V}}$

Step-3 :Applied Given rotation to converge bidiagonal matrix and Golub Kahan to make decreasing off diagonal matrix in bidiagonal matrix $\bar{\mathbf{S}}$.Result of this step is $\breve{\mathbf{U}}\breve{\mathbf{S}}\breve{\mathbf{V}}$ .Given Rotation result is $\breve{\mathbf{U}}\breve{\mathbf{S}}\breve{\mathbf{V}}$,meanwhileGolub Kahan step result is $\hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}$

For downdating based USV decomposition same like updating, three step has to be performed. It is three step for downdating

Step-1 :Reconsturctdowndating for USV decomposition,for each nullify V, we have to multiply with given rotation to ensure that for $\breve{\mathbf{S}}$ decomposition, will be upper triangular

Step-2 :Using Householder algorithm for make $\hat{\mathbf{S}}$ result from step 1 to be bidiagonal matrix $\bar{\mathbf{S}}$ and get new $\bar{\mathbf{U}}\bar{\mathbf{V}}$

Step-3 :Applied Given rotation to converge bidiagonal matrix and Golub Kahan to make decreasing off diagonal matrix in bidiagonal matrix $\bar{\mathbf{S}}$ .Given Rotation result is $\breve{\mathbf{U}}\breve{\mathbf{S}}\breve{\mathbf{V}}$,meanwhile Golub Kahan step result is $\hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}$

Before clearly explanation about updating and downdating, there are formula which is important to undertstand and will used in both updating and downdating for this research

3.4.1 Given Rotation

Golub Kahanet all [11]presented a rotation in the plane spanned by two coordinates axes. Denoted that given rotation for any complex number is $\mathbf{G}$,

$$\mathbf{G} = \begin{bmatrix} \tilde{c} & -\tilde{s} \\ \tilde{s}* & \tilde{c}* \end{bmatrix}, \quad (3\text{-}10)$$

when we multiply with one vector $\mathbf{f} = \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{bmatrix}$,where

$$\tilde{c} = \frac{\tilde{\alpha}*}{\sqrt{|\tilde{\alpha}|^2 + |\tilde{\beta}|^2}} \quad (3\text{-}11)$$

$$\tilde{s} = \frac{\tilde{\alpha}*}{\sqrt{|\tilde{\alpha}|^2 + |\tilde{\beta}|^2}} \quad (3\text{-}12)$$

$$\tilde{c}* = \frac{\tilde{\alpha}}{\sqrt{|\tilde{\alpha}|^2 + |\tilde{\beta}|^2}} \quad (3\text{-}13)$$

$$\tilde{s}* = \frac{-\tilde{\beta}}{\sqrt{|\tilde{\alpha}|^2 + |\tilde{\beta}|^2}} \quad (3\text{-}14)$$

Result after multiply **G** and **f**,it will be nullify *b* and change value *a*in vector **f** , it will be

$$\begin{bmatrix} \tilde{c} & -\tilde{s} \\ \tilde{s}* & \tilde{c}* \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{bmatrix} = \begin{bmatrix} \sqrt{\tilde{\alpha}^2 + \tilde{\beta}^2} \\ 0 \end{bmatrix} \quad (3\text{-}15)$$

3.4.2 Householder Transform

Householder transform provided in Golub Kahanet all [11] which is developed for complex number by venkaiah et all [5] and Kuo –Liang et all [8]. Householder which is extended by Kuo –Liang has simple formula and proven that had good result. Householder transform **Q** from Kuo Liang as

$$\mathbf{Q} = \mathbf{I} - \frac{\mathbf{zz}*}{\mathbf{z}*\hat{\mathbf{s}}} \quad (3\text{-}16)$$

where $\hat{\mathbf{s}}, \mathbf{b} \in \mathbb{C}^n$ and $\mathbf{z} = \hat{\mathbf{s}} - \mathbf{b}$ and then expressed that $\mathbf{Q}\,\hat{\mathbf{s}} = \mathbf{b}$ and **Q** is unitary matrix. The Housholder transform can be applied to make bidiagonal matrix that explained by Alan Kayloret all [10]. Householder for row and column alternately multiplied by $\hat{\mathbf{S}}$ to create a bidiagonal form matrix.,. For making bidiagonal matrix, we can follow this six step, three step for Bidigonalization column and three step other for row .

3.4.2.1 Householder Transform for Column

Step-1 :Find Householder matrix that should have property:

$$\mathbf{Q}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_{o-1,o} \\ s_{o,o} \\ s_{o+1,o} \\ \vdots \\ s_{k,o} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ s_{o-1,o} \\ s \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Left multiplication by **Q** which has component 1,…..o-1 alike then,

where
$$\bar{s} = \pm\sqrt{\sum_{i=0}^{k} s_{i,k}^2} \quad (3\text{-}17)$$

Step-2 :Denote new S with multiply householder $\mathbf{Q}$ with $\hat{\mathbf{S}}_1$

Step-3 :To make matrix same like previous, it has to multiply $\hat{\mathbf{U}}$ with $\mathbf{Q}^H$ as $\bar{\mathbf{U}} = \hat{\mathbf{U}}\mathbf{Q}^H$

3.4.2.2 Householder Transform for Row

Step-1 :if $o \le k-2$, determine Householder matrix $\mathbf{P}$ that should have property:

Left multiplication by $\mathbf{P}$ which has component 1,….k alike then,

$$\mathbf{Q} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\hat{\mathbf{s}}} \quad (3\text{-}18)$$

where $\hat{\mathbf{s}}, \mathbf{b} \in \mathbb{C}^n$ and $\mathbf{z} = \hat{\mathbf{s}} - \mathbf{b}$ and then expressed that $\mathbf{Q}\,\hat{\mathbf{s}} = \mathbf{b}$ and $\mathbf{Q}$ is unitary matrix. Where

$$\bar{s} = \pm\sqrt{\sum_{j=o+1}^{k} \bar{s}_{o,j}^2} \quad (3\text{-}19)$$

$$\begin{bmatrix} 0 & \cdots & 0 & \bar{s}_{o,o} & \bar{s}_{o,o+1} & \bar{s}_{o,o+2} & \cdots & \bar{s}_{o,n} \end{bmatrix} \mathbf{P}_{o+1} = \begin{bmatrix} 0 & \cdots & 0 & \bar{s}_{o,o} & \bar{s} & 0 & \cdots & 0 \end{bmatrix}$$

Step-2 :Denote new $\bar{\mathbf{S}}$ with multiply householder $\mathbf{P}$ with previous $\hat{\mathbf{S}}$ result

Step-3 :To make matrix same like previous, it has to multiply $\hat{\mathbf{V}}$ (resut of URV Decomposition[4]) with $\mathbf{P}^H$

After following step above, USV decomposition which $\hat{\mathbf{S}}$ has bidiagonal form will constructed.

3.4.3 Bidiagonal Matrix to be Diagonal Matrix

In this section, a flowchart is shown which explains how the biadiagonal form matrix is transformed into a diagonal matrix, first is to converge the bidiagonal matrix using given rotation, then subtract off the diagonal using the Golub Kahan Algorithm.*Figure 3.1* show flowchart to get diagonal form matrix.

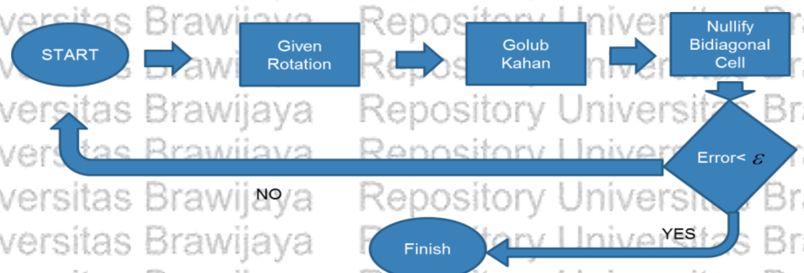

Figure 3.1 Flowchart Diagonalization

3.4.3.1 Given Rotation for Converge Bidiagonal Matrix

The main purpose for usingGiven rotation is to have output that $\breve{\mathbf{S}}$ has smaller off diagonal elements compare that previous $\bar{\mathbf{S}}$.The explain that Given Rotatiionalgorithm has some step as

Step-1 : Fori=1,2,…K-1 $\alpha_1 = s_{i,i}$ and $\beta_1 = s_{i,i+1}$,determine, with property that

$$\begin{bmatrix}\alpha_1 & \beta_1\end{bmatrix}\begin{bmatrix}\tilde{c}_1 & \tilde{s}_1{}^* \\ -\tilde{s}_1 & \tilde{c}_1{}^*\end{bmatrix} = \begin{bmatrix}r_1 & 0\end{bmatrix} \tag{3-20}$$

Step-2 :Denote $\breve{\mathbf{S}}$ with right multiplication Given rotation above $\mathbf{G}_1$ with previous $\bar{\mathbf{S}}$ (result from householderalgorithm)

Step-3 :To make matrix same like previous, it has to do left multiplication $\bar{\mathbf{V}}$ with hermitian of given rotation above $\mathbf{G}_1^H$, $\breve{\mathbf{V}} = \mathbf{G}_1{}^H\bar{\mathbf{V}}$

Step-4 :For i=1,2,…K-1 $\alpha_1 = s_{i,i}$ and $\beta_1 = s_{i,i+1}$,determine, with property that:

$$\begin{bmatrix}\tilde{c}_2 & -\tilde{s}_2 \\ \tilde{s}_2{}^* & \tilde{c}_2\end{bmatrix}\begin{bmatrix}\alpha_2 \\ \beta_2\end{bmatrix} = \begin{bmatrix}r_2 \\ 0\end{bmatrix} \tag{3-21}$$

Step-5 :To make matrix same like previous, it has to do right multiplication $\bar{\mathbf{U}}$ with hermitian of given rotation above $\mathbf{G}_2^H$, $\breve{\mathbf{U}} = \bar{\mathbf{U}}\mathbf{G}_2^H$ (3-22)

3.4.3.2 Golub Kahan SVD Algorithm

The aim to applied Golub Kahan SVD is will get $\breve{\mathbf{S}}_2$ has smaller off diagonal elements compare that previous $\breve{\mathbf{S}}_1$.Alan Kayloret all [10] explain that Golub Kahan Algorithm have some step

Step-1 : Set C = 2 x2 submatriks of $\breve{\mathbf{S}}$ For i=1,2,…K-1, $\breve{s}_{i,i}, \breve{s}_{i,i+1}, \breve{s}_{i+1,i+1}, \breve{s}_{i+1,i+1}$

C

$$\begin{pmatrix} \breve{s}_{11} & \breve{s}_{12} & 0 \\ 0 & \breve{s}_{22} & \breve{s}_{23} \\ 0 & 0 & \breve{s}_{33} \end{pmatrix}$$

Step-2 :For each C, Obtain eigenvalues λ_1, λ_2 of c ,set μ whichever closer to $c_{i+1,i+1}$ of C

Step-3 :For $\alpha_3 = s_{i,i}^2 - \mu$ and $\beta_3 = s_{i,i}.s_{i,i+1}$,determine with property that :

$$\begin{bmatrix} \alpha_3 & \beta_3 \end{bmatrix} \begin{bmatrix} \tilde{c}_3 & \tilde{s}_3 * \\ -\tilde{s}_3 & \tilde{c}_3 * \end{bmatrix} = \begin{bmatrix} r_3 & 0 \end{bmatrix} \quad (3\text{-}23)$$

Step-4 :Denote new $\hat{\mathbf{S}}$ with right multiplication Given rotation above $\mathbf{G}_3$ with previous $\breve{\mathbf{S}}$ (result of Given Rotation Algorithm).

Step-5 :To make matrix same like previous, it has to do left multiplication $\breve{\mathbf{V}}$ with hermitian of given rotation above $\mathbf{G}_3^H$,gets $\hat{\mathbf{V}} = \mathbf{G}_3^H \breve{\mathbf{V}}$,

Step 6 until 8 has to be done,because to make ensure that $\hat{\mathbf{S}}$ is upper bidiagonal form.

Step-6 :For $\alpha_4 = \hat{s}_{i,i}$ and $\beta_4 = \hat{s}_{i+1,i}$ determine, with propertythat :

$$\begin{bmatrix} \tilde{c}_4 & -\tilde{s}_4 \\ \tilde{s}_4 * & \tilde{c}_4 \end{bmatrix} \begin{bmatrix} \alpha_4 \\ \beta_4 \end{bmatrix} = \begin{bmatrix} r_4 \\ 0 \end{bmatrix} \quad (3\text{-}24)$$

Step-7 :Denote new $\hat{\mathbf{S}}$ with Left multiplication Given rotation above $\mathbf{G}_4^H$ with previous $\hat{\mathbf{S}}$ (Result Golub Kahan Step 1-4)

Step-8 :To make matrix same like previous, it has to do left multiplication $\breve{\mathbf{U}}$ with hermitian of given rotation above $\mathbf{G}_4^H$,So we get $\hat{\mathbf{U}} = \breve{\mathbf{U}}\mathbf{G}_4^H$

3.4.4 Updating User

When user added in base station range, new channel is added in last column of previous channel. Suppose, new channel $\breve{\mathbf{H}}$ is combined form previous channel $\mathbf{H}$ with new vector channel from new added user $\mathbf{h}_{K+1}$ .There are three step like written above for updating user in base station but explained more details

Step-1 :Reconsturct update for USV decomposition

a. If we have previous channel $\mathbf{H} = \mathbf{USV}$, and new channel vector from a user is $\mathbf{h}_{k+1}$

b. Calculate

$$\mathbf{r} = \mathbf{U}^{H}\mathbf{h}_{k+1}, \tag{3-25}$$

$$\mathbf{t} = \mathbf{h}_{k+1} - \mathbf{Ur}, \tag{3-26}$$

$$\boldsymbol{\alpha} = \|\mathbf{t}\| \tag{3-27}$$

c. Obtain $\tilde{\mathbf{u}} = \frac{1}{\alpha}\left(\mathbf{I} - \mathbf{U}\ \ \mathbf{U}^{H}\right)\mathbf{h}_{K+1}$

d. If we express new channel as $\breve{\mathbf{H}} = \breve{\mathbf{U}}\breve{\mathbf{S}}\breve{\mathbf{V}}$

$$\breve{\mathbf{H}} = \underbrace{\begin{bmatrix}\mathbf{U}_1 & \tilde{\mathbf{u}}\end{bmatrix}}_{\breve{\mathbf{U}}} \underbrace{\begin{bmatrix}\mathbf{S} & \mathbf{U}^{H}\mathbf{h}_{K+1} \\ 0 & \alpha\end{bmatrix}}_{\mathbf{R}} \underbrace{\begin{bmatrix}\mathbf{V}^{H} & 0 \\ 0 & 1\end{bmatrix}}_{\breve{\mathbf{V}}^{H}} \tag{3-28}$$

Step-2 :Using Householder algorithm for make $\hat{\mathbf{S}}$ from step 1 to be bidiagonalmatrix,because of $\hat{\mathbf{S}}$ upper bidiagonal matrix, so for the first time householder algorithm , Directly apply the householder to nullify the row because the first column is zero .Result of this step is $\bar{\mathbf{U}}\bar{\mathbf{S}}\bar{\mathbf{V}}$

Step-3 :Applied Given rotation,Golub Kahanrepeatedlyto make bidiagonal matrix to be diagonal matrix until get error which is wanted.

3.4.5 Downdating User

When a user is leave range of base station, column of channel which is contain information about user has to remove. Suppose, new channel $\tilde{\mathbf{H}}$ is channel which already reduced the user's channel which leave base station. Suppose vector channel from user who leaving is $\mathbf{h}_{K-1}$. There are three step like written above for updating user in base station.There are three step like written above for downdating user in base station but explained more details .

Step-1 :Reconsturctdowndating for USV decomposition,for each nullify V, we have to multiply with given rotation to ensure that for S decomposition, will be upper triangular

For i=1:k-1

a. Nullify last column of $\mathbf{V}$ as $\begin{bmatrix}\mathbf{V} & 0 \\ 0 & 1\end{bmatrix}$ using backward of given rotation $\mathbf{G}_3$,which is follow

$$\begin{bmatrix}\tilde{s}* & \tilde{c}* \\ \tilde{c} & \tilde{d}\end{bmatrix}\begin{bmatrix}v_i \\ v_{i+1}\end{bmatrix} = \begin{bmatrix}0 \\ \sqrt{v_i^{\,2} + v_{i+1}^{\,2}}\end{bmatrix} \tag{3-29}$$

b. To make matrix same like previous, it has to do right multiplication $\mathbf{S}$ with hermitian of given rotation above $\mathbf{G}_3^H$ as result $\widehat{\mathbf{S}} = \mathbf{S}\mathbf{G}_3^H$

c. To make $\widehat{\mathbf{S}}$ hold upper triangular ,for each given rotation above ,need to multiply $\widehat{\mathbf{S}}$ for make each of $\mathbf{S}_{i+1,i}$ zero. For $\widehat{\alpha} = s_{i,i}$ and $\widehat{\beta} = s_{i+1,i}$ determine $\mathbf{G}_4^H$ with property that :

$$\begin{bmatrix} \tilde{c} & -\tilde{s} \\ \tilde{s}* & \tilde{c}* \end{bmatrix} \begin{bmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{bmatrix} = \begin{bmatrix} \sqrt{\widehat{\alpha}^2 + \widehat{\beta}^2} \\ 0 \end{bmatrix} \quad (3\text{-}30)$$

d. To make matrix same like previous, it has to do left multiplication $\mathbf{U}$ with hermitian of given rotation above $\mathbf{G}_4^H$ to get $\widehat{\mathbf{U}} = \mathbf{U}\mathbf{G}_4^H$

Step-2 :Using Householder algorithm for make $\widehat{\mathbf{S}}$ from step 1 to be bidiagonalmatrix,because of $\widehat{\mathbf{S}}$ upper bidiagonal matrix, so for the first time householder algorithm , Directly apply the householder to nullify the row because the first column is zero.Result of this step is $\overline{\mathbf{U}}\overline{\mathbf{S}}\overline{\mathbf{V}}$

Step-3 :Applied Given rotation,Golub Kahan repeatedly to make bidiagonal matrix to be diagonal matrix until get error which is wanted.

3.5 Computational Complexitity

To show that our proposed method has less complexity than conventional method, we provide a table which compare conventional zero forcing with proposed one. Meanwhile, we provide also comparison of computational complexity MMSE using conventional method with MMSE using proposed method one. For updating method there are three step to construct new approximate transmitted signal. The computational complexity of the this algorithm can be sum up from this three step

1. Step 1, computational complexity to reconstruct added new channel
2. Step 2, For householder method,
3. Step 3, For Given rotation Golub Kahan

For downdating method there are three step to construct new approximate transmitted signal. The computational complexity of this algorithm can be sum up from this three step ,there are

1. Step 1, computational complexity to reconstruct added new channel
2. Step 2, For householder method,
3. Step 3, For Given rotation Golub Kahan

In calculations obtained computational complexity calculated from each algorithm used. *Table 3.1* shows the computational complexity Downdating Algorithm calculated from each algorithm

Algorithm	Computational Complexity
URV Decomposition	$MK + 2K$
House Holder Matrix	$M + 2M^2 + \sum_{i=2}^{K-1} K(M-i)^2 + (M-i)^2 M + \sum_{i=2}^{K-1} M(M-i)^2$
Given Rotation	$4M(M + K + 3)$
Golub Kahan	$4M(M + K + 3)$
$MK + 2K + M + 2M^2 + \sum_{i=2}^{K-1} K(M-i)^2 + (M-i)^2 + \sum_{i=2}^{K-1} M(M-i)^2 + 8M(M+K+3)$	

Table 3.1 Downdating Algorithm Computational Complexity

While *table 3.2* shows computational complexity calculated from each algorithm in Updating Algorithm . The different with downdating only in URV Decomposition.

Table 3.2 Updating Algorithm Computational Complexity

Algorithm	Computational Complexity
URV Decomposition	$4M(M + K + 3)$
$M + 2M^2 + \sum_{i=2}^{K-1} K(M-i)^2 + (M-i)^2 + \sum_{i=2}^{K-1} M(M-i)^2 + 12M(M+K+3)$	

CHAPTER 4
RESULT AND DISCUSSION

In this chapter show comparison zero forcing using conventional method with zero forcing using proposed method. In other hand, this section also compare MMSE using conventional method with using proposed method. Because of that, it can explained and show clearly that this proposed method can used for both MMSE and Zero Forcing equalizer. This chapter denote computational complexity for each method and show simulation result of Bit Error Rate (BER) versus Signal Noise Ratio (SNR). In the simulation result we are using number of user K=16 transmitting qpsk signals, each user using one antenna. Meanwhile, number of antenna in Base Station , M= 128 over rayleigh fading channel.

4.1 Simulation Result

In this section, provided some simulation result the consist of BER and SNR to verified the validity of our algorithm against conventional algorithm.We divided to be two Figure, *Figure 4-1* is comparison BER and SNR for conventional and proposed algorithm in updating method. Whereas *Figure 4.2* show comparison BER and SNR proposed algorithm against conventional one in downdating user.

In updating method, proposed algorithm used in two equalization method. Figure 4.1 show updating proposed method used in Zero Forcing equalization. *Figure 4.2* denote updating proposed method used MMSE equalization

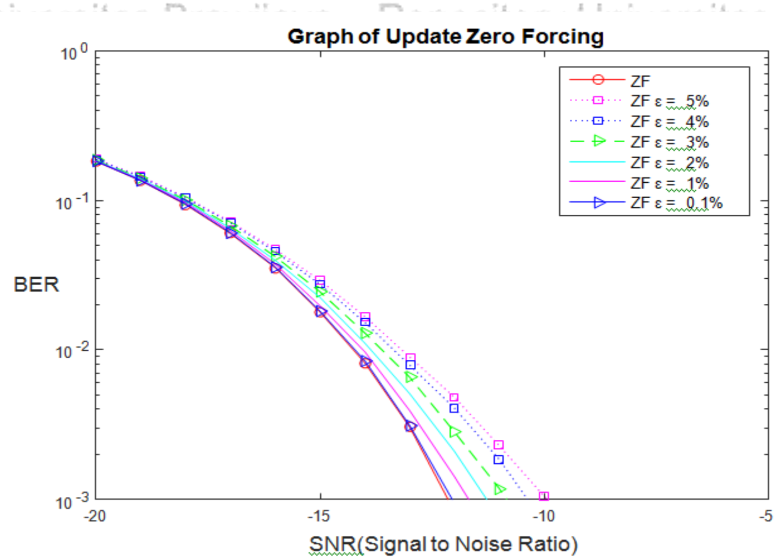

Figure 4.1 BER performance comparison after updating one user in Zero Forcing

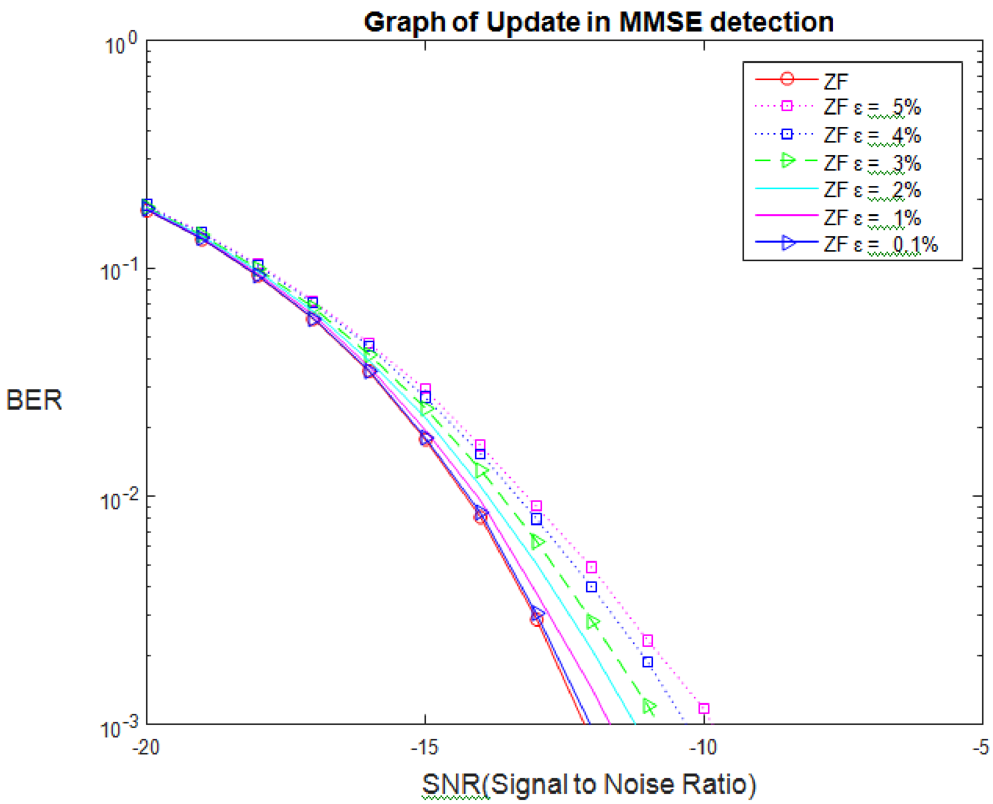

Figure 4.2 BER performance comparison after updating one user in MMSE

In downdating method, proposed algorithm used in two equalization method. *Figure 4.3* show downdating proposed method used in Zero Forcing equalization.

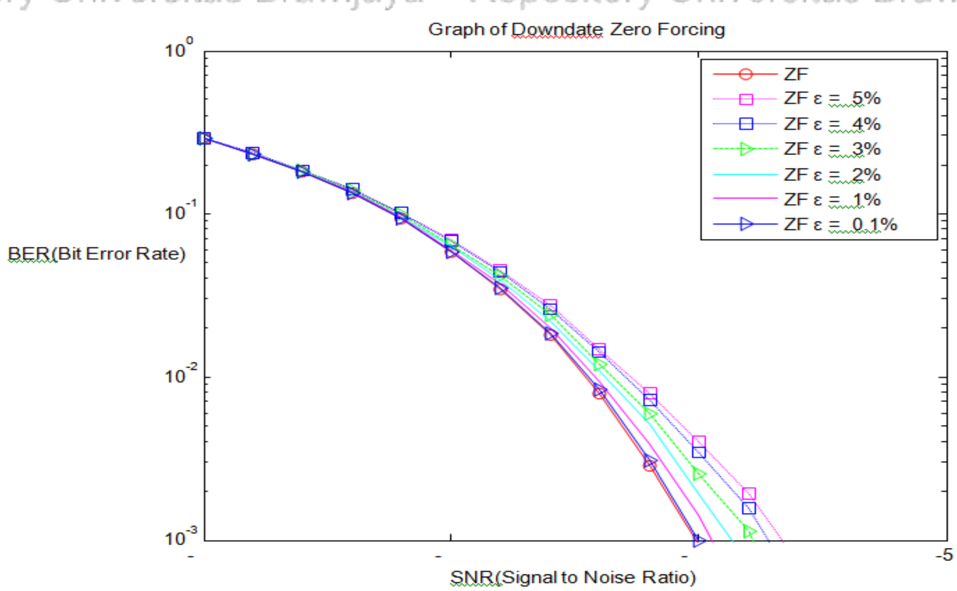

Figure 4.4 Downdating proposed method used MMSE equalization.

Figure 4.3 BER performance comparison after dowandating one user in Zero Forcing

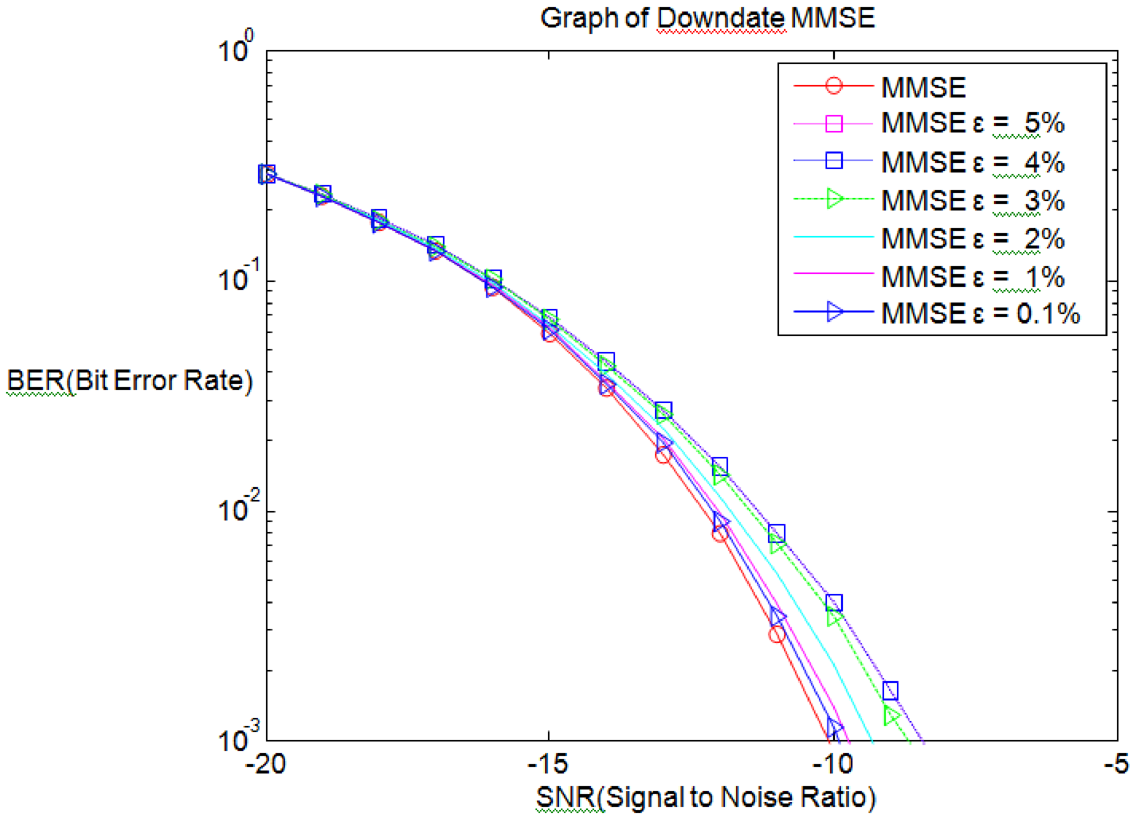

Figure 4.4 BER performance comparison after downdating one user in MMSE

For know error analysis , in next section we provide error analysis for each iteration. For all figure, show BER and SNR with different error compared with exact inverse Zero Forcing or MMSE detection.

4.2 Error Analysis

In this section, we provide Cumulative Distribution Function (CDF) for MMSE method and Zero forcing method for each iteration. From that CDF figure and analysis computational complexity table for each iteration, we can see which iteration has less error and less computational complexity.In all figure, that show $\varepsilon = error$ for each iteration. We can denoted how many iteration to ensure each error. *Figure 4.5* show CDF of Updating method using Given Rotation. In the x axis show number of iteration ,while y axis show how many percent error of BER for each iteration.

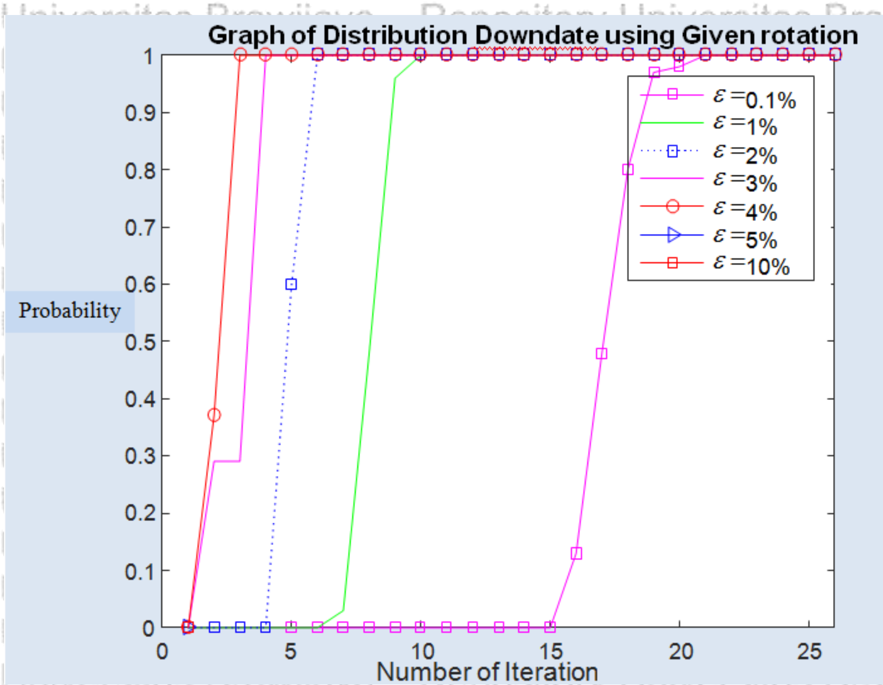

Figure 4.5 CDF of Updating method using Given Rotation

Meanwhile *figure 4.6* show CDF of Updating method without using Given Rotation. This graph show that if this didn't use given rotation, iteration more complex.

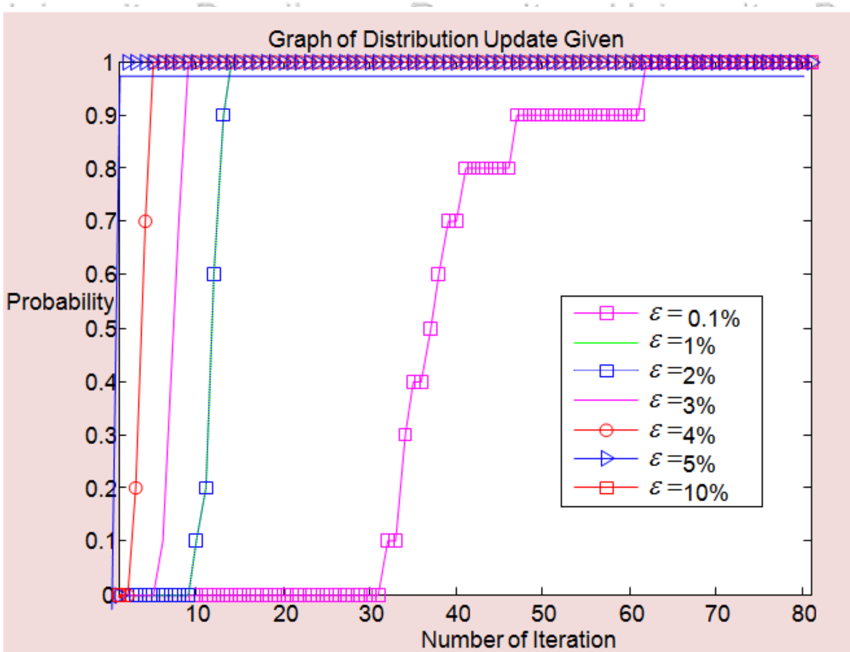

Figure 4.6 CDF of Updating method without using Given Rotation

In *figure 4.7* show CDF of Downdating method using Given Rotation. In other hand, *figure 4.8* This graph show that if this didn't use given rotation, iteration has higher number.

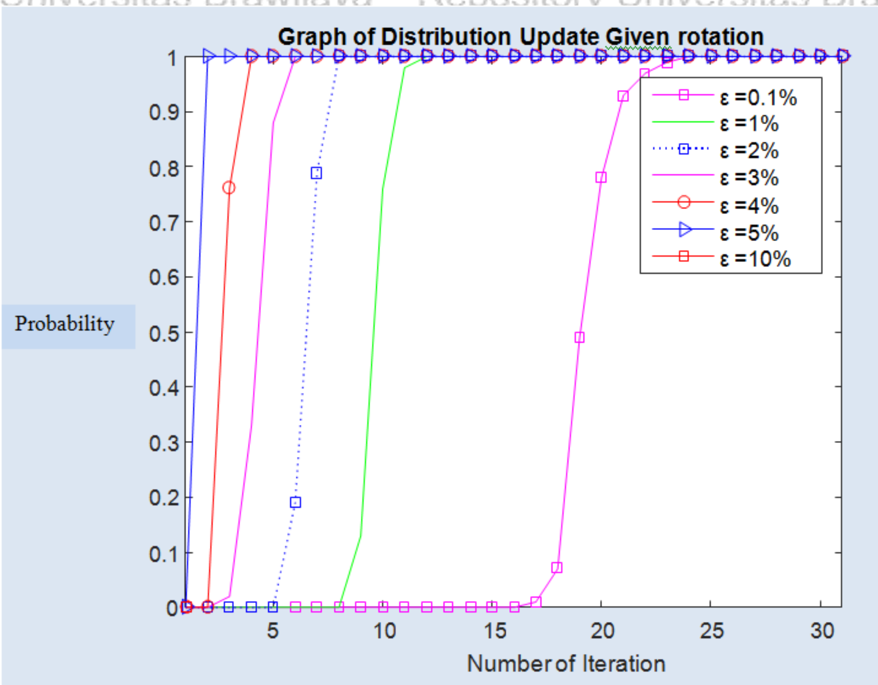

Figure 4.7 CDF of Downdating method using Given Rotation

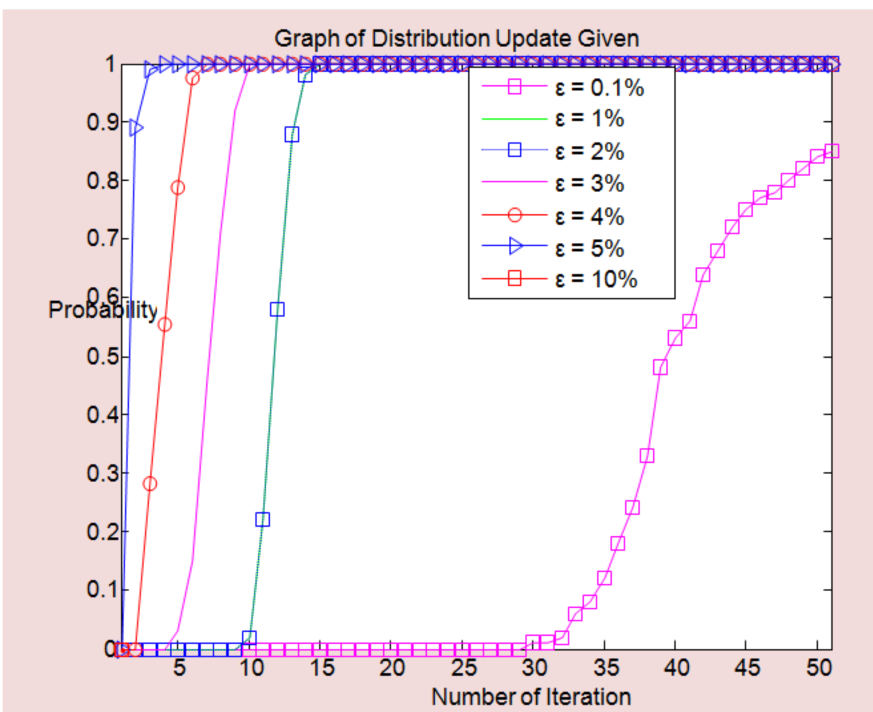

Figure 4.8 CDF of Downdating method without using Given Rotation

4.3 Comparison Result

In this section, we provide BER comparison in MMSE detection with the other research.We consider QPSK modulation scheme.comparison with Neumann Series (NS),Gauss Seidel(GS) and Newton Iteration. Both in Updating and Downdating method outperform other algorithm. Fig 4.9 show that Ber comparison with other method in Updating and Fig 4.10 show that Ber comparison with other method in Downdating.

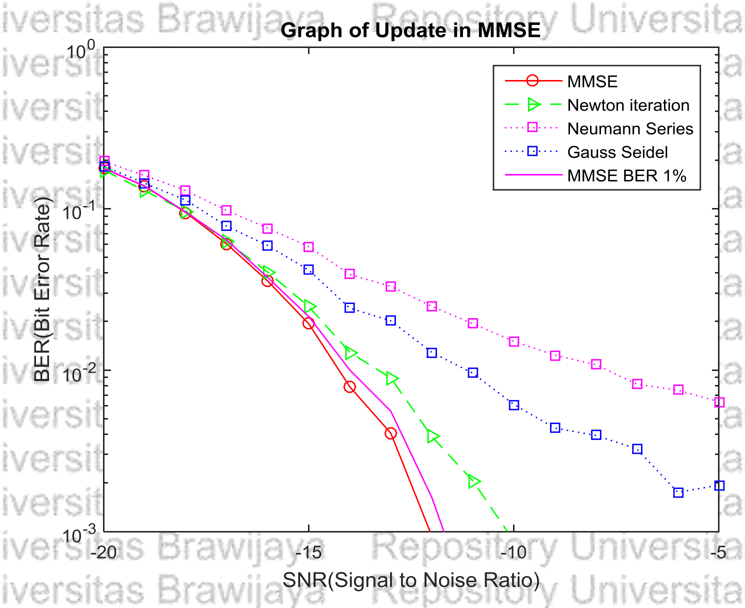

Figure 4.9 BER comparison for Updating with other method

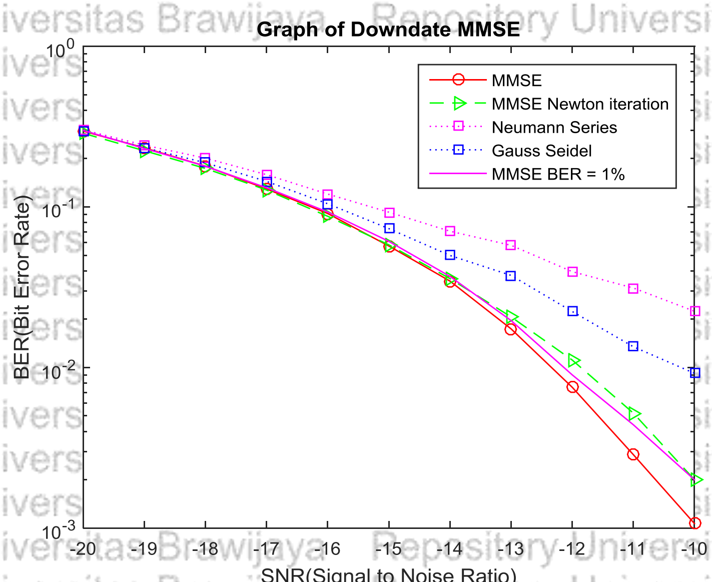

Figure 4.10 BER comparison for Downdatingwith other method

In the *Figure 4.11* show that Downdating when 2,3 or 4 user leaving base station area using Zero Forcing.. In this case show that if more user , so error higher too. Otherwise,*Figure 4.12* show that Updating when 2,3 or 4 user leaving base station area using Zero Forcing.

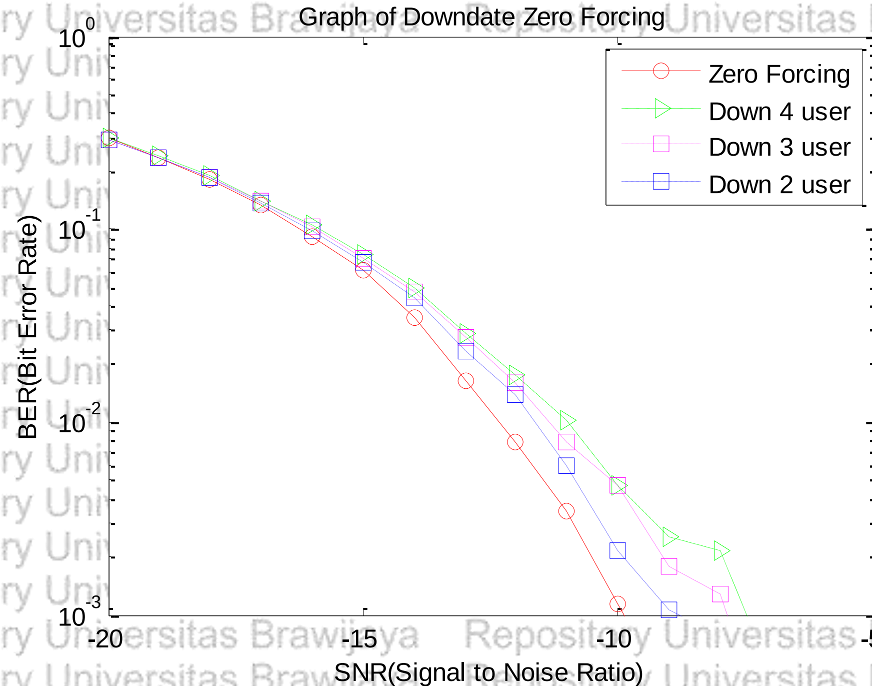

Figure 4.11 BER Downdating when 2,3 or 4 user leaving base station

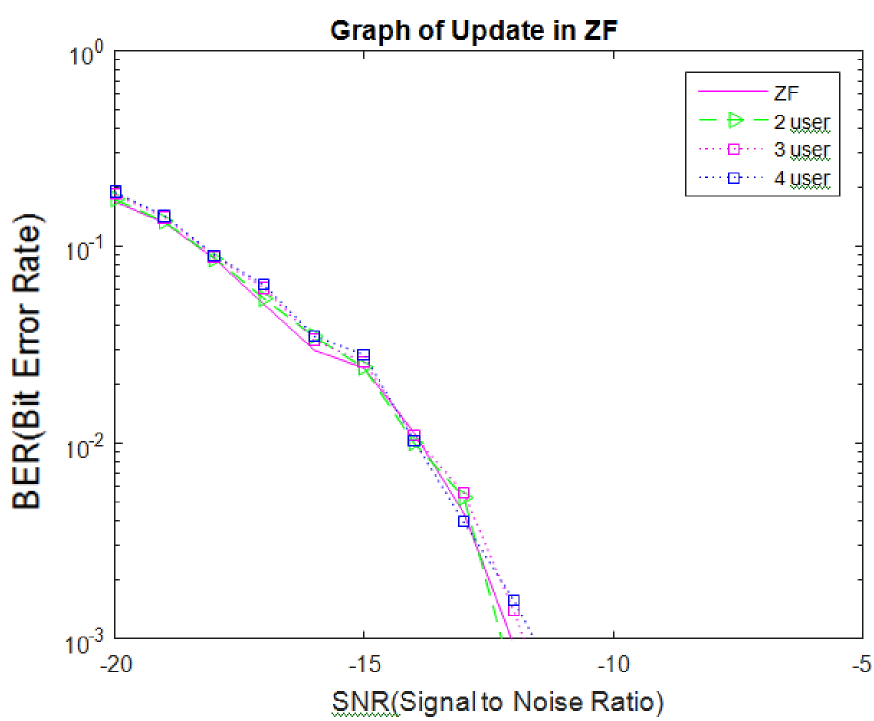

Figure 4.12 BER Updating when 2,3 or 4 user leaving base station

CHAPTER 5
CONCLUSION

This chapter explain conclusion of this research.Result show that this method can used for Zero Forcing and MMSE because result matrix in this rearchhase SVD algorithm. This method have better result than other method when error 1%, showed in BER peroformance in chapter 4.

The complexity of this method less than other method..It s proved in Computational Complexity less than other method as Neumann Series, Gauss Seidel and Newton Iteration which have to multiple whole matrix H.

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