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LONG-RUN INFLATION-GROWTH RELATIONSHIP IN AN OPEN ECONOMY: NOMINAL RIGIDITIES, UNEMPLOYMENT, AND FINANCIAL FRICTIONS

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Introduction

Laguna and Sanso (2020) questioned whether the maximization of the growth rate for a negative inflation of around 2–3% in a closed economy, obtained by Amano et al. (2009) and Amano, Carter and Moran (2012), was valid for any engine of endogenous growth with sticky prices and wages. They proved that a wage setting process defined in terms of wage per hour (or worker) is the key factor for obtaining a negative trend inflation maximizing growth, being this inflation zero if the process takes place on the wage per unit of human capital.

Prior to this article, Laguna (2019) proved, in the context of a closed economy too, that the introduction of unemployment allowed the finding that the maximization of both the labor force participation and employment happened for the inflation rate maximizing growth. However, unemployment was not affected by long-term inflation. Subsequently, he introduced the financial sector in the economy and proved that he had a negative effect on growth, but that the leverage was maximum for the inflation maximizing growth in the human capital model while in the Schumpeterian model was minimum.

In this thesis, we follow the same approach as in Laguna (2019) with two modifications. The first is that we want to see to what extent the results are maintained or not in the context of open economies and what new findings are derived from this extension of the economic perspective. Secondly, applying the result of Laguna and Sanso (2020) we consider directly that the wage setting process is carried out in terms units of effective labor. Thereby no more maximization of growth for negative inflation appears but for zero inflation.

Like in Laguna (2019), the first chapter considers the long-term inflation-growth relationship with equality between supply and demand for labor, the second introduces unemployment and the third the financial sector to end with an empirical application of the latter model, which is the most general of those considered, to six countries. The novelties of the results derived from all these additional developments entail a remarkable wealth, even from the point of view of the public policy.

Chapter 1

Long-run inflation-growth relationship in large open economies with wage rigidity: a perspective from three growth engines

Abstract

The long-run relationship between inflation and growth in New Keynesian models for two large open economies and three alternative engines of growth shows a wide diversity of growth rates because of different monetary policies, intertemporal discount rates and wage stickiness severities.
When there exists wage stickiness, the optimal policy is always null inflation for either country. A greater intertemporal discount rate in a country has a negative effect on its own growth rate and positive or negative on the other country's one depending on the engine of growth.
The severity of the wage stickiness has a spillover effect on the autonomy of the monetary policies named "contagion of non-neutrality": the lesser (greater) the level of wage stickiness in a country, the lesser (greater) the influence of its monetary policy on the own and the other country's growth rate.
Finally, a "price stability premium" arises in the human capital model in that a country with zero or very low inflation or deflation could attain a greater growth rate than in the case of price and global wage flexibility provided the other country has a nonzero inflation rate.

1.1. Introduction

Laguna and Sanso (2020), focusing on the study of the relationship between trend inflation and long-run growth, studied whether the maximization of the growth rate for a negative inflation of around 2–3% in a closed economy[1] is valid for any engine of endogenous growth with sticky prices and wages. They proved that a wage setting process defined in terms of wage per hour (or worker) is the key factor for obtaining a negative trend inflation maximizing growth, being this inflation zero if the process takes place on the wage per unit of human capital.

[1] A result obtained by Amano et al. (2009) and Amano, Carter and Moran (2012).

This first chapter continues this line of research by assuming that the second of the two wage setting approaches is the appropriate in models with production function characterized by Harrod's neutral technological progress. Even in those where the identification of effective employment is not as clear as in human capital model. The use of wage per effective unit of labor and its corresponding demand for labor is a central piece of the steady state system of equations. The conclusion about the optimal trend inflation should be the same in different models with similar technological features and the verification of this property is our first aim.

Our second objective is the extension of the study carried out on the long-run inflation-growth relationship for a closed economy by Laguna and Sanso (2020) to an open economy context characterized by large countries.

The seemingly straightforward task of extending the Ramsey model to an open economy world with exogenous growth, allowing countries to lend and borrow, leads to some counterfactual results. One of them is that the most patient country will own all the wealth —capital and loans— and will consume almost all the world's output (Blanchard and Fischer, 1999; Barro and Sala-i-Martín, 2009). An alternative to generalize the framework avoiding these consequences is to assume, as did Hayashi (1982), a world economy represented by countries with the same intertemporal discount rate —the international interest rate— and the existence of investment adjustment costs. This context allows a clear derivation of the short- and long-term dynamics of an open economy. Unfortunately, as the model did not consider economic growth, not even exogenous, steady state variables, internal and external, are constant magnitudes.

Rebelo (1992) provided a survey on the implications of the earlier endogenous growth models on the explanation of the cross-country diversity in rates of economic growth, showing that these models could only generate differences in growth rates in the absence of international capital markets. In the other case, they implied that the growth rate would be the same all over the world, without allowing ways to differentiate it according to the data. The integration of endogenous growth in DSGE New Keynesian models favored the explanation of a greater diversity in growth rates. In particular, Obstfeld and Rogoff (2002), Benigno and Benigno (2003), Corsetti and Pesenti (2005) and Bergin and Corsetti (2013) find the existence spillover effects implying strategic interactions among countries.

Chu et al. (2015) is a previous study of the relationship between trend inflation and long-run growth in a two-country Schumpeterian DSGE model. The author highlight that it is the first study that analyzes monetary policy in a growth-theoretic framework featuring R&D and innovation in an open economy. They find that increasing domestic inflation reduces domestic R&D investment and the growth rate of domestic technology and, given that economic growth in a country depends on both domestic and foreign technologies, increasing foreign inflation affects the domestic economy with an inflation bias, not present in our paper. These results capture spillover effects, which are novel channels in the outcome of monetary policy across countries implying new strategic interactions among them. Extending the closed-economy model in Chu and Cozzi (2014) to an open economy, Chu et al. (2015) find that by affecting innovation and technologies, inflation has international spillover effects through trade, influencing the outcome of monetary policy competition across countries. Nash equilibrium inflation rates in the two countries are higher than their globally optimal inflation rates, and the degree of this inflationary bias is increasing (decreasing) in the market power of firms under the CIA constraint on R&D (consumption). Using a cross-country panel data to estimate the effects of inflation on R&D, they calibrate moments from their theoretical model to this empirical estimate and other data in the Euro Area and the US. In summary, they find a significant welfare gain from monetary coordination between the two regions.

Chu et al. (2019) also consider monetary policy across countries with North-South product cycles and international technology transfer via foreign direct investment (FDI). Calibrating the model to data in China and the US, they find an asymmetric implication that monetary policy in the US has a significant effect on the welfare of households in China, but not vice versa.

Taking into account these antecedents on open economy models, the basic elements of our approach are three. On the one hand, from Hayashi (1982) we take the homogeneity of a single good in all the economies, with no difference between traded and non-traded goods, and the possibility of investment adjustment costs. On the other hand, we introduce labor-supply decisions and the possibility of wage and price rigidities from the New Keynesian models. Three key references for open economies in this last approach are Smets and Wouters (2003), Gali and Monacelli (2005) and Marcellino and Rychalovska (2012). As the third basic element, we introduce three different endogenous growth engines: a physical capital externality as in Romer (1986), Schumpeterian growth as in Aghion and Howitt (1992) and human capital growth as in Lucas (1988).

However, we dismiss other features of these references in order to enhance the results from the open-economy perspective. The intertemporal discount rate and the international interest rate can take any value and may be different between countries, unlike Hayashi (1982). From the New Keynesian models, we do not adopt the hypothesis of the existence of two types of goods (traded and non-traded), nor the final-good differentiation. In fact, these are fundamentally short-term aspects, and we are interests in the long run ones.

We obtain clear long-term results for the three considered endogenous growth engines in a world of two large countries with the previously indicated characteristics carrying out computing simulations using the Dynare toolbox. We mainly study the inflation-growth relationship. The main results are the same in the three growth engines and only some secondary aspects of them differ. We highlight these differences and their explanation when they appear.

As in Laguna and Sanso (2020) for a closed economy, we find long-term neutrality of the monetary policies with flexibility in prices and wages and with only price stickiness in both countries, as trend inflation and long-run growth rates are independent. However, non-neutrality arises when we introduce wage stickiness. Then each country's long-term growth rate depends first on the two intertemporal discount factors, secondly on each country's own inflation target, thirdly on the other country's inflation target, and finally on the severity of the wage stickiness. In other words, there is no autonomy in monetary policies given the presence of spillover effects.

When the intertemporal discount factors are the same in the two countries, we obtain results with sticky wages specific of the open economy context. The growth rates for every pair of trend inflation may be the same for the two countries (Romer and Aghion-Howitt models) or different (Lucas model) and they always depend not only on each country's inflation target but also on the other country's target. In this context, the maximum growth rates with sticky wages for two of the three types of growth engines (Romer, 1986; Aghion and Howitt, 1992) coincide when trend inflation in the two countries is null. This result comes from considering wages per unit of effective labor, leaving behind the result of Amano et al. (2009) and Amano, Carter and Moran (2012). These maximum growth rates are the same as the obtained for wage flexibility, a result already characterized in Laguna and Sanso (2020) for a closed economy.

However, in the model based on Lucas (1988) the growth rate of a country with null trend inflation could be greater than the corresponding to flexibility in the two countries. This possibility appears if the inflation target of the other country (trend inflation) is different from zero, being greater the increase of the growth rate the greater the distance from zero in either direction. Notwithstanding, the behavior of the average growth rate of the two countries has the same shape of the other two models with its maximum equal to the rate corresponding to price and wage flexibility in the two countries.

In summary, the maximum of the international growth rate with wage stickiness in the three models is only attainable when the two countries choose null trend inflation. These maximum growth rates are the same as the obtained for global wage flexibility. In addition, this choice of monetary policy is the optimal in any strategic situation, cooperative or not.

When the intertemporal discount factors differ, the long-run growth rates are no longer the same in the two countries, being always lower in the country with the highest discount rate. The achievement of the maximum growth rates for the two countries requires the choice of a specific combination of inflation targets.

Moreover, if one country has price and wage flexibility and the other wage stickiness, the growth of the former is independent of its own monetary policy but not of the latter's monetary policy. A flexible economy thus will not always be at the value of the growth rate attainable when the two countries are in this situation. It might suffer from a "contagion of non-neutrality" caused by the wage stickiness of the other country. The effect of this contagion on the growth rate of the country with flexibility is not the same in the three models. While in the models of Romer and Aghion and Howitt itis negative, in the model of Lucas is positive.

A corollary of the contagion of non-neutrality is that it is greater the greater is the level of the wage stickiness. In other words, if the time interval of each wage revision in the country with sticky wages is two periods instead of four the consequences of the contagion on the growth rate of the country with flexibility will not be so negative.

Section 1.2 presents the characterization of the three models used. Section 1.3 describes the long-term relationship between inflation and growth when both economies have the same intertemporal discount rate. Section 1.4 shows the consequences on this relationship when the intertemporal discount rates differ in the two countries. Section 1.5 explains the "contagion of non-neutrality" with different severity of wage stickiness. Finally, Section 1.6 summarizes the main findings.

1.2. Open-economy New Keynesian models with endogenous growth and staggered wage and price setting

This section presents three open-economy New Keynesian models with different growth engines: physical capital externality as in Romer (1986), Schumpeterian growth as in Aghion and Howitt (1992) and human capital growth as in Lucas (1988). The following subsections present the behavior of the economic agents for each of these three models, the source of growth and the elements that characterize the external sector.

1.2.1. Physical capital externality model

Five economic agents are present in the physical capital externality model: households, intermediate good producers, capital producers, final good producers and the central bank. This first model introduces endogenous growth through an externality —stock of knowledge originated from the accumulation of capital— in the production function of the intermediate good producers.

Households

There is a continuum of households over the interval $[0, 1]$. Household members offer labor to the intermediate goods producers in exchange for a wage. They consume the final good, sold by the retailers, and hold an external financial position. They maximize their expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 L_{st}^{1+\nu} ds \right) \tag{1.1.1}$$

where $\beta \in (0,1)$ is the discount factor, C_t is consumption, $\nu > 0$, and L_{st} is the supply of labor service s, with $s \in [0, 1]$.

The household must satisfy the budget constrain

$$C_t + \frac{B_t}{P_t R_t^{st}} = \frac{B_{t-1}}{P_t} + \mathrm{D_t} + \int_0^1 \frac{W_{st}}{P_t} L_{st} K_t ds \tag{1.1.2}$$

where B_t is the nominal value of the external financial position, R_t^{st} is the nominal gross interest rate, D_t represents the firms' real profits, W_{st} is the nominal wage per unit of effective labor service s, and P_t is the final good's price. The NPG condition $\lim_{T \to \infty} E_t(B_t) \geq 0$ avoid the presence of Ponzi-like schemes.

The first-order conditions of household's maximization problem are

$$\frac{\beta^t}{C_t} = \lambda_t \tag{1.1.3}$$

$$\frac{W_{st}}{L_{st}^{\nu} P_t} = C_t \tag{1.1.4}$$

$$\beta E_t \left[\frac{C_t}{C_{t+1}} \frac{R_t^{st}}{\Pi_{\mathrm{t+1}}} \right] = 1 \tag{1.1.5}$$

where λ_t is the Lagrange multiplayer corresponding to the budget constrain and Π_{t+1} is the quotient P_{t+1}/P_t.

Intermediate good producers

Intermediate good producers rent labor from households' members, setting the wage in accordance with Taylor contracts (Taylor, 1980), and acquire capital from capital producers to obtain their output. They operate in a perfectly competitive scenario. Each intermediate good producer $j \in [0,1]$ has a Cobb-Douglas-type production function, following the expression

$$Y_{jt}^{i} = K_{jt}^{\alpha}\left[K_t L_{jt}\right]^{1-\alpha} = K_{jt}^{\alpha}\left[\left(\int_0^1 \left(K_t L_{sjt}\right)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}\right]^{1-\alpha} \qquad (1.1.6)$$

where Y_{jt}^{i} is the output, K_{jt} the capital, $K_t = \int_0^1 K_{jt} dj$ the aggregate capital, L_{sjt} the amount of labor service s used and $L_{jt} = \left(\int_0^1 L_{sjt}^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}$ the aggregate labor services. This production function generates economic growth through an externality caused by the stock of knowledge fueled by the aggregate capital stock K_t (Romer, 1986).

The intermediate good producers aim to maximize their profits

$$F_{jt}^{K} = P_t^{i} K_{jt}^{\alpha} {L_{jt}^{K}}^{1-\alpha} - \int_0^1 W_{st} L_{sjt}^{K} ds - r_t^{Q} Q_t K_{jt} + (Q_{t+1} - \delta) K_{jt} \qquad (1.1.7a)$$

$$L_{jt}^{K} = \left(\int_0^1 \left(L_{sjt}^{K}\right)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}} \qquad (1.1.7b)$$

$$L_{sjt}^{K} = K_t L_{sjt} \qquad (1.1.7c)$$

where r_t^{Q} is the cost of using capital, P_t^{i} the price of the intermediate good, Q_t the price of the capital and δ the capital depreciation rate. The demand function for labor service s that maximizes the intermediate goods producer's profits is

$$L_{sjt}^{K} = \left(\frac{(1-\alpha)Y_{jt}^{i}}{W_{st}/P_t^{i}}\right)^{\sigma} {L_{jt}^{K}}^{1-\sigma} \qquad (1.1.8a)$$

$$L_{sjt} = \frac{1}{K_t}\left(\frac{(1-\alpha)Y_{jt}^{i}}{W_{st}/P_t^{i}}\right)^{\sigma} {L_{jt}^{K}}^{1-\sigma} = \left(\frac{(1-\alpha)Y_{jt}^{i}/K_t}{W_{st}/P_t^{i}}\right)^{\sigma} L_{jt}^{1-\sigma} \qquad (1.1.8b)$$

and the aggregate expression, common to all producers is

$$L_{jt} = \frac{(1-\alpha)\, Y_{jt}^{i}/K_t}{\Delta_{W_t}} \Rightarrow L_t = \frac{(1-\alpha)\, Y_t^{i}/K_t}{\Delta_{W_t}} \qquad (1.1.9a)$$

where Δ_{W_t} is the average real wage per unit of effective work:

$$\Delta_{W_t} = \left[\int_0^1 \left(\frac{W_{st}}{P_t^i}\right)^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}} \qquad (1.1.9b)$$

As the ratio Y_{jt}^i / L_{jt} depends only on market elements, it is common to all the producers. Because the production functions for every intermediate good producer are identical, the same capital-labor ratio implies that the total output of intermediate goods is:

$$Y_t^i = K_t L_t^{1-\alpha} \qquad (1.1.10)$$

Intermediate good producers set equilibrium nominal wages W_{st}^* for each interval of J periods from t (Taylor, 1980) according to, given the absence of unemployment, the household members' preferences

$$\max_{W_{st}} E_t \sum_{\tau=0}^{J-1} \beta^{t+\tau} \left(log C_{t+\tau} - \frac{1}{1+\nu} \int_0^1 L_{st+\tau}^{1+\nu} ds \right) +$$
$$+ \sum_{\tau=0}^{J-1} \lambda_{t+\tau} \left(-C_{t+\tau} - \frac{B_{t+\tau}}{P_{t+\tau} R_{t+\tau}^{st}} + \frac{B_{t+\tau-1}}{P_{t+\tau}} + D_{t+\tau} + \int_0^1 \frac{W_{st+\tau}}{P_{t+\tau}} L_{st+\tau} K_{t+\tau} ds \right) \quad (1.1.11)$$

where L_{st} is the sum, for all j, of the demand for the labor service s.

The optimal nominal wage per unit of effective labor W_{st}^*, which will remain fixed for J periods from t, will be

$$W_{st}^* = \left(\frac{\sigma}{\sigma - 1} \frac{(1-\alpha)^\nu \left(Y_t^i / K_t\right)^\nu}{\Delta_{W_t}^\nu} \frac{\sum_{\tau=0}^{J-1} \beta^\tau \frac{1}{(1/P_{t+\tau})^{\sigma(1+\nu)}}}{\sum_{\tau=0}^{J-1} \frac{\beta^\tau}{C_{t+\tau}} \frac{1}{(1/P_{t+\tau})^\sigma} K_{st+\tau} P_{t+\tau}^{-1}} \right)^{\frac{1}{1+\nu\sigma}}$$

which in the steady state, for $\mathrm{T} = [0, \dots, J-1]$:

$$\frac{W_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{-\mathrm{T}}} \left(\frac{\sigma}{\sigma - 1} \frac{(1-\alpha)^\nu \left(Y^{i^K}\right)^\nu C^K}{\Delta_W^\nu} \frac{\sum_{\tau=0}^{J-1} \beta^\tau \Pi^{\sigma(1+\nu)\tau}}{\sum_{\tau=0}^{J-1} \beta^\tau \Pi^{(\sigma-1)\tau}} \right)^{\frac{1}{1+\nu\sigma}} \qquad (1.1.12)$$

where $Y^{iK} = \frac{Y^i}{K}$ and $C^K = \frac{C}{K}$.

In addition, from the profit maximization problem, as capital producers are competitive, the following expression for the cost of capital utilization can be obtained:

$$r_t^q = \frac{\alpha P_t^i \frac{Y_{jt}^i}{K_{jt}} + Q_{t+1} - \delta}{Q_t} \tag{1.1.13}$$

Lastly, given the absence of financial frictions, we have

$$r_t^q = R_t \tag{1.1.14}$$

where R_t is the gross real interest rate.

Capital producers

Capital producers behave following an investment function with adjustment costs and sell their output to intermediate good firms, as in Gertler and Karadi (2011). The main relationships in the capital accumulation process are

$$K_{t+1} = K_t + I_t^n \tag{1.1.15}$$

$$I_t = I_t^n \left[1 + f\left(\frac{I_t^n}{K_t}\right)\right] + \delta K_t \tag{1.1.16}$$

$$g_t = \frac{K_{t+1}}{K_t} = 1 + \frac{I_t^n}{K_t} \tag{1.1.17}$$

where I_t and I_t^n are gross and net investment and $f(I_t^n/K_t)$ is the adjustment cost function.

The maximization of the present net value of the investment flow

$$\max_{I_t^n} E_0 \sum_{t=0}^{\infty} \beta^t \left[Q_t I_t^n - \left(I_t^n + f\left(\frac{I_t^n}{K_t}\right) I_t^n\right)\right] \tag{1.1.18}$$

provides the price Q_t of the capital:

$$Q_t = 1 + f\left(\frac{I_t^n}{K_t}\right) + \frac{I_t^n}{K_t} f'\left(\frac{I_t^n}{K_t}\right) \tag{1.1.19}$$

We assume the function

$$f\left(\frac{I_t^n}{K_t}\right) = \frac{\zeta}{2}\left(\frac{I_t^n}{K_t} - \frac{I^n}{K}\right)^2 \tag{1.1.20}$$

where $\zeta > 0$, I^n/K is the investment-capital ratio in the steady state and $f(I^N/K) = f'(I^n/K) = 0$. Consequently, $Q = 1$ in the steady state.

Final good producers

Final good producers, or retailers, acquire and differentiate intermediate goods to sell them to the households. The final output is the aggregate of a continuum of retail final goods

$$Y_t = \left(\int_0^1 Y_{rt}^{\frac{\varepsilon-1}{\varepsilon}} dr\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.1.21}$$

and if households minimize costs, the demand for each final good r is

$$Y_{rt} = \left(\frac{P_{rt}}{P_t}\right)^{-\varepsilon} Y_t \tag{1.1.22a}$$

$$P_t = \left(\int_0^1 P_{rt}^{1-\varepsilon} dr\right)^{\frac{1}{1-\varepsilon}} \tag{1.1.22b}$$

where Y_{rt} is the output of retailer $r \in [0,1]$, P_{rt} is the price of variety r and P_t is the price index of the final output.

Retailers follow Taylor contracts, in which they set for each interval of I periods from t the price P_t^* that maximizes their profits:

$$\max_{P_t^*} \sum_{\tau=0}^{I-1} E_t\left[\frac{\lambda_{t+\tau}}{\lambda_t} Y_{rt}\left(\frac{P_t^*}{P_{t+\tau}} - P_{t+\tau}^i\right)\right] \tag{1.1.23}$$

Solving this problem, the optimal price is

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1}\frac{E_t \sum_{\tau=0}^{I-1} \beta^\tau P_{t+\tau}^{\varepsilon} Y_{t+\tau} C_{t+\tau}^{-1} P_{t+\tau}^i}{E_t \sum_{\tau=0}^{I-1} \beta^\tau P_{t+\tau}^{\varepsilon-1} Y_{t+\tau} C_{t+\tau}^{-1}} \tag{1.1.24}$$

and, in the steady state, the relative price, for $\mathrm{T} = [0, \dots, I-1]$, will be:

$$\frac{P_{-\mathrm{T}}^{*}}{P} = \frac{1}{\Pi^{\mathrm{T}}} \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon})^{\tau} P^{i}}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^{\tau}} \qquad (1.1.24')$$

From expression $(1.1.22a)$, assuming a simplified technology that converts one unit of intermediate good into another unit of final good, the total output weighted by the price dispersion is the intermediate good producers' output

$$Y_t^i = \Delta_{P_t} Y_t \qquad (1.1.25a)$$

where Y_t is the total output of the economy, Y_t^i the total output of the intermediate good producers, and Δ_{P_t} the price dispersion:

$$\Delta_{P_t} = \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{t-\tau}^{*}}{P_t} \right)^{-\varepsilon} \qquad (1.1.25b)$$

Central bank

We maintain the assumption that there is no money, following the "cashless economy" hypothesis [Woodford (2003), Galí (2008)] typically adopted in New Keynesian macroeconomic models, agents have deposits in banks, and the banks in the central bank. The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate, following the Taylor rule

$$R_t^{st} = R \Pi_t \left(\frac{\Pi_\mathrm{t}}{\Pi} \right)^{\phi_\Pi} \qquad (1.1.26)$$

where R is the steady state real interest rate, Π the steady state gross inflation target, and ϕ_Π measures the central bank's reaction to deviations of inflation from the target.

Lastly, the relationship between real and nominal interest rates follows the Fisher equation:

$$R_t^{st} = R_t E_t \Pi_{t+1} \qquad (1.1.27)$$

Equilibrium conditions and external sector

The aggregate equilibrium of the economy is defined as:

$$Y_t = C_t + I_t + X_t - X_t^\dagger \qquad (1.1.28a)$$

where, assuming two economies, henceforth "Nation" (N) and "Rest of the World" (ROW) —denoted with a dagger symbol [†]—, X_t are exports and $X_t^\dagger$ are imports. For the sake of simplicity, no public sector will be considered. Consequently, ROW's equation will be:

$$Y_t^\dagger = C_t^\dagger + I_t^\dagger + X_t^\dagger - X_t \qquad (1.1.28b)$$

N's and ROW's exports depend on the real exchange rate e_t and the other economy's production, following the expressions $K^\dagger/K$

$$X_t = \rho K_t^\dagger + \Omega e_t^\omega Y_t^\dagger \qquad (1.1.29a)$$

$$X_t^\dagger = \rho^\dagger K_t + \Omega^\dagger \left(\frac{1}{e_t}\right)^{\omega^\dagger} Y_t \qquad (1.1.29b)$$

where $\rho > 0$, $\rho^\dagger > 0$, $\Omega > 0$, $\Omega^\dagger > 0$, $\omega > 0$ and $\omega^\dagger > 0$.

After normalizing the equation through K, as all the growing variables must be normalized in the steady state, and considering the relationship between the productions of both economies $l_t = K_t^\dagger/K_t$ in order to link the normalization of the two countries, we get

$$Y^K = C^K + I^K + X^K - lX^{\mathrm{K}\dagger} \qquad (1.1.28a')$$

$$X^K = (\rho + \Omega e^\omega Y^{\mathrm{K}\dagger})l \qquad (1.1.29a')$$

$$Y^{\mathrm{K}\dagger} = C^{\mathrm{K}\dagger} + I^{\mathrm{K}\dagger} + X^{\mathrm{K}\dagger} - \frac{1}{l}X^K \qquad (1.1.28b')$$

$$X^{\mathrm{K}\dagger} = \left[\rho^\dagger + \Omega^\dagger \left(\frac{1}{e}\right)^{\omega^\dagger} Y^K\right]\frac{1}{l} \qquad (1.1.29b')$$

in the steady state.

N's and ROW's real external financial positions at time t, b_t and $b_t^\dagger$ respectively, are defined as the position at time $t-1$ plus interest payments minus net exports at time t, analytically we can define

$$b_t = R_t^i b_{t-1} - X_t + e_t X_t^\dagger \tag{1.1.30a}$$

$$b_t^\dagger = R_t^i b_{t-1}^\dagger - X_t^\dagger + \frac{1}{e_t} X_t \tag{1.1.30b}$$

where R_t^i is the appropriate gross real interest rate. If N lends to ROW, the interest rate will be N's interest rate, and vice versa:

$$R_t^i = \begin{cases} R_t, & b_t > 0 \\ R_t^\dagger, & b_t < 0 \end{cases} \tag{1.1.31}$$

Normalizing considering the lagged variables and noticing the other economy's variables, in the steady state we get the expressions:

$$b = R^i \frac{b_{-1}}{g} - X + elX^\dagger \tag{1.1.30a'}$$

$$b^\dagger = R^i \frac{b_{-1}^\dagger}{g^\dagger} - \mathrm{X}^\dagger + \frac{1}{el} X \tag{1.1.30b'}$$

The relationship between N's and ROW's external financial position at time t is

$$b_t^\dagger - \left(\frac{b_{t-1}^\dagger}{\Pi_\mathrm{t}^\dagger}\right) = -\left[\left(\frac{b_t}{e_t} - \frac{b_{t-1}}{\Pi_t^\dagger e_{t-1}}\right)\right] \tag{1.1.32}$$

which normalized in the steady state takes the form:

$$b^\dagger - \left(\frac{b_{-1}^\dagger}{\Pi^\dagger g^\dagger}\right) = -\left[\left(\frac{b_t}{e_t l} - \frac{b_{t-1}}{\Pi_t^\dagger e_{-1} lg}\right)\right] \tag{1.1.32'}$$

Lastly, we assume the condition of uncovered interest parity:

$$(R_t - 1) = \left(R_t^\dagger - 1\right) + \frac{e_{t+1} - e_t}{e_t} \tag{1.1.33}$$

Steady state

The system of equations required to determine the steady state values of the endogenous variables are presented in Appendix 1.1. The number of each equation of the system corresponds to a previous one, that number appearing with the superscript [$'$], because they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through the capital stock, each economy by its own. Also, as explained, it is necessary to define the ratio for this variable $l = K^{\dagger}/K$ to link the normalizations of the two economies. Normalized variables are denoted with superscript K, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. N's equations described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have a system of equations composed of forty-five equations and forty-five endogenous variables.

1.2.2. Schumpeterian model

The model presented in this section is a modification of the standard New Keynesian model with price and wages rigidities including Schumpeterian endogenous growth following Aghion and Howitt (1992). This model considers four types of agents: households, intermediate good producers, final goods producers, and the central bank.

Households

Household's members work, consume goods produced by the final good producers, buy bonds, and receive interest payments from public debt. This agent is composed of infinite horizon individuals uniformly distributed in a continuum $[0,1]$. Each member offers at t an amount L_{st} of differentiated labor service s. L_t is the aggregate labor. Household's members obtain utility from consumption and disutility from labor supply, as we have seen in the previous model:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 L_{st}^{1+\nu} ds \right) \tag{1.2.1}$$

The households must satisfy the budget constrain

$$C_t + \frac{B_t}{P_t R_t^{st}} + R\&D_t = \frac{B_{t-1}}{P_t} + \mathrm{D_t} + \int_0^1 \frac{W_{st}}{P_t} L_{st} A_t ds \qquad (1.2.2)$$

which now includes $R\&D_t$, the investment in research and development, and the productivity A_t explained below because W is the wage per unit of effective labor. The NPG condition $\lim_{T\to\infty} E_t(B_t) \geq 0$ is also considered.

Final good producers

Final goods producers buy intermediate goods and rent labor force to produce their goods, setting wages according to Taylor contracts. They operate in a perfectly competitive scenario. Following Aghion and Howitt (1992), the final goods production function is

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di \qquad (1.2.3)$$

where A_{it} is the productivity, x_{it} is the intermediate good i used at t, and $0 < \alpha < 1$. L_t is the same variable used in the model of Romer.

The solution for the maximization problem for the profits function:

$$F_{Yt} = P_t \int_0^1 (A_{it} L_t)^{1-\alpha} x_{it}^{\alpha} di - \int_0^1 W_{st} \left(\int_0^1 A_{it} di \right) L_{st} ds - \int_0^1 P_{it} x_{it} di \qquad (1.2.4a)$$

$$F_{Y_t}^{A} = P_t \int_0^1 {L_t^{A_i}}^{1-\alpha} x_{it}^{\alpha} di - \int_0^1 W_{st} L_{st} A_t ds - \int_0^1 P_{it} x_{it} di \qquad (1.2.4b)$$

$$L_t^{A_i} = \left(\int_0^1 (L_{st} A_{it})^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \qquad (1.2.4c)$$

$$A_t = \int_0^1 A_{it} di \qquad (1.2.4d)$$

provides both the intermediate goods and aggregate labor demand functions

$$x_{it} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{P_{it}}{P_t} \right)^{-\frac{1}{1-\alpha}} L_t^{A_i} \qquad (1.2.5)$$

$$L_t = \frac{(1-\alpha) Y_t / A_t}{\Delta_{W_t}} \qquad (1.2.6)$$

$$\Delta_{W_t} = \left[\int_0^1 \left(\frac{W_{st}}{P_t}\right)^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}} \tag{1.2.7}$$

where Δ_{W_t} represents the average real wage per unit of effective labor.

Final goods producers set a wage W_{st}^* at t for J periods according to the households' members' preferences. As in the previous model, we obtain an expression for the optimal W_{st}^* by maximizing the utility function subject to the budget constraints:

$$W_{st}^* = \left(\frac{\sigma}{\sigma-1}\frac{\sum_{\tau=0}^{J-1}\beta^t L_{st+\tau}^{1+\nu}}{\sum_{\tau=0}^{J-1}\lambda_{t+\tau}\frac{L_{st+\tau}A_{t+\tau}}{P_{t+\tau}}}\right)^{\frac{1}{1+\nu\sigma}} \tag{1.2.8}$$

Evaluated in the steady state, the expression of the real wage per unit of effective labor is, for $\mathrm{T} = [0, \dots, J-1]$:

$$\frac{W_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{-\mathrm{T}}}\left(\frac{\sigma}{\sigma-1}\frac{C^Y(1-\alpha)^\nu}{A^{Y^{1+\nu}}\Delta_W^{(1-\sigma)\nu}}\frac{\sum_{\tau=0}^{J-1}\beta^\tau \Pi^{\sigma(1+\nu)\tau}}{\sum_{\tau=0}^{J-1}\beta^\tau \Pi^{(\sigma-1)\tau}}\right)^{\frac{1}{1+\nu\sigma}} \tag{1.2.8'}$$

where $A = \frac{A}{Y}$ and $C^Y = \frac{C}{Y}$.

Additionally, aggregating and introducing (1.2.5) in (1.2.3):

$$A_t = \frac{Y_t}{\left(\alpha^{\frac{1}{1-\alpha}}\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{t-s}^*}{P_t}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha} L_t} \tag{1.2.9}$$

Intermediate good producers

Monopolistically competitive intermediate good producers sell their goods to the final goods producers and set the prices according to Taylor contracts each I period. The profits function for each intermediate good producer is

$$F_{it} = \left(\frac{P_{it}}{P_t} - 1\right)x_{it} \tag{1.2.10a}$$

where P_{it} is the price of the intermediate good i at t and the unit cost is 1. The intermediate producers obtain one unit of intermediate good from one unit of final good.

Following Aghion and Howitt (1992), considering the intermediate goods demand function $(1.2.5)$, the profits function is:

$$F_{it} = \left(\frac{P_{it}}{P_t} - 1\right) \alpha^{\frac{1}{1-\alpha}} \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{1-\alpha}} A_{it} L_t \qquad (1.2.10b)$$

Introducing price rigidities for I periods, the average expected profits are:

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} - 1\right) \left(\frac{P_{t-s}^*}{P_t}\right)^{-\frac{1}{1-\alpha}} \qquad (1.2.11)$$

Assuming the dismissing returns probability function $\Phi(\mathrm{n_{it}}) = n_{it}^{\chi}$ where $0 < \chi < 1$, $n_{it} = R_{it}/A_{it}^*$, R_{it} is the amount of final goods destined to the innovation process and A_{it}^* the intermediate goods productivity if innovation is successful, and with $\Phi'(n_{it}) = \chi n_{it}^{\chi-1} > 0$ and $\Phi''(n_{it}) = \chi(\chi - 1)\mathrm{n_{it}^{\chi-2}} < 0$, if the innovation is successful, expected profit of the research and development process that could generate innovation will be:

$$\Phi\left(\frac{R_{it}}{A_{it}^*}\right) VF_{it}^* - R_{it} \qquad (1.2.12)$$

As a result, the resources R_{it} must meet the condition

$$\Phi'\left(\frac{R_{it}}{A_{it}^*}\right) \frac{VF_{it}^*}{A_{it}} - 1 = 0 \qquad (1.2.13)$$

and considering $(1.2.11)$, an expression for the optimal value of n_{it} can be obtained that will be common to all the intermediate good firms since it depends only on market elements:

$$n = n_{it} = \left[\chi \alpha^{\frac{1}{1-\alpha}} \mathrm{L_t} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} - 1\right) \left(\frac{P_{t-s}^*}{P_t}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}} \qquad (1.2.14)$$

Considering the law of large numbers, the proportion of successful innovators will be $\mu = \Phi(n)$. The technological level will be

$$A_t = \mu\gamma A_{t-1} + (1-\mu)A_{t-1} \tag{1.2.15}$$

which, considering $g_t = A_t/A_{t-1} = Y_t/Y_{t-1}$, can be rewritten as:

$$g = \mu(\gamma - 1) + 1 \tag{1.2.16}$$

Considering $\mu = \Phi(n) = n^{\chi}$, the gross growth rate in the steady state is:

$$g = \left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^{*}}{P} - 1\right)\left(\frac{P_{-\tau}^{*}}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}}(\gamma - 1) + 1 \tag{1.2.16'}$$

Finally, the optimal price that maximizes the expected profits function (1.2.11) is:

$$P_t^{*} = \frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\lambda_{t+\tau}\lambda_t^{-1}x_{it+\tau}}{\sum_{\tau=0}^{I-1}\lambda_{t+\tau}(\lambda_t P_{t+\tau})^{-1}x_{it+\tau}} \tag{1.2.17}$$

The optimal relative price P^{*}/P evaluated in the steady state is, for $\mathrm{T} = [0, \dots, I-1]$:

$$\frac{P_{-\mathrm{T}}^{*}}{P} = \frac{1}{\Pi^{\mathrm{T}}}\frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{1}{1-\alpha}}\right)^{\tau}}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{\alpha}{1-\alpha}}\right)^{\tau}} \tag{1.2.17'}$$

Central bank

The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate. Trend inflation is considered as given.

Equilibrium conditions and external sector

The aggregate equilibrium of the economy will be defined as

$$Y_t = C_t + R\&D_t + \int_{i=0}^{1} x_{it}di + X_t - X_t^{\dagger} \tag{1.2.18a}$$

where, assuming two economies, henceforth "Nation" (N) and "Rest of the World" (ROW) —denoted with a dagger symbol [†]—, X_t are exports and $X_t^\dagger$ are imports. For the sake of simplicity, no public sector will be considered. Consequently, ROW's equation will be:

$$Y_t^\dagger = C_t^\dagger + R\&D_t^\dagger + \int_{i=0}^{1} x_{it}^\dagger di + X_t^\dagger - X_t \tag{1.2.18b}$$

N's and ROW's exports depend on the real exchange rate e_t and the other economy's production, following the expressions

$$X_t = \rho + \Omega e_t^\omega Y_t^\dagger \tag{1.2.19a}$$

$$X_t^\dagger = \rho^\dagger + \Omega^\dagger \left(\frac{1}{e_t}\right)^{\omega^\dagger} Y_t \tag{1.2.19b}$$

where $\rho > 0$, $\rho^\dagger > 0$, $\Omega > 0$, $\Omega^\dagger > 0$, $\omega > 0$ and $\omega^\dagger > 0$.

From equations $(1.2.18)$ we can obtain an expression for consumption considering $(1.2.5)$, and $R\&D_t = A_t n_{it}$. After normalizing the equation through Y, as all the growing variables must be normalized in the steady state, and considering the relationship between the productions of both economies $l_t = Y_t^\dagger / Y_t$ in order to link the normalization of the two countries, we get for N in the steady state

$$C = 1 - \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{-\tau}^*}{P} - 1\right) \left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}} A - \\ - \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}} A - X + l X^\dagger \tag{1.2.18a'}$$

$$X = (\rho + \Omega e^\omega) l \tag{1.2.19a'}$$

and for ROW

$$C^{\dagger} = 1 - \left[\chi^{\dagger} \alpha^{\dagger \frac{1}{1-\alpha^{\dagger}}} L^{\dagger} \frac{1}{I^{\dagger}} \sum_{\tau=0}^{I^{\dagger}-1} \left(\frac{P_{-\tau}^{\dagger *}}{P^{\dagger}} - 1 \right) \left(\frac{P_{-\tau}^{\dagger}}{P^{\dagger}} \right)^{-\frac{1}{1-\alpha^{\dagger}}} \right]^{\frac{1}{1-\chi^{\dagger}}} A^{\dagger} -$$

$$-\alpha^{\dagger \frac{1}{1-\alpha^{\dagger}}} L^{\dagger} \frac{1}{I^{\dagger}} \sum_{\tau=0}^{I^{\dagger}-1} \left(\frac{P_{-\tau}^{\dagger *}}{P^{\dagger}} \right)^{-\frac{1}{1-\alpha^{\dagger}}} A^{\dagger} - X^{\dagger} + \frac{1}{l} X \qquad (1.2.18b')$$

$$X^{\dagger} = \left[\rho^{\dagger} + \Omega^{\dagger} \left(\frac{1}{e} \right)^{\omega^{\dagger}} \right] \frac{1}{l} \qquad (1.2.19b')$$

Expressions $(1.1.31a)$ and $(1.1.31b)$ for the economies' real external financial positions, $(1.1.32)$ for the appropriate gross real interest rate, $(1.1.33)$ for the relationship between economies' external financial positions and $(1.1.34)$ for the uncovered interest parity are also part of the model.

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 1.2. The number of each equation of the system corresponds to a previous one, that number appearing with the superscript [$'$], because they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through production level of the final good Y, each economy by its own. In addition, as explained, it is necessary to define the ratio for this variable $l = Y^{\dagger}/Y$ to link the normalizations of the two economies. Normalized variables are denoted with superscript Y, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. N's equations described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have thirty-one equations and thirty-one endogenous variables.

1.2.3. Human capital model

The model presented in this section introduces the accumulation of human capital as the source of endogenous growth, according to Lucas (1988). The idea behind this model is that individuals divide their time between work —therefore generating income today and consuming— and training —giving up part of work income but raising future wages as their productivity increases—. Accumulation of human capital generates raises in productivity of both labor and physical capital. As in the previous two models, this model includes price and wage rigidities based on Taylor staggered mechanism. The model considers four types of agents: households, intermediate good producers, final good producers or retailers and the central bank.

Households

Household's members offer labor to intermediate good producers, consume the final good, accumulate human capital and hold bonds. Households are composed of infinite horizon individuals and are uniformly distributed in a continuum $[0,1]$. Their discounted expected utility is the same as in the previous models:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+\nu} \int_0^1 L_{st}^{1+\nu} ds \right) \tag{1.3.1}$$

but the budget constraint now includes the dynamics associated to the stock of physical capital K owned by the households:

$$C_t + \frac{B_t}{P_t} + K_{t+\tau+1} = \frac{B_{t-1}}{P_t} R_t^{st} + \mathrm{D}_t + \int_0^1 \frac{W_{st}}{P_t} L_{st}\, ds + (1 + R_{t+\tau} - \delta) K_{t+\tau} \tag{1.3.2}$$

Households keep a stock of physical capital and rents it to the intermediate good producers. Assuming no investment adjustment costs, following Christiano, Eichenbaum and Evans (2005), the law of motion of physical capital is

$$K_{t+1} = (1-\delta) K_t + I_t \tag{1.3.3}$$

with δ the depreciation rate and I_t the gross investment.

Households' members choose the total time devoted to non-leisure activities —including both working and human capital accumulation— N_{st} and the fraction of time unit devoted to the production activity $u_{st} \in [0,1]$, which implies that $(1 - u_{st})$ is the fraction devoted to human capital accumulation. Considering that h_{st} is the stock of human capital, the effective labor demand for service s will be defined as:

$$L_{st} = u_{st} N_{st} h_{st} \tag{1.3.4}$$

Assuming ξ as a productivity parameter, the accumulation process of human capital follows:

$$h_{st+1} = [1 + \xi(1 - u_{st}) N_{st}] h_{st} \tag{1.3.5}$$

From the optimal control problem developed in Appendix 1.3b, we obtain a constant value over time, common to all services s, for labor supply N. Considering wage flexibility, this value will be

$$N = \frac{1}{\xi}\left(1 - \frac{\beta}{1 + g}\right) \tag{1.3.6}$$

in which g is the growth rate in the steady state. The growth rate is also derived from the solution of this dynamic optimization problem. Final output, intermediate goods production, physical capital stock and effective labor will all grow at the same rate in the steady state, the growth rate of the human capital accumulation process

$$g = \frac{\beta}{1 + \delta - R(1 - X^{\dagger})} - 1 \tag{1.3.7}$$

in which R is the real interest rate and $X^{\dagger}$ are the exports of ROW.

In the case of wage rigidities two values for N are obtained, one representing labor services that change wages N^0, and another for those that do not N^1. These values are

$$N^0 = \frac{1}{\xi}\left(1 - \frac{\beta\Pi^{J-1}}{1+g}\right) \qquad (1.3.8a)$$

$$N^1 = \frac{1}{\xi}\left(1 - \frac{\beta\Pi^{-1}}{1+g}\right) \qquad (1.3.8b)$$

while the growth rate will still be the expression (1.3.7).

Intermediate good producers

Intermediate good producers operate in a perfectly competitive scenario and are uniformly distributed in a continuum, indexed by $j \in [0,1]$. They have a Cobb-Douglas-type production function defined as

$$Y_{jt}^{i} = AK_{jt}^{\alpha}L_{jt}^{1-\alpha} \qquad (1.3.9)$$

where Y_{jt}^{i} is the homogeneous output of an intermediate good producer, A is the total factor productivity, K_{jt} the stock of physical capital and L_{jt} a composite index of all labor services as in the previous models.

From profit maximization, we obtain the labor demand of service s for the producer j

$$L_{sjt} = \left[(1-\alpha)A{K_{jt}}^{\alpha}\right]^{\sigma}\left(\frac{W_{st}}{P_t^i}\right)^{-\sigma} {L_{jt}}^{1-\sigma\alpha} \qquad (1.3.10)$$

which aggregating all labor services, and considering homogeneous producers, can be rewritten as the aggregate expression

$$L_t = \left[\left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{(1-\alpha)A}{\Delta_{W_t}}\right]^{\frac{1}{\alpha}} K_t \qquad (1.3.11)$$

where Δ_{W_t} represents again the average real wage:

$$\Delta_{Wt} = \left[\int_0^1 \left(\frac{W_{st}}{P_t}\right)^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}} \qquad (1.3.12)$$

Unlike the Schumpeterian model, intermediate good producers are the economic agents that set wages for J periods in the models with human capital, according to households' preferences, given the absence of unemployment. In a similar process explained in the corresponding section of the physical capital externality model, an expression for the optimal wage can be obtained:

$$W_{st}^{*} = \left(\frac{\sigma}{\sigma - 1} \frac{\sum_{\tau=0}^{J-1} \beta^{t} L_{st+\tau}^{1+\nu}}{\sum_{\tau=0}^{J-1} \lambda_{t+\tau} \frac{L_{st+\tau}}{P_{t+\tau}}} \right)^{\frac{1}{1+\nu\sigma}} \tag{1.3.13}$$

Evaluated in the steady state, the expression of the real wage is, for $\mathrm{T} = [0, \dots, J-1]$:

$$\frac{W_{-\mathrm{T}}^{*}}{P} = \frac{1}{\Pi^{-\mathrm{T}}} \left[\left(\frac{\sigma}{\sigma - 1} \right) \left(\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{\Delta_{W}{}^{1-\alpha\sigma}}{(1-\alpha)A} \right)^{\frac{1}{\alpha}} C^{K} \frac{\sum_{\tau=0}^{J-1} \beta^{\tau} N_{\tau}^{1+\nu}}{\sum_{\tau=0}^{J-1} \beta^{\tau} \Pi^{(\sigma-1)\tau}} \right]^{\frac{1}{1-\sigma}} \tag{1.3.13'}$$

Also, from profit maximization, an expression for the real interest rate can be obtained:

$$R_{t} = \alpha \left[A \left(\frac{\varepsilon - 1}{\varepsilon} \right) \right]^{\frac{1}{\alpha}} \left[\frac{1 - \alpha}{\Delta_{Wt}} \right]^{\frac{1-\alpha}{\alpha}} \tag{1.3.14}$$

Final good producers

An infinite number of final good producers, or retailers, are defined over a continuum $[0,1]$. Retailers repackage the intermediate good and sell the final output to the households. Equations from $(1.1.21)$ to $(1.1.25)$ defined for the physical capital externality model are also valid for the human capital model.

Central bank

The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate. Trend inflation is considered as given.

Equilibrium conditions and external sector

An expression for the consumption to physical capital ratio can be obtained from the aggregate equilibrium of the economy, composed of consumption, investment and the external sector. This ratio is

$$\frac{C_t}{K_t} = \frac{Y_t}{K_t} - \frac{K_{t+1} + K_t}{K_t} - \frac{\delta K_t}{K_t} + \frac{X_t}{K_t} - l_t \frac{X_t^{\dagger}}{K_t^{\dagger}} \tag{1.3.15}$$

where, assuming two economies, henceforth "Nation" (N) and "Rest of the World" (ROW) —denoted with a dagger symbol [†]—, X_t are exports, $X_t^{\dagger}$ are imports, and l_t the quotient $K_t^{\dagger}/K_t$ needed to link both economies consumption-physical capital ratios.

Considering that the production functions for every intermediate goods producer are identical —which implies $Y_t^i = AK_t^{\alpha}L_t^{1-\alpha}$—, $(1.1.25a)$, $(1.3.11)$ and that physical capital stock grows in the steady state at the same rate as the human capital accumulation process rate $(1.3.7)$, we can obtain the expression in the steady state for N's ratio

$$C^K = \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{\Delta_{\mathrm{W}}} \right]^{\frac{1-\alpha}{\alpha}} - g - \delta + X^K - lX^{\dagger^{K^{\dagger}}} \tag{1.3.15a'}$$

which for ROW is:

$$C^{\dagger^{K^{\dagger}}} = \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}} \left[\left(\frac{\varepsilon^{\dagger} - 1}{\varepsilon^{\dagger}} \right) \frac{1 - \alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}} \right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} - g^{\dagger} - \delta^{\dagger} + X^{\dagger^{K^{\dagger}}} - \frac{1}{l} X^K \tag{1.3.15b'}$$

N's and ROW's exports depend on the real exchange rate e_t and the other economy's production, following the expressions

$$X_t = \Omega e_t^{\omega} Y_t^{\dagger} \tag{1.3.16a}$$

$$X_t^{\dagger} = \Omega^{\dagger} \left(\frac{1}{e_t} \right)^{\omega^{\dagger}} Y_t \tag{1.3.16b}$$

where $\Omega > 0$, $\Omega^{\dagger} > 0$, $\omega > 0$ and $\omega^{\dagger} > 0$, which normalized by the physical capital stock in the steady state:

$$X^K = \Omega e^{\omega} l \, \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}} \left[\left(\frac{\varepsilon^{\dagger} - 1}{\varepsilon^{\dagger}} \right) \frac{1 - \alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}} \right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} \tag{1.3.16a'}$$

$$X^{\dagger^{K^{\dagger}}} = \Omega^{\dagger} \left(\frac{1}{e} \right)^{\omega^{\dagger}} \frac{1}{l} \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{\Delta_{\mathrm{W}}} \right]^{\frac{1-\alpha}{\alpha}} \tag{1.3.16b'}$$

Expressions $(1.1.31a)$ and $(1.1.31b)$ for the economies' real external financial positions, $(1.1.32)$ for the appropriate gross real interest rate, $(1.1.33)$ for the relationship between economies' external financial positions and $(1.1.34)$ for the uncovered interest parity are also considered for this model.

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 1.3. The number of each system's equation corresponds to a previous one, with the superscript [$'$] stating that they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through physical capital stock K, each economy by its own. Also, it is necessary to define the ratio for this variable $l = K^{\dagger}/K$ to link the normalizations. Normalized variables are denoted with superscript K, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. N's equations described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have thirty-three equations and thirty-three endogenous variables.

1.3. Long-term inflation-growth relationship: economies with the same intertemporal discount rate

Once identified the normalized systems of equations required to determine the steady state values of the endogenous variables in each model, simulations are carried out through Dynare in order to get the response of growth to changes in trend inflation. The current section will present the results when the intertemporal discount rates are the same in both economies, therefore two identical economies. The following section will carry out simulations for different discount rates, while Section 1.5 will address the "contagion effect of non-neutrality", defining two economies with different price and wage settings and the same or different intertemporal discount rate for consumption. Table 1.3.1 presents the values for the parameters used in our simulations.

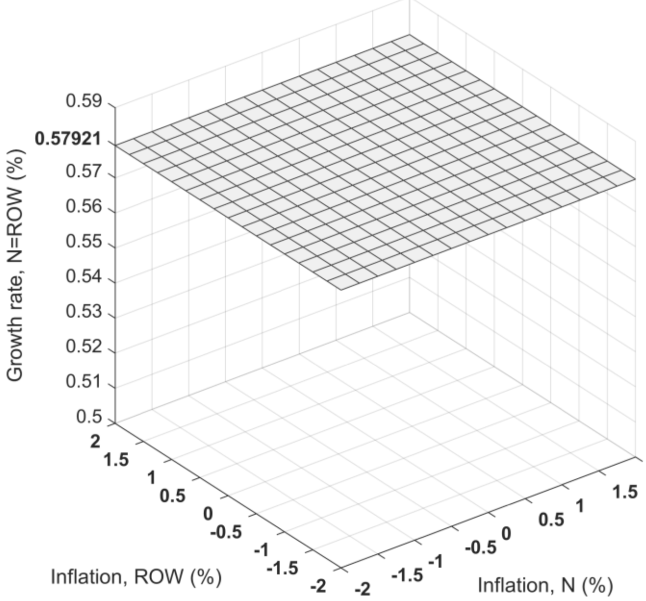
Table 1.3.1. Parameter values

Parameters	Description	Capital externality model	Schumpeterian model	Human capital model
$\beta = \beta^{\dagger}$	Utility discount factor	0.995	0.999	0.999
$\alpha = \alpha^{\dagger}$	Output-capital elasticity	0.332	0.332	0.332
$\sigma = \sigma^{\dagger}$	Elasticity of substitution for labor services	12	10	10
$v = v^{\dagger}$	Relative utility weight of labor	3.5	1	1
$\rho = \rho^{\dagger}$	Exports parameter	0.2	0.1	0
$\Omega = \Omega^{\dagger}$	Exports-exchange elasticity	0.5	0.5	0.5
$\omega = \omega^{\dagger}$	Exports-exchange elasticity	0.5	0.5	0.5
$\delta = \delta^{\dagger}$	Capital depreciation rate	0.03	-	0.04
$\varepsilon = \varepsilon^{\dagger}$	Elasticity of substitution between retail or intermediate goods	1.3	-	1.4
$\Phi_{\pi} = \Phi_{\pi}^{\dagger}$	Taylor rule's inflation reaction coefficient	2.05	-	-
$\gamma = \gamma^{\dagger}$	Productivity increase after innovation	-	1.009	-
$\chi = \chi^{\dagger}$	Elasticity of the probability of successful innovation with respect to the investment	-	0.1	-
$\xi = \xi^{\dagger}$	Human capital accumulation productivity	-	-	0.013
$A = A^{\dagger}$	Total factor productivity	-	-	1

Physical capital externality model

This section presents the results for the physical capital externality model when the economies are identical —same utility discount rate— and both have the same combination of rigidities. In order to analyze the impact of monetary policy on the steady-state growth rate, we start studying the case of flexibility in prices and wages in both economies ($I = I^{\dagger} = J = J^{\dagger} = 1$). The results show that the growth rate remains constant regardless of the inflation target, at an annual rate $g = g^{\dagger} = 2.317\%$ —quarterly $g = g^{\dagger} = 0.57921\%$—, as can be seen in Figure 1.3.1. In other words, monetary policies in both countries are neutral, meaning that the inflation target adopted by one country does not affect its growth rate, the growth rate of the other country, or the international growth rate.

Figure 1.3.1. Inflation-growth relationship. Physical capital externality model.
Nation $(\boldsymbol{I} = \mathbf{1}, \boldsymbol{J} = \mathbf{1})$ Rest of World $(\boldsymbol{I}^{\dagger} = \mathbf{1}, \boldsymbol{J}^{\dagger} = \mathbf{1})$ $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$

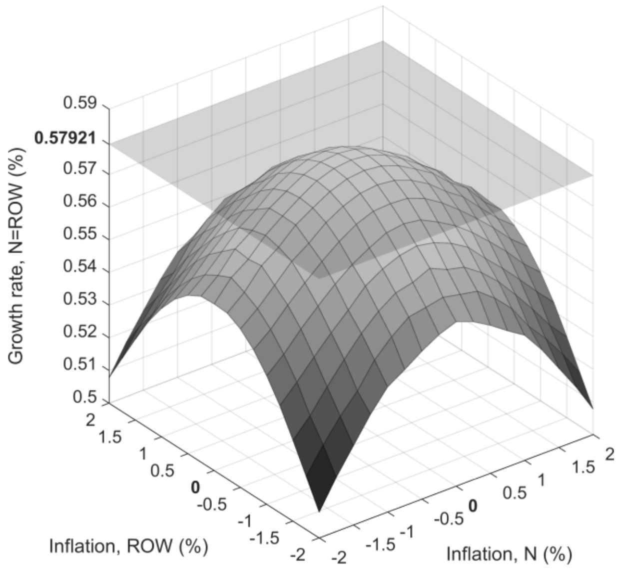


If we calibrate the model with only price rigidity in both economies $I = I^{\dagger} = 2, J = J^{\dagger} = 1$ the maximum growth rate that can be obtained is the same that we have obtained for flexibility in prices and wages —quarterly $g = g^{\dagger} = 0.57921\%$— and it is compatible with either deflation or inflation in both economies, within a wide range. This means that the situation is equivalent to the previous one. As inflation targets move away from these points, the growth rate for both economies declines slightly, but the difference is negligible. For admissible values of Π and $\Pi^{\dagger}$ the conclusion found is neutrality.

The model with price and wage rigidities in both economies $I = I^{\dagger} = 2, J = J^{\dagger} = 4$ shows the relationship between Π and g represented in Figure 1.3.2 below the horizontal plane corresponding to flexibility. The maximum achievable global growth remains at a quarterly rate of $g = g^{\dagger} = 0.57921\%$, the same as with flexibility, but it is only achieved at the specific point of null inflation in both economies $(\Pi, \Pi^{\dagger}) = (0\%, 0\%)$, whatever the order of wage rigidity J. As we move away from that point, the growth rate decreases more the greater the distance to that point. Moreover, the greater the wage rigidity, the higher the slope at any point except the corresponding to the maximum growth.

Laguna and Sanso (2020) already found this coincidence between the maximum growth rate with wage stickiness and that obtained with wage flexibility in the case of a closed economy. However, it is only achieved at point $(\Pi, \Pi) = (0\%, 0\%)$, a result that breaks the typical result in closed economies with sticky wages and endogenous growth in which the optimal trend inflation is negative[2]. As we move away from this point, the growth rate decreases, the fall being greater the greater the distance. Moreover, the greater the wage stickiness, the higher the slope at any point except that corresponding to the maximum growth rate.

Figure 1.3.2. Inflation-growth relationship. Physical capital externality model.
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^\dagger = 2, J^\dagger = 4}$) $\boldsymbol{\beta = \beta^\dagger}$



The mechanism behind the point of maximum growth is clear. Wage stickiness introduces a distortion that has an effect equivalent to a negative shock of productivity. Nevertheless, at this point that distortion does not exist, because the values of the inflation and growth rates are such that the periodic revision of wages is not necessary in either. Then the result is equivalent to flexibility.

[2] Amano et al. (2009) state the result of negative inflation to achieve the maximum growth rate in a closed economy with exogenous growth, while Amano, Carter, and Moran (2012) do the same for an endogenous growth model of the Romer (1990) type. In our simulations we found that, for open economies with a different endogenous growth engine (Romer, 1986) and wages per unit of effective work instead of wages per hour, these results are no longer confirmed, as maximum growth rate is found in our simulations at the point of null inflation.

This circumstance only happens at this point of maximum growth. Any deviation from it in at least one of the countries introduces a distortion mainly in the average wage, which decreases the labor demand, the production, the imports, the own growth, and the growth of the other country.

If we remove price rigidities in both economies, we obtain results akin to this case. The maximum growth $g = 0.57921\%$ continues to be reached at null inflation. Although some values are slightly modified, the effect is negligible. The conclusion we obtain is that non-neutrality arises because of wage rigidity, not price rigidity.

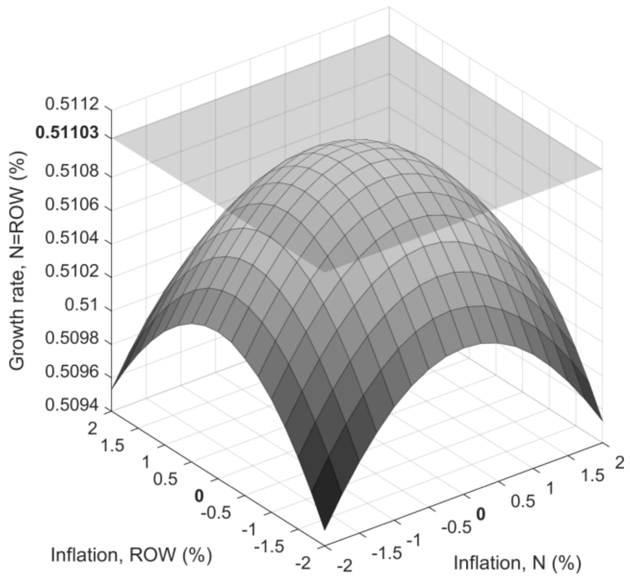
Schumpeterian model

The results in the Schumpeterian model for the case of flexibility in prices and wages in both economies and for only price stickiness show that the growth rate in each country is always the same, regardless of the inflation targets. This result matches what we have seen in our previous model. The growth rate in both countries is $g = g^{\dagger} = 0.51103\%$ quarterly. The model with price and wage rigidities in both economies $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ implies the relationships between the gross trend inflation rates Π and $\Pi^{\dagger}$ and the long-run growth rates g and $g^{\dagger}$ represented in Figure 1.3.3 below the horizontal plane corresponding to flexibility.

The maximum achievable global growth remains at a quarterly rate of $g = g^{\dagger} = 0.51103\%$, the same as with flexibility, achieved again at the point of null inflation. As in the previous model, moving away from that point decreases the growth rate. The greater the wage rigidity, the higher the slope for pair of inflation rates.

Figure 1.3.3. Inflation-growth relationship. Schumpeterian model.

Nation ($\boldsymbol{I = 2, J = 4}$) **Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$)** $\boldsymbol{\beta = \beta^{\dagger}}$



In conclusion, the results repeat the behavior of the previous model. Notice, however, the difference in scale that characterizes this model: we can only see quarterly differences in thousandths percent for a reasonable range of inflation rates. As Figure 1.3.3 shows, for $(\Pi, \Pi^{\dagger}) = (-2\%, -2\%)$ the growth rate in both countries is $g = g^{\dagger} = 0.509566\%$ quarterly. A change of just $\Delta g = \Delta g^{\dagger} = -0.001471\%$ compared to the maximum achievable growth rate obtained at point $(\Pi, \Pi^{\dagger}) = (0\%, 0\%)$, where L and $L^{\dagger}$ are maximum and, consequently, g and $g^{\dagger}$.

Human capital model: price stability premium

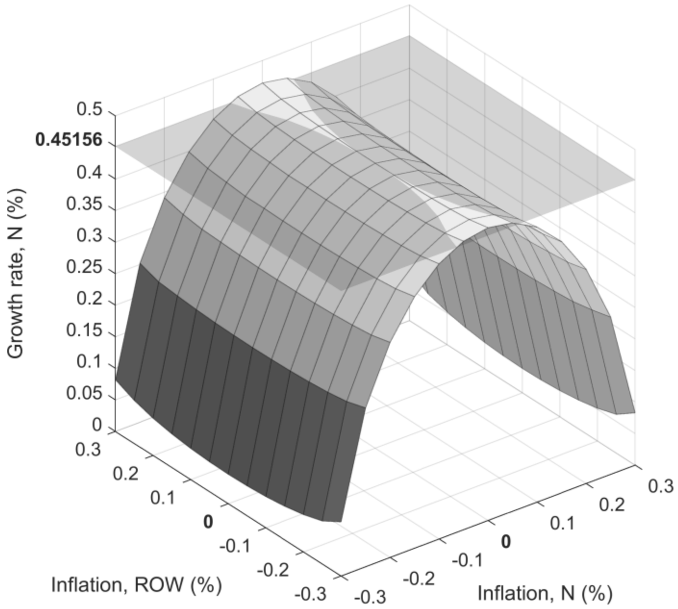
As in the two previous models, results for the case of price and wage flexibility and only price stickiness in both economies show that the inflation targets of the two economies do not affect the growth rate in in both countries that coincide in the value $g = g^{\dagger} = 0.45156\%$ quarterly.

Results start to differ sharply from those obtained in the other two models with the introduction of price and wage stickiness in both economies $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$. The two economies will not share anymore the same growth rate for every combination of inflation target in this case, with the only exception of the point $(\Pi, \Pi^{\dagger}) = (0, 0)$ where the growth rate value is the corresponding to price and wage flexibility for both countries. However, this value is not the maximum, a second difference with the other two models.

In the previous two models, the expression $(1.1.5')$, along with the uncovered interest parity $(1.1.33')$, is responsible for the result in the steady state. They imply together $g\beta = g^{\dagger}\beta^{\dagger}$, from where the growth rates are the same in steady state if their discount factor coincide. This is no longer the case in the human capital model where the first equation is not present.

In this case, the country's own monetary policy is what mainly affects each economy's growth rate, as can be seen in Figure 1.3.4, unlike the other two models where ROW's inflation target affects Nation's growth rate to the same extent as its own.

Figure 1.3.4. Inflation-growth relationship for N. Human capital model.
Nation ($\boldsymbol{I} = 2, \boldsymbol{J} = 4$) Rest of World ($\boldsymbol{I}^{\dagger} = 2, \boldsymbol{J}^{\dagger} = 4$) $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$



However, this does not mean that monetary policy decisions from ROW does not affect Nation. As expression $(1.3.7)$ shows, Nation's growth rate directly depends on the real interest rate[3] and exports from ROW. It therefore depends on the real exchange rate and ROW's output to physical capital ratio.

The second difference is that the value corresponding to flexibility, and to point $(\Pi, \Pi^\dagger) = (0{,}0)$ with wage stickiness, is not the maximum growth rate attainable for each country. When one country adopts null inflation as its target but the other country not, the growth rate of the former is greater the greater the distance of the inflation target of the latter from zero in either direction. Table 1.3.2 reflects the movements on exchange rate as we move the inflation target of ROW from zero while keeping Nation's target fixed at null inflation rate, as well as the consequences on exports and growth. The simulation shows that the dynamic of the trade balance is the responsible for the divergence in the growth rates.

Table 1.3.2. Inflation-Exchange-Exports-Growth values. Human capital model.
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^\dagger = 2, J^\dagger = 4}$) $\boldsymbol{\beta = \beta^\dagger}$

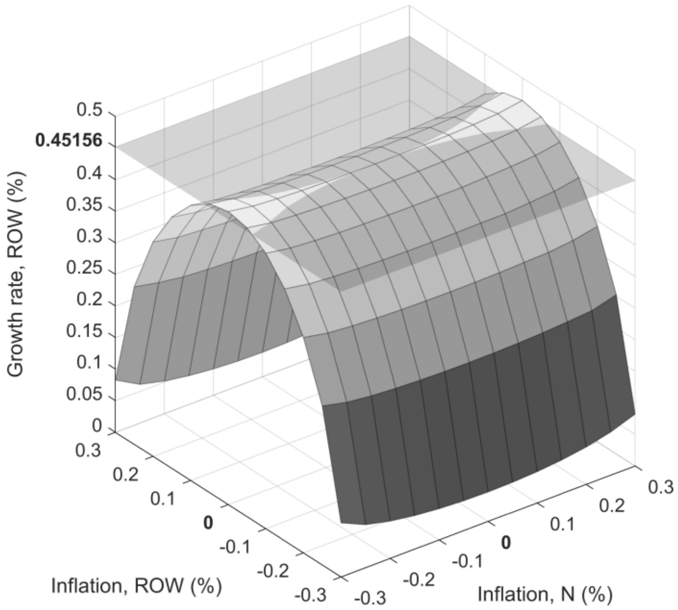
Infl. N	Infl. ROW	Exchange	Exports N	Growth N	Exports ROW	Growth ROW
0%	-0,30%	1,10112	0,4821	0,4670%	0,4382	0,0440%
0%	-0,25%	1,05206	0,4809	0,4596%	0,4571	0,2319%
0%	-0,20%	1,02931	0,4804	0,4561%	0,4667	0,3251%
0%	-0,15%	1,01527	0,4801	0,4540%	0,4729	0,3847%
0%	-0,10%	1,00646	0,4799	0,4526%	0,4768	0,4230%
0%	-0,05%	1,00157	0,4798	0,4519%	0,4791	0,4446%
0%	0,00%	1,00000	0,4797	0,4515%	0,4799	0,4515%
0%	0,05%	1,00160	0,4798	0,4519%	0,4791	0,4444%
0%	0,10%	1,00655	0,4799	0,4527%	0,4768	0,4226%
0%	0,15%	1,01543	0,4801	0,4540%	0,4728	0,3840%
0%	0,20%	1,02961	0,4804	0,4562%	0,4666	0,3238%
0%	0,25%	1,05265	0,4810	0,4597%	0,4569	0,2296%
0%	0,30%	1,10328	0,4825	0,4673%	0,4374	0,0361%

If ROW deviates from the optimal null inflation, it will benefit the growth of Nation in a greater extent the more ROW deviates from null inflation, as it causes an increase in the exchange rate. At the point of null inflation for both economies, growth rate of both

[3] That only depends now on the real average wage as reflected in $(1.3.14')$ and ends up being the same in both economies ($R = R^\dagger$) due to the uncovered interest parity in the steady state.

countries is the same as in flexibility $g = g^{\dagger} = 0.45156\%$. Nevertheless, this will no longer be the optimal point for either, as the further away from null inflation, the more the other's growth rate will improve. For example, at point $(\Pi, \Pi^{\dagger}) = (0\%, -0.3\%)$ a quarterly growth rate for Nation of $g = 0.4670\%$ is reached, while ROW decreases to a mere $g^{\dagger} = 0.044\%$ as it is heavily affected by its own monetary policy decision considering the existence of wage rigidities, mainly due to the variation of the wage per unit of human capital. This improvement in the growth rate is a "price stability premium" for the country with zero or very low inflation or deflation.

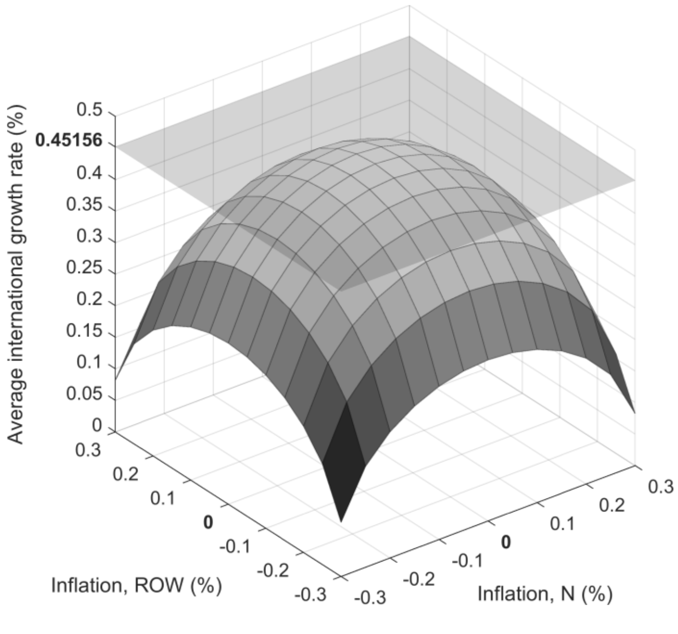
Figure 1.3.5. Inflation-growth relationship for ROW. Human capital model.
Nation $(\boldsymbol{I} = 2, \boldsymbol{J} = 4)$ Rest of World $(\boldsymbol{I}^{\dagger} = 2, \boldsymbol{J}^{\dagger} = 4)$ $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$



In any case, from the point of view of each economy, null inflation is still the optimal policy, and then it will always depend on the other's deviation from the optimum to achieve a higher growth. In addition, it will be the equilibrium in a cooperative and no cooperative equilibrium. The fact that one economy deviating from the optimum improves the growth of the other economy is an interesting and distinctive result of the human capital model in presence of wage stickiness. The possibility of obtaining with wage stickiness a country's growth rate above the flexibility value is a result not obtained in the other two models nor in the four models for a closed economy in Laguna and Sanso (2020).

Moreover, the familiar bell-shaped figure obtained in the other two models returns if we represent the behavior of the global average growth rate in the human capital model. The average growth rate reaches its maximum value, the same as with wage and price flexibility, at the point of null inflation for the two countries. Then it falls as the inflation target pairs move further away from null inflation. Notice the large drop in the average growth rate within reasonable variations of the inflation targets, unlike the results found in the Schumpeterian model and more in line with those obtained for the model with physical capital externality.

Figure 1.3.6. Inflation-world average growth relationship. Human capital model.
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta = \beta^{\dagger}}$



1.4. Long-term inflation-growth relationship: economies with different intertemporal discount rates

This section presents the results for the simulations calibrated with different intertemporal discount rates in the two economies. The values for the parameters displayed in Table 1.3.1

are still valid for these simulations, with the only exception of the intertemporal discount factors β and $\beta^{\dagger}$, that now take the values presented in Table 1.4.1.

Table 1.4.1. Values for the intertemporal discount factors

Param.	Description	Physical	Schump.	Human
β	Discount factor for N	0.995	0.9990	0.999
$\beta^{\dagger}$	Discount factor for ROW	0.990	0.9989	0.998

Physical capital externality model

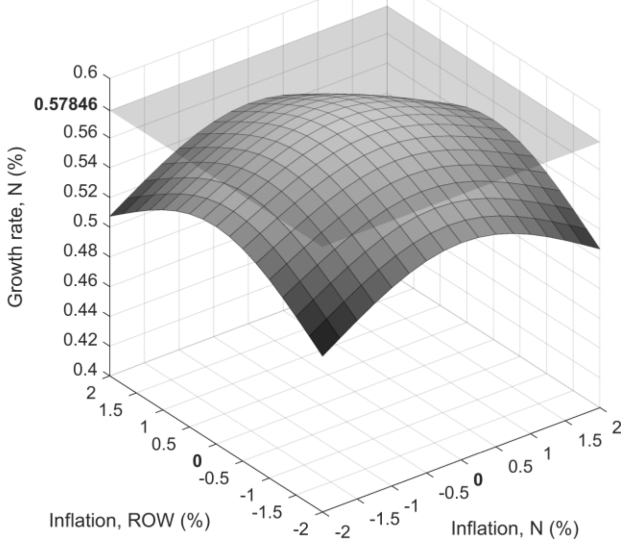
We start again with the case of flexibility in prices and wages in both economies $I = I^{\dagger} = J = J^{\dagger} = 1$. The results show that each country growth is always the same, regardless of the inflation targets. In the case simulated —$\beta = 0.995$ and $\beta^{\dagger} = 0.990$—, N grows at the quarterly rate $g = 0.57846\%$ and ROW at the rate $g^{\dagger} = 0.07304\%$. The monetary policies are neutral, because the inflation target of a country does not affect its growth rate, the growth rate of the other country, or the international growth rate. The same conclusion holds with only price stickiness.

The values of this parameter in the two countries play a key role in the results. When the countries had the same discount factor, $\beta = \beta^{\dagger} = 0.995$, the quarterly growth rate in both countries was $g = g^{\dagger} = 0.57921\%$. We can see that a higher discount rate in a country, ROW in our previous simulation, means lower growth for it. The other country with lower discount factor experiences a slight reduction.

The model with price and wage stickiness implies the two relationships between the gross trend inflation rates —Π and $\Pi^{\dagger}$— and the long-run growth rates —g and $g^{\dagger}$— represented in Figures 1.4.1 and 1.4.2 for N and ROW, respectively, below the horizontal plane corresponding to price and wage flexibility. The maximum achievable growth remains at rate

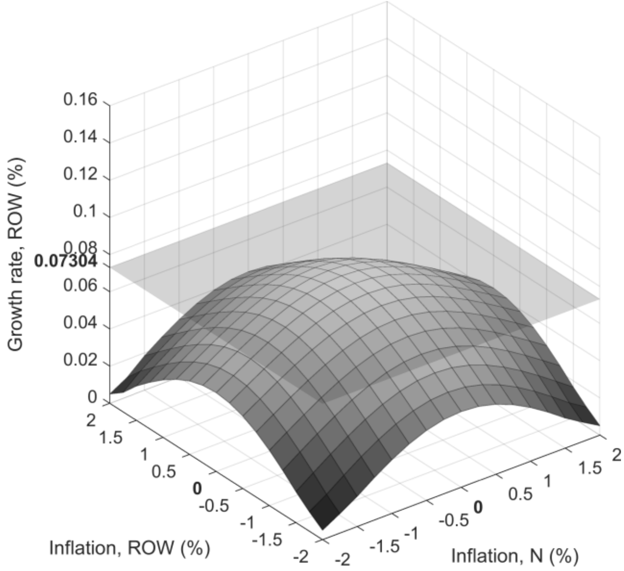
$g = 0.57846\%$ quarterly for N and $g^{\dagger} = 0.07304\%$ for ROW, the same as with flexibility in both cases. The same results are obtained without price stickiness in both economies.

Figure 1.4.1. Inflation-growth relationship for N. Physical capital externality model
Nation ($\boldsymbol{I = 2, J = 4}$) $\boldsymbol{\beta = 0.995}$
Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta^{\dagger} = 0.990}$



The main reason why the growth rate decreases in the two countries is that the real interest rate falls. The drop in the case of the country with lower intertemporal discount factor is greater because the fall in β increase the magnitude of the previous effect.

Figure 1.4.2. ROW's Inflation-growth relationship. Physical capital externality model
Nation ($\boldsymbol{I = 2, J = 4}$) $\boldsymbol{\beta = 0.995}$
Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta^{\dagger} = 0.990}$

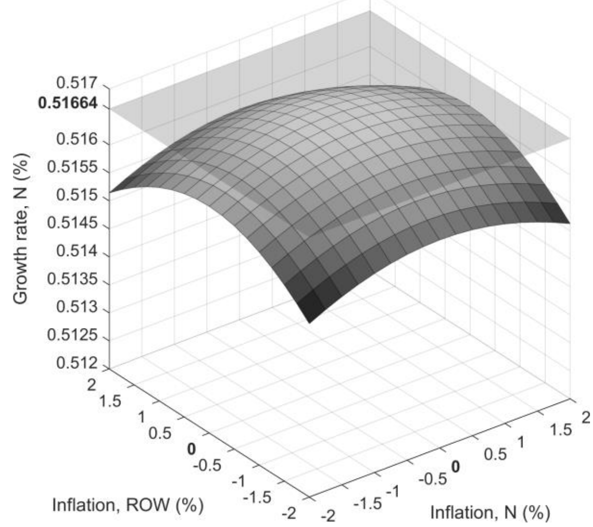


Falls in the price of the use of capital, as well as in the production of intermediate goods and in L (due to an increase in the average real wage) are the main reasons. This is the mechanism behind Figures 1.4.1 and 1.4.2. Whether the growth rate is the same or different is the consequence of an identical or different discount rate in the two countries.

Schumpeterian model

When we consider flexible prices and wages in both economies in the Schumpeterian model, the growth rate in each country is always the same regardless of the inflation targets. In the case simulated with different intertemporal discount factors, $\beta = 0.9990$ and $\beta^{\dagger} = 0.9989$, N grows at the quarterly rate $g = 0.51664\%$ and ROW at rate $g^{\dagger} = 0.50657\%$.

Figure 1.4.3. Inflation-growth relationship for N. Schumpeterian model
Nation ($\boldsymbol{I} = 2, \boldsymbol{J} = 4$) $\boldsymbol{\beta} = 0.9990$
Rest of World ($\boldsymbol{I}^{\dagger} = 2, \boldsymbol{J}^{\dagger} = 4$) $\boldsymbol{\beta}^{\dagger} = 0.9989$

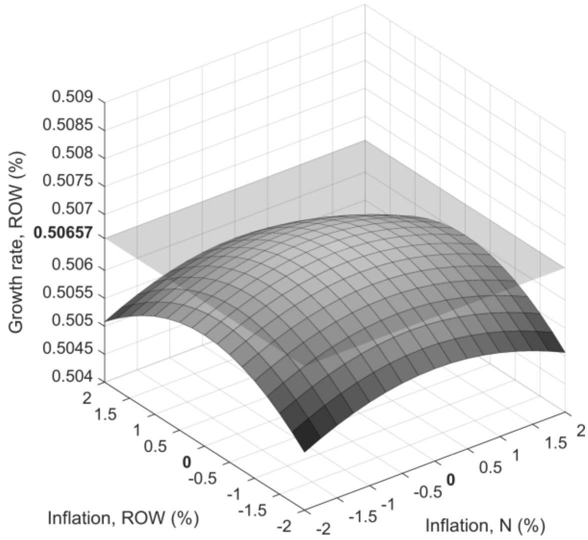


Considering that we obtained $g = g^{\dagger} = 0.51103\%$ for $\beta = \beta^{\dagger} = 0.999$ in the previous section, we can see that the most "impatient" economy has a decrease in its own growth rate, while the "patient" one has an increase in a greater extent, in contrast to the previous model with physical capital externality in which both decreased.

The model with price and wage rigidities in both economies implies the relationship between the gross trend inflation rates and the long-run growth rates represented in Figures 1.4.3 and

1.4.4 under the horizontal plane corresponding to flexibility. As in the previous model, the maximum achievable growth remains at the rate of flexibility, once again only at the point of null inflation for the two countries. As we move away from this point, the growth rate decreases slightly, noticing again the small differences that characterize this model.

Figure 1.4.4. Inflation-growth relationship for ROW. Schumpeterian model
Nation ($I = 2, J = 4$) $\quad \beta = 0.9990$
Rest of World ($I^{\dagger} = 2, J^{\dagger} = 4$) $\quad \beta^{\dagger} = 0.9989$



The main reason why the maximum growth rate increases in the country that keep constant the intertemporal discount factor and decreases in the country experiencing the fall in this rate is that L increase in the former and decrease in the latter. This is also coherent with the increase in the interest rate in the two countries and the different effect on L (it increases in N and decreases in ROW).

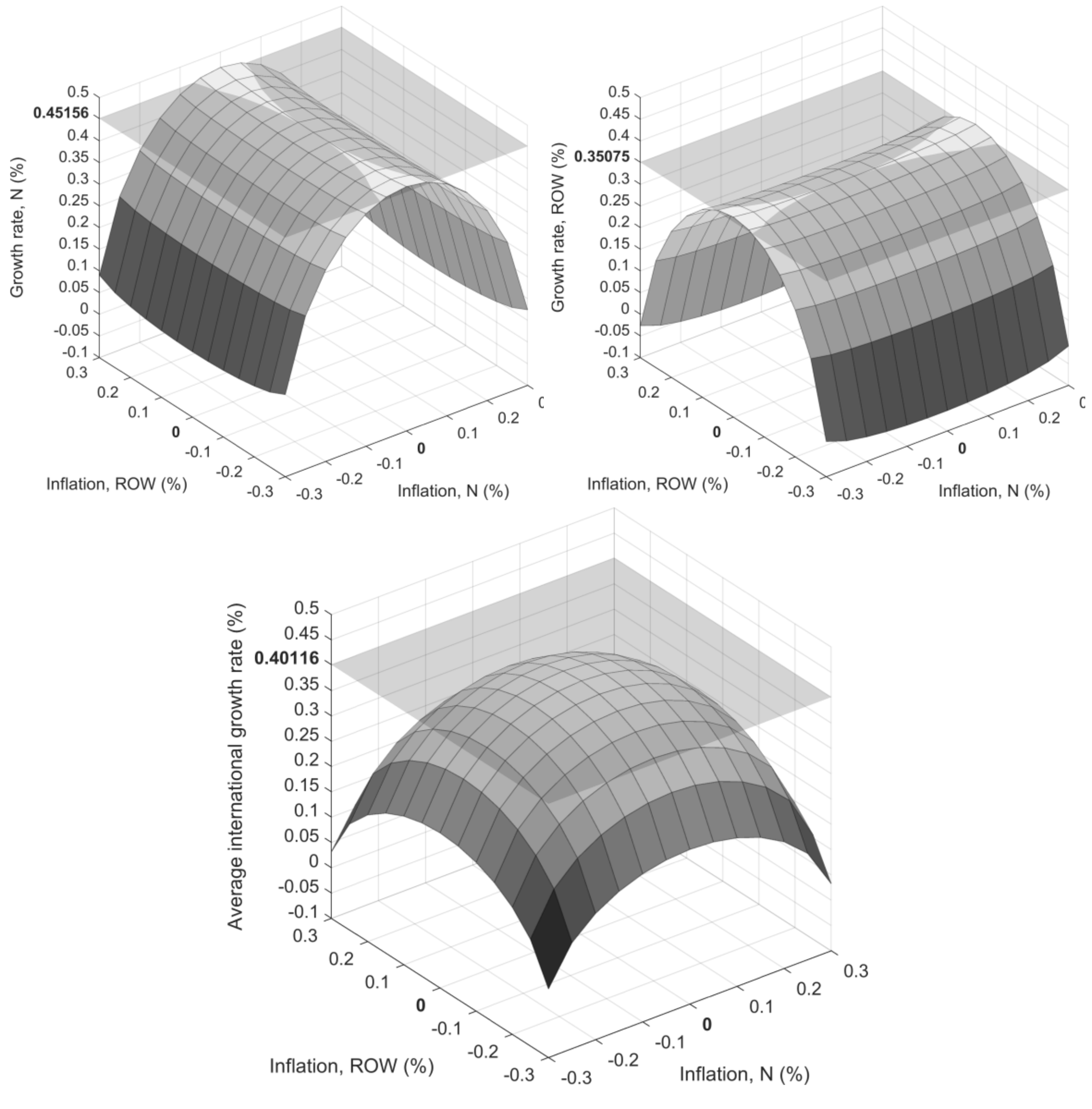
Human capital model

We also obtain in the human capital model a growth rate for each country that is always the same regardless of the inflation targets if we consider flexible prices and wages or price stickiness only in both economies, now at $(g, g^{\dagger}) = (0.45156\%, 0.35075\%)$ for a different discount factors $(\beta, \beta^{\dagger}) = (0.999, 0.998)$. The growth rate increases slightly in the economy that maintains the discount factor and, as in the previous models, decreases in the most "impatient" economy. The international average growth is $\Delta g = 0.40116\%$, down from $\Delta g = 0.4515\%$ when $\beta = \beta^{\dagger} = 0.999$.

In the case of price and wage rigidities in both economies, everyone reaches again its flexibility growth rate when both economies have null inflation, as Figure 1.4.5 shows.

Figure 1.4.5. Inflation-growth relationship for N, ROW and average. Human capital model.

Nation ($I = 2, J = 4$) $\beta = 0.999$
Rest of World ($I^{\dagger} = 2, J^{\dagger} = 4$) $\beta^{\dagger} = 0.998$



If ROW (Nation) deviates from the optimal null inflation, it benefits the growth rate of Nation (Rest of the World) providing in its favor a "price stability premium". Considering the international (global) average growth rate for every pair of inflation rates, the bell-shaped result of the other two models returns, reaching the maximum growth in the flexibility value only for null inflation in the two countries.

The main reason why the maximum growth rate increases in the country that keep constant the intertemporal discount factor and decreases in the country experiencing the fall in this rate is that exports decrease in both countries, but more in the former, adding the effects of the fall of β in ROW. All this in spite of the fall in the interest rate of much less magnitude. This is also coherent with the different effect on L (it increases in N and decreases in ROW).

All this has allowed us to know that the long-run effects of the intertemporal discount rate of a country not only has consequences on its own economy but also on the other country in the case of large economies. The effects of a lower discount factor for a country are always negative for its own growth rate, whatever the model but, on the growth rate of the other country, it depends on the model. While in a model with the technology of Romer (1986) the effect is also negative, when it is of the type of Aghion y Howitt (1992) or Lucas (1988) it is positive.

1.5. Different severity of wage stickiness: contagion of non-neutrality

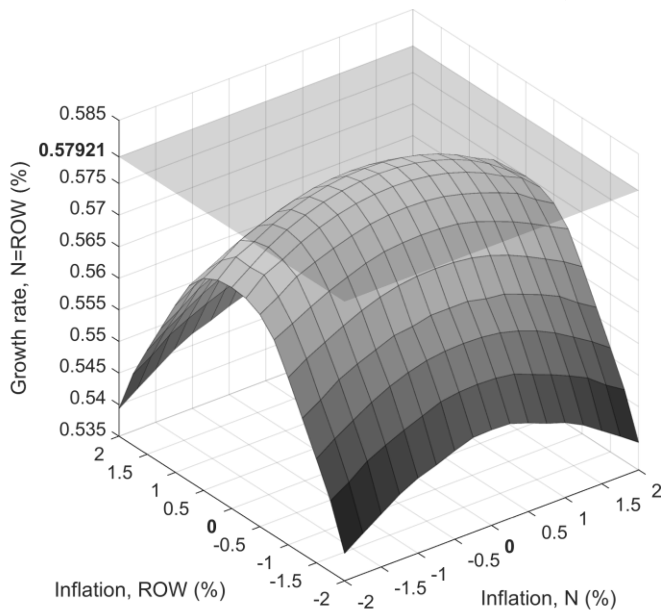
In this section, we consider different wage stickiness in Nation and ROW. The values for the parameters summarized in Table 1.3.1 are still valid, and we will differentiate the intertemporal discount rates as in Table 1.4.1 after presenting first the results for the same discount rate as in Table 1.3.1. Simulations in this section allow us to conclude that a flexible

N might not be able to maximize its growth, because it could be suffering a "contagion of non-neutrality" caused by ROW.

Physical capital externality model

If we introduce wage stickiness of different order in the two countries, for example of order two $(I = 2, J = 2)$ in Nation and of order four $I^{\dagger} = 2, J^{\dagger} = 4$ in ROW, we obtain the results displayed in Figure 1.5.1. Comparing with Figure 1.3.2, we can conclude that a decrease in Nation's wage stickiness leads to a reduction in the negative influence of Nation's monetary policy on ROW growth, while the influence of ROW's monetary policy remains the same.

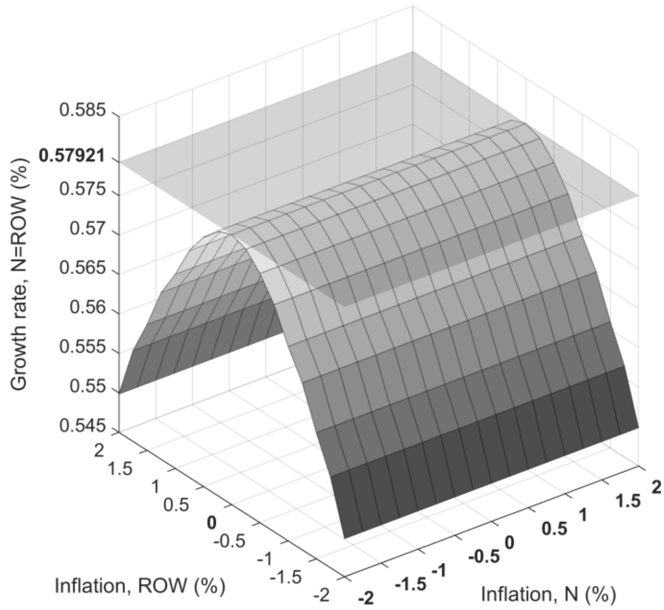
Figure 1.5.1. Inflation-growth relationship. Physical capital externality model.
Nation $(\boldsymbol{I = 2, J = 2})$ Rest of World $(\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4})$ $\boldsymbol{\beta = \beta^{\dagger} = 0.995}$



Therefore, international achievable growth increases at each point except for the maximum value, which remains at $g = 0.57921\%$ quarterly and is reached at the point $(\Pi, \Pi^{\dagger}) = (0\%, 0\%)$. The conclusion is that the lesser (greater) the wage stickiness in a country the lesser (greater) the influence of the own monetary policy on the own and the other country's growth rate.

In an additional simulation, we consider the extreme situation of price and wage flexibility in N ($I = J = 1$, lack of stickiness) and price and wage rigidity $I^{\dagger} = 2,\ J^{\dagger} = 4$ in ROW. Results are shown in Figure 1.5.2. The main conclusion we can draw from this simulation is that in the country with price and wage flexibility the long-run growth is independent from its own monetary policy Π, but not from ROW's monetary policy $\Pi^{\dagger}$. This means that, as long as ROW's inflation level is kept constant, the internationally achievable economic growth will be the same whatever Nation's monetary policy, but a change in ROW's inflation will have consequences on Nation's growth. This way, the attainable growth is only maximum if the country with rigidities adopts the inflation rate that maximizes growth.

Figure 1.5.2. Inflation-growth relationship. Physical capital externality model.
Nation ($\boldsymbol{I = 1, J = 1}$) Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta = \beta^{\dagger} = 0.995}$



The maximum international growth reaches, as previously, the flexibility level but, instead for a single point, it is reached for $\Pi^{\dagger} = 0\%$ whatever the value of Π. The consideration of two economies leads to a conclusion that cannot be drawn from closed economy approaches. By placing limits on flexibility —specifically wage rigidity— the internationally achievable growth depends not only on the own inflation target but also on the other country's target.

Therefore, wage rigidity implies not only that monetary policy is non-neutral but also that it implies a "contagion of non-neutrality".

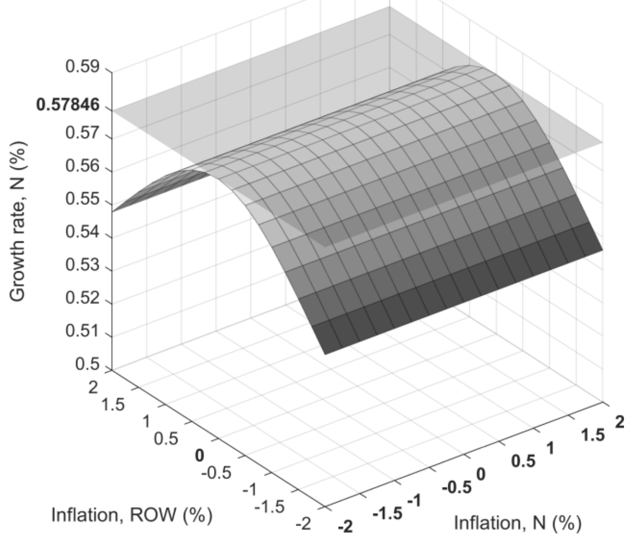
The figure 1.5.2 is in fact a repetition of a particular case of Figure 1.3.2, in the sense that the mechanism at work for every inflation value of N in the first is a repetition of the vertical cut for the trend inflation value $\Pi = 0\%$ of N in the second. The reason is the following: when the long-run gross inflation rate $\Pi^{\dagger}$ in ROW is 0%, the growth rate in N for any value of Π will be the maximum because at all the points the conditions of flexibility in the two countries are fulfilled. However, if $\Pi^{\dagger}$ is different from 0%, the wage stickiness distortion produces the same effect as in the section in the surfaces of Figure 1.3.2 for $\Pi = 0\%$. Therefore, the resulting surfaces have the same behavior for every value of Π in Figure 1.5.2.

We consider finally price and wage flexibility $I = J = 1$ in N and price and wage stickiness $I^{\dagger} = 2, J^{\dagger} = 4$ in ROW with the different discount factors, shown in Table 1.4.1. The results for the relationships between trend inflation and long-run growth are shown in Figures 1.5.3 and 1.5.4 for N and ROW respectively.

In these simulations the long-run growth in N is again independent of its monetary policy but not of ROW's monetary policy. If ROW's inflation target is kept constant, the achievable economic growth will be constant for the two countries, regardless of N's monetary policy. However, a change in ROW's inflation target will have consequences not only for its own growth rate but also for N's growth rate.

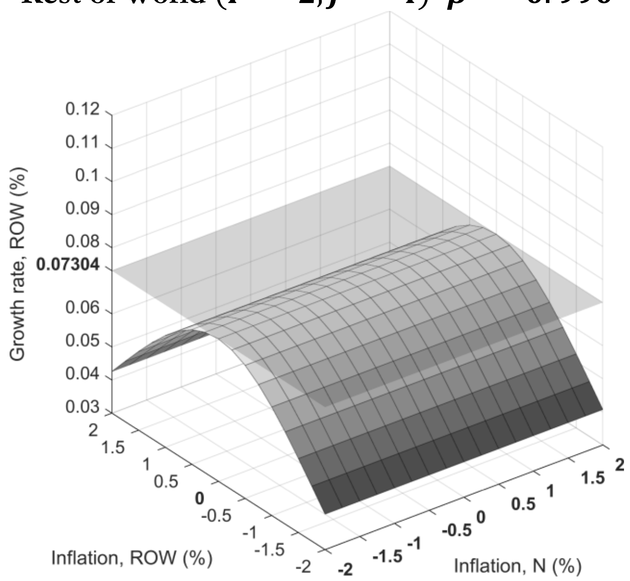
The comparison of Figures 1.5.3 and 1.5.4 clearly shows the nature of the contagion effect of non-neutrality. On one side, we have country N with a lower intertemporal discount rate and price and wage flexibility. This situation enables N to achieve a quarterly growth rate of 0.57846% quarterly. On the other side, ROW has a higher intertemporal discount rate and price and wage stickiness, and it can reach at most 0.07304% growth rate. However, the differences are not limited to these facts.

Figure 1.5.3. Inflation-growth relationship for N. Physical capital externality model
Nation $(\boldsymbol{I}=\mathbf{1},\boldsymbol{J}=\mathbf{1})$ $\boldsymbol{\beta}=\mathbf{0.995}$
Rest of World $(\boldsymbol{I}^{\dagger}=\mathbf{2},\boldsymbol{J}^{\dagger}=\mathbf{4})$ $\boldsymbol{\beta}^{\dagger}=\mathbf{0.990}$



As in the case of figure 1.5.2, figures 1.5.3 and 1.5.4 are in fact a repetition of a particular case of Figures 1.4.1 and 1.4.2. Therefore, the resulting surfaces have the same behavior for every value of Π.

Figure 1.5.4. Inflation-growth relationship for ROW. Physical capital externality model
Nation $(\boldsymbol{I}=\mathbf{1},\boldsymbol{J}=\mathbf{1})$ $\boldsymbol{\beta}=\mathbf{0.995}$
Rest of World $(\boldsymbol{I}^{\dagger}=\mathbf{2},\boldsymbol{J}^{\dagger}=\mathbf{4})$ $\boldsymbol{\beta}^{\dagger}=\mathbf{0.990}$

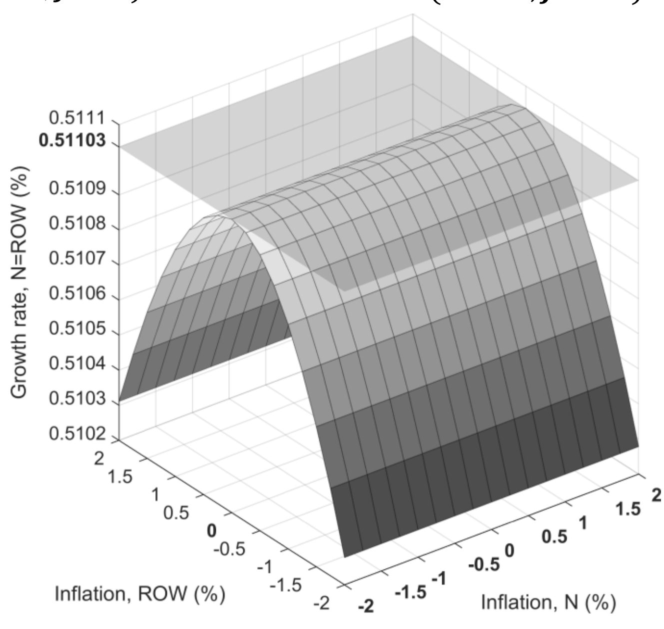


The reason for the use of the term "contagion effect" is because the country with full flexibility cannot finally perform its best if the country with wage stickiness choses a value of trend inflation —target— that is not coincident with that rendering its maximum growth. As this result is a consequence of the non-neutrality originated by the existence of nominal wage stickiness in the other country, we call the phenomenon as "contagion of non-neutrality" for the country with price and wage flexibility given that it is a spillover effect coming from the other country.

Schumpeterian model

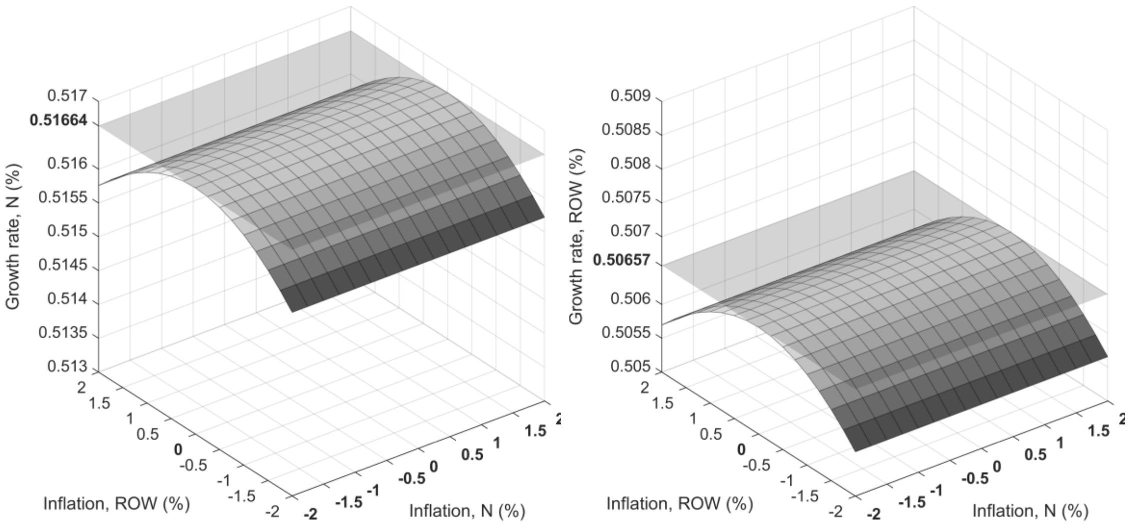
The Schumpeterian model generates a very similar outcome. Figure 1.5.5 shows the results for the simulations considering price and wage flexibility in one economy and price and wage rigidity in ROW with the same intertemporal discount rate. Figure 1.5.6 shows the relationship for N and ROW, with a differentiated discount factor.

Figure 1.5.5. Inflation-growth relationship. Schumpeterian model.
Nation ($\boldsymbol{I=1, J=1}$) Rest of World ($\boldsymbol{I^{\dagger}=2, J^{\dagger}=4}$) $\boldsymbol{\beta=\beta^{\dagger}}$



The simulations are consistent with those obtained for our previous model but, again, in a smaller scale. These figures show that the long-run growth in N is independent of its monetary policy but not of ROW's monetary policy, implying again the possibility of a "contagion effect of non-neutrality", because the country with price and wage flexibility will not always be able to achieve its maximum growth rate. The attainable growth will be the maximum in both countries, at the level of price and wage flexibility, only if ROW adopts the inflation target $\Pi^{\dagger} = -0\%$ whatever the value of Π. A decrease in the utility discount factor for ROW causes again to both an increase in N's growth and a decrease in ROW's growth. Maximum growth rate for the same intertemporal discount rate in both economies $g = g^{\dagger} = 0.51103\%$ is between the growth for N $g = 0.51664\%$ and the growth for ROW $g^{\dagger} = 0.50657\%$ when these discount rates are different.

Figure 1.5.6. Inflation-growth relationship for N and ROW. Schumpeterian model.
Nation ($\boldsymbol{I = 1, J = 1}$) $\boldsymbol{\beta = 0.9990}$
Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta^{\dagger} = 0.9989}$



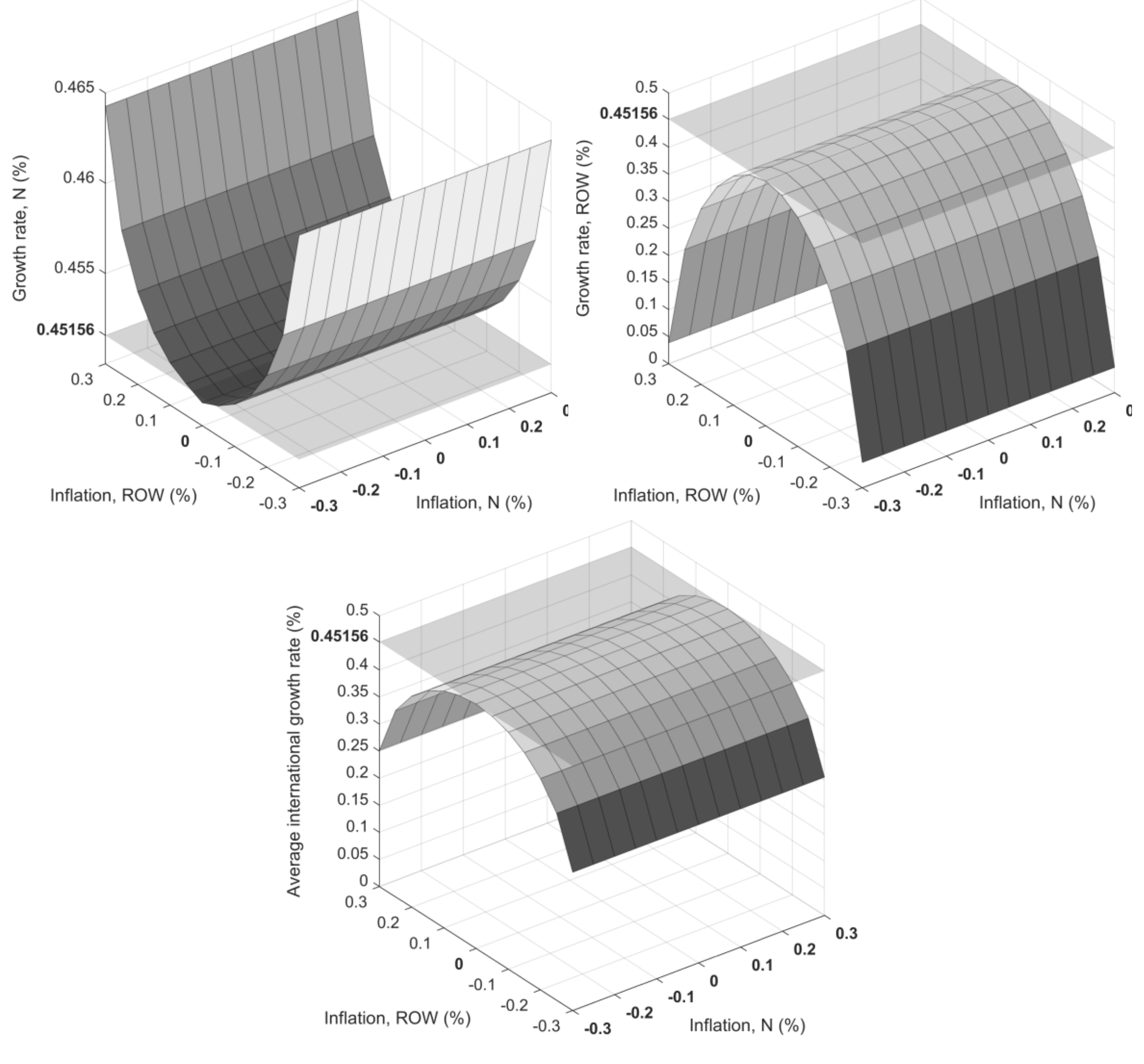
Human capital model

This section presents the results for the human capital model of considering price and wages flexibility in Nation and both rigidities in ROW, for the same or different consumption discount rate. Figure 1.5.7 contains three simulations representing the long-run inflation-growth relationship for Nation, for ROW and for the average international growth with the same discount factor. The same representation, but for a different discount factor between economies, is presented in Figure 1.5.8.

The results for these simulations show a clear difference from those analyzed in the two previous models, a difference consistent with the peculiarities of this model analyzed in the previous two sections. This distinction is reflected in the first quadrant of each figure, that corresponds to the result for the economy with both price and wage flexibility.

Figure 1.5.7. Inflation-growth relationship for N, ROW and average. Human capital model.

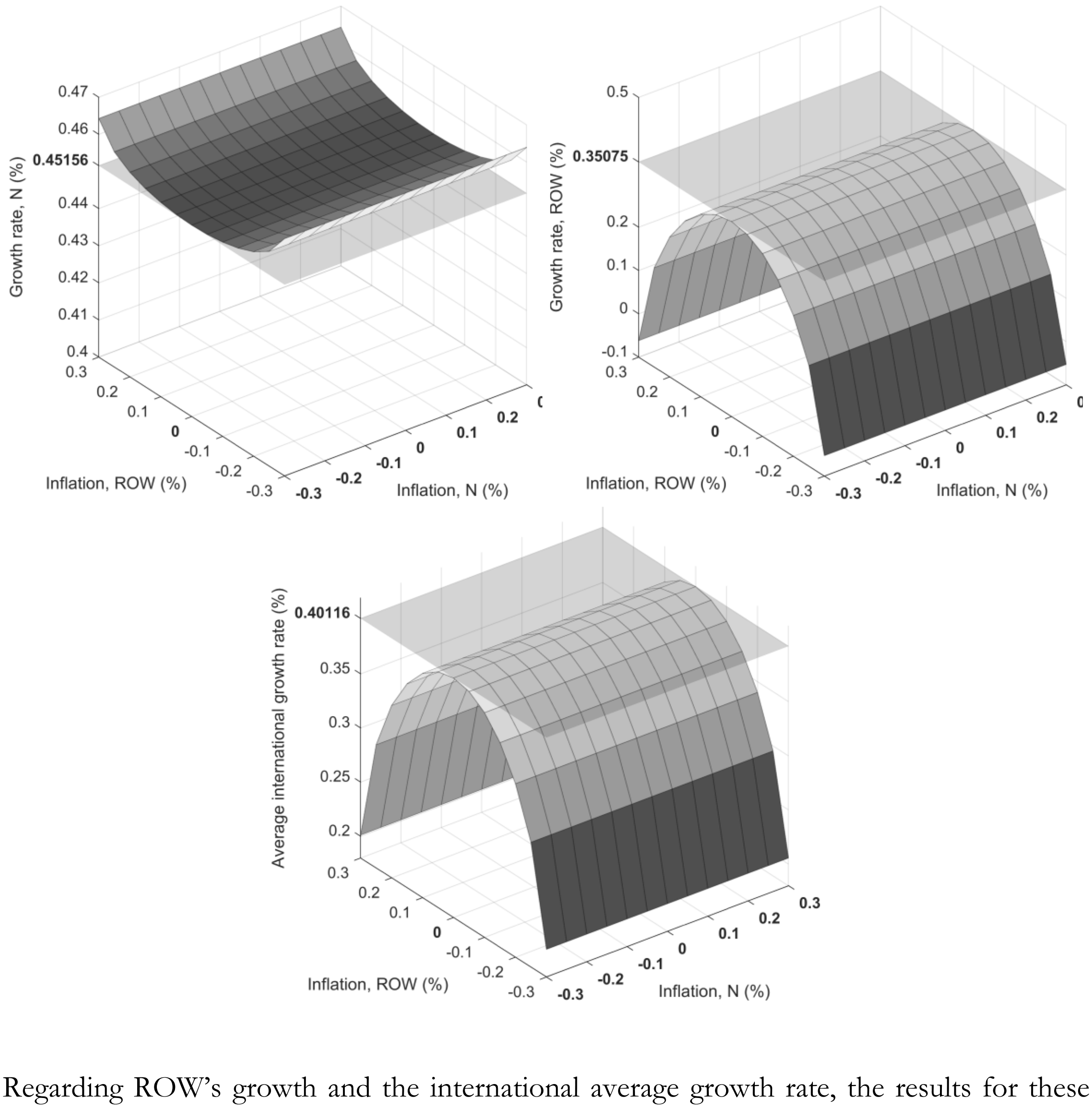
Nation ($\boldsymbol{I = 1, J = 1}$) $\boldsymbol{\beta = 0.999}$
Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta^{\dagger} = 0.999}$



With flexibility, Nation's long-term growth is independent of its own inflation target, but there is now a reversed "contagion of non-neutrality" of monetary policy. In this case, the effect of the contagion on the flexible economy's growth is fully positive since, when the sticky Rest of the World deviates from the optimal null inflation point, it improves Nation's growth, as we discussed in the previous sections, generating this U-shape.

Figure 1.5.8. Inflation-growth relationship for N, ROW and average. Human capital model.

Nation ($\boldsymbol{I = 1, J = 1}$) $\boldsymbol{\beta = 0.999}$
Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta^{\dagger} = 0.998}$



Regarding ROW's growth and the international average growth rate, the results for these simulations are consistent with those obtained for the other two models. Also, the movements in the inflation-growth relationship to movements in the discount factor shown in Figure 1.5.8 are coherent with those obtained so far, with lower growth for the more "impatient" economy.

1.6. Conclusions

This paper has studied the long-term relationship between inflation and growth in three new Keynesian open-economy models with different types of growth engines and price and wage stickiness. The simulations carried out for a world composed of two large countries with the same economic structure, except for the intertemporal discount factor, provided the following main conclusions:

(i) For all three models, the neutrality of the monetary policies holds when price and wage flexibility or only price stickiness is present in both countries, regardless of the values of the intertemporal discount factor.

(ii) However, non-neutrality arises when wages are sticky if the intertemporal discount factors are equal, being null inflation rates in the two countries the policy providing the maximum global growth rates in the three models. This maximum growth rate is the same as when there is wage flexibility in the two countries.

(iii) In the presence of non-neutrality, long-term growth depends not only on each country's inflation target but also on the other country's target. There is no autonomy in monetary policies due to the existence of spillover effects that introduce strategic interaction. In any case, the best policy is null inflation for the two countries.

(iv) Growth rate in (ii) is the same for the two countries in the models of Romer and Aghion and Howitt. In the model of Lucas, it only coincides in the point of null inflation in the two countries, where the value coincides with the corresponding to flexibility, being increasing for null inflation with the distance from zero of the other country's inflation. This possibility entails a "price stability premium" for the country with null or very low inflation.

(v) When the intertemporal discount factors are different, the long-run growth rates for all pairs of inflation rates differ, being the maximum attainable growth in the country with the highest discount rate the lowest of the two countries in all three models: the most "impatient" economy has the lowest growth rate. This highest discount factor has a negative effect on the growth rate of the other country in the model of Romer and positive in the models of Aghion and Howitt and Lucas.

(vi) The severity of the wage stickiness in a country affects in general the attainable growth possibilities given its monetary policy. The lesser (greater) the wage stickiness in a country, the lesser (greater) the influence of its monetary policy on the own and the other country's growth rate. We have named this result "contagion of non-neutrality".

(vi) Finally, an extreme case of this contagion of non-neutrality appears if price and wage flexibility exist (or only wage flexibility) in one of the countries while wages are sticky in the other, the economic growth of the former is independent from its own monetary policy but not from the latter's monetary policy. Therefore, a flexible country will not always be able to reach its maximum attainable growth, because it might suffer the contagion of non-neutrality caused by the wage stickiness of the other country.

In summary, the models presented allows us to show a wide range of circumstances that can cause divergence of growth rates in contrast to Hayashi (1983) and Rebelo (1992), in spite of having maintained mostly the same preference and technology structures. In addition, although the convenience of cooperation between the monetary authorities of the countries is clear, given the presence of important spillover effects affecting the growth rate, the situation is not the same as in Chu et al. (2015) because the optimal policy for all the countries in cooperative and no cooperative situations is zero trend inflation.

Chapter 2

Trend inflation-growth relationship in large economies with unemployment: an extended long-term Phillips curve

Abstract

We extend in this chapter two of the models of Chapter 1, introducing unemployment in the labor market through efficiency wages as in Shapiro and Stiglitz (1984). Our main findings are the following. Monetary policy is neutral for growth and labor market variables with wage flexibility in both economies. They are not able of reaching the growth rate value of wage flexibility when wages are sticky where, according to Friedman's criticism of the Phillips curve, unemployment rate is independent of monetary policy for identical economies or with difference in only the discount rate. Moreover, labor force participation and employment rates behave in response to changes in trend inflation in a similar way to growth rate, with a maximum at null inflation for both economies. With different discount rates, maximum growth, unemployment, LFP and employment rates increase (decrease) in the economy with the higher (lower) discount rate in the Schumpeterian model and the opposite happens in the human capital model.

We confirm the existence of "contagion of non-neutrality", being also extended to the labor market given that, even if an economy has wage flexibility, wage rigidity in the rest of the world ends up causing that foreign monetary policy decisions have an effect on nation's LFP and employment rates, while the national policy still has no effect. In the Schumpeterian model, even though maximum growth are not different between the two economies, we appreciate differences between the values for unemployment, labor force participation and employment rates for each economy. In the economy with sticky wages, unemployment and LFP are higher, while employment is the same. In the human capital model, neither the growth rate nor the labor market variables are the same in the two economies, the four variables are higher in the flexible economy than in the rigid one. With the introduction of unemployment, human capital model no longer presents the positive "contagion effect of non-neutrality" described in Chapter 1 where a flexible nation could improve its growth rate in the presence of deviations from null inflation in ROW. The contagion effect is now clearly negative and makes this model more coherent with the results of the Schumpeterian model.

When the structural differences of the two economies affect more parameters in the human capital model with sticky wages, the relationship "trend inflation-unemployment rate" changes having its minimum for zero inflation, while the relationship "trend inflation-growth" is significantly steeper.

2.1. Introduction

The characterization of the external sector in Chapter 1 has allowed us the determination of the long-term inflation-growth relationship in a world of two large economies with the same or different intertemporal discount rates and different severity of price and wage stickiness within and between the economies. The results confirm that non-neutrality arises when we introduce wage stickiness. Each country's long-term growth rate depends first on the two intertemporal discount factors, secondly on each country's own inflation target, and finally on the other country's inflation target.

With three selected growth engines —Romer (1986), Aghion and Howitt (1992) and Lucas (1988)— and considering wages per effective unit of labor, instead of per hour of worker, we obtained that the maximum long-term growth rate with wage stickiness was reached at the point of null inflation in both economies, with the same or different intertemporal discount rates. When the discount rates of the two countries were different, the country with the highest discount rate always had the lowest growth rate. A lower discount factor in one of the economies also influenced the other's country growth rate, which decreased with Romer (1986) and increased with Aghion and Howitt (1992) and Lucas (1988). In every case, the maximum growth rate with wage stickiness always reached the flexibility rate at null inflation.

Furthermore, we also confirmed a result that we have defined as "contagion of non-neutrality": if one country has price and wage flexibility and the other wage stickiness, the growth of the former is independent of its own monetary policy but not of the latter's monetary policy. Therefore, a flexible economy will not always be able to maximize its attainable growth, because it might be suffering from a "contagion of non-neutrality" caused by the other country's wage rigidity.

Through Chapter 2, we are going to deepen the study of the long-term inflation-growth relationship by integrating new variables that enriches the models and gives them a more general perspective. Particularly, the models proposed in the first chapter assume a hypothesis that, although it is standard in this kind of long-term dynamic models, is not realistic. This hypothesis is that the labor market is in equilibrium, i.e. labor demand equals labor supply and therefore there is no unemployment.

This chapter, thus, introduces the possibility of unemployment within an open-economy framework, assuming again a world of two large economies, each one with their own labor force and without the possibility to migrate from one country to another. The main objective is to analyze the impact on the quoted results after introducing unemployment, to determine if Chapter 1's conclusions continue to be fulfilled within this more realistic scenario, and to measure in what extent the existence of unemployment modifies them. We aim to study the behavior of growth, unemployment, the labor force participation (LFP) and employment to changes in monetary policies in the long-term, with the same and different intertemporal discount rates for the economies, and if the results found come to confirm the Friedman's criticism of the Philips curve within our open-economy framework.

We find precedents in the literature that try to explain the relationship between unemployment and economic growth in the long-term. Bean and Pissarides (1993), and later Eriksson (1997), established a negative relationship between these two variables, but both assumed an exogenous labor force in a closed economy. Later, Chen, Hsu and Lai (2016) consider an endogenous labor force defining a perspective in which changes in labor market institutions can increase or decrease long-term economic growth. Schubert and Turnovsky (2018) deepen in the long-term relationship between growth and unemployment by considering the role of job search and wage bargaining, concluding that long-term trade-offs between unemployment and growth are small: while in the short-term, an increase in productivity would lead to an increase in growth and a decrease in unemployment, in the long-term, unemployment would return to its equilibrium value, fully neutralizing the effect.

In order to address the analysis of this second chapter, we will take two of the three models developed in Chapter 1, the Schumpeterian model of Aghion and Howitt (1992) and the human capital model of Lucas (1988), and introduce a friction in the labor market. We have considered for that purpose the theory of efficiency wages as in Shapiro and Stiglitz (1984) that introduces a distortion involving incentive problems that reduce labor demand and, thus, generates unemployment that acts as a discipline mechanism for workers.

The introduction of unemployment allows us to confirm the Friedman's extension of the long-term Phillips curve, but we also analyze the behavior of two other variables of great macroeconomic importance in response to trend inflation: the LFP and the employment rate.

Under our open-economy framework, these variables behave similarly in the long-term growth rate in response to changes in the trend inflation rate, results that are compatible with a constant value of unemployment rate. This result implies that monetary policy decisions can be adopted to affect these two variables and, thus, the "contagion of non-neutrality" affects LFP and employment rates.

We can highlight two particular results from the simulations carried out for the two models. The first one corresponds to Schumpeterian model, where it can be appreciated that, when both economies have wage stickiness, ROW's monetary policy has a greater impact on N's LFP and employment rates than N's policy itself. The second one is a novelty in the human capital model: the loss of the positive "contagion of non-neutrality" observed in Chapter 1. If N is flexible and ROW has sticky wages, the contagion is always negative: if the sticky rest of world deviates from null inflation, both long-term growth rate and labor market variables —LFP and employment rate— decrease. The introduction of unemployment with efficiency wages, thus, makes the human capital model's results consistent with those obtained in the Schumpeterian model.

Characterization of both models is developed in Section 2.2. Section 2.3 presents the relationship between growth, unemployment, LFP, employment and long-term inflation when both economies have the same consumption intertemporal discount rate, while Section 2.4 presents these relationships when rates differ between countries. Section 2.5 explains the "contagion of non-neutrality" with different severity of price and wage between the economies, detailing the particularities of these two models compared to those presented in Chapter 1. Section 2.6 presents a sensitivity analysis for growth, unemployment, labor force participation and employment to changes in efficiency wages parameters. Finally, Section 2.7 summarizes the main results.

2.2. Two open-economy New Keynesian models with endogenous growth, unemployment and staggered wage and price setting

In this section we will take the Schumpeterian model and the human capital model developed in Chapter 1 and introduce the rigidities of the labor market, no longer considering equilibrium in this market.

For that purpose, we have considered efficiency wages as in Shapiro and Stiglitz (1984), allowing us to define the labor supply, labor demand and therefore, unemployment rate. The following subsections introduce the appropriate modifications in the equations for each agent, first for Aghion and Howitt (1992) and second for Lucas (1988).

2.2.1. Schumpeterian model

Households

Assuming supply and demand are no longer equal in the labor market requires the modification of the expected intertemporal utility of the households, taking now the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{1}{1+v} \int_0^1 N_{st}^{1+v} ds \right) \tag{2.1.1}$$

where now N_{st} represents the labor supply for service $s \in [0,1]$, instead of the term L_{st} that appeared in the previous chapter. In this model L_{st} will represent only the labor demand and will not necessarily have the same value as N_{st}, generating unemployment.

Households will also have changes in their budget constraint. If we assume that the unemployment subsidy is fully funded with taxes on wages, the expression will be

$$C_t + \frac{B_t}{P_t} + R\&D_t = \frac{B_{t-1}}{P_t} R_t^{st} + D_t + (1 - d_t) \int_0^1 \frac{W_{st}}{P_t} N_{st} ds \tag{2.1.2}$$

where d_t is the unemployment rate.

The solution to the decision problem provides an expression for labor supply for service s and the aggregation of services:

$$N_{st} = \left(\frac{1}{C_t}(1-d_{st})w_{st}\right)^{\frac{1}{v}} \qquad N_t = \int_0^1 N_{st}ds \tag{2.1.3}$$

$$\beta E_t\left[\frac{C_t}{C_{t+1}}\frac{R_t^{st}}{\Pi_{t+1}}\right] = 1 \tag{2.1.4}$$

Final good producers

Final good producers operate in a perfectly competitive scenario, each producer choosing the inputs that maximize the profits function. The profits function is

$$F_{Y_t} = P_t\int_0^1 (A_{it}L_t)^{1-\alpha}x_{it}^{\alpha}di - \int_0^1 W_{st}L_{st}ds - \int_0^1 P_{it}x_{it}di \tag{2.1.5}$$

where the first term is the production function according to Aghion and Howitt (1992).

The solution for the maximization problem provides us the labor demand for service s:

$$L_{st} = \left(\frac{(1-\alpha)Y_tL_t^{\frac{1-\sigma}{\sigma}}}{\frac{W_{st}}{P_t}}\right)^{\sigma} \qquad L_t = \int_0^1 L_{st}ds \tag{2.1.6}$$

Aggregating we obtain the labor demand function

$$L_t = \frac{(1-\alpha)Y_t}{\Delta_t^W} \tag{2.1.7}$$

$$\Delta_t^W = \left[\int_0^1 (w_{st})^{1-\sigma}ds\right]^{\frac{1}{1-\sigma}} \tag{2.1.8}$$

where (2.1.8) represents the average real wage.

The solution also provides the intermediate goods demand function:

$$x_{it} = \alpha^{\frac{1}{1-\alpha}}\left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{1-\alpha}}A_{it}L_t \tag{2.1.9}$$

Considering that labor supply is not equal to labor demand anymore, we can define the expression for the unemployment rate from the expressions (2.1.3) and (2.1.6):

$$d_{st} = \frac{N_{st} - L_{st}}{N_{st}} \tag{2.1.10}$$

$$d_t = \frac{\int_0^1 (N_{st} - L_{st}) ds}{\int_0^1 N_{st} ds} \tag{2.1.11}$$

Final goods producers rent labor force to produce their goods. In Chapter 1 wages were set according to the household's members' preferences with Taylor contracts each I periods. To introduce the labor market friction that generates unemployment we must retool this framework. As explained, we are going to introduce efficiency wages as in Shapiro and Stiglitz (1984), which involves incentive problems such as asymmetric information, moral hazard, and adverse selection. The existence of unemployment under this theory works as a discipline mechanism towards employees.

Within this framework employees have a dichotomy: not making effort (shirking), at cost 0, or making effort, at cost e. Employers cannot know the effort made by the employees, so to generate incentives, they must set wages that ensures that employees will choose making effort instead of shirking. For wage flexibility, the arbitrage equations to consider are

$$RV_E^S = w + (b + q)(V_d - V_E^S) \tag{2.1.12}$$

$$RV_E^N = w - e + b(V_d - V_E^N) \tag{2.1.13}$$

$$RV_d = z + d(V_E - V_d) \tag{2.1.14}$$

where V_E^S is the discounted present value for shirking, V_E^N for making effort and V_d for the unemployed. The parameter b represents the probability of employment loss, q the probability of get caught shirking and being fired, z is the utility of unemployment benefits, and $d = \frac{N-L}{N}$ is the job-finding rate.

The employers will set a wage that fulfills the condition $V_E^N = V_E^S$. Considering (2.1.10), and defining the cost of making effort e and the unemployment benefits z per effective unit, we can obtain the effective wage expression

$$w = \frac{z}{A} + \frac{e}{A} + \left(R + \frac{b}{d}\right)\frac{e}{qA} \tag{2.1.15}$$

where we can easily deduce that a higher level of unemployment requires a lower wage to fulfill the condition, hence the discipline mechanism.

In order to introduce wage stickiness —we consider four periods in our analysis—, conditions (2.1.12), (2.1.13) and (2.1.14) are adapted according to a Taylor-type staggering process

$$RV_E^S = \frac{1}{4}\left[w\Delta_w^{bq} + \left(b\Delta_b + q\Delta_q\right)(V_d - V_E^S)\right] \tag{2.1.12′}$$

$$RV_E^N = \frac{1}{4}[w\Delta_w^b - e\Delta_b + b\Delta_b(V_d - V_E^N)] \tag{2.1.13′}$$

$$RV_d = \frac{1}{4}[z\Delta_d + d\Delta_d(V_E - V_d)] \tag{2.1.14′}$$

where $\Delta_b = \sum_{J=0}^{3}(1-b)^J$ is the cumulative probability of being employed, $\Delta_q = \sum_{J=0}^{3}(1-q)^J$ of being employed while shirking and $\Delta_d = \sum_{J=0}^{3}(1-d)^J$ of being unemployed. Also, $\Delta_w^{bq} = \sum_{J=0}^{3}\left(\frac{(1-b)(1-q)}{\Pi}\right)^J$ differences the four values of steady state net wages that coincide each quarter revision for shirking workers, and $\Delta_w^b = \sum_{J=0}^{3}\left(\frac{1-b}{\Pi}\right)^J$ for workers that make effort. Wage flexibility implies $\Delta_b = \Delta_q = \Delta_d = \Delta_w^{bq} = \Delta_w^b = 1$.

Fulfilling again the condition $V_E^N = V_E^S$, we obtain the following expression for the steady state effective wage after each revision:

$$w = \frac{\frac{e}{A}\Delta_b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R + b\Delta_b)}{4R + b\Delta_b + d\Delta_d}\frac{z}{A}\Delta_d}{\Delta_w^b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] - \Delta_w^{bq}(4R + b\Delta_b)} \tag{2.1.15′}$$

Additionally, for both economies:

$$A = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^{*}}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha} L} \tag{2.1.16}$$

Intermediate good producers

The profits function for the producers that generate an intermediate good i under a monopolistically competitive scenario is the same as in Chapter 1:

$$F_{it} = \left(\frac{P_{it}}{P_t} - 1\right) x_{it} \tag{2.1.17}$$

The intermediate producers obtain one unit of intermediate good from one unit of final good. The intermediate output is sold to the final good producers, setting prices for I periods.

Considering the intermediate goods demand function (2.1.9), the profits function can be rewritten as:

$$F_{it} = \left(\frac{P_{it}}{P_t} - 1\right) \alpha^{\frac{1}{1-\alpha}} \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{1-\alpha}} A_{it} L_t \tag{2.1.17'}$$

Introducing price rigidities for I periods, the average expected profits is:

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^{*}}{P_t} - 1\right) \left(\frac{P_{t-s}^{*}}{P_t}\right)^{-\frac{1}{1-\alpha}} \tag{2.1.18}$$

As explained in Chapter 1, from the expected profits if innovation is successful, we obtain the steady state gross growth rate:

$$g = \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{-\tau}^{*}}{P} - 1\right) \left(\frac{P_{-\tau}^{*}}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}} (\gamma - 1) + 1 \tag{2.1.19}$$

Finally, the optimal price $\frac{P^*}{P}$ that maximizes the expected profits function $(2.1.18)$ evaluated in the steady stat is, for $\mathrm{T} = [0, \dots, I-1]$:

$$\frac{P^*_{-\mathrm{T}}}{P} = \frac{1}{\Pi^{\mathrm{T}}} \frac{1}{\alpha} \frac{\sum_{\tau=0}^{I-1} \left(\beta\Pi^{\frac{1}{1-\alpha}}\right)^{\tau}}{\sum_{\tau=0}^{I-1} \left(\beta\Pi^{\frac{\alpha}{1-\alpha}}\right)^{\tau}} \qquad (2.1.20)$$

Central bank

The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate. Trend inflation is considered as given.

Equilibrium conditions and external sector

The aggregate equilibrium of the economy will be defined as

$$Y_t = C_t + R\&D_t + \int_0^1 x_{it} di + X_t - X_t^{\dagger} \qquad (2.1.21a)$$

where, assuming two economies, henceforth "nation" (N) and "rest of the world" (ROW) —denoted with a dagger symbol [†]—, X_t are exports and $X_t^{\dagger}$ are imports. For the sake of simplicity, no public sector will be considered. Consequently, ROW's equation will be:

$$Y_t^{\dagger} = C_t^{\dagger} + R\&D_t^{\dagger} + \int_{i=0}^1 x_{it}^{\dagger} di + X_t^{\dagger} - X_t \qquad (2.1.21b)$$

N's and ROW's exports depend on the real exchange rate e_t and the other economy's production, following the expressions

$$X_t = \rho + \Omega e_t^{\omega} Y_t^{\dagger} \qquad (2.1.22a)$$

$$X_t^{\dagger} = \rho^{\dagger} + \Omega^{\dagger} \left(\frac{1}{e_t}\right)^{\omega^{\dagger}} Y_t \qquad (2.1.22b)$$

where $\rho > 0, \rho^{\dagger} > 0, \Omega > 0, \Omega^{\dagger} > 0, \omega > 0$ and $\omega^{\dagger} > 0$.

As in Chapter 1, from equation $(2.1.21)$ we can obtain an expression for consumption considering $(2.1.9)$, and $R\&D_t = A_t n_{it}$. After normalizing the equation through Y, as all the growing variables must be normalized in the steady state, and considering the relationship between the production of both economies $l_t = Y_t^\dagger / Y_t$ in order to link the normalization of the two countries, we get for N in the steady state

$$C = 1 - \left[\chi \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} - 1 \right) \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right]^{\frac{1}{1-\chi}} A -$$

$$- \alpha^{\frac{1}{1-\alpha}} L \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} A - X + l X^\dagger \qquad (2.1.21a')$$

$$X = (\rho + \Omega e^\omega) l \qquad (2.1.22a')$$

and for ROW

$$C^\dagger = 1 - \left[\chi^\dagger \alpha^{\dagger \frac{1}{1-\alpha^\dagger}} L^\dagger \frac{1}{I^\dagger} \sum_{s=0}^{I^\dagger - 1} \left(\frac{P_{-s}^{\dagger *}}{P^\dagger} - 1 \right) \left(\frac{P_{-s}^\dagger}{P^\dagger} \right)^{-\frac{1}{1-\alpha^\dagger}} \right]^{\frac{1}{1-\chi^\dagger}} A^\dagger -$$

$$- \alpha^{\dagger \frac{1}{1-\alpha^\dagger}} L^\dagger \frac{1}{I^\dagger} \sum_{s=0}^{I^\dagger - 1} \left(\frac{P_{-s}^{\dagger *}}{P^\dagger} \right)^{-\frac{1}{1-\alpha^\dagger}} A^\dagger - X^\dagger + \frac{1}{l} X \qquad (2.1.21b')$$

$$X^\dagger = \left[\rho^\dagger + \Omega^\dagger \left(\frac{1}{e} \right)^{\omega^\dagger} \right] \frac{1}{l} \qquad (2.1.22b')$$

N's and ROW's real external financial positions at time t are defined as the position at time $t-1$ plus interest payments minus net exports at time t, analytically we can define

$$b_t = R_t^i b_{t-1} - X_t + e_t X_t^\dagger \qquad (2.1.24a)$$

$$b_t^\dagger = R_t^i b_{t-1}^\dagger - X_t^\dagger + \frac{1}{e_t} X_t \qquad (2.1.24b)$$

where R_t^i is the appropriate gross real interest rate. If N lends to ROW, the interest rate will be N's interest rate, and vice versa:

$$R_t^i = \begin{cases} R_t, & b_t > 0 \\ R_t^\dagger, & b_t < 0 \end{cases} \qquad (2.1.25)$$

Normalizing considering the lagged variables and noticing the other economy's variables, in the steady state we get the expressions:

$$b = R^i \frac{b_{-1}}{g} - X + elX^{\dagger} \qquad (2.1.24a')$$

$$b^{\dagger} = R^i \frac{b^{\dagger}_{-1}}{g^{\dagger}} - \mathrm{X}^{\dagger} + \frac{1}{el} X \qquad (2.1.24b')$$

The relationship between N's and ROW's external financial position at time t is

$$b^{\dagger}_t - \left(\frac{b^{\dagger}_{t-1}}{\Pi^{\dagger}_{\mathrm{t}}}\right) = -\left[\left(\frac{b_t}{e_t} - \frac{b_{t-1}}{\Pi^{\dagger}_t e_{t-1}}\right)\right] \qquad (2.1.26)$$

which normalized in the steady state takes the form:

$$b^{\dagger} - \left(\frac{b^{\dagger}_{-1}}{\Pi^{\dagger} g^{\dagger}}\right) = -\left[\left(\frac{b_t}{e_t l} - \frac{b_{t-1}}{\Pi^{\dagger}_t e_{-1} l g}\right)\right] \qquad (2.1.26')$$

Lastly, we assume the condition of uncovered interest parity:

$$(R_t - 1) = \left(R^{\dagger}_{t-1} - 1\right) + \frac{e_{t+1} - e_t}{e_t} \qquad (2.1.27)$$

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 2.1. The number of each equation of the system corresponds to a previous one, that number appearing with the superscript [$'$], because they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through production level of the final good Y, each economy by its own. Also, as explained, it is necessary to define the ratio for this variable $l = Y^{\dagger}/Y$ to link the normalizations of the two economies. Normalized variables are denoted with superscript Y, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. N's equations described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have sixty-one equations and sixty-one endogenous variables.

2.2.2. Human capital model

Households

In order to introduce labor supply into the human capital model, we will modify the expected intertemporal utility of the households in the same way as in (2.1.1) and the budget constraint will also now take into account the unemployment subsidy. As this subsidy is fully funded with taxes on wages, the budget constrain will take the form:

$$C_t + \frac{B_t}{P_t} + K_{t+1} = \frac{B_{t-1}}{P_t} R_t^{st} + \mathrm{D}_t + (1 - d_t) \int_0^1 \frac{W_{st}}{P_t} L_{st}\, ds + (1 + R_t - \delta) K_t \quad (2.2.1)$$

From the optimal control problem developed in Appendix 2.2b, we obtain a constant value over time, common to all services s, for labor supply N. With wage flexibility, this will be

$$N = \frac{1}{\xi(1-d)} \left(1 - \frac{\beta}{1+g}\right) \quad (2.2.2)$$

in which g is the growth rate in the steady state

$$g = \frac{\beta}{1 + \delta - R(1 - X^{\dagger})} - 1 \quad (2.2.3)$$

where R is the real interest rate and $X^{\dagger}$ are the exports of the rest of the world.

For wage rigidities, two values for N are obtained, one representing labor services that change wages N^0, and another for those that do not N^1. These values are

$$\begin{aligned} N^0 &= \frac{1}{\xi(1-d)} \left(1 - \frac{\beta \Pi^{J-1}}{1+g}\right) \\ N^1 &= \frac{1}{\xi(1-d)} \left(1 - \frac{\beta \Pi^{-1}}{1+g}\right) \\ N &= \frac{1}{J}\left(N^0 + (J-1) N^1\right) \end{aligned} \quad (2.2.2')$$

while the growth rate will still be the same (2.2.3) expression.

The fraction of time unit devoted to the production activity, considering $g = h_{st+1}/h_{st}$, can be obtained from the accumulation process of human capital

$$h_{st+1} = [1 + \xi(1 - u_{st}(1 - d_{st}))N_{st}]h_{st} \tag{2.2.4}$$

which, as described in Appendix 2.2b, aggregated with wage flexibility is

$$u_t = \frac{1}{1 - d_t}\left(1 - \frac{g}{\xi N_t}\right) \tag{2.2.5}$$

and, with sticky wages, will be

$$\begin{aligned} u^0 &= \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^0}\left[(1+g)\frac{1}{\Pi^3}\left(\frac{N^1}{N^0}\right)^{\nu} - 1\right]\right\} \\ u^{01} &= \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\Pi\left(\frac{N^0}{N^1}\right)^{\nu} - 1\right]\right\} \\ u^1 &= \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\frac{1}{\Pi} - 1\right]\right\} \end{aligned} \tag{2.2.5'}$$

with u^0 the decision for the services that will set the wage the next period, u^{01} for the services that fixed the wage in $J-2$, and u^1 for those in $s \in [0, J-2)$.

Intermediate good producers

Intermediate good producers operate in a perfectly competitive scenario and are uniformly distributed in a continuum, indexed by $j \in [0,1]$. They have a Cobb-Douglas-type production function defined as

$$Y_{jt}^{i} = AK_{jt}^{\alpha}L_{jt}^{1-\alpha} \tag{2.2.6}$$

where Y_{jt}^{i} is the homogeneous output of an intermediate good producer, A is the total factor productivity, K_{jt} the stock of physical capital and L_{jt} a composite index of all labor services.

The optimal conditions are:

$$L_{jt} = \left[\left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{(1-\alpha)A}{\Delta_{W_t}}\right]^{\frac{1}{\alpha}} K_t \tag{2.2.7}$$

$$\Delta_{Wt} = \left[\int_0^1 \left(\frac{W_{st}}{P_t}\right)^{1-\sigma} ds\right]^{\frac{1}{1-\sigma}} \tag{2.2.8}$$

$$R_t = \alpha \left[A\left(\frac{\varepsilon - 1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}} \left[\frac{1-\alpha}{\Delta_{Wt}}\right]^{\frac{1-\alpha}{\alpha}} \tag{2.2.9}$$

The labor demand in the human capital model considering wage flexibility takes the form

$$L_t = (1 - d_t)u_t N_t \tag{2.2.10}$$

which, with sticky wages, as developed in Appendix 2.2b, will be

$$\begin{gathered} L^0 = (1-d)u^0 N^0 \\ L^{01} = (1-d)u^{01} N^{01} \\ L^1 = (1-d)u^1 N^1 \\ L = \frac{1}{J}(L^0 + L^{01} + (J-2)L^1) \end{gathered} \tag{2.2.10'}$$

with L^0 the decision for the services that will set the wage the next period, L^{01} for the services that fixed the wage in $J-2$, and L^1 for those in $s \in [0, J-2)$; and where the unemployment is:

$$d_{st} = \frac{N_{st} - L_{st}}{N_{st}} \tag{2.2.11}$$

$$d_t = \frac{\int_0^1 (N_{st} - L_{st}) ds}{\int_0^1 N_{st} ds} \tag{2.2.12}$$

Employers will set a wage that fulfills the same conditions $(2.1.12)$, $(2.1.13)$ and $(2.1.14)$ explained in the Schumpeterian model. The effective wage expression will be

$$w = z + e + \left(R + \frac{b}{d}\right)\frac{e}{q} \tag{2.2.13}$$

which considering wage stickiness is:

$$w = \frac{e\Delta_b \left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R + b\Delta_b)}{4R + b\Delta_b + d\Delta_d} z\Delta_d}{\Delta_w^b \left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] - \Delta_w^{bq}(4R + b\Delta_b)} \tag{2.2.13'}$$

Final good producers

An infinite number of final good producers, or retailers, are defined over a continuum $[0,1]$. Retailers repackage the intermediate good and sell the final output to the households. Equations from $(1.1.21)$ to $(1.1.25)$ defined for the physical capital externality model are also valid for the human capital model. The optimal price for $\mathrm{T} = [0, \dots, I-1]$ is:

$$\frac{P_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{\mathrm{T}}} \frac{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon})^{\tau}}{\sum_{\tau=0}^{I-1} (\beta \Pi^{\varepsilon-1})^{\tau}} \tag{2.2.15}$$

$$\Delta_{P_t} = \frac{1}{I} \sum_{\tau=0}^{I-1} \left(\frac{P_{t-\tau}^*}{P_t}\right)^{-\varepsilon} \tag{2.2.16}$$

Central bank

The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate. Trend inflation is considered as given.

Equilibrium conditions and external sector

As in Chapter 1, expressions for the consumption to physical capital ratio can be obtained in both economies from the aggregate equilibrium of the economy:

$$\frac{C_t}{K_t} = \frac{Y_t}{K_t} - \frac{K_{t+1} + K_t}{K_t} - \frac{\delta K_t}{K_t} + \frac{X_t}{K_t} - l_t \frac{X_t^{\dagger}}{K_t^{\dagger}} \tag{2.2.17}$$

These expressions, evaluated in the steady state, are:

$$C^K = \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}} \left[\left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{1-\alpha}{\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} - g - \delta + X^K - l {X^{\dagger}}^{K^{\dagger}} \tag{2.2.17a'}$$

$${C^{\dagger}}^{K^{\dagger}} = \frac{{A^{\dagger}}^{\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}} \left[\left(\frac{\varepsilon^{\dagger} - 1}{\varepsilon^{\dagger}}\right) \frac{1-\alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} - g^{\dagger} - \delta^{\dagger} + {X^{\dagger}}^{K^{\dagger}} - \frac{1}{l} X^K \tag{2.2.17b'}$$

Exports will also be:

$$X_t = \Omega e_t^{\omega} Y_t^{\dagger} \qquad X_t^{\dagger} = \Omega^{\dagger} \left(\frac{1}{e_t}\right)^{\omega^{\dagger}} Y_t \tag{2.2.18}$$

$$X^K = \Omega e^{\omega} l \, \frac{A^{\dagger \frac{1}{\alpha^\dagger}}}{\Delta_P^\dagger} \left[\left(\frac{\varepsilon^\dagger - 1}{\varepsilon^\dagger} \right) \frac{1 - \alpha^\dagger}{\Delta_W^\dagger} \right]^{\frac{1-\alpha^\dagger}{\alpha^\dagger}} \qquad (2.2.18a')$$

$$X^{\dagger K^\dagger} = \Omega^\dagger \left(\frac{1}{e} \right)^{\omega^\dagger} \frac{1}{l} \frac{A^{\frac{1}{\alpha}}}{\Delta_P} \left[\left(\frac{\varepsilon - 1}{\varepsilon} \right) \frac{1 - \alpha}{\Delta_W} \right]^{\frac{1-\alpha}{\alpha}} \qquad (2.2.18b')$$

Expressions $(2.1.24a)$ and $(2.1.24b)$ for the economies' real external financial positions, $(2.1.25)$ for the appropriate gross real interest rate, $(2.1.26)$ for the relationship between economies' external financial positions and $(2.1.27)$ for the uncovered interest parity are also considered for this model.

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 2.2. The number of each system's equation corresponds to a previous one, with the superscript [$'$] stating that they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through physical capital stock K, each economy by its own. Also, it is necessary to define the ratio for this variable $l = K^\dagger/K$ to link the normalizations. Normalized variables are denoted with superscript K, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. N's equations described in this section have their counterpart in ROW. In sum, for $I = I^\dagger = 2$ and $J = J^\dagger = 4$ we have forty-nine equations and forty-nine endogenous variables.

2.3. Long-term inflation-growth relationship: economies with the same intertemporal discount rate

Once identified the normalized systems of equations required to determine the steady state values of the endogenous variables for both models, we carried out simulations through

Dynare in order to get these values and the response of growth, unemployment, LFP and employment rates to changes in trend inflation. The current Section 2.3 will present the results when the intertemporal discount rate is the same in both economies. The following Section 2.4 carry out simulations for different discount rates, while Section 2.5 address the contagion of non-neutrality, with different price and wage settings and the same or different intertemporal discount rates. Table 2.3.1 presents the values for the parameters used in our simulations. A complete sensitivity analysis for every efficiency wage parameter is presented in Section 2.6.

Table 2.3.1. Parameter values

Parameter	Description	Schumpeterian	Human
$\beta = \beta^{\dagger}$	Utility discount rate	0.97	0.975
$\alpha = \alpha^{\dagger}$	Output-capital elasticity	0.332	0.332
$\sigma = \sigma^{\dagger}$	Elasticity of substitution for labor services	12	10
$v = v^{\dagger}$	Relative utility weight of labor	1	0.004
$\rho = \rho^{\dagger}$	Exports parameter	0.1	0
$\Omega = \Omega^{\dagger}$	Exports-exchange rate elasticity	0.5	0.5
$\omega = \omega^{\dagger}$	Exports-exchange rate elasticity	0.5	0.5
$\gamma = \gamma^{\dagger}$	Productivity upgrade after every innovation	1.009	-
$\chi = \chi^{\dagger}$	Elasticity of the probability of success in the innovation respect to the relative investment	0.1	-
$\xi = \xi^{\dagger}$	Human capital accumulation productivity	-	1.5
$A = A^{\dagger}$	Total factor productivity	-	1
$b = b^{\dagger}$	Probability of job loss	0.1	0.1
$q = q^{\dagger}$	Probability of being fired if caught shirking	0.9	0.9
$z = z^{\dagger}$	Utility of leisure and unemployment benefits	0.2	0.1
$e = e^{\dagger}$	Labor effort cost	0.04	0.05

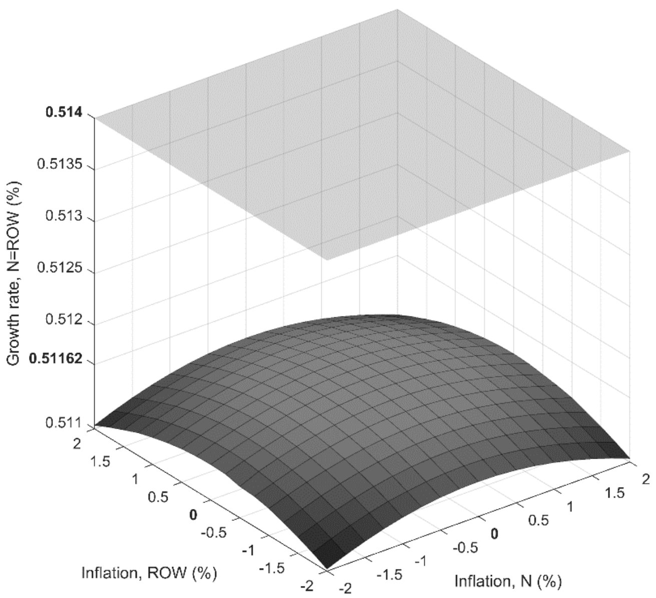
Schumpeterian model results

This section presents the results for the Schumpeterian model when the economies are identical —same utility discount rate— and both have the same combination of wage and price rigidities. We start our analysis with the case of flexibility in prices and wages in both economies ($I = I^{\dagger} = J = J^{\dagger} = 1$). The results show that growth, unemployment, LFP and employment rate are always the same in the two countries, regardless of the inflation targets.

For $\beta = \beta^{\dagger} = 0.97$ and the chosen combination of efficiency wage values, both economies grow at a quarterly rate of $g = g^{\dagger} = 0.514\%$, unemployment is $d = d^{\dagger} = 0.185\%$, the LFP rate is $N = N^{\dagger} = 87.2692\%$ and employment $LL = LL^{\dagger} = 87.107\%$. The monetary policies are neutral because the inflation targets of neither country affect these variables. The results are the same for price rigidity only in both economies.

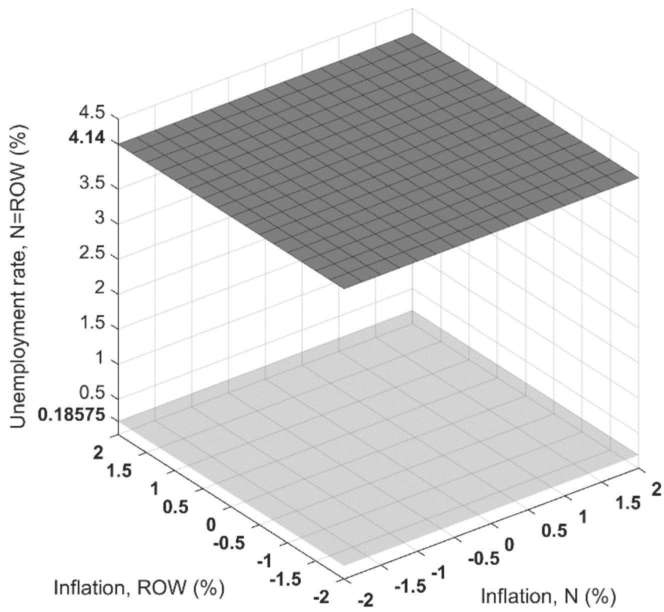
When wages are sticky in both economies, results show at first glance a major difference when compared to those of the previous chapter: the existence of a growth loss, as both economies are incapable of reaching the value of flexibility, like in the previous chapter. At least, for the chosen combination of efficiency wages values, not being possible to exclude yet the possibility that, for other combinations, the flexibility value or even a higher one is reached. A sensitivity analysis is presented in Section 2.6 for these parameters in order to solve this question. Maximum growth rate for both economies is $g = g^{\dagger} = 0.51162\%$, achieved at null trend inflation for both economies $\Pi = \Pi^{\dagger} = 0\%$. Figure 2.3.1 shows the long-term inflation-growth relationship for price and wage stickiness in comparison to the value for flexibility.

Figure 2.3.1. Inflation-growth relationship, Schumpeterian model.
Nation $(\boldsymbol{I} = 2, \boldsymbol{J} = 4)$ Rest of World $(\boldsymbol{I}^{\dagger} = 2, \boldsymbol{J}^{\dagger} = 4)$ $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$



Regarding unemployment, results confirm the Friedman's criticism of the Philips curve within our open-economy framework, as unemployment rate is independent from trend inflation for both wage flexibility and wage stickiness. Figure 2.3.2. shows the long-term inflation-unemployment relationship. For wage stickiness we confirm a notable increase of unemployment for both economies, at $d = d^{\dagger} = 4.14\%$. In short, the results confirm the irrelevance of unemployment in the long-term inflation-growth relationship.

Figure 2.3.2. Inflation-unemployment relationship, Schumpeterian model.
Nation $(\boldsymbol{I} = \mathbf{2}, \boldsymbol{J} = \mathbf{4})$ Rest of World $(\boldsymbol{I}^{\dagger} = \mathbf{2}, \boldsymbol{J}^{\dagger} = \mathbf{4})$ $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$

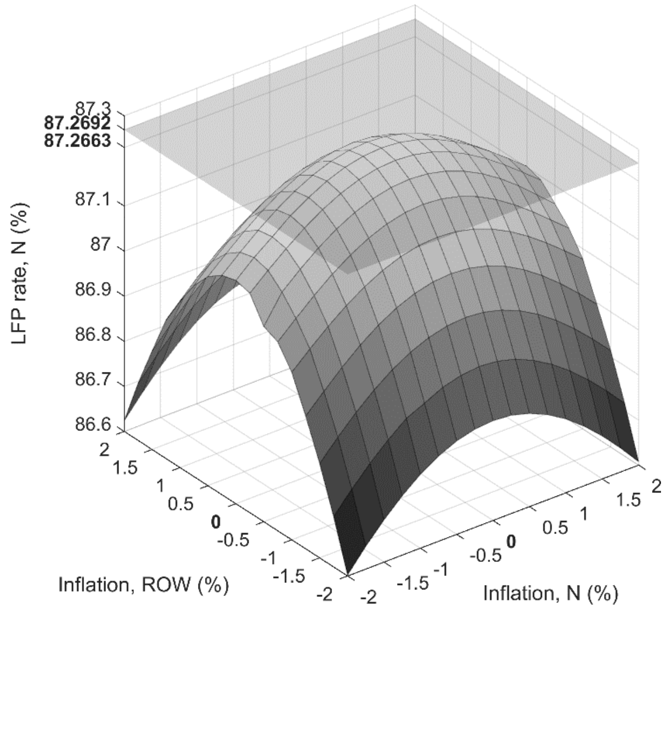


However, we obtain an interesting result analyzing the long-term inflation-LFP and inflation-employment relationships, shown in Figure 2.3.3 and Figure 2.3.4 respectively. An inverted paraboloid shape can be seen with the introduction of wage stickiness for both variables, with a maximum point at null inflation, the exact same point that maximizes growth. In the case of the two variables, it is not possible to reach the value of flexibility. Maximum value for LFP is a just little lower, at $N = N^{\dagger} = 87.2663\%$, and employment presents a greater decrease, at $LL = LL^{\dagger} = 83.6535\%$. This difference in the magnitude of the decrease is the reason for a higher unemployment with wage stickiness.

It might seem that both variables present the same shape we have seen for the growth rate, but a more in-depth analysis shows that this is not exactly the case: the figures, while maintaining their maximum at null inflation, present a more elongated shape. This means that, when both economies have wage stickiness, ROW's monetary policy has a greater impact on N's LFP and employment rates than N's policy itself, and vice versa.

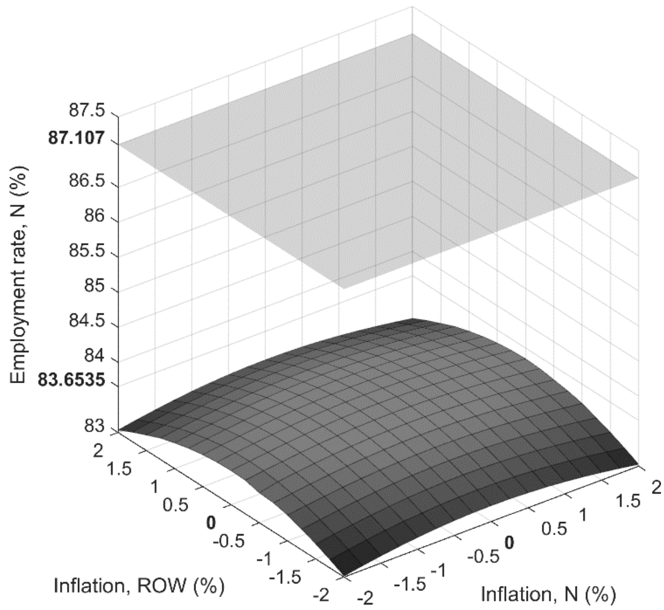
This result, which cannot be appreciated in the model of Lucas, as will be explained below, is mainly a consequence of the movements in consumption levels of each nation due to movements in the exchange rate and, therefore, exports. As expression (2.1.3) reflects, labor supply directly depends on consumption. N's consumption increases as ROW's trend inflation deviates from null inflation, while it decreases when its own inflation target deviates from null inflation. The same is true for consumption in ROW. This has an effect on each nation's long-term results for these two key labor market variables, softening the effect of each nation's own monetary policy with sticky wages.

Figure 2.3.3. Inflation-LFP relationship for N, Schumpeterian model.
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^\dagger = 2, J^\dagger = 4}$) $\boldsymbol{\beta = \beta^\dagger}$



Constant long-term unemployment rates, a result consistent with Friedman's criticism, are therefore compatible with different values for the LFP and employment rates in both countries. This result implies that there is a relationship between growth and these two key labor market variables in the long-term when we introduce wage stickiness. Our results show that a monetary policy decision of a country —inflation targeting— has a direct effect on both its own and the other country's long-term labor force participation and employment, and this effect is transferred and has a direct contribution to the long-term growth dynamics.

Figure 2.3.4. Inflation-employment relationship for N, Schumpeterian model.
Nation $(\boldsymbol{I} = 2, \boldsymbol{J} = 4)$ Rest of World $(\boldsymbol{I}^{\dagger} = 2, \boldsymbol{J}^{\dagger} = 4)$ $\boldsymbol{\beta} = \boldsymbol{\beta}^{\dagger}$

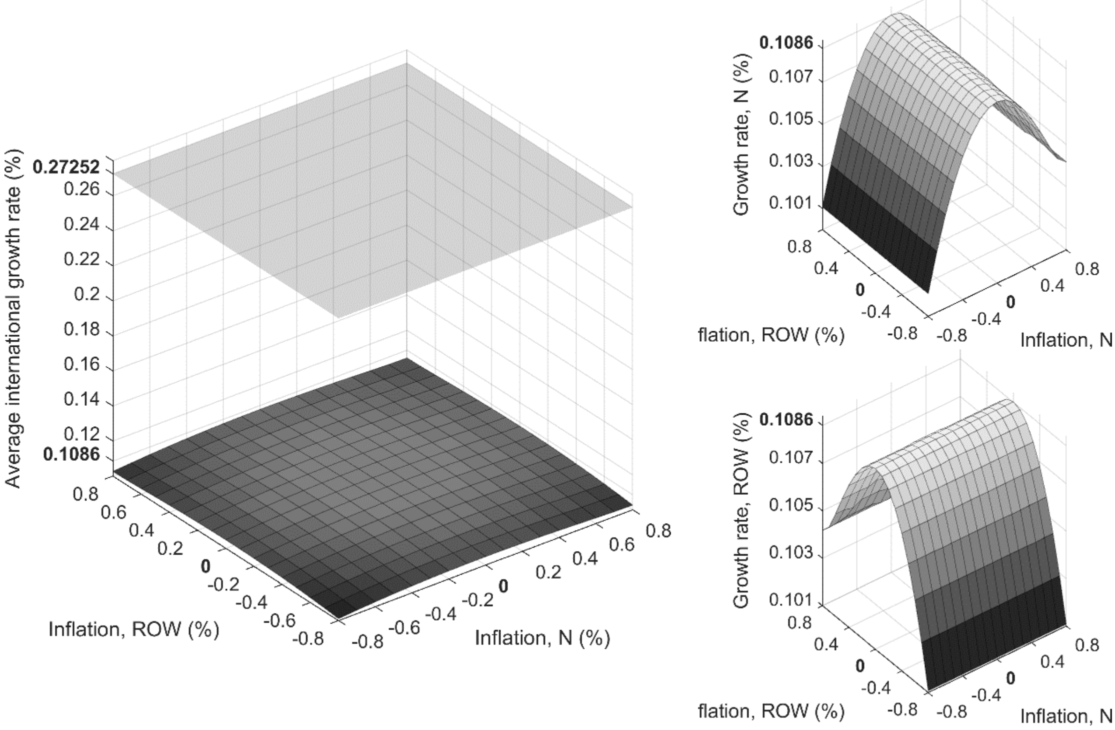


Human capital model results

Let us start our analysis once more with the case of price and wage flexibility and only price stickiness in both economies. As expected, inflation targets do not affect neither long-run growth rates nor the labor market variables. For $\beta = \beta^{\dagger} = 0.975$ and the chosen combination of efficiency wage values, both economies grow at a quarterly rate of $g = g^{\dagger} = 0.27252\%$, unemployment is $d = d^{\dagger} = 8.97199\%$, the LFP rate is $N = N^{\dagger} = 43.3931\%$ and employment $LL = LL^{\dagger} = 39.4999\%$.

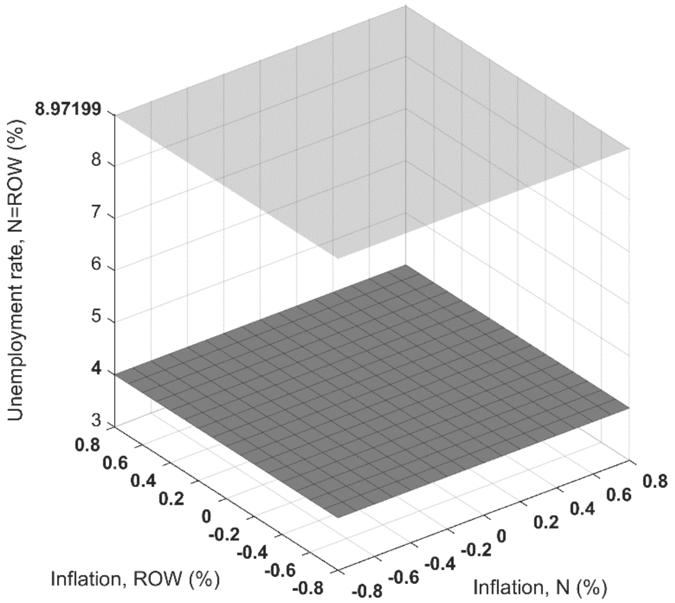
If we introduce wage stickiness in both economies, results show in these case two major differences when compared to those of Chapter 1. The first one is that a loss in growth, LFP and employment can be appreciated, as in the Schumpeterian model. However, unemployment now has a sharp decrease. This is a result already found and detailed in Laguna and Sanso (2020) for a closed economy, which is also translated to our open economy framework.

Figure 2.3.5. Inflation-growth relationship, human capital model.
Nation ($I = 2, J = 4$) Rest of World ($I^{\dagger} = 2, J^{\dagger} = 4$) $\beta = \beta^{\dagger}$



The second result is the loss of the positive "contagion of non-neutrality". When both economies have sticky wages, there is hardly any movement in growth, the LFP or the employment rate to changes in trend inflation from the other economy. We can conclude that there is no contagion when both economies are identical, i.e. same wage stickiness severity or, of course, flexibility. The reason for this result can be understood by analyzing the movements in the interest rate —that must be the same between economies in the long-run, in accordance to the UIRP— and the exchange rate to changes in trend inflations. As seen in expression $(2.2.9)$, the interest rates of each economy depend only on their respective average wages, and efficiency wages of each economy present almost no variations to changes in the trend inflation of the rest of the world for the human capital model, unlike Chapter 1, where wages had a minimum at null inflation.

Figure 2.3.6. Inflation-unemployment relationship, human capital model.
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta = \beta^{\dagger}}$



The two economies will not share, as in Chapter 1, the same growth rate for every combination of inflation target, with the only exception of the point of null inflation. This result is extended also to the LFP and the employment rate, but not to the unemployment rate as it remains constant. An inverted bell shape with a maximum at null inflation can be appreciated if we define an international average growth, LFP and employment rate. These average international rates for the labor market are symmetrical with respect to the null inflation point, not presenting the elongation seen in the Schumpeterian model. This effect disappears because $(2.2.2)$ no longer directly depends on consumption. Maximum growth rate decreases to $g = g^{\dagger} = 0.1086\%$, unemployment decreases to $d = d^{\dagger} = 4\%$, the LFP decreases until $N = N^{\dagger} = 38.777\%$ and employment also decreases to $LL = LL^{\dagger} = 37.2254\%$. Figures 2.3.5 to 2.3.8 collect all these results.

Figure 2.3.7. Inflation-LFP relationship, human capital model.

Nation ($\boldsymbol{I = 2, J = 4}$)   Rest of World ($\boldsymbol{I^\dagger = 2, J^\dagger = 4}$) $\boldsymbol{\beta = \beta^\dagger}$

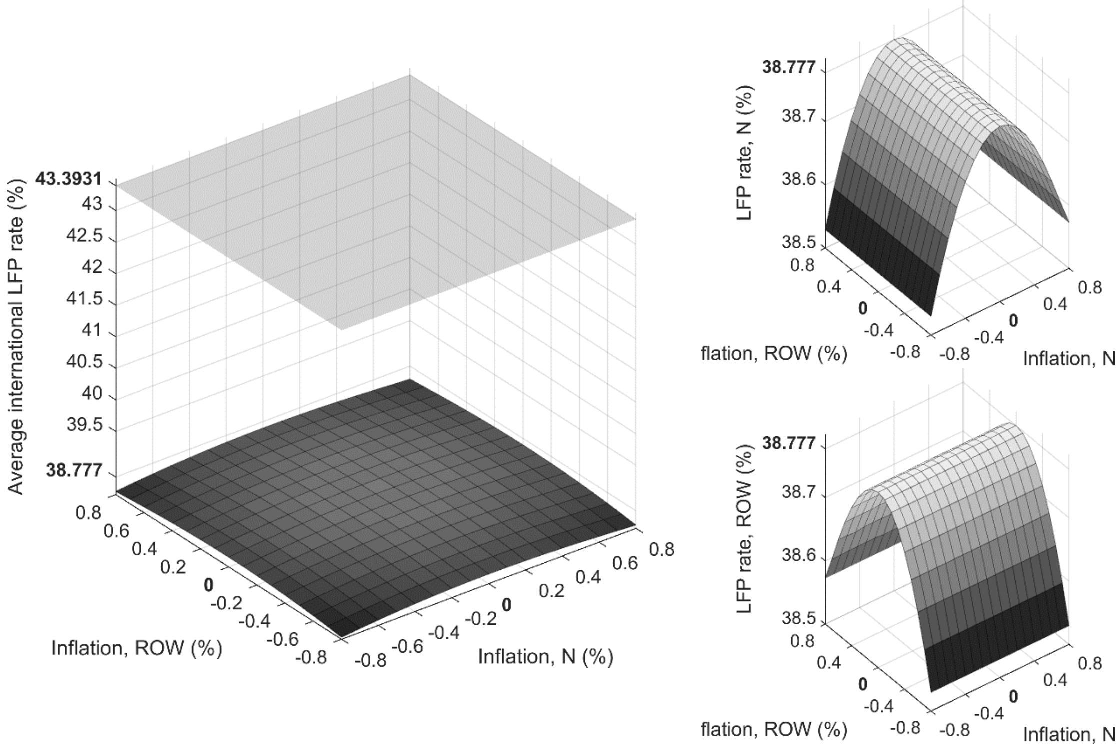
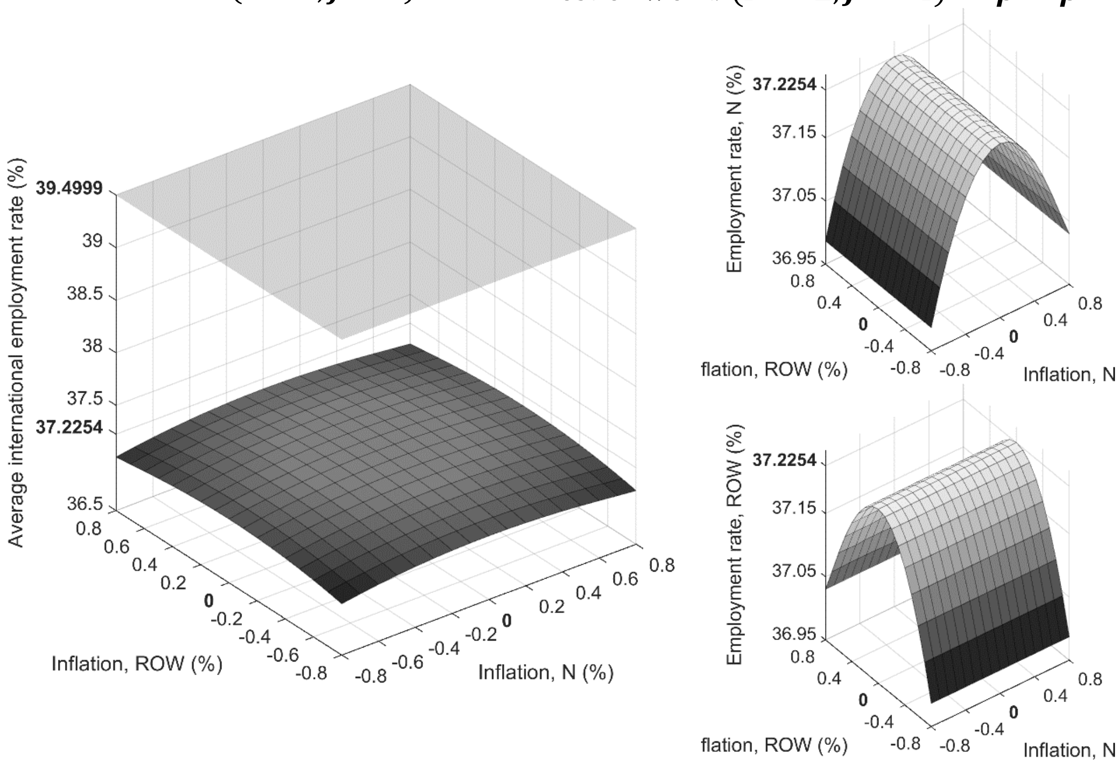


Figure 2.3.8. Inflation-employment relationship, human capital model.

Nation ($\boldsymbol{I = 2, J = 4}$)   Rest of World ($\boldsymbol{I^\dagger = 2, J^\dagger = 4}$) $\boldsymbol{\beta = \beta^\dagger}$



2.4. Long-term inflation-growth relationship: economies with different intertemporal discount rate for consumption

This section presents the results for the simulations carried out with different intertemporal discount rates between the two economies and compares the findings with those found in both the previous section and in Chapter 1.

The values for the parameters summarized in Table 2.3.1 are still valid for these simulations, with the only exception of the utility discount rates β and $\beta^{\dagger}$, that now take the values presented in Table 2.4.1.

Table 2.4.1. Values for the utility discount rates

Parameter	Description	Schumpeterian	Human
β	Utility discount rate for N	0.97	0.975
$\beta^{\dagger}$	Utility discount rate for ROW	0.9699	0.97

Schumpeterian model

Let us start again our analysis with the case of flexibility in prices and wages in both economies. The results are the expected: with flexibility, each country growth, unemployment, LFP rate and employment are the same regardless of the inflation targets. A different intertemporal discount rate gives us different values for each of these variables between the economies.

As in Chapter 1, a decrease in the utility discount rate for ROW leads to both an increase in N's growth and a decrease in ROW's growth. For every value of trend-inflation, growth for N is at $g = 0.52\%$ and for ROW at $g^{\dagger} = 0.509\%$. When wages are sticky, both economies are again incapable of reaching the maximum value of flexibility, with N growing at $g = 0.51741\%$ and ROW at $g^{\dagger} = 0.50705\%$, confirming this gap with the introduction of efficiency wages. Both maximum values are reached at point $\Pi = \Pi^{\dagger} = 0\%$.

Table 2.4.2. Maximum rates with different intertemporal discount factors. Schumpeterian model.

Variable	$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi}\ \ \forall\boldsymbol{\Pi}^\dagger$		$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{2},\mathbf{4})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\boldsymbol{\Pi} = \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$		Variation	
	$\boldsymbol{\beta} = \mathbf{0,97}$ $\boldsymbol{\beta}^\dagger = \mathbf{0,9699}$					
Growth	g	0.52000%	g	0.51741%	Δg	-0.00259%
	$g^\dagger$	0.50900%	$g^\dagger$	0.50705%	$\Delta g^\dagger$	-0.00195%
Unemployment	d	0.23238%	d	4.13834%	Δd	3.90596%
	$d^\dagger$	0.15578%	$d^\dagger$	4.14097%	$\Delta d^\dagger$	3.98519%
LFP	N	96.5650%	N	96.5628%	ΔN	-0.00220%
	$N^\dagger$	80.4939%	$N^\dagger$	80.4908%	$\Delta N^\dagger$	-0.00310%
Employment	LL	96.3406%	LL	92.5667%	ΔLL	-3.77390%
	$LL^\dagger$	80.3685%	$LL^\dagger$	77.1577%	$\Delta LL^\dagger$	-3.21080%

For unemployment, results still confirm the Friedman's criticism of the Philips curve, but now, the unemployment rate of N jumps to $d = 0{,}23238\%$ and for ROW decreases to $d^\dagger = 0{,}15578\%$. The increase (decrease) in N's (ROW's) growth is, considering that long-term interest rates are equal between economies, a consequence of the increase (decrease) in labor demand for N (ROW). This increase (decrease) is done by a decrease (increase) in the average wage which, for flexibility, is equivalent to the real wage. In order for the real wage to fall (rise), considering $(2.1.15)$, an increase (decrease) in the unemployment rate is needed, which is what the results are reflecting. With the introduction of wage rigidities, a notable increase in both countries is registered, reducing the gap between the unemployment rate of the two economies, as N registers $d = 4.13834\%$ and ROW $d^\dagger = 4.14097\%$.[4]

[4] With wage rigidity, the country with the lowest beta is the one that has more unemployment. Contrary to flexibility. However, they are very close to each other and to the value of the same beta. They could be considered as equal, given the close they are.

Finally, both LFP rate and employment increases (decreases) in the nation with a higher (lower) value of the intertemporal utility discount rate. For flexibility, we have a notable jump of LFP rate in the nation, $N = 96.565\%$, and a fall for the rest of the world, $N^{\dagger} = 80.4939\%$. Employment behaves similarly, with $LL = 96.3406\%$ and $LL^{\dagger} = 80.3685\%$.

The inverted paraboloid shape returns if we analyze the long-term inflation-LFP and inflation-employment relationships with wage rigidities. Maximum values for LFP and employment also do not reach the flexibility value for both economies. LFP is slightly under the flexibility value at $N = 96.5628\%$ for the nation and $N^{\dagger} = 80.4908\%$ for the rest of the world, whereas for the employment a larger loss is registered introducing wage rigidity, with $LL = 92.5667\%$ for N and $LL^{\dagger} = 77.1577\%$ for ROW. Movements caused by the introduction of a different utility discount factor are registered in Table 2.4.2.

Human capital model

With flexible prices and wages or only price stickiness in both economies, growth, unemployment, LFP rate and employment are always the same regardless of the inflation targets. A different intertemporal discount rate gives us different values for each variable between the economies, but these movements are not in the same direction as in the Schumpeterian model studied in the previous subsection. Movements in the growth rate with unemployment also are in the opposite direction to those studied in Chapter 1 for the model of Lucas with labor market equilibrium.

For a differentiated discount factor of $(\beta, \beta^{\dagger}) = (0.975, 0.970)$, growth rates are now at $(g, g^{\dagger}) = (0.30985\%, 0.37128\%)$. The country with the highest discount rate has now a higher growth rate. Movements in the same direction can be appreciated for the LFP, with $(N, N^{\dagger}) = (44.4433\%, 53.2872\%)$ and the employment rate, now at $(LL, LL^{\dagger}) = (40.0168\%, 47.9831\%)$. The main conclusion is that, since labor supply is higher, human capital accumulation is also higher and, therefore, the growth rate increases more, particularly because the unemployment rate is very similar[5] despite the different values for the intertemporal discount factor. This unemployment rate interacts with the higher labor supply.

Table 2.4.3. Maximum rates with different intertemporal discount factors, human capital model.

Variable	$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi}\quad\forall\boldsymbol{\Pi}^\dagger$		$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{2},\mathbf{4})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\boldsymbol{\Pi} = \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$		Variation	
	$\boldsymbol{\beta} = \mathbf{0,975}$ $\boldsymbol{\beta}^\dagger = \mathbf{0,970}$					
Growth	g	0.30985%	g	0.10841%	Δg	-0.20144%
	$g^\dagger$	0.37128%	$g^\dagger$	0.13007%	$\Delta g^\dagger$	-0.24121%
Unemployment	d	9.95988%	d	3.99443%	Δd	-5.96544%
	$d^\dagger$	9.95379%	$d^\dagger$	3.99463%	$\Delta d^\dagger$	-5.95899%
LFP	N	44.4433%	N	38.7714%	ΔN	-5.6719%
	$N^\dagger$	53.2872%	$N^\dagger$	46.5154%	$\Delta N^\dagger$	-6.7718%
Employment	LL	40.0168%	LL	37.2227%	ΔLL	-2.7941%
	$LL^\dagger$	47.9831%	$LL^\dagger$	44.6572%	$\Delta LL^\dagger$	-3.3259%

With the introduction of sticky wages in both economies, a loss can be appreciated for every variable, including the unemployment rate —that increased in the Schumpeterian model—, which is consistent with Section 2.3. Results for both cases are compiled in Table 2.4.3.

2.5. Different severity of wage stickiness: contagion of non-neutrality

In Chapter 1, we have confirmed the existence of a "contagion of non-neutrality". This effect caused that a flexible economy will not always be able to maximize its growth, because if the other country had price and wage rigidity, we found that the growth of the former was independent of its own monetary policy but not of the latter's monetary policy.

[5] They are very close to each other and could be considered as equal.

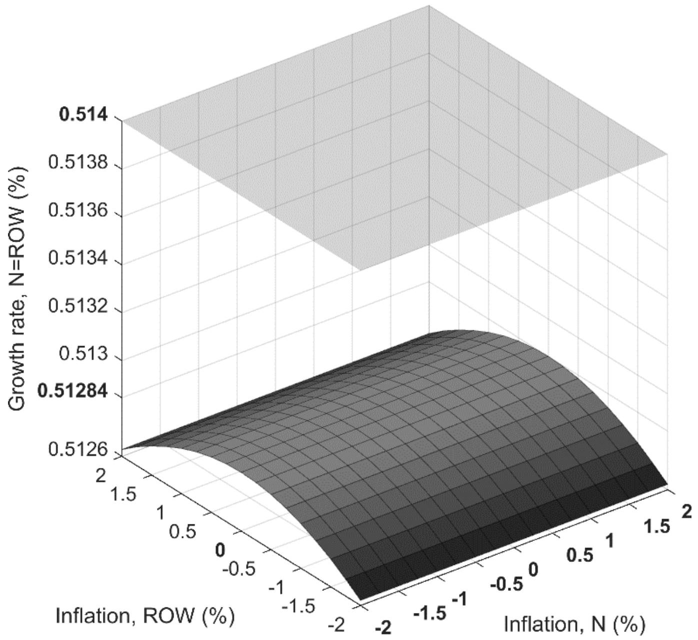
For the simulations in this section, we consider price and wage flexibility in N and price and wage rigidity in ROW. The values for the parameters summarized in Table 2.3.1 are still valid, and we will also differentiate the utility discount rate the same way as in Table 2.4.1.

Schumpeterian model

The result for the relationship between trend inflation and long-term growth considering N has flexible wages and ROW has sticky wages —and both economies have the same values for the parameters shown in Table 2.3.1— is shown in Figure 2.5.1.

Figure 2.5.1. Inflation-growth relationship, Schumpeterian model.

Nation ($\boldsymbol{I=1, J=1}$) Rest of World ($\boldsymbol{I^{\dagger}=2, J^{\dagger}=4}$) $\boldsymbol{\beta=\beta^{\dagger}}$



Once again, we confirm our findings: the long-term growth in N is independent of its monetary policy but not for ROW's monetary policy. As long as ROW's inflation target is kept constant, the achievable economic growth will be constant for the two countries, regardless of N's monetary policy. A change in ROW's inflation target, however, has consequences for ROW's growth but also for N's growth rate. The maximum achievable growth $g=g^{\dagger}=0{,}51284\%$ cannot reach the value of flexibility in both economies, a result consistent with our previous simulations for the Schumpeterian model. Notice that this value for growth rate is between the flexibility in both economies value ($g = g^{\dagger} = 0{,}514\%$) and the stickiness in both economies one ($g = g^{\dagger} = 0{,}51162\%$). We conclude that wage stickiness in the rest of the world has an impact —a contagion of non-neutrality— on a flexible economy's growth greater the greater the severity of the stickiness.

The novelty is that, while unemployment still remains completely independent to inflation targets of both economies, employment and the labor force participation rate also experiment this contagion, with results that show the same shape —a repeated vertical cut of the inverted paraboloid shape at the optimal point for the cases of wage rigidities in both economies— as for growth in Figure 2.5.1. That is, even if an economy has wage flexibility, wage rigidity in the rest of the world ends up causing that the foreign monetary policy decisions have an effect on national's labor market variables, while the national policy still has no effect. Contagion of non-neutrality is, thus, also transferred to the labor market.

Even though maximum growth rates are not different between the two economies, since for two identical economies —same utility discount rate— the maximum long-term growth is the same even with different wage settings, differences can be appreciated between the values for unemployment, the labor force participation and employment rates.

Table 2.5.1 shows the values for these variables at the point of maximum growth $\Pi^{\dagger} = 0\%$ in the case of wage stickiness. The introduction of stickiness in the rest of the world causes the LFP to fall in the nation and rise in the rest of the world since, although the average wage rises in both economies, and rises more in the nation, the effect of consumption, which rises in N while in ROW, neutralizes the effect. Consumption falls due to the effect of exports, which falls in the nation and rises in the rest of the world due to the dynamics of the exchange rate, which decreases in the long term. The employment ends up falling in both economies, due to the increase in average wages in both economies. Movements in unemployment, which falls in the nation and rises in the rest of the world, are a consequence of the movements in LFP and employment.

Table 2.5.1 also includes simulations for economies with different utility discount rates at the points of maximum growth. Movements in all four variables are in the same direction as for the previous scenario, but magnitudes are slightly higher. Growth rates of both economies never reach the value obtained considering wage flexibility.

Table 2.5.1. Effects of introducing sticky wages on ROW, Schumpeterian model.

Variable	$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi},\forall\boldsymbol{\Pi}^\dagger$		$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\forall\boldsymbol{\Pi},\boldsymbol{\Pi}^\dagger = \mathbf{0}\%$*		**Variation**	
			$\boldsymbol{\beta} = \boldsymbol{\beta}^\dagger = \mathbf{0.97}$			
Growth	g	0.51400%	g	0.51284%	Δg	-0.00115%
	$g^\dagger$	0.51400%	$g^\dagger$	0.51284%	$\Delta g^\dagger$	-0.00115%
Unemployment	u	0.18575%	u	0.17812%	Δu	-0.00762%
	$u^\dagger$	0.18575%	$u^\dagger$	4.13970%	$\Delta u^\dagger$	3.95394%
LFP	N	87.2692%	N	85.6140%	ΔN	-1.65520%
	$N^\dagger$	87.2692%	$N^\dagger$	89.1521%	$\Delta N^\dagger$	1.88290%
Employment	LL	87.1070%	LL	85.4615%	ΔLL	-1.64550%
	$LL^\dagger$	87.1070%	$LL^\dagger$	85.4615%	$\Delta LL^\dagger$	-1.64550%
			$\boldsymbol{\beta} = \mathbf{0.97} \quad \boldsymbol{\beta}^\dagger = \mathbf{0.9699}$			
Growth	g	0.52000%	g	0.51832%	Δg	-0.00167%
	$g^\dagger$	0.50956%	$g^\dagger$	0.50796%	$\Delta g^\dagger$	-0.00103%
Unemployment	u	0.23245%	u	0.22022%	Δu	-0.01223%
	$u^\dagger$	0.15578%	$u^\dagger$	4.14082%	$\Delta u^\dagger$	3.98503%
LFP	N	96.5650%	N	94.2565%	ΔN	-2.30850%
	$N^\dagger$	80.4939%	$N^\dagger$	81.8059%	$\Delta N^\dagger$	1.31200%
Employment	LL	96.3406%	LL	94.0489%	ΔLL	-2.29170%
	$LL^\dagger$	80.3685%	$LL^\dagger$	78.4185%	$\Delta LL^\dagger$	-1.95000%

* Except for unemployment where it is $\forall\Pi,\forall\Pi^\dagger$.

Human capital model

With the introduction of the efficiency wage in the human capital model, we obtain remarkably different results from those obtained in Chapter 1. In the previous chapter, when we considered a flexible nation and the rest of the world had sticky wages, we found the possibility of a positive contagion effect of non-neutrality. The nation could only improve its growth rate due to deviations from null inflation in the rest of the world.

Table 2.5.2. Inflation-Exchange-Exports-Growth values, human capital model.
Nation $(\boldsymbol{I = 1, J = 1})$ Rest of World $(\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4})$ $\boldsymbol{\beta = \beta^{\dagger}}$

Infl. N	Infl. ROW	Exchange	Exports N	Growth N	Exports ROW	Growth ROW
0%	-0,8%	1,028190	0,5202280	0,2634910%	0,5060320	0,1010190%
0%	-0,7%	1,027880	0,5202370	0,2635830%	0,5061800	0,1028260%
0%	-0,6%	1,027620	0,5202450	0,2636630%	0,5063060	0,1043810%
0%	-0,5%	1,027400	0,5202520	0,2637300%	0,5064120	0,1056860%
0%	-0,4%	1,027220	0,5202570	0,2637840%	0,5064970	0,1067420%
0%	-0,3%	1,027080	0,5202610	0,2638250%	0,5065610	0,1075490%
0%	-0,2%	1,026980	0,5202640	0,2638550%	0,5066040	0,1081200%
0%	-0,1%	1,026930	0,5202650	0,2638720%	0,5066280	0,1084520%
0%	0,0%	1,026910	0,5202660	0,2638770%	0,5066320	0,1085550%
0%	0,1%	1,026930	0,5202650	0,2638710%	0,5066170	0,1084390%
0%	0,2%	1,026980	0,5202630	0,2638550%	0,5065850	0,1081170%
0%	0,3%	1,027070	0,5202610	0,2638280%	0,5065370	0,1076050%
0%	0,4%	1,027180	0,5202570	0,2637930%	0,5064740	0,1069260%
0%	0,5%	1,027320	0,5202540	0,2637530%	0,5064020	0,1061330%
0%	0,6%	1,027470	0,5202500	0,2637090%	0,5063250	0,1052770%
0%	0,7%	1,027600	0,5202460	0,2636680%	0,5062520	0,1044700%
0%	0,8%	1,027680	0,5202420	0,2636440%	0,5062100	0,1040070%

This result was actually an exception neither Romer (1986) nor Aghion and Howitt (1992) generated something similar. Now, with the introduction of unemployment in Lucas (1988), this result disappears. The contagion effect is clearly negative and, thus, this change makes the human capital model's results consistent with those obtained in the Schumpeterian model. In order to understand why this happens, let us take a look at Table 2.5.2, which provides an analysis similar to Table 1.3.2 in Chapter 1, where we explained the main reason for the positive contagion effect.

Table 2.5.3. Effects of introducing sticky wages on ROW, human capital model.

Variable	$(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (1,1)$ $\forall\Pi, \forall\Pi^\dagger$		$(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (2,4)$ $\forall\Pi, \Pi^\dagger = 0\%$*		Variation	
			$\beta = \beta^\dagger = 0.975$			
Growth	g	0.27252%	g	0.26387%	Δg	-0.00864%
	$g^\dagger$	0.27252%	$g^\dagger$	0.10855%	$\Delta g^\dagger$	-0.16397%
Unemployment	u	8.97193%	u	8.73626%	Δu	-0.23567%
	$u^\dagger$	8.97193%	$u^\dagger$	3.99942%	$\Delta u^\dagger$	-4.97251%
LFP	N	43.3931%	N	43.1497%	ΔN	-0.24340%
	$N^\dagger$	43.3931%	$N^\dagger$	38.7755%	$\Delta N^\dagger$	-4.61760%
Employment	LL	39.4999%	LL	39.3800%	ΔLL	-0.11990%
	$LL^\dagger$	39.4999%	$LL^\dagger$	37.2247%	$\Delta LL^\dagger$	-2.27520%
			$\beta = 0.975 \quad \beta^\dagger = 0.97$			
Growth	g	0.30985%	g	0.29418%	Δg	-0.01567%
	$g^\dagger$	0.37128%	$g^\dagger$	0.12999%	$\Delta g^\dagger$	-0.24129%
Unemployment	u	9.95988%	u	9.55085%	Δu	-0.40903%
	$u^\dagger$	9.95379%	$u^\dagger$	3.99250%	$\Delta u^\dagger$	-5.96129%
LFP	N	44.4433%	N	44.0024%	ΔN	-0.44090%
	$N^\dagger$	53.2872%	$N^\dagger$	46.5132%	$\Delta N^\dagger$	-6.77400%
Employment	LL	40.0168%	LL	39.7998%	ΔLL	-0.21700%
	$LL^\dagger$	47.9831%	$LL^\dagger$	44.6561%	$\Delta LL^\dagger$	-3.32700%

* Except for unemployment where it is $\forall\Pi, \forall\Pi^\dagger$.

There is a significant difference with respect to the movements in the nation's growth and exports to movements in ROW's inflation target, which are both in the opposite direction compared to Table 1.3.2. However, notice that this happens even though the exchange rate continues to increase when ROW deviates from null inflation. The main conclusion we can draw is that the responsible for the change in the direction is the other component of the nation's exports: with the introduction of a sticky efficiency wage in the rest of the world while keeping the nations' wage flexible, ROW's production ends up falling to such an extent that it ends up lowering N's exports and, as a consequence, N's growth rate, despite a favorable exchange rate.

Table 2.5.3 summarizes the effects of introducing sticky wages with the same or different discount factors. All the variables, including unemployment, fall with the introduction of sticky wages in ROW, and they fall to a greater extent if we reduce ROW's discount factor.

2.6. A sensitivity analysis for efficiency wages parameters

This section presents a sensitivity analysis for the main macroeconomic variables —growth, unemployment, labor force participation and employment— to changes in efficiency wages parameters.

As explained in the previous sections, the results obtained in our simulations correspond to a specific set of values of the four efficiency wage parameters shown in Table 2.3.1. In order to complete and close our analysis, a sensitivity analysis is required to determine to what extent the results are consistent with other values of the parameters. In particular, we want to verify if the fact that the maximum growth, LFP and employment rates are lower with wage stickiness than with flexibility is coherent with the most likely values of the parameters.

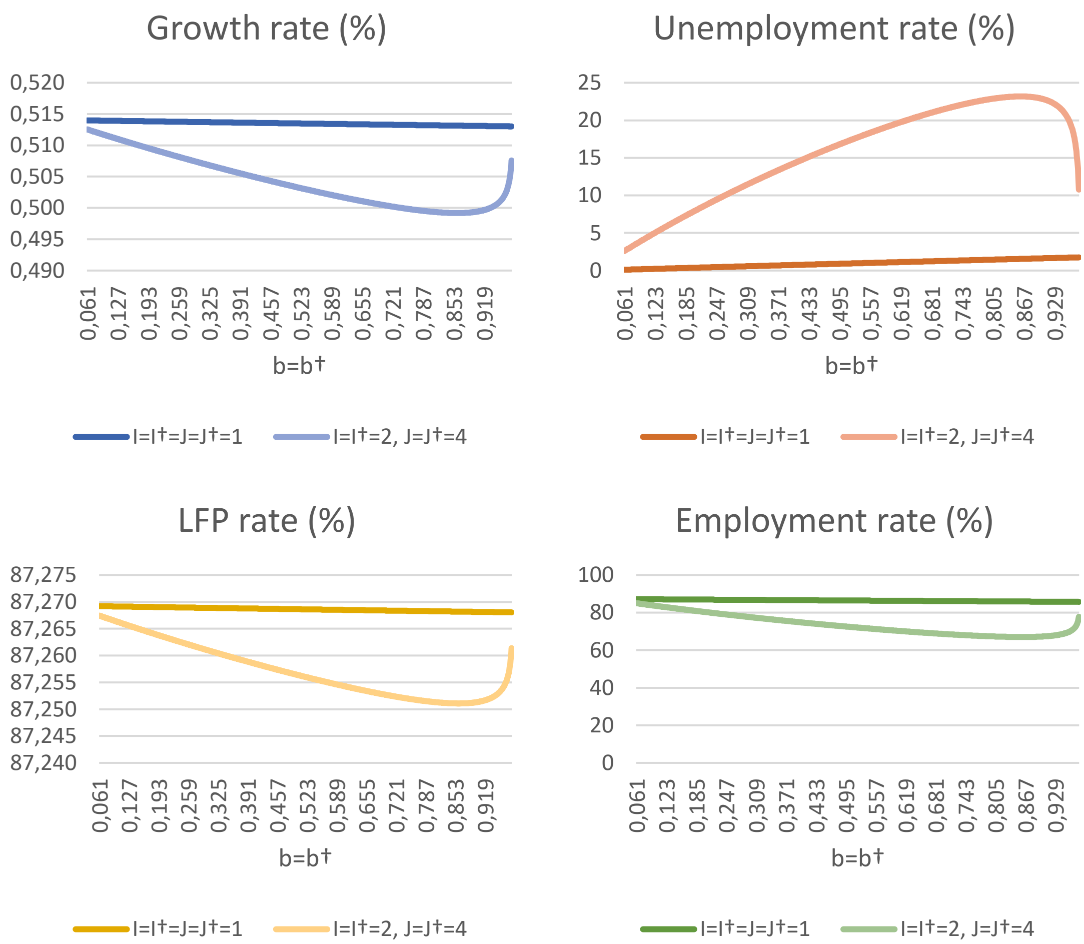
Notice that, even though for wage flexibility in both economies it is irrelevant, for wage rigidities the inflation rate used will be the one that maximizes growth, LFP and employment. In addition, the sensitivity analysis takes place varying each parameter while the rest maintain the value of Table 2.3.1.

2.6.1. Schumpeterian model

Sensitivity analysis for parameter b

The sensitivity analysis for parameter b shows that growth, LFP, and employment with wage flexibility in both economies are always higher than the values for stickiness in both economies. Unemployment for flexibility is always lower than for stickiness also. This result means that our findings in Section 2.4 and 2.5 always hold for any value of this parameter. Figure 2.6.1 shows the response for the four main macroeconomic variables to changes in parameter b for both cases, wage flexibility and stickiness.

Figure 2.6.1. Sensitivity analysis to changes in $b = b^{\dagger}$, Schumpeterian model



Under flexible wages, a minimal change in the four variables can be appreciated. Real wages grow when we increase the parameter, so a slight decrease can be seen in growth, LFP and employment rate, while unemployment increases at the same pace. A bigger impact can be seen under wage stickiness as b grows. This way, the differences between the flexible rates and the sticky rates becomes bigger as the parameters grow until a decline is observed as it approaches $b = b^{\dagger} = 1$.

Sensitivity analysis for parameter q

The sensitivity analysis for parameter q shows that growth, LFP and employment rates with flexible wages are higher than those of stickiness provided that it is greater than $q^{\dagger} = 0.041$. Unemployment rate for flexibility will also be lower from this value. This point, or any lower one, is far from a reasonable value for the probability of being fired if caught shirking. Then we can say that results of the previous sections hold for likely values of the parameter q.

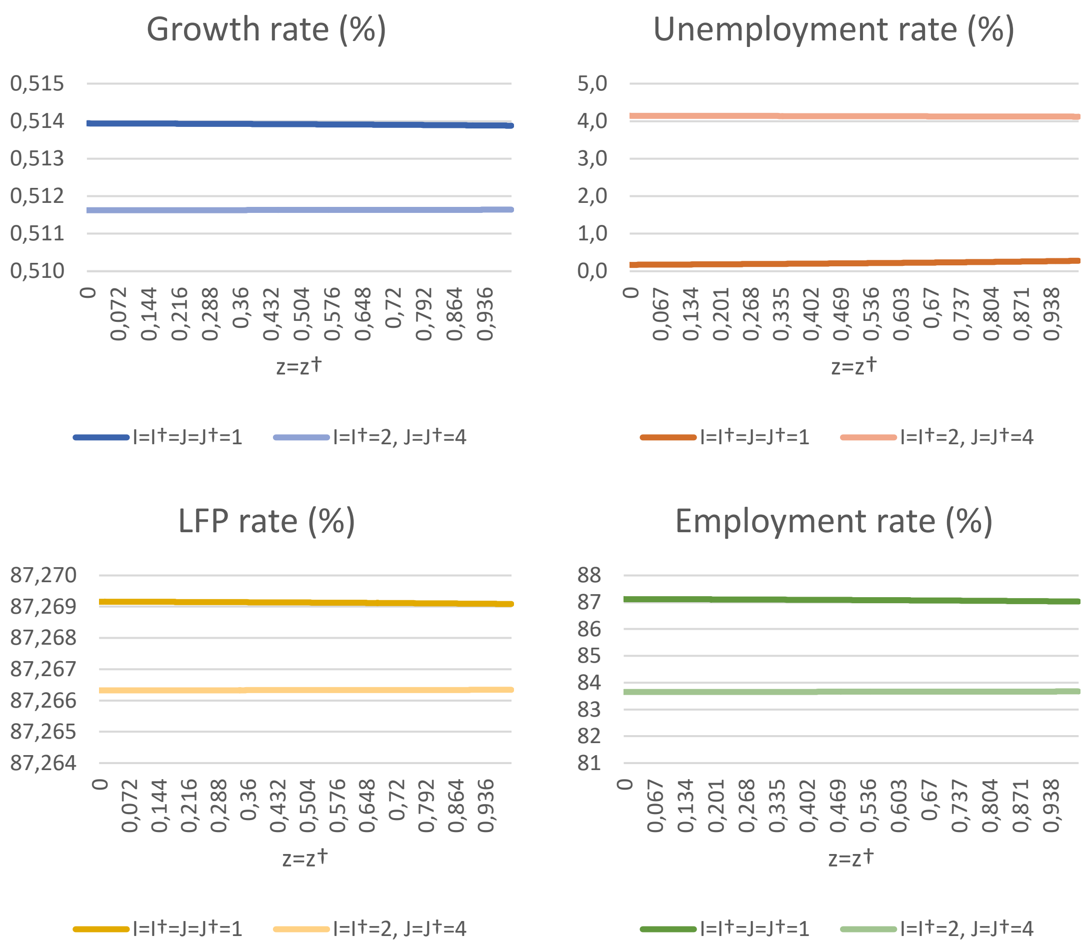
Figure 2.6.2. Sensitivity analysis to changes in $q = q^{\dagger}$, Schumpeterian model

Flexible wages decrease with parameter q, increasing growth, labor force participation and employment rates, while decreasing the unemployment rate. The results are mirrored when sticky wages are considered as they grow with q due to the effect of a constant trend inflation and the cross-effects of the probabilities. This way, the differences between flexible rates and sticky ones becomes bigger as the parameter grows.

Sensitivity analysis for parameter z

For parameters z, flexibility growth, LFP and employment rates are always higher than the sticky ones. The opposite result is obtained for unemployment.

Figure 2.6.3. Sensitivity analysis to changes in $z = z^{\dagger}$, Schumpeterian model

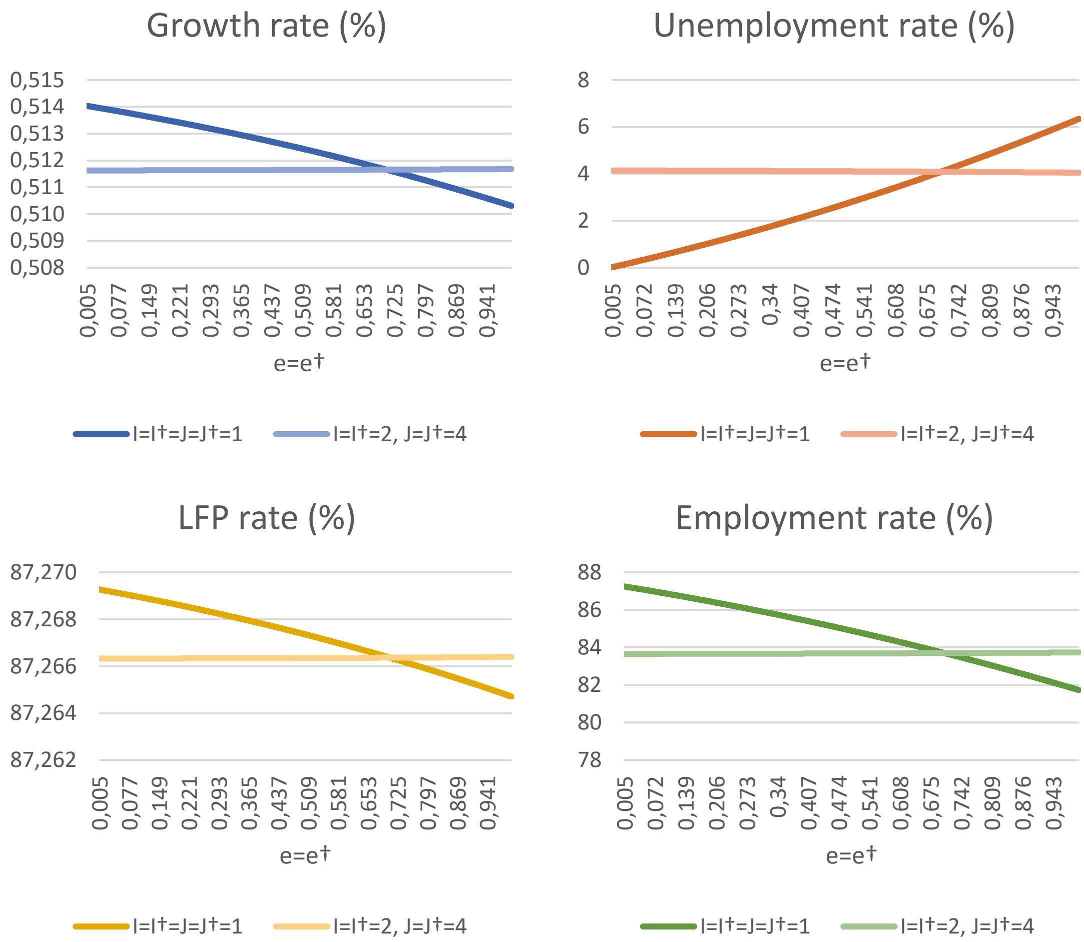


Flexibility values for growth, LFP and employment rates are always slightly above the sticky values, with unemployment slightly increasing under the sticky values. Notice how values for sticky wages almost have no changes at all, due to the effect of a fixed trend inflation and the lack of wage revision.

Sensitivity analysis for parameter e

Finally, for parameter e, the sensitivity analysis confirms that our findings in Section 2.4 and 2.5 hold until these parameter reaches the value $e = e^{\dagger} = 0{,}707$.

Figure 2.6.4. Sensitivity analysis to changes in $e = e^{\dagger}$, Schumpeterian model



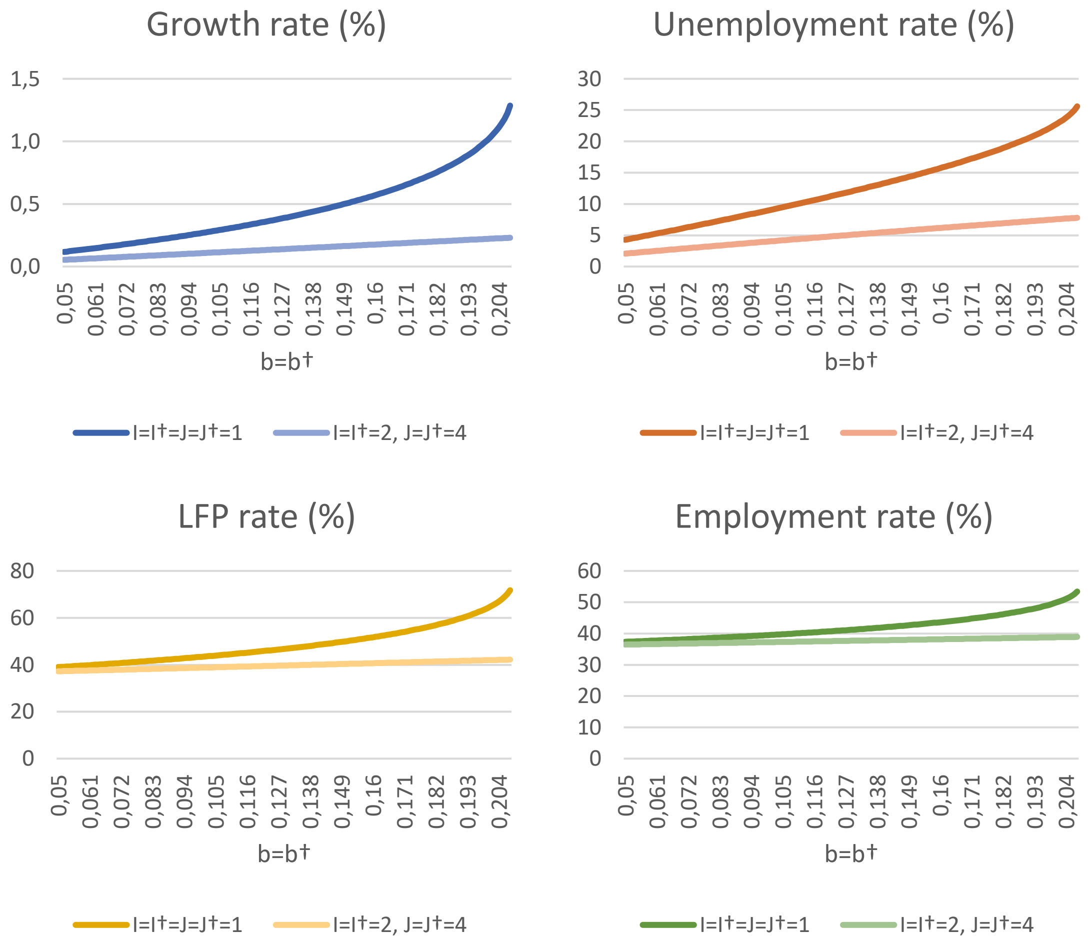
For $e = e^{\dagger} < 0{,}707$ the flexibility average wage is lower than with stickiness. Growth, labor force participation and employment rates are thus higher with flexibility, being the opposite for the unemployment rate. Notice again that, just as we found with parameters z, the variations for the stickiness values to changes in e are almost negligible. Under wage flexibility we have a more substantial reaction to changes in e, significantly decreasing growth, LFP and employment and increasing unemployment. At $e = e^{\dagger} = 0{,}707$ flexibility reaches the level of stickiness and for $e = e^{\dagger} > 0{,}707$ our findings are no longer correct. Reasonable values for labor effort cost are under this value.

2.6.2. Human capital model

Sensitivity analysis for parameter b

The sensitivity analysis for parameter b shows that all the four main variables that we study in this chapter —growth, LFP, employment and unemployment rates— are always higher in the case of flexibility in both economies than with stickiness, no matter the values for parameters b.

Figure 2.6.5. Sensitivity analysis to changes in $b = b^{\dagger}$, human capital model.

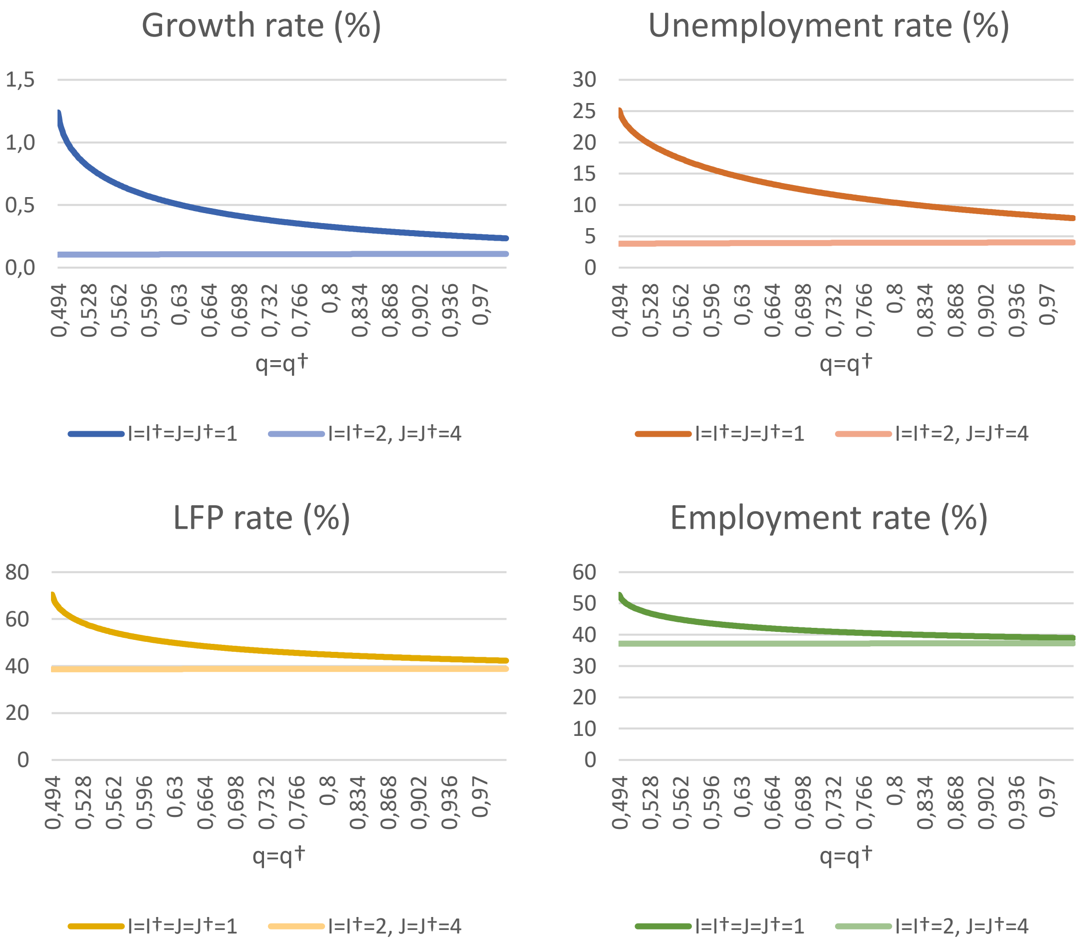


While with stickiness a slight increase can be appreciated in all four variables as the value of b increases, the flexibility values increase in a very pronounced way. While the sticky efficiency wages hardly experience variations with movements in parameters b, the flexible wages rapidly drop as the parameters approach 0.2. We conclude that the results presented in the previous sections for the model of Lucas are valid whatever the values of b in both economies.

Sensitivity analysis for parameter q

The sensitivity analysis for parameter q shows that the values of growth, LFP, employment and unemployment rates for flexibility are always higher than those that we obtain for sticky wages.

Figure 2.6.6. Sensitivity analysis to changes in $q = q^{\dagger}$, human capital model.

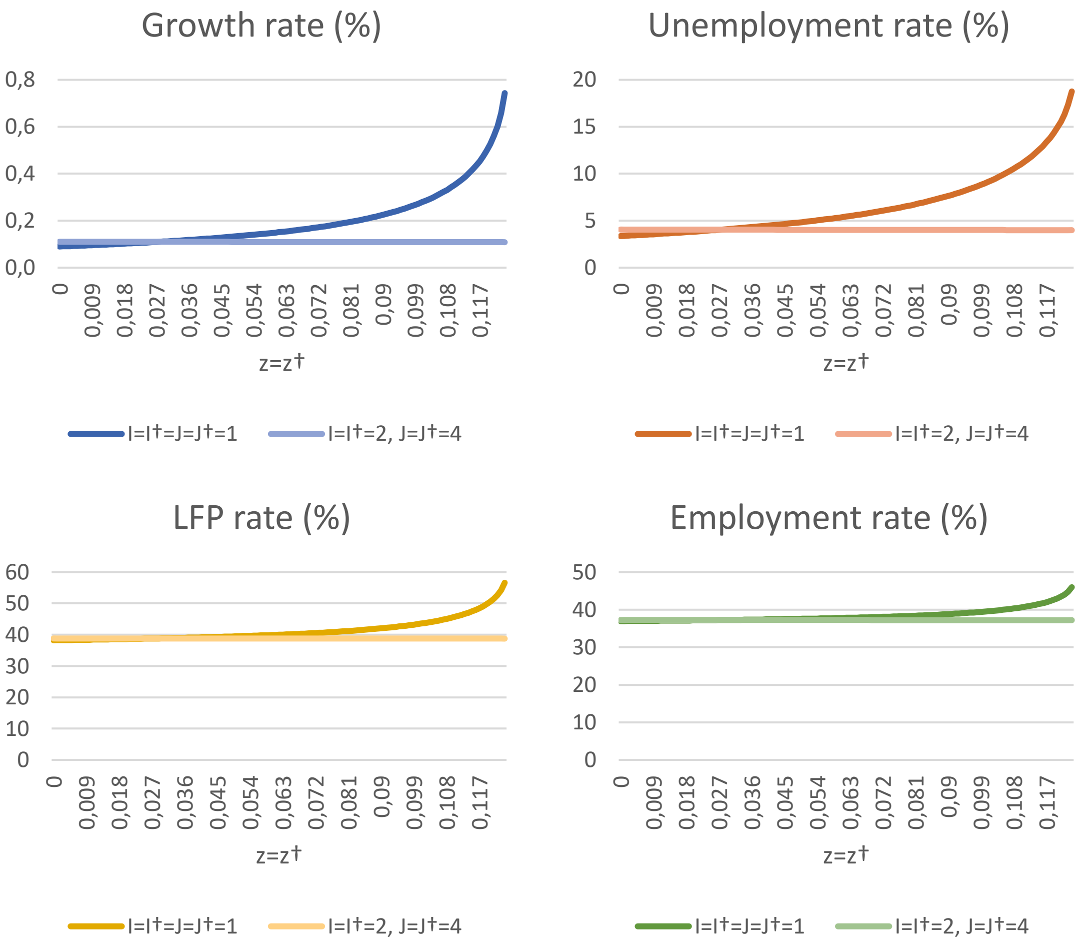


While the sticky efficiency wages are almost constant with movements in parameter q, the flexible wage sharply increase as the parameter increases to 1. For any value of q in both economies, the results presented in this chapter are valid.

Sensitivity analysis for parameter z

As in the previous analysis carried out for the other parameters, the sticky efficiency wage of both economies is hardly affected by movements in the parameters z. For that reason, all four variables remain almost constant in Figure 2.6.7. This is not the case, however, with flexible wages in both economies. As we increase the parameters z, flexibility growth, unemployment, LFP and employment rates are higher, increasing the distance to the stickiness value.

Figure 2.6.7. Sensitivity analysis to changes in $z = z^{\dagger}$, human capital model.

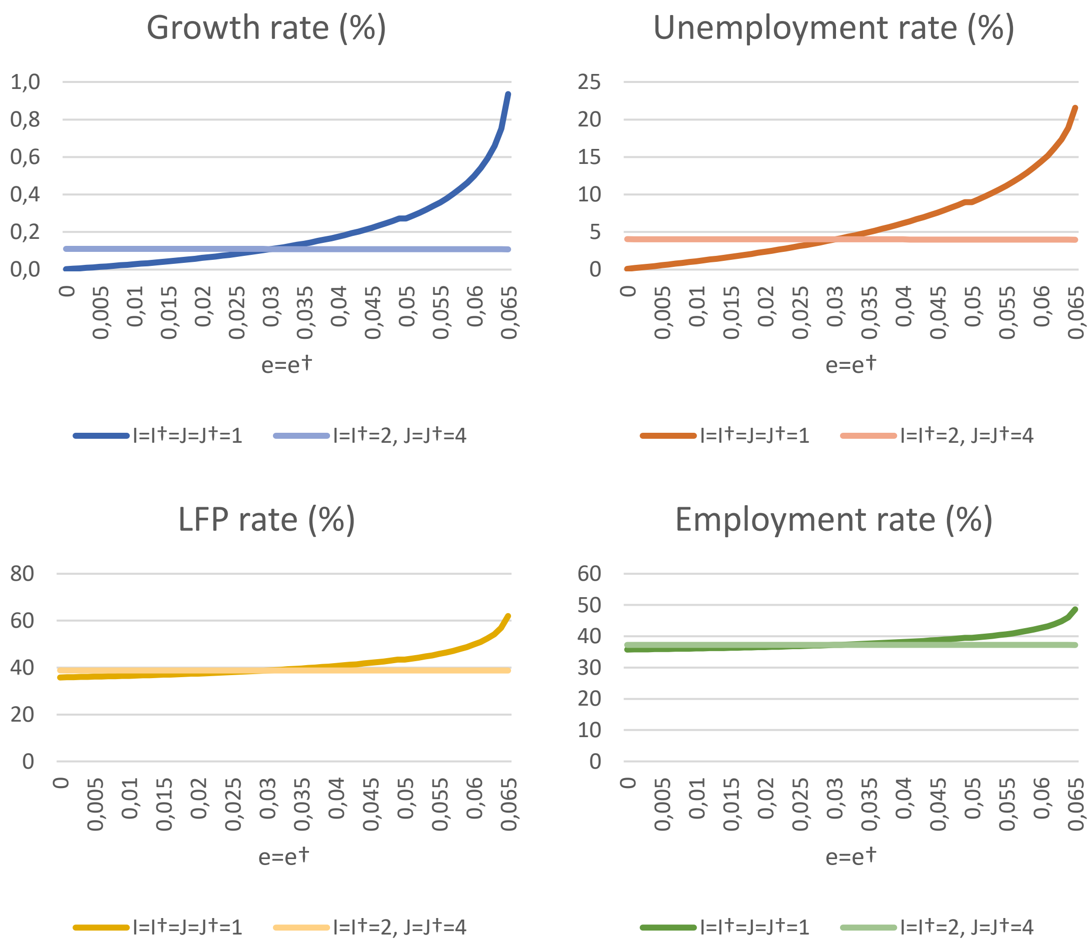


However, in this case we have a threshold at point $z = z^{\dagger} = 0.028$. If we keep lowering the values of parameters z, the flexibility values for all four variables will be under those that are obtained for stickiness. This is no problem, as a reasonable value for the utility of leisure and unemployment benefits should always be higher than this value. We can conclude that the results presented in this chapter are correct whatever the values of parameters z over 0.028.

Sensitivity analysis for parameter e

While the values for all four variables with wage stickiness in both economies remain constant, this is not the case if we introduce flexible wages in both economies, as in the previous sensitivity analysis.

Figure 2.6.8. Sensitivity analysis to changes in $e = e^{\dagger}$, human capital model.



For this parameter, however, there is also a threshold, this time at point $e = e^{\dagger} = 0.031$. As long as the value of these parameters is higher than that point, our results will hold. Most plausible values are usually higher than this value.

The flexible efficiency wage sharply decreases as parameter e increases in both economies and this is the main reason for an increasing difference between the flexible and sticky values as the parameter increases. The unemployment also increases, as there is a higher impact on the LFP rate than in the employment rate.

2.7. A wider set of results as consequence of greater structural differences between economies

So far, the sections of this chapter have analyzed the results provided by the model simulations under specific assumptions. Section 2.3 presented the results for two identical large nations, Section 2.4 went a step further and presented the scenario where the nations differ in a key structural element such as the discount rate, and finally Section 2.5 introduced different combinations of wage stickiness severity between the two economies, allowing us to detect the contagion of non-neutrality.

It is clear that the possibilities of structural differences between the two countries are very wide, so it is very interesting to wonder what results will follow if we deepen even further into the structural differences between their economies. What will happen to the previous results if we calibrate the models for economies with deep structural differences, not even sharing the value of anyone of their parameters? This question is what this section aims to explore in order to show a wider set of results because of greater structural differences between economies. As we are not going to examine all the possibilities, our conclusions will not be exhaustive, but we will try that they are as general as possible.

Inspired in Chapter 3, which introduces an empirical application for six developed countries, this section shows a different perspective of the previous results of this chapter for the model of Lucas and compare them with those obtained after calibrating the model with the parameter values of Spain as N and OECD average as ROW.

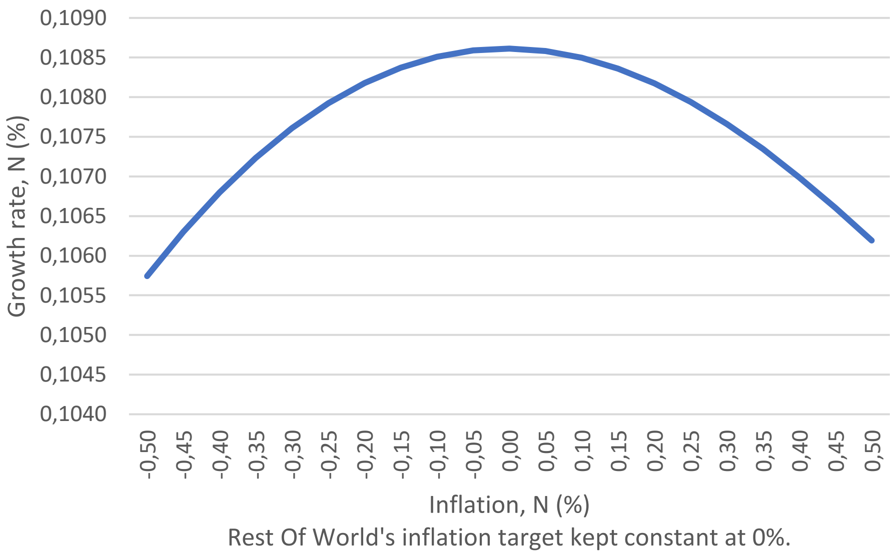
From among a set of six countries selected in Chapter 3 for the empirical application, we have chosen Spain in this section because it is the country with the greatest structural differences related to the rest of the world. This greater differentiation provides more sensitive results than in any other selected country, but overall, the results are also applicable, to varying degrees of intensity, to other countries in the aforementioned set; in particular, to all EMU countries as we will see in Chapter 3.

The simulations presented in this section correspond exclusively to the human capital model. There are several reasons for excluding the Schumpeterian model from this analysis. The first reason is, as we have observed so far, the small variability of the response to changes in the inflation target. The second reason is the also limited variability obtained after increasing the structural difference between economies: movements in the growth rate are still very small and the unemployment rate remains constant. And thirdly, whereas in the human capital model the results that we are going to present show different degrees of intensity depending on the degree of structural differences between economies —the more different the structures are, the greater the gain from moving to the point of null inflation—, this result is not clearly observed in the Schumpeterian model.

Therefore, let us analyze the consequences, in the human capital model, on growth, LFP, employment and unemployment rates, of the fact that the nation has significant structural differences with respect to the rest of the world.

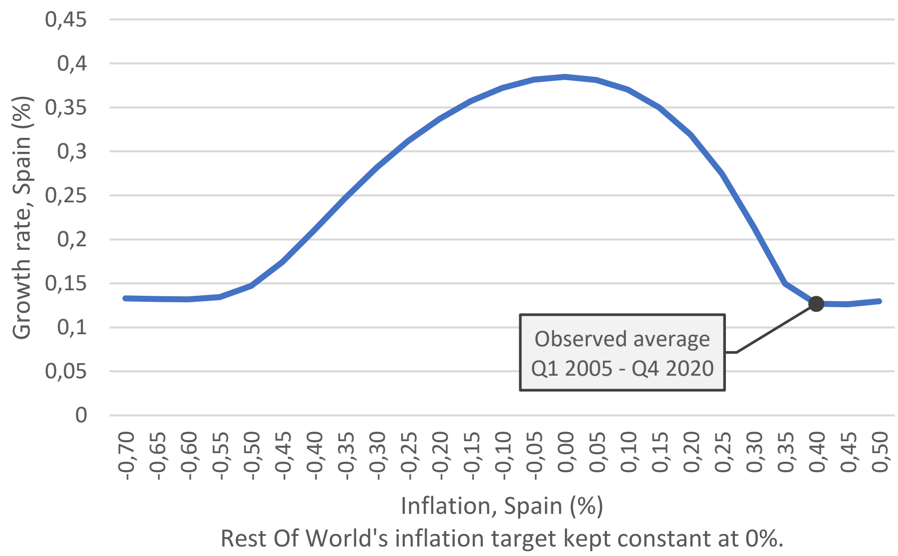
Beginning the analysis with the growth rate, we shall re-examine the results of the simulations for the calibration used in Table 2.3.1 where economies are identical and both have price and wage stickiness, but from a different point of view. Let us take Figure 2.3.5 —the part of the figure that corresponds to the nation— and let us perform a cross section inflation-growth at the point of null inflation for the rest of the world. We obtain Figure 2.7.1a as a result. We will repeat this exercise with the labor market variables as well, in such a way that we hold the rest of the world constant at the point of null inflation while performing movements in the nation's inflation target and, thus, obtaining now a two-dimensional representation of a particular case of the previous simulations.

Figure 2.7.1a. Quarterly growth rate, Ch. 2 calibration (%)



Now, let us calibrate the same model but for the case where the nation is Spain, and the rest of the world is OECD average. We again keep the rest of the world at its point of null inflation and simulate movements in Spain's inflation target, obtaining Figure 2.7.1b. The growth rate increases when Spain moves to the point of null inflation are substantially larger than what we have seen so far in this chapter. From the first quarter of 2005 to the last quarter of 2020, the observed average value for Spain's quarterly growth rate has been 0.12%, and its inflation rate 0.39%[6]. We appropriately identify this point in Figure 2.7.1b. According to the simulations of the human capital model with unemployment, this country could increase its growth rate 0.26 percentage points by moving from its observed average value to the null inflation target. When we carry out the same exercise with the parameters of Table 2.3.1, where both economies are identical, this increase is just 0.002%. Spain provides the extreme case of structural difference among the countries selected in our empirical analysis. We have therefore selected it in this section to illustrate this comparison between high and low structural difference results, but there is a whole range of intensities depending on the structural differences between the economies, as we will see more in depth in the empirical analysis in Chapter 3.

Figure 2.7.1b. Quarterly growth rate w/o financial friction, Spain (%)

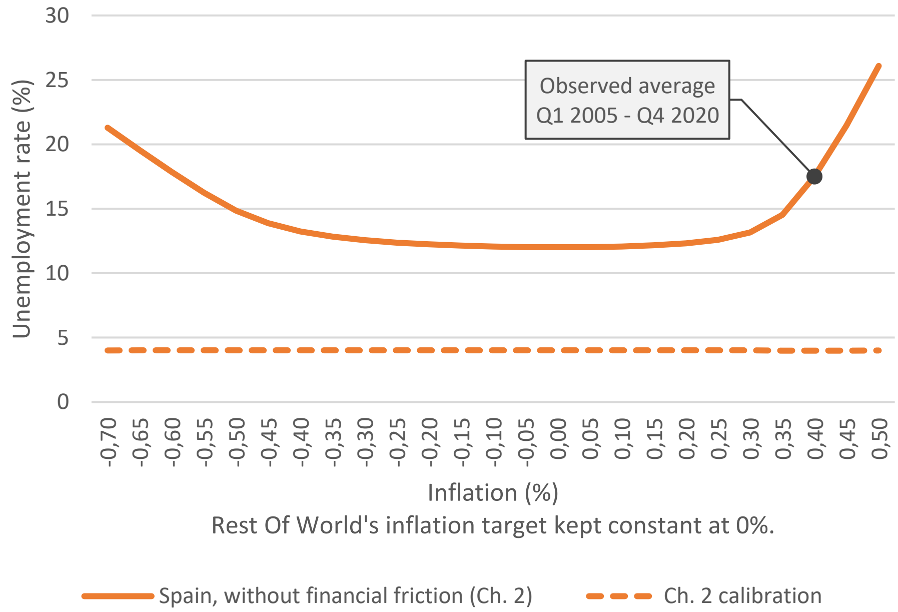


Now, let us analyze what happens with the unemployment rate for the case of Spain. Figure 2.7.2 compares the cross section of Figure 2.3.6 at the point of null inflation in the rest of the world along with the representation of the unemployment rate obtained for each level of inflation in the simulations calibrating the model for Spain. While until now we have seen that the unemployment rate remained constant for any pair of inflation targets, we now see

[6] Data from OECD Statistics [https://stats.oecd.org/].

how this result does not hold when the difference between the structures of the economies is substantially larger. In other words, Friedman's criticism of the Phillips curve is not satisfied, as the results reveal a positive relationship between the long-run inflation and unemployment rates if the first separates sufficiently from zero, in spite of being constant in the surroundings of that value. The simulations for the human capital model calibrated for Spain show then that the unemployment rate has a minimum at null inflation and that any deviation from that point generates a higher unemployment. Once again, the intensity of this result depends on the degree of differences between the structures of the economies. In this case, the observed average value for the unemployment rate in Spain is 17.38%. According to the simulations carried out with the human capital model, the unemployment rate could be lowered to 12% by adopting a zero-inflation policy.

Figure 2.7.2. Unemployment, Ch. 2 calibration vs Spain w/o financial friction (%)



Finally, let us examine the relationship between LFP and employment rates and the inflation rate. Figure 2.7.3a represents the results corresponding to the calibration of Table 2.3.1, while Figure 2.7.3b shows the simulations for the structure corresponding to Spain. This profile shows, for the particular case of Spain, a local maximum at null inflation and, for both the LFP and the employment rate, an increase in both rates starting at a specific threshold of inflation or deflation, with a significantly higher slope for the former. LFP and employment can present a wide diversity of profiles because of the degree of structural difference between the economies.

Figure 2.7.3a. LFP and employment rate, Ch. 2 calibration (%)

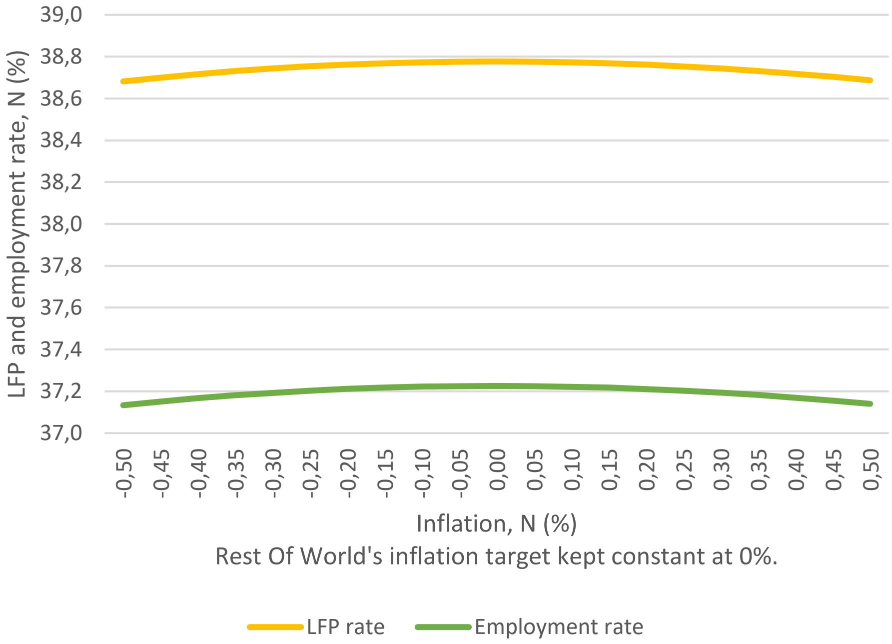
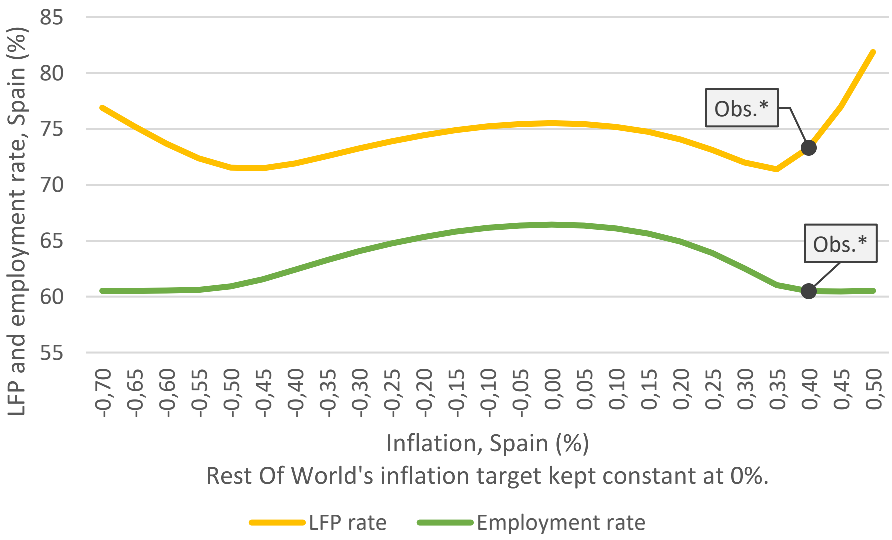


Figure 2.7.3b. LFP and employment rate w/o financial friction, Spain (%)



*Observed average, Q1 2005 - Q4 2020.

In summary, the results provided by the human capital model calibrated for Spain as nation and OECD average as rest of the world show the existence of a wider set of result than those obtained in the previous sections because of substantial structural differences between the economies. We have found, first, a greater variability in the growth rate in response to changes in trend inflation and, secondly, that Friedman's long term Phillips curve does not necessarily hold. In such circumstances, as we have seen with the calibration for Spain, significant reductions in the long-term unemployment rate and increases in the growth rate when adopting a zero-inflation policy are possible, given that the main conclusion provided by these great structural differences show that the maximum growth rate and the minimum unemployment rate take place with null inflation.

2.8. Conclusions

In this chapter, we extended two open-economy New Keynesian models with endogenous growth and nominal rigidities from Chapter 1 —the Schumpeterian model of Aghion and Howitt (1992), and the human capital model of Lucas (1988)— and introduced the possibility of unemployment in the labor market with efficiency wages as in Shapiro and Stiglitz (1984), in order to study the long-term inflation-growth relationship.

Our main findings are the following:

(i) With flexibility in both economies, growth, unemployment, LFP and employment are constant regardless of the inflation targets of the two economies. Monetary policies are neutral from growth and labor market variables point of view.

(ii) Both economies are not able of reaching the growth rate value of flexibility when wages are sticky. The existence of this growth loss is a notable difference compared to Chapter 1.

(iii) Friedman's criticism of the Phillips curve is confirmed in our open-economy framework, as unemployment rate is independent from monetary police with wage stickiness in both countries. A notable increase of unemployment is confirmed with the introduction of these rigidities in the Schumpeterian model and a decrease in the human capital model.

(iv) The labor force participation and employment rates behave in response to changes in trend inflation in a way similar to the case of the growth rate, with a maximum at null inflation for both economies. These results are compatible with the constant value for unemployment.

(v) In the Schumpeterian model it can be appreciated than the rest of the world's trend inflation ends up having a greater impact on the nation's labor force participation and employment rates than the national's own monetary policy itself. This is a consequence of movements in consumption levels of each nation due to movements in the exchange rate. This result is not present in the model of Lucas.

(vi) Different utility discount rates give different values for the variables in the two economies: maximum growth, unemployment, LFP and employment rates increase (decrease) in the economy with the higher (lower) discount rate in the Schumpeterian model. This result is reversed for the human capital model.

(vii) It is also confirmed the existence of the "contagion of non-neutrality" defined in Chapter 1. This effect is also extended to the labor market as, even if an economy has wage flexibility, the wage rigidity from the rest of the world ends up causing that the foreign monetary policy decisions have an effect on national's LFP and employment rates, while the national policy still has no effect. In the Schumpeterian model, even though maximum growth are not different between the two economies, differences can be appreciated between the values for unemployment, the labor force participation and employment rates for each economy as, in the economy with sticky wages, unemployment and LFP are higher, while employment is the same. In the human capital model, neither the growth rate nor the labor market variables are the same between the economies. All four variables are higher in the flexible economy than in the rigid one.

(viii) With the introduction of unemployment, the human capital model no longer presents the positive "contagion effect of non-neutrality" described in Chapter 1 where a flexible nation could only improve its growth rate in the presence of deviations from null inflation in ROW. The contagion effect is now clearly negative and makes this model more coherent with the results obtained in the Schumpeterian model. The main reason is that, with the introduction of a sticky efficiency wage in the rest of the world, their production falls so much that it ends up dragging down the nation's exports and, thus, growth rate.

(ix) Finally, when the structural differences of the two economies affect more parameters in the human capital model with sticky wages, the relationship "trend inflation-unemployment rate" changes having its minimum for zero inflation, while the relationship "trend inflation-growth" is significantly steeper.

Chapter 3

Open economy inflation-growth relationship with unemployment and financial frictions: an empirical application

Abstract

We add to the models of Chapter 2 a financial sector with the same distortion of Gertler and Karadi (2011) and carry out an empirical application of the Lucas model to six countries.

The introduction of the financial sector supposes a fall in growth, supply of labor and employment with flexible as well as rigid wages. If one country has wage flexibility and the other rigidity, the results on the growth are the same in the Schumpeterian model, while in the human capital model the country with flexibility increases growth, labor supply, employment and unemployment and the country with rigidity experiences a fall. The effect on unemployment is not so clear because in the human capital model there is a decrease while in the Schumpeterian one it falls when wages are flexible and increases when they are sticky, although in both cases slightly.

Leverage is independent of the inflation rate in both models with wage flexibility, while with rigidity it is minimum in the Schumpeterian model and maximum in the human capital model for null inflation, although the external finance premium is minimized in both cases. When one country has wage flexibility and the other wage rigidity, deviations from the minimum in the rigidity case induce increases of the leverage in the Schumpeterian model and a reduction in the human capital model.

When the structural differences of the two economies affect more parameters in the human capital model with sticky wages, the relationship “trend inflation-unemployment rate” changes having its minimum for zero inflation, while the relationship “trend inflation-growth” is significantly steeper.

The conclusions of the empirical application leads to a classification of the countries into two groups. The first, which contains the European countries (Germany, France and Spain), presents clear gains in growth and unemployment when moving to zero inflation, while the second (United States, Australia and Japan) barely improves in both indicators. It is a very interesting conclusion because it highlights the European countries facing the rest and because it highlights the relevance of the structural divergence because the reason why moving to the optimal policy is worthwhile is the existence of important structural differences with the rest of the world.

3.1. Introduction

We complete our analysis introducing a new type of agents, financial intermediaries, into the Schumpeterian and human capital open-economy models presented in Chapter 2. The two previous chapters have allowed us to understand the long-term relationships between trend inflation, growth, and key labor market variables within an open-economy framework, showing two large economies that may differ in their long-term behavior because of differences in key economic parameters, such as the intertemporal discount rate or their wage settings processes, or wider structural differences. One of the most outstanding result revealed is the “contagion of non-neutrality”, already found in Chapter 1 for the inflation-growth relationship and confirmed in Chapter 2 for the inflation-LFP and inflation-employment relationships in the long-term. The conclusion of Chapter 2 that the maximum

LFP and employment take place for the inflation rate that maximize the growth rate deserve a special consideration, as well as the extension of the results as consequence of wide structural differences between countries.

This chapter measures the impact that the introduction of financial frictions on both economies has on the results obtained in the two previous chapters and concludes providing an empirical application by estimating and calibrating the human capital model for a set of six developed countries and showing the potential gains of moving to null inflation in the long-run.

The current literature provides us several precedents addressing the relationship between the financial system and the long-term economic growth, particularly between growth and the leverage ratio. Goldsmith (1969) was the first to consider that the financial structure could influence economic growth. However, although he successfully documented the evolution of national financial systems, he was unable to provide much empirical evidence due to data limitations. Subsequently, Auerbach (1985) found a positive relationship between firm's profits growth and the leverage ratio in the long-term. Lang, Ofek and Stulz (1996) found that the leverage ratio does not reduce growth for firms considered as good investments, yet a negative relationship appears for those that are not so well considered by capital markets. Arestis, Luintel and Luintel (2008), in contrast to earlier results such as Levine (2002) or Beck and Levine (2002), found that the financial structure influences economic growth, but the effect of leverage differs country-to-country. Gambacorta, Yang and Tsatsaronis (2014) maintain that, up to a certain point, banks and markets drive economic growth, but, beyond that limit, bank loans or financing through the market no longer provide real growth. Finally, Laguna (2019) shows a non-conclusive influence of the leverage ratio on the grow rate, since the results depend on the type of financial friction introduced. The consensus so far is, thus, that there is no clear relationship between growth and leverage ratio in the long-term that is generally applicable to all scenarios, even the absence of this relationship cannot be dismissed.

Two are the main objectives of this chapter. The first one is to find out whether a distortion in the financial markets has an impact on the results obtained in the previous two chapters and to verify whether the leverage ratio has any relationship to the long-term growth and labor market variables that can be generalized under our open-economy framework. For this purpose, we have selected asymmetric information as in Gertler and Karadi (2011) in order to introduce the financial distortions in both the Schumpeterian and the human capital model. As in previous chapters, all simulations are computed with the Dynare toolbox.

Once the long-term simulations are complete, our second objective of this chapter is to explore the empirical implications of the models. Our analysis is carried out for six developed economies —Australia, France, Germany, Japan, Spain and the United States— in order to propose policies that might let them achieve their long-term economic growth, labor force participation and employment potential. The statistical information is obtained from the OECD database. The empirical application has been done through a calibration after the estimation process, which adjusts the parameters of the models in such a way that each nation achieves its observed values for growth, inflation and labor market variables in the long run, while, at the same time, the rest of the world approximates to the OECD average observed values.

After integrating the new agent and incorporating the moral hazard problem between the financial intermediaries and the depositors as in Gertler and Karadi (2011), we found that the simulations show in both models a drop in economic growth, LFP and employment when both economies have either flexibility or nominal rigidities. The effect on the unemployment rate is less clear. While there is a decrease in both wage settings in the human capital model, in the Schumpeterian model there is a slight decrease with flexibility in both economies and a slight increase with rigidities.

When the nation is flexible and the rest of the world is rigid, in the Schumpeterian model both nations have a decrease in growth, LFP and employment after the introduction of the financial intermediaries. Unemployment falls very slightly in the flexible nation and rises slightly in the rigid nation. In contrast, in the human capital model, while the flexible nation experiences an increase in growth, unemployment, LFP and employment, the rigid rest of the world experiences a slight fall. This is due to the increase in the exchange rate after the introduction of the financial intermediaries, which benefits the exports of the flexible nation.

In the models considering price and wage flexibility, leverage is constant whatever the inflation targets, as well as growth and labor market variables. When we consider wage rigidities, while in the Schumpeterian model we obtain that growth, employment and LFP are maximized at the point where leverage is minimized, in the human capital model, leverage is maximized. When the nation has wage flexibility and the rest of the world is rigid, a contagion effect is also found in the leverage, with the rest of the world's monetary policy influencing the nation's leverage. Deviations from the point of null inflation in the rest of the world generate increases in the nation's leverage in the Schumpeterian model and decreases in the human capital model.

Therefore, no clear effect of the leverage ratio on economic growth can be inferred since, although the same financial friction has been introduced in both models —Gertler and Karadi (2011)—, growth maximization is achieved with the minimization of leverage in Aghion and Howitt (1992) but with the maximization of leverage in Lucas (1988).

For the empirical analysis, we focus our attention exclusively on analyzing the results obtained for the human capital model since, as we have seen in the developments so far, the improvements in the key macroeconomic variables are very small with the model of Aghion and Howitt (1992). The results for the human capital model allow us to divide the countries into two groups. The first group (Germany, Spain, France) shows clear increases in economic growth and falls in unemployment when these countries move to null inflation while the rest of the world does not change its policy, and the second group (Australia, Japan, United States) shows very moderate or insignificant improvements. The reasons for this division is the degree of the structural heterogeneity between each country and the rest of the world.

Section 3.2 will introduce the appropriate changes on both models in order to introduce the financial sector in both economies. Section 3.3 presents the relationship between long-term growth and the labor and financial variables, with the same or different discount rates and wage settings. Section 3.4 explains the effects of considering financial frictions on maximum growth rates on both models. Section 3.5 presents a sensitivity analysis for growth, unemployment, labor force participation, employment and leverage to changes in efficiency wages parameters and to the proportion of assets diverted by the financial intermediary. Section 3.6 provides the empirical analysis for the selected six developed countries and, finally, Section 3.7 summarizes the main results.

3.2. Two open-economy New Keynesian models with endogenous growth, unemployment, financial frictions, staggered wage, and price setting

The following subsections introduce a new agent —the financial sector— and the appropriate modifications in the equations for each of the previous agents, first for the Schumpeterian model based on Aghion and Howitt (1992) and second for the human capital modes based on Lucas (1988). We have selected asymmetric information as in Gertler and

Karadi (2011) as the way of introducing financial distortions in both the Schumpeterian and the human capital model.

3.2.1. Schumpeterian model

Households

Households will now provide funds to the financial intermediaries with their deposits. As households' members do not need funding, their decision problem is not affected by the introduction of the financial friction. The expected intertemporal utility of the households and the budget constrain that must be satisfied are the same as in Chapter 2.

Financial intermediaries

Gertler and Karadi (2011) introduce financial frictions by defining a model based on asymmetric information where the financial intermediaries have more information than their depositors —the households—. Financial intermediaries lend their funds to the non-financial firms.

Let T_{jt} be the amount of wealth —net worth— that a financial intermediary has at the end of period t, B_{jt} the deposits the intermediary gets from households, and S_{jt} the total credit, the intermediary balance is:

$$S_{jt} = T_{jt} + B_{jt} \tag{3.1.1}$$

Over time, net wealth evolves as the difference between earnings on assets and interest payments on liabilities, expressed as

$$T_{jt+1} = R_{t+1}^{k} S_{jt} - R_{t+1} B_{jt} = \left(R_{t+1}^{k} - R_{t+1}\right) S_{jt} + R_{t+1} T_{jt} \tag{3.1.2}$$

where R^k is the return of the intermediary assets. Any growth in net wealth will involve a premium $(R^k - R)$ earned on the intermediary assets.

As the intermediary will not fund assets with a discounted return lower than the discounted cost, the inequality

$$E_t \beta^{i}\left(R^k_{t+1+i} - R_{t+1+i}\right) \geq 0 \tag{3.1.3}$$

must be satisfied, introducing the financial frictions, where β is the discount factor.

The intermediary's objective is to maximize its expected wealth:

$$V_{jt} = E_t \sum_{i=0}^{\infty} \beta^i T_{jt+i} = E_t \sum_{i=0}^{\infty} \beta^i \left[\left(R^k_{t+1+i} - R_{t+1+i}\right) S_{jt+i} + R_{t+1+i} T_{jt+i}\right] \tag{3.1.4}$$

At each period, the financial intermediary might decide to divert a fraction λ of funds. The cost will be that depositors will origin a bank run and the bankrupt the intermediary trying to recover the remaining $(1-\lambda)$ rate of the funds. Consequently, the households will deposit in the financial intermediary if the following incentive constraint is satisfied:

$$V_{jt} \geq \lambda S_{jt} \tag{3.1.5}$$

The expected wealth can be simplified as

$$V_{jt} = \nu_t S_{jt} + \eta_t T_{jt} \tag{3.1.4'}$$

where ν_t is the marginal return of an additional unit of investment and η_t the marginal return of an additional unit of wealth, and can be expressed as

$$\nu_t = E_t\left(R^k_{t+1} - R_{t+1}\right) + \beta E_t\left(G(S)_{t+1}\, \nu_{t+1}\right) \tag{3.1.6}$$

$$\eta_t = E_t R_{t+1} + \beta E_t\left(G(T)_{t+1} \eta_{t+1}\right) \tag{3.1.7}$$

where $G(S)_{t+1} = S_{jt+1}/S_{jt}$ is the gross growth rate in assets and $G(T)_{t+1} = T_{jt+1}/T_j$ is the gross growth rate of net worth.

The incentive constrain can be expressed as:

$$\nu_t S_{jt} + \eta_t T_{jt} \geq \lambda S_{jt} \tag{3.1.5'}$$

From the previous expression, we can obtain the leverage ratio, fulfilling the condition

$$S_{jt} = \frac{\eta_t}{\lambda - \nu_t} T_{jt} = \phi_t T_{jt} \tag{3.1.8}$$

where $\phi_t = \eta_t/(\lambda - \nu_t) = S_{jt}/T_{tj}$ is the leverage ratio.

Considering ϕ_t does not depend on elements specific to each financial intermediary, we can rewrite it as

$$S_t = \phi_t T_t \tag{3.1.9}$$

where $S_t = \sum_j S_{jt}$ and $T_t = \sum_j T_{jt}$.

We can now express the growth rates of T and S as:

$$G(T)_{t+1} = \left(R_{t+1}^k - R_{t+1}\right)\phi_t + R_{t+1} \tag{3.1.10}$$

$$G(S)_{t+1} = \frac{\phi_{t+1} T_{jt+1}}{\phi_t T_{jt}} = \frac{\phi_{t+1}}{\phi_t} G(T)_{t+1} \tag{3.1.11}$$

Finally, the total wealth of the financial intermediaries can be expressed as:

$$T_t = \Gamma\left[\left(R_t^k - R_t\right)\phi_{t-1} + R_t\right]T_{t-1} + \psi R_t^k \phi_{t-1} T_{t-1} \tag{3.1.12}$$

$$G(T)_t = \Gamma\left[\left(R_t^k - R_t\right)\phi_{t-1} + R_t\right] + \psi R_t^k \phi_{t-1} \tag{3.1.13}$$

where Γ is the banker's survival rate and ψ the wealth proportion of the new bankers.

Final good producers

As in previous chapters, the final goods producers operate in a perfectly competitive scenario, each producer choosing the inputs that maximize the profit function. Now, the final goods producers' profit function will consider the financial cost. In order to do that, the expression is rewritten as:

$$F_{Y_t} = P_t \int_0^1 (A_{it} L_t)^{1-\alpha} x_{it}^{\alpha} di - \left(1 + R_t^k\right)\left(\int_0^1 W_{st} L_{st} ds + \int_0^1 P_{it} x_{it} di\right) \tag{3.1.14}$$

From the solution to the maximization problem, we obtain the demand function for labor services and intermediate goods, as in previous chapters, now taking the form:

$$L_{st} = \left(\frac{(1-\alpha) Y_t L_t^{\frac{1-\sigma}{\sigma}}}{R_t^k \frac{W_{st}}{P_t}} \right)^{\sigma} \tag{3.1.15}$$

$$x_{it} = \alpha^{\frac{1}{1-\alpha}} \left(\frac{(1+R_t^k) P_{it}}{P_t} \right)^{-\frac{1}{1-\alpha}} A_{it} L_t \tag{3.1.16}$$

If we aggregate (3.1.15), we obtain the new expression for the labor demand function:

$$L_t = \frac{(1-\alpha) Y_t}{R^k \Delta_t^W} \tag{3.1.17}$$

Additionally:

$$A = \frac{1}{\left(\left(\frac{\alpha}{R^k} \right)^{\frac{1}{1-\alpha}} \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{(1+R^k) P_{-s}^*}{P} \right)^{-\frac{1}{1-\alpha}} \right)^{\alpha} L} \tag{3.1.18}$$

All the expressions introduced in Chapter 2 regarding the labor market friction generating unemployment with efficiency wages maintain the same expressions.

Intermediate good producers

The intermediate producers obtain one unit of intermediate good from one unit of final good. The intermediate output is sold to the final good producers, setting prices for I periods. Considering the intermediate goods demand function (3.1.16), the profits function is rewritten as:

$$F_{it} = \left(\frac{P_{it}}{P_t} - 1 \right) \alpha^{\frac{1}{1-\alpha}} \left(\frac{(1+R_t^k) P_{it}}{P_t} \right)^{-\frac{1}{1-\alpha}} A_{it} L_t \tag{3.1.19}$$

Introducing price rigidities for I periods, the average expected profits is:

$$VF_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} L_t \frac{1}{I} \sum_{s=0}^{I-1} \left(\frac{P_{t-s}^*}{P_t} - 1 \right) \left(\frac{(1+R_t^k) P_{t-s}^*}{P_t} \right)^{-\frac{1}{1-\alpha}} \tag{3.1.20}$$

As in previous chapters, assuming the success of the innovation probability function $\Phi(n_{it}) = n_{it}^{\chi}$ where $0 < \chi < 1$, $\Phi'(n_{it}) = \chi n_{it}^{\chi-1} > 0$ and $\Phi''(n_{it}) = \chi(\chi-1)n_{it}^{\chi-2} < 0$, if the innovation is successful, the expected profits will be $\Phi(n_{it})VF_{it}^{*}$ where $n_{it} = S_{it}/A_{it}^{*}$ and now S_{it} is the credit utilized to fund the $R\&D$ activity. Taking this into consideration, the expected profit of the innovation activity will be now:

$$\Phi\left(\frac{S_{it}}{A_{it}^{*}}\right)VF_{it}^{*} - \left(1 + R_t^k\right)S_{it} \tag{3.1.21}$$

Following Chapter 1, the optimal value for n, common to all innovators, will be:

$$n_{it} = n = \left[\frac{\chi}{1 + R_t^k}\alpha^{\frac{1}{1-\alpha}}L_t\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{t-s}^{*}}{P_t} - 1\right)\left(\frac{\left(1 + R_t^k\right)P_{t-s}^{*}}{P_t}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}} \tag{3.1.22}$$

Therefore, the steady state gross growth rate will now take the expression:

$$g = \left[\frac{\chi}{R^k}\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{P_{-s}^{*}}{P} - 1\right)\left(\frac{(1 + R^k)P_{-s}^{*}}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}}(\gamma - 1) + 1 \tag{3.1.23}$$

Equilibrium conditions and external sector

The aggregate equilibrium of the economy will be defined as

$$Y_t = C_t + R\&D_t + \int_0^1 x_{it}di + X_t - X_t^{\dagger} \tag{3.1.24}$$

where, assuming two economies, henceforth "nation" (N) and "rest of the world" (ROW) —denoted with a dagger symbol [†]—, X_t are exports and $X_t^{\dagger}$ are imports.

As in Chapter 1 and 2, from equation (3.1.24) we can obtain an expression for consumption considering (3.1.16), and $R\&D_t = A_t n_{it}$. After normalizing the equation by Y, as all the growing variables must be normalized in the steady state, and considering the relationship between the production of both economies $l_t = Y_t^{\dagger}/Y_t$ in order to link the normalization of the two countries, we get for N in the steady state:

$$C^{\dagger Y^{\dagger}} = 1 - \left[\frac{\chi^{\dagger}}{R^{k\dagger}} \alpha^{\dagger \frac{1}{1-\alpha^{\dagger}}} L^{\dagger} \frac{1}{I^{\dagger}} \sum_{\tau=0}^{I^{\dagger}-1} \left(\frac{P_{-\tau}^{\dagger *}}{P^{\dagger}} - 1 \right) \left(\frac{P_{-\tau}^{\dagger}}{P^{\dagger}} \right)^{-\frac{1}{1-\alpha^{\dagger}}} \right]^{\frac{1}{1-\chi^{\dagger}}} A^{\dagger Y^{\dagger}} -$$

$$- \left(\frac{\alpha^{\dagger}}{R^{k\dagger}} \right)^{\frac{1}{1-\alpha^{\dagger}}} L^{\dagger} \frac{1}{I^{\dagger}} \sum_{\tau=0}^{I^{\dagger}-1} \left(\frac{P_{-\tau}^{\dagger *}}{P^{\dagger}} \right)^{-\frac{1}{1-\alpha^{\dagger}}} A^{\dagger Y^{\dagger}} - X^{\dagger Y^{\dagger}} + \frac{1}{l} X^{Y} \qquad (3.1.26b')$$

Expressions $(2.1.22a)$ and $(2.1.22b)$ for exports, $(2.1.23a)$ and $(2.1.23b)$ for the economies' real external financial positions, $(2.1.24)$ for the appropriate gross real interest rate, $(2.1.25)$ for the relationship between economies' external financial positions and $(2.1.26)$ for the uncovered interest parity are also considered for this model.

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 3.1. The number of each equation of the system corresponds to a previous one, appearing with the superscript [$'$] because they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through production level of the final good Y, each economy by its own. In addition, as explained, it is necessary to define the ratio for this variable $l = Y^{\dagger}/Y$ to link the normalizations of the two economies. Normalized variables are denoted with superscript Y, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. Equations corresponding to N described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have seventy-five equations and seventy-five endogenous variables.

3.2.2. Human capital model

Households

As in the previous model, households will now provide funds to the financial intermediaries with their deposits. As households' members do not need funding, their decision problem is not affected by the introduction of the financial friction, except for the allocation of deposits. There are no relevant changes to the developments explained in Chapter 2.

Financial intermediaries

We also introduce the financial friction developed by Gertler and Karadi (2011) in the human capital model. The developments explained in the previous section for the Schumpeterian model are still valid. However, it must be taken into consideration that R^k is the gross return. All expressions are appropriately modified in Appendix 3.2 with respect to those presented in the previous section for the Schumpeterian model.

Intermediate good producers

Intermediate good producers will now consider the opportunity cost of financing their working and physical capital with their own funds in their profits function. In a similar way as the final good producers in the Schumpeterian model, the profits will be:

$$F_{Y_t} = P_{jt} A K_{jt}^{\alpha} L_{jt}^{1-\alpha} - \left(1 + R_t^k\right)\left(\int_0^1 W_{st} L_{st} ds + R_t P_t K_t\right) \tag{3.2.1}$$

From profits maximization, the optimal conditions are:

$$L_{jt} = \left[\left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\alpha)A}{(1 + R_t^k)\Delta_{W_t}}\right]^{\frac{1}{\alpha}} K_t \tag{3.2.2}$$

$$R_t = \alpha \left[\frac{A}{1 + R_t^k}\left(\frac{\varepsilon - 1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}} \left[\frac{1-\alpha}{(1 + R_t^k)\Delta_{W_t}}\right]^{\frac{1-\alpha}{\alpha}} \tag{3.2.3}$$

Expressions for labor demand considering flexible or sticky efficiency wages and the unemployment rate are the same as those explained in Chapter 2. The effective efficiency wage expression will also be the same but now it will consider R^k as reflected in the expression that appears in Appendix 3.2.

Final good producers

An infinite number of final good producers, or retailers, are defined over a continuum $[0,1]$. Retailers repackage the intermediate good and sell the final output to the households. Equations defined in Chapter 2 are still valid for this model.

Central bank

The central bank is responsible for implementing the monetary policy through the modification of the short-term nominal interest rate. Trend inflation is considered as given as in previous models.

Equilibrium conditions and external sector

As in Chapter 1 and 2, expressions for the consumption to physical capital ratio can be obtained in both economies from the aggregate equilibrium of the economy. These expressions, considering $(3.2.2)$ and evaluated in the steady state, are

$$C^K = \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{(1+R^k)\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} - g - \delta + X^K - lX^{\dagger K^\dagger} \tag{3.2.7a}$$

$$C^{\dagger K^\dagger} = \frac{A^{\dagger\frac{1}{\alpha^\dagger}}}{\Delta_P^\dagger}\left[\left(\frac{\varepsilon^\dagger-1}{\varepsilon^\dagger}\right)\frac{1-\alpha^\dagger}{(1+R^{k\dagger})\Delta_{\mathrm{W}}^\dagger}\right]^{\frac{1-\alpha^\dagger}{\alpha^\dagger}} - g^\dagger - \delta^\dagger + X^{\dagger K^\dagger} - \frac{1}{l}X^K \tag{3.2.7b}$$

in which exports for each country are:

$$X^K = \Omega e^{\omega} l\,\frac{A^{\dagger\frac{1}{\alpha^\dagger}}}{\Delta_P^\dagger}\left[\left(\frac{\varepsilon^\dagger-1}{\varepsilon^\dagger}\right)\frac{1-\alpha^\dagger}{(1+R^{k\dagger})\Delta_{\mathrm{W}}^\dagger}\right]^{\frac{1-\alpha^\dagger}{\alpha^\dagger}} \tag{3.2.8a}$$

$$X^{\dagger K^\dagger} = \Omega^\dagger\left(\frac{1}{e}\right)^{\omega^\dagger}\frac{1}{l}\frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{(1+R^k)\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} \tag{3.2.8b}$$

Expressions $(2.1.22a)$ and $(2.1.22b)$ for exports, $(2.1.23a)$ and $(2.1.23b)$ for the economies' real external financial positions, $(2.1.24)$ for the appropriate gross real interest rate, $(2.1.25)$ for the relationship between economies' external financial positions and $(2.1.26)$ for the uncovered interest parity are also considered for this model.

Steady state

The system of equations required to determine the steady state values of the endogenous variables is presented in Appendix 3.2. The number of each system's equation corresponds to a previous one, with the superscript [$'$] stating that they are modified by the normalization and the properties of the steady state. All the growing variables are normalized through physical capital stock K, each economy by its own. In addition, it is necessary to define the ratio for this variable $l = K^{\dagger}/K$ to link the normalizations. Normalized variables are denoted with superscript K, and the time subscript does not appear —except for expectations or lagged variables— because the variables are constant in the steady state. Equations corresponding to N described in this section have their counterpart in ROW. In sum, for $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$ we have sixty-one equations and sixty-one endogenous variables.

3.3. Influence of trend inflation on growth, labor market variables and leverage: same or different discount rates and wage setting processes

As in previous chapters, once we have identified the normalized systems of equations for both models, we carry out simulations through Dynare in order to get the steady state of the variables and the response of growth, unemployment, LFP, employment and leverage to changes in trend inflation. This section analyzes six cases for each model, each with a different combination of price and wage settings for the two economies and with the same or different discount rate. Table 3.3.1 presents the values for the parameters.

Table 3.3.1. Parameter values

Parameter	Description	Schumpeterian	Human
$\beta = \beta^{\dagger}$	Utility discount factor	0.97	0.98
β	Utility discount factor N (differentiating)	0.97	0.981
$\beta^{\dagger}$	Utility discount factor ROW (differentiating)	0.9699	0.98
$\alpha = \alpha^{\dagger}$	Output-capital elasticity	0.332	0.65
$\sigma = \sigma^{\dagger}$	Elasticity of substitution for labor services	12	10
$v = v^{\dagger}$	Relative utility weight of labor	1	0.004
$\rho = \rho^{\dagger}$	Exports parameter	0.1	0
$\Omega = \Omega^{\dagger}$	Exports-exchange rate elasticity	0.5	0.5
$\omega = \omega^{\dagger}$	Exports-exchange rate elasticity	0.5	0.5
$\gamma = \gamma^{\dagger}$	Productivity upgrade after every innovation	1.009	-
$\chi = \chi^{\dagger}$	Elasticity of the probability of success in the innovation respect to the relative investment	0.1	-
$\varepsilon = \varepsilon^{\dagger}$	Elasticity of substitution among retail or intermediate goods	-	1.48
$\delta = \delta^{\dagger}$	Capital depreciation rate	-	0.0519
$\xi = \xi^{\dagger}$	Human capital accumulation productivity	-	0.07
$A = A^{\dagger}$	Total factor productivity	-	1
$b = b^{\dagger}$	Probability of job loss	0.1	0.1
$q = q^{\dagger}$	Probability of being fired if caught shirking	0.9	0.9
$z = z^{\dagger}$	Utility of leisure and unemployment benefits	0.2	0.1
$e = e^{\dagger}$	Labor effort cost	0.04	0.05
$\Gamma = \Gamma^{\dagger}$	Banker's survival rate	0.94	0.4045
$\psi = \psi^{\dagger}$	Wealth proportion of the new bankers	0.01	0.0575
$\lambda = \lambda^{\dagger}$	Proportion of diverted assets	0.382	0.27

Schumpeterian model

Results of the simulations obtained in this chapter maintain essentially the main conclusions of the previous one when economies are identical or differ only in the discount rate, adding specific conclusions of the consequences provided by the introduction of the financial sector. With flexibility, each country growth, unemployment, LFP and employment rates are constant regardless of the value of trend inflation. A different utility discount rate gives us different values for each of these variables between the economies, with changes in the same direction as in Chapter 2.

When wages are sticky, both economies are again incapable of reaching growth, LFP and employment rates of flexibility. Except for unemployment, again independent of the inflation rate with the same economic structure in the two economies or differing only in the discount rate, maximum values are reached at point $\Pi = \Pi^{\dagger} = 0$. Moving away from this point, the values decrease, and the fall is greater the greater the distance, in the same way as in Chapter 2. In Section 3.6 we consider consequences of a wide structural difference.

Results considering that N has flexible wages and ROW sticky wages confirm once again the existence of a "contagion of non-neutrality", as changes in the inflation target of the sticky economy have an effect on long-term growth and labor market variables of the flexible economy. Different values of the labor market variables between economies take place even with the same discount factor, in the same directions as we found in Chapter 2. Table 3.3.2 presents maximum values for six cases simulated, with multiple combinations of price and wage settings for the two economies and with the same or different discount rate.

Finally, we must analyze if there is a relationship between trend inflation and leverage ratio. We start our analysis with the case of flexibility in prices and wages in both economies $I = I^{\dagger} = J = J^{\dagger} = 1$. The results show that leverage ratio is always constant, regardless of the inflation targets. For $\beta = \beta^{\dagger} = 0.97$, both economies have a leverage ratio of $\phi = \phi^{\dagger} = 6.17831$, and for $\beta = 0.97$ and $\beta^{\dagger} = 0.9699$ we find a decrease on N, $\phi = 6.17794$, and an increase in ROW, $\phi^{\dagger} = 6.17858$.

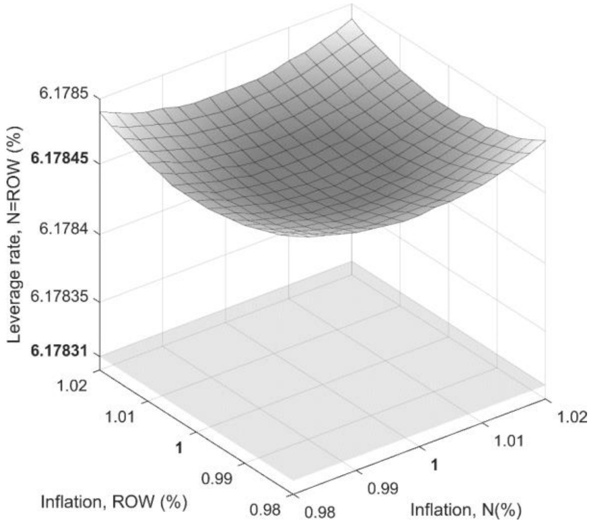
Table 3.3.2. Maximum values for the variables, Schumpeterian model

Variable	$(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (1,1)$ $\forall\Pi, \forall\Pi^\dagger$		$(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (2,4)$ $\forall\Pi, \Pi^\dagger = 0\%$*		$(I,J) = (2,4)$ $(I^\dagger,J^\dagger) = (2,4)$ $\Pi = \Pi^\dagger = 0\%$*	
			$\beta = \beta^\dagger = 0.97$			
Growth	g	0.51027%	g	0.50925%	g	0.50804%
	$g^\dagger$	0.51027%	$g^\dagger$	0.50925%	$g^\dagger$	0.50804%
Unemployment	u	0.18065%	u	0.17325%	u	4.14%
	$u^\dagger$	0.18065%	$u^\dagger$	4.14%	$u^\dagger$	4.14%
LFP	N	85.2980%	N	83.6799%	N	85.2962%
	$N^\dagger$	85.2980%	$N^\dagger$	87.1426%	$N^\dagger$	85.2962%
Employment	LL	85.1439%	LL	83.535%	LL	81.7648%
	$LL^\dagger$	85.1439%	$LL^\dagger$	83.535%	$LL^\dagger$	81.7648%
Leverage	ϕ	6.17831	ϕ	6.17837	ϕ	6.17845
	$\phi^\dagger$	6.17831	$\phi^\dagger$	6.17837	$\phi^\dagger$	6.17845
			$\beta = 0,97 \quad \beta^\dagger = 0.9699$			
Growth	g	0.51612%	g	0.51474%	g	0.51383%
	$g^\dagger$	0.50576%	$g^\dagger$	0.50438%	$g^\dagger$	0.50347%
Unemployment	u	0.22632%	u	0.21441%	u	4.14%
	$u^\dagger$	0.15141%	$u^\dagger$	4.14%	$u^\dagger$	4.14%
LFP	N	94.4530%	N	92.1905%	N	94.4529%
	$N^\dagger$	78.6362%	$N^\dagger$	79.9171%	$N^\dagger$	78.6334%
Employment	LL	94.2392%	LL	91.9928%	LL	90.5443%
	$LL^\dagger$	78.5172%	$LL^\dagger$	76.6078%	$LL^\dagger$	75.3770%
Leverage	ϕ	6.17794	ϕ	6.17802	ϕ	6.17808
	$\phi^\dagger$	6.17858	$\phi^\dagger$	6.17866	$\phi^\dagger$	6.17872

* Except for unemployment rate that is $\forall\Pi, \forall\Pi^\dagger$.

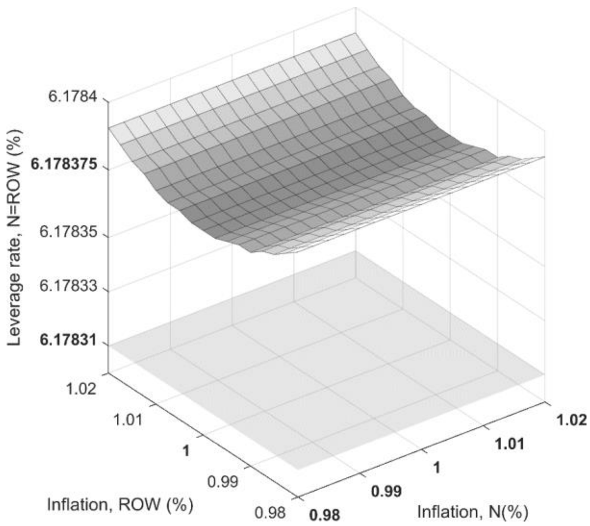
When wages are sticky in both economies, $I = I^\dagger = 2$ and $J = J^\dagger = 4$, leverage ratio adopts a low-sloped paraboloid shape, with a minimum at the point of null inflation. For $\beta = \beta^\dagger = 0.97$ minimum leverage ratio in both economies is $\phi = \phi^\dagger = 6.17845$ —a higher value than with flexibility— and for $\beta = 0.97$ and $\beta^\dagger = 0.9699$, at $\phi = 6.17808$ for N and $\phi^\dagger = 6.17872$ for ROW, a higher value for the economy with lower discount factor and growth rate. Leverage ratio slightly increases as we introduce stickiness in the economies, since for flexibility in N and stickiness in ROW, leverage ratio for the same discount factor $\beta = \beta^\dagger = 0.97$ is for both economies $\phi = \phi^\dagger = 6.17837$. Monetary policy of a sticky economy can also slightly affect the leverage ratio of a flexible one, while its own monetary policy is neutral.

Figure 3.3.1. Inflation-leverage relationship, Schumpeterian model
Nation $(I = 2, J = 4)$ Rest of World $(I^{\dagger} = 2, J^{\dagger} = 4)$ $\beta = \beta^{\dagger}$



In short, we obtain that, under stickiness, leverage ratio is minimized when long-term growth rate is maximized for both economies. A contagion is also present for the financial sector and has an impact on the long-term growth. Figure 3.3.1 shows the simulations for flexibility and for stickiness when both economies have the same intertemporal discount rate, and Figure 3.3.2. represents the leverage ratio behavior of the nation with flexible wage to changes in both monetary policies, reflecting the contagion of non-neutrality extended to the financial sector.

Figure 3.3.2. Contagion on leverage ratio, Schumpeterian model
Nation $(I = 2, J = 4)$ Rest of World $(I^{\dagger} = 2, J^{\dagger} = 4)$ $\beta = \beta^{\dagger}$



Human capital model

After the introduction of financial intermediaries into the human capital model, growth and labor market variables response to changes in the inflation targets of both economies maintain also essentially the main conclusions of Chapter 2 when economies are identical or differ only in the discount rate, adding specific conclusions of the consequences provided by the introduction of the financial sector. With wage flexibility, the rates of these variables are constant regardless the value of trend inflation, and with wage stickiness, average international growth, LFP and employment takes a bell-shaped form with a maximum at null inflation. Average international unemployment is also constant for every combination of trend inflation with or without wage stickiness. The contagion effect also appears in these variables for a flexible nation and a rigid rest of the world. Table 3.3.3 presents the maximum values for these variables and the leverage ratio with different price and wage settings for the two economies and with the same or different discount rate.

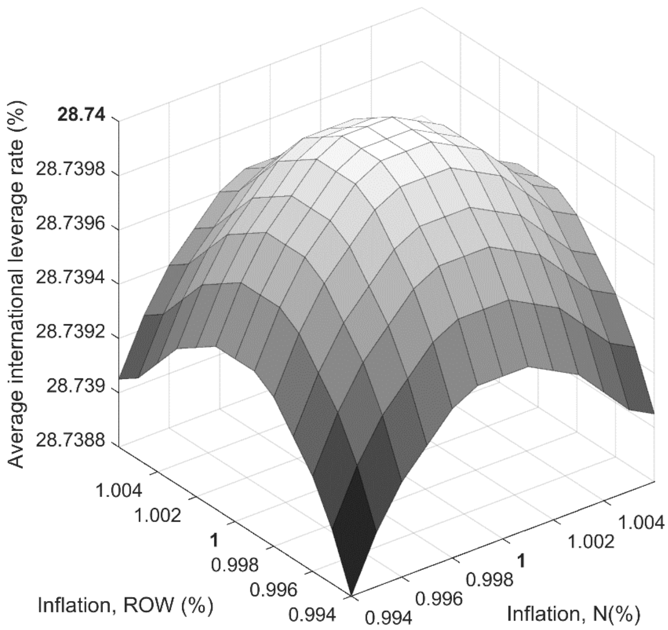
Table 3.3.3. Maximum values for the variables, human capital model

Variable	$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi}\ \ \forall\boldsymbol{\Pi}^\dagger$		$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\forall\boldsymbol{\Pi}, \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$*		$(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{2},\mathbf{4})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\boldsymbol{\Pi} = \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$*	
			$\boldsymbol{\beta} = \boldsymbol{\beta}^\dagger = \mathbf{0.98}$			
Growth	g	0.45787%	g	0.23151%	g	0.08643%
	$g^\dagger$	0.45787%	$g^\dagger$	0.08631%	$g^\dagger$	0.08643%
Unemployment	u	15.7640%	u	9.41926%	u	3.98114%
	$u^\dagger$	15.7640%	$u^\dagger$	3.97622%	$u^\dagger$	3.98114%
LFP	N	41.4935%	N	35.1124%	N	31.0152%
	$N^\dagger$	41.4935%	$N^\dagger$	31.0119%	$N^\dagger$	31.0152%
Employment	LL	34.9524%	LL	31.8048%	LL	29.7804%
	$LL^\dagger$	34.9524%	$LL^\dagger$	29.7788%	$LL^\dagger$	29.7804%
Leverage	ϕ	28.8561	ϕ	28.7855	ϕ	28.74
	$\phi^\dagger$	28.8561	$\phi^\dagger$	28.74	$\phi^\dagger$	28.74
			$\boldsymbol{\beta} = \mathbf{0.981}\quad \boldsymbol{\beta}^\dagger = \mathbf{0.98}$			
Growth	g	0.57794%	g	0.19497%	g	0.08219%
	$g^\dagger$	0.60882%	$g^\dagger$	0.08642%	$g^\dagger$	0.08651%
Unemployment	u	19.0010%	u	8.52938%	u	3.98489%
	$u^\dagger$	19.0147%	$u^\dagger$	3.98093%	$u^\dagger$	3.98472%
LFP	N	43.4521%	N	32.6552%	N	29.4680%
	$N^\dagger$	45.7409%	$N^\dagger$	31.0151%	$N^\dagger$	31.0176%
Employment	LL	35.1958%	LL	29.8699%	LL	28.2938%
	$LL^\dagger$	37.0434%	$LL^\dagger$	29.7804%	$LL^\dagger$	29.7816%
Leverage	ϕ	28.8615	ϕ	28.742	ϕ	28.7068
	$\phi^\dagger$	28.9034	$\phi^\dagger$	28.74	$\phi^\dagger$	28.74

* Except for unemployment rate that is $\forall\boldsymbol{\Pi}, \forall\boldsymbol{\Pi}^\dagger$.

Let us focus on the relationship between trend inflation and leverage ratio, since it presents a different profile to that obtained in the Schumpeterian model. With wage flexibility, leverage is constant whatever the values of trend inflation of the two economies. For $\beta = \beta^{\dagger} = 0.98$, both economies have a leverage ratio of $\phi = \phi^{\dagger} = 28.8561$, and for $\beta = 0.981$ and $\beta^{\dagger} = 0.98$ we have an increase of both, $\phi = 28.8615$ and $\phi^{\dagger} = 28.9034$.

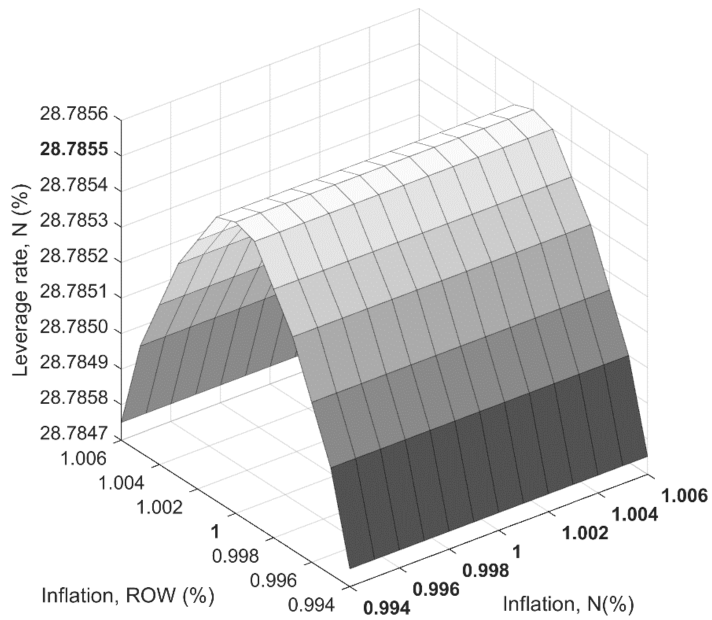
Figure 3.3.3. Inflation-average inter. leverage relationship, human capital model
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta = \beta^{\dagger}}$



With wage stickiness, in the Schumpeterian model we obtained a paraboloid shape form with a minimum at null inflation, and now we obtain for the average international leverage rate a familiar bell-shape form, with a maximum at that same point, which is the combination of inflation targets that maximize the growth rate of both economies. For $\beta = \beta^{\dagger} = 0.98$ maximum leverage ratio in both economies is $\phi = \phi^{\dagger} = 28.74$ —a lower value than with flexibility— and for $\beta = 0.981$ and $\beta^{\dagger} = 0.98$, $\phi = 28.7068$ for N and $\phi^{\dagger} = 28.74$ for ROW, a lower value for the economy with higher discount factor and lower growth rate. The reason for this opposite behavior is the sector financed in each of the models. While in the Schumpeterian model credit goes to the innovation sector that optimizes its results when growth is maximum, in the human capital model working capital it finances the cost of labor and physical capital, which is at its highest level when growth is maximum.

Leverage ratio slightly decreases as we introduce wage stickiness in the economies as Table 3.3.3 reflects. Figure 3.3.3 represents this relationship under wage stickiness and Figure 3.3.4 shows the contagion of non-neutrality, representing the leverage ratio of the flexible nation to changes in both monetary policies when the rest of the world has wage stickiness.

Figure 3.3.4. Contagion on leverage ratio, human capital model
Nation ($\boldsymbol{I = 2, J = 4}$) Rest of World ($\boldsymbol{I^{\dagger} = 2, J^{\dagger} = 4}$) $\boldsymbol{\beta = \beta^{\dagger}}$



3.4. Main consequences of considering the financial friction

Until now, we have obtained the main features of the long run behavior of the two economies in different conditions and have resumed the influence of the financial sector in the leverage ratio. Our aim in this section is answering two questions related to the consequences of having introduced the financial friction. The first question is about the consequences on the optimal behavior of the main variables and the second the influence of the parameter λ, a measure of the friction, on these variables.

3.4.1. Comparison of steady states with and without financial friction

Once we have simulated the models, we must measure and explain what consequences have introducing a financial friction on the relationship between trend inflation, from one side, and growth and labor market variables, for the other.

Schumpeterian model

The values presented in Table 3.4.1 allow us to infer the effects of the introduction of the financial sector on the main macroeconomic variables at the optimal point when compared with those obtained in Chapter 2.

Table 3.4.1. Maximum rates, Schumpeterian model

Variable		No financial friction $(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (1,1)$ $\forall\Pi$ $\forall\Pi^\dagger$		Financial friction $(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (1,1)$ $\forall\Pi$ $\forall\Pi^\dagger$		Variation
Growth		0.51400%		0.51027%		-0.00373%
Unemployment		0.18575%		0.18065%		-0.00510%
LFP		87.2692%		85.2980%		-1.97120%
Employment		87.1070%		85.1439%		-1.96310%
Leverage		0		6.17831		6.17831
Variable		**No financial friction** $(I,J) = (2,4)$ $(I^\dagger,J^\dagger) = (2,4)$ $\Pi = \Pi^\dagger = 0\%$		**Financial friction** $(I,J) = (2,4)$ $(I^\dagger,J^\dagger) = (2,4)$ $\Pi = \Pi^\dagger = 0\%$		**Variation**
Growth		0.51162%		0.50804%		-0.00358%
Unemployment		4.13997%		4.14016%		0.00%
LFP		87.2663%		85.2962%		-1.9701%
Employment		83.6535%		81.7648%		-1.8887%
Leverage		0		6.17845		6.17845
Variable		**No financial friction** $(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (2,4)$ $\forall\Pi$ $\Pi^\dagger = 0\%$		**Financial friction** $(I,J) = (1,1)$ $(I^\dagger,J^\dagger) = (2,4)$ $\forall\Pi$ $\Pi^\dagger = 0\%$		**Variation**
Growth	g	0.51284%	g	0.50925%	Δg	-0.0035%
	$g^\dagger$	0.51284%	$g^\dagger$	0.50925%	$\Delta g^\dagger$	-0.0035%
Unemployment	u	0.17812%	u	0.17315%	Δu	-0.0049%
	$u^\dagger$	4.13966%	$u^\dagger$	4.13988%	$\Delta u^\dagger$	0.00%
LFP	N	85.6140%	N	83.6799%	ΔN	-1.9341%
	$N^\dagger$	89.1521%	$N^\dagger$	87.1426%	$\Delta N^\dagger$	-2.0095%
Employment	LL	85.4615%	LL	83.535%	ΔLL	-1.9265%
	$LL^\dagger$	85.4615%	$LL^\dagger$	83.535%	$\Delta LL^\dagger$	-1.9265%
Leverage	ϕ	0	ϕ	6.17837	$\Delta\phi$	6.17837
	$\phi^\dagger$	0	$\phi^\dagger$	6.17837	$\Delta\phi^\dagger$	6.17837

These movements can be summarized, for both economies, in a loss of economic growth, labor supply and labor demand, which is compatible with a slight reduction in the unemployment rate when we consider flexible wages and a constant unemployment rate with sticky wages.

The growth loss is, as equation (3.1.23) indicates, a consequence of the increase in the production costs, as the financial friction represents an additional cost for the final good producers, leading to a decrease in labor demand, as (3.1.17) reflects. This growth loss appears regardless of the combination of wage setting processes in the two economies. Both labor force participation and employment are also down in all three cases, the first one due to an increase in consumption. The movements found for unemployment are a consequence of the movements in these two variables, with a null effect when wages is sticky.

Human capital model

Table 3.4.2 summarizes the optimal values of the variables for each of the six simulated cases after introducing the financial intermediaries in the human capital model and compares them with those obtained for the models in Chapter 2. Note that the values presented in this table for the models without financial friction do not correspond exactly to those obtained in the previous chapter because the models have been recalibrated with the parameter values in Table 3.3.1.

When both economies are flexible, the introduction of the financial friction generates a significant drop in economic growth, labor supply, labor demand and unemployment in both economies. The financial friction represents an additional cost for the intermediate good producers. The efficiency wage expression now considers R^k, generating a cost increase and a notable decrease in labor demand in both countries. A lower growth interacts with labor supply, which falls. Because of these interactions, the unemployment rate also falls.

Table 3.4.2. Maximum rates, human capital model

Variable	No financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi}\ \ \forall\boldsymbol{\Pi}^\dagger$	Financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{1},\mathbf{1})$ $\forall\boldsymbol{\Pi}\ \ \forall\boldsymbol{\Pi}^\dagger$	Variation
Growth	1.06903%	0.45787%	-0.61116%
Unemployment	26.0384%	15.7640%	-10.2744%
LFP	58.6516%	41.4935%	-17.1581%
Employment	43.3796%	34.9524%	-8.4272%
Leverage	0	28.8561	28.8561

Variable	No financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{2},\mathbf{4})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\boldsymbol{\Pi} = \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$	Financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{2},\mathbf{4})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\boldsymbol{\Pi} = \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$	Variation
Growth	0.08647%	0.08643%	-0.00%
Unemployment	3.98268%	3.98114%	-0.00154%
LFP	31.0163%	31.0152%	-0.00110%
Employment	29.7810%	29.7804%	-0.00059%
Leverage	0	28.74	28.74

Variable	No financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\forall\boldsymbol{\Pi}\ \ \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$		Financial friction $(\boldsymbol{I},\boldsymbol{J}) = (\mathbf{1},\mathbf{1})$ $(\boldsymbol{I}^\dagger,\boldsymbol{J}^\dagger) = (\mathbf{2},\mathbf{4})$ $\forall\boldsymbol{\Pi}\ \ \boldsymbol{\Pi}^\dagger = \mathbf{0}\%$		Variation	
Growth	g	0.20722%	g	0.23151%	Δg	0.0242%
	$g^\dagger$	0.08640%	$g^\dagger$	0.08631%	$\Delta g^\dagger$	-0.00009%
Unemployment	u	8.59907%	u	9.41926%	Δu	0.82019%
	$u^\dagger$	3.97993%	$u^\dagger$	3.97622%	$\Delta u^\dagger$	-0.00371%
LFP	N	34.427%	N	35.1124%	ΔN	0.6854%
	$N^\dagger$	31.014%	$N^\dagger$	31.0119%	$\Delta N^\dagger$	-0.0021%
Employment	LL	31.4666%	LL	31.8051%	ΔLL	0.3385%
	$LL^\dagger$	29.78%	$LL^\dagger$	29.7788%	$\Delta LL^\dagger$	-0.0012%
Leverage	ϕ	0	ϕ	28.7855	$\Delta\phi$	28.7855
	$\phi^\dagger$	0	$\phi^\dagger$	28.7399	$\Delta\phi^\dagger$	28.7399

When both economies are rigid, movements are in the same direction as when both are flexible but notice the smaller amount of the variations compared to flexibility. Even the effect on growth is null. However, when the nation is flexible and the rest of the world is rigid, the economies experience movements in opposite directions after the introduction of financial intermediaries in both countries. While the flexible nation experiences an increase in growth, labor supply, labor demand and unemployment, the rigid rest of the world experiences a drop in all four variables of a much smaller, almost negligible, magnitude. This result can be explained because, as long as both economies are identical, the steady state equilibrium exchange rate remains at $e = 1$, with or without financial frictions., but when the nation is flexible and the rest of the world is rigid, then $e > 1$, increasing the exports of the flexible country. This exchange rate increases even more when we introduce the financial friction. A higher exchange rate benefits the exports of the flexible nation and thus increases its growth.

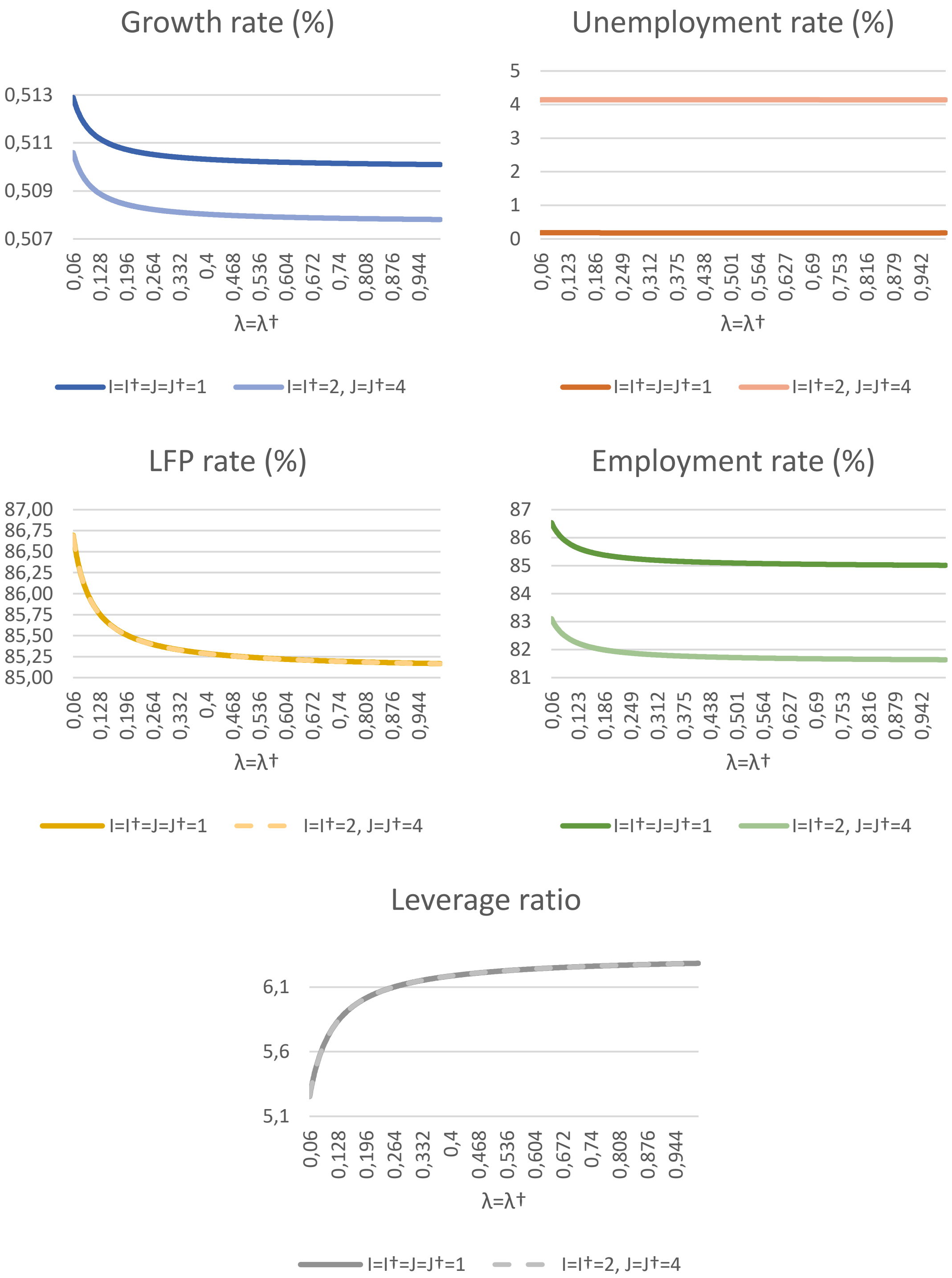
3.4.2. Effects of the financial friction size

The parameter λ measure the size of the friction introduced in the financial sector, given that it indicates the importance of the distortion that is creating in the market the asymmetry of information (potential proportion of diverted assets by the financial intermediaries). Then, it is interesting to know how this parameter affects to the variables we are interested in. In what follows, we consider successively the two models.

Schumpeterian model

Figure 3.4.1 shows the evolution of the main variable with the values of the parameter λ. The behavior is very similar regardless wages are flexible or sticky but in a different level. Nominal rigidity leads to lower growth, LFP and employment, and higher unemployment and leverage. Intuitively, as parameter λ increases, the information becomes more asymmetric between depositors and financial intermediaries in both countries and therefore economic growth, LFP and employment decrease, as the financial cost to achieve incentives compatibility is higher. The unemployment rate remains constant whatever the value of the parameter and leverage increases as we increase lambda. The conclusions obtained in the previous sections about the relative performance of flexible and rigid economies are valid for the entire range of the parameter.

Figure 3.4.1. Sensitivity analysis to changes in $\lambda = \lambda^{\dagger}$, Schumpeterian model



But the additional information we obtain is that the effect of λ is negative for growth, LFP and employment, and does not affect unemployment in the two possible wage regimes and that this distortion has the same effect in LFP whatever the wage setting process.

Human capital model

Figure 3.4.2 shows initially that the sensitivity to this parameter is greater with wage flexibility than with wage rigidity. Moreover, growth, LFP, employment and unemployment rate all have a different shape with or without nominal rigidity.

Figure 3.4.2. Sensitivity analysis to changes in $\lambda = \lambda^{\dagger}$, human capital model

While this parameter has no effect with wage rigidity, with wage flexibility has an inverted-U shape for all variables except for leverage, with a maximum at 0,275. Intuitively, as the value of λ increases the cost of achieving incentive compatibility is higher, but in the two models negative effects appear very limited, even being positive in the human capital model up to 0.275. Leverage rises as the values of λ increases in both cases at the same rate. The main conclusion we can draw from this analysis is that the results presented in the previous sections of this chapter are also valid for virtually the entire range of this parameter.

3.5. A sensitivity analysis for efficiency wage parameters

As in Chapter 2, this section presents a sensitivity analysis for the main macroeconomic variables —growth, unemployment, labor force participation, employment, and now leverage— to changes in efficiency wage parameters in order to determine to what extent the results are consistent with other values for these parameters.

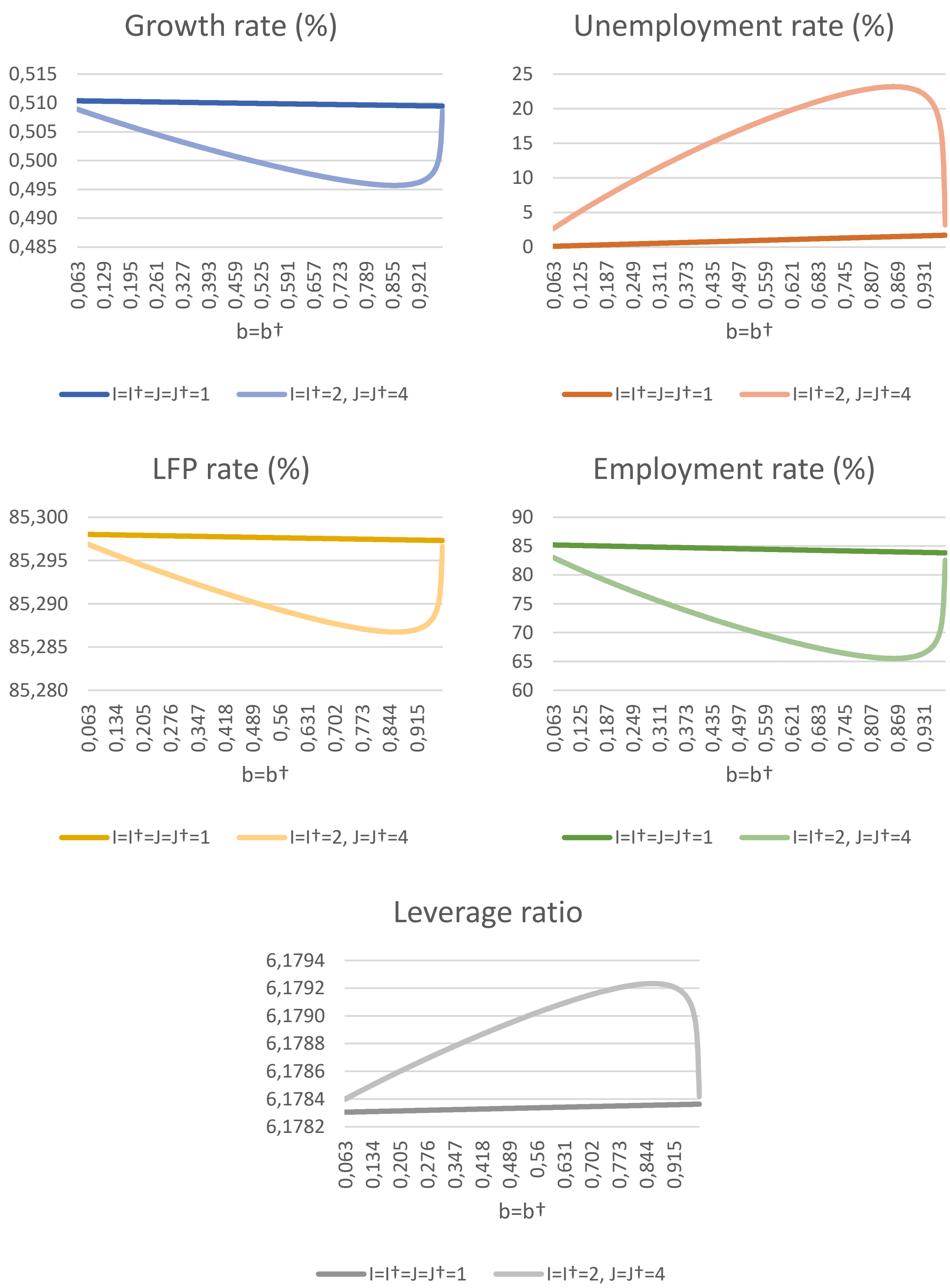
While results for flexibility in both economies are independent of trend inflation, for wage stickiness the inflation target will be the one that maximizes growth, 0% for both models. The sensitivity analysis is carried out for each parameter, while the other parameters keep the values of Table 3.3.1. As a general conclusion, we confirm that the results obtained in the previous sections are valid for a wide range of values for these parameters.

3.5.1. Schumpeterian model

Sensitivity analysis for parameter b

Similar to Chapter 2, the sensitivity analysis for parameter b, representing the probabilities of employment loss, shows that growth, LFP, and employment with flexibility in both economies are always higher than when stickiness is considered in both. Unemployment and leverage ratio for flexibility are always lower than for stickiness. Again, for this parameter, we can appreciate more sensitivity on sticky wages to changes in the parameters value. Under flexible wages, a minimal variation for the variables can be appreciated. Our findings in Section 3.3 and Section 3.4 will always be met.

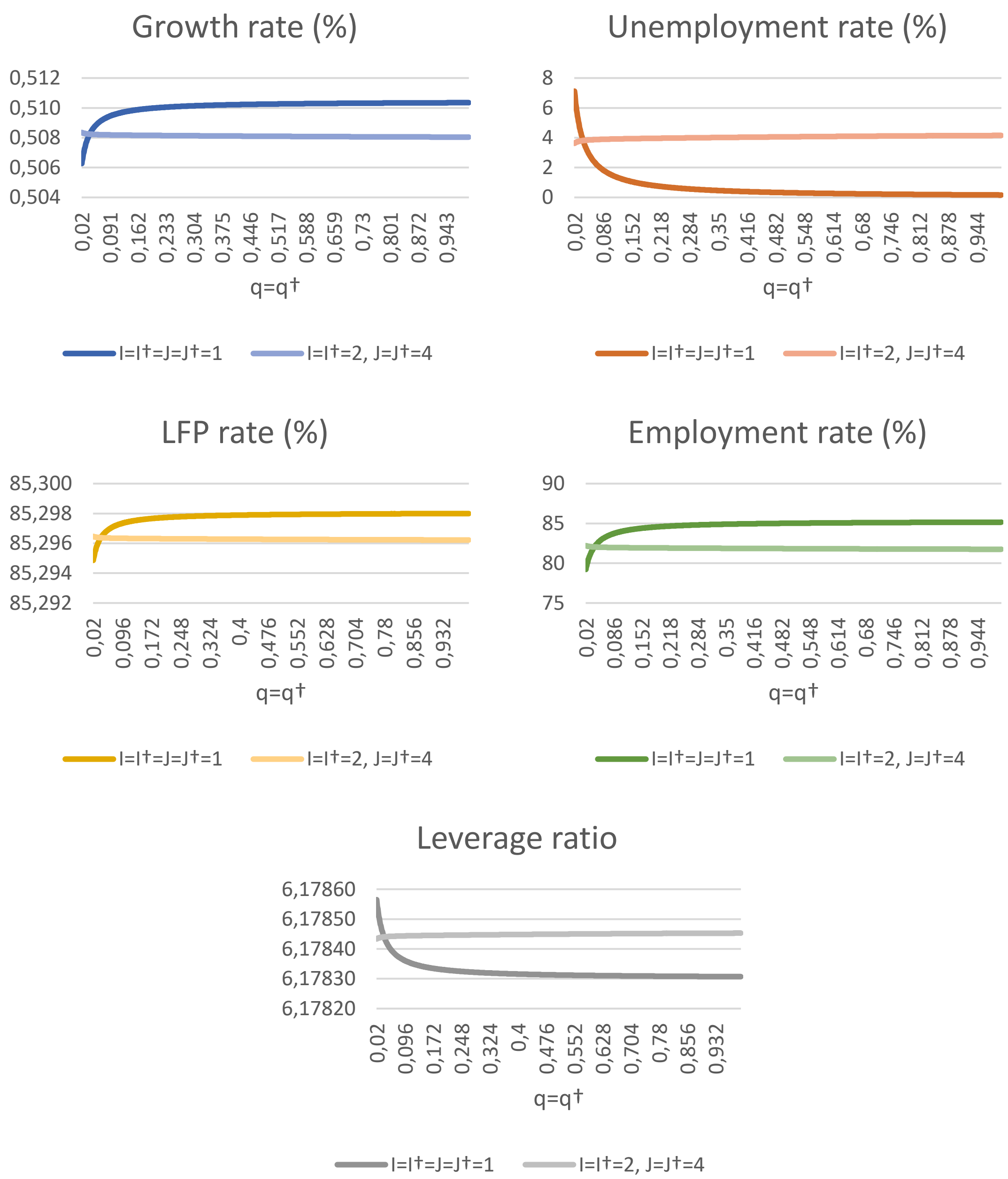
Figure 3.5.1. Sensitivity analysis to changes in $b = b^{\dagger}$, Schumpeterian model



Sensitivity analysis for parameter q

As in Chapter 2, there is a threshold from where the conclusions of the previous sections are valid. This point is in now $q = q^{\dagger} = 0.04$. From that point, the differences between the flexible and the sticky rates become bigger as the probability of being caught shirking and being fired grow in both economies, since flexible wages decrease with increments in parameters q while sticky wages slightly increase. Fortunately, this point, or lower, is far from a reasonable value for this parameter.

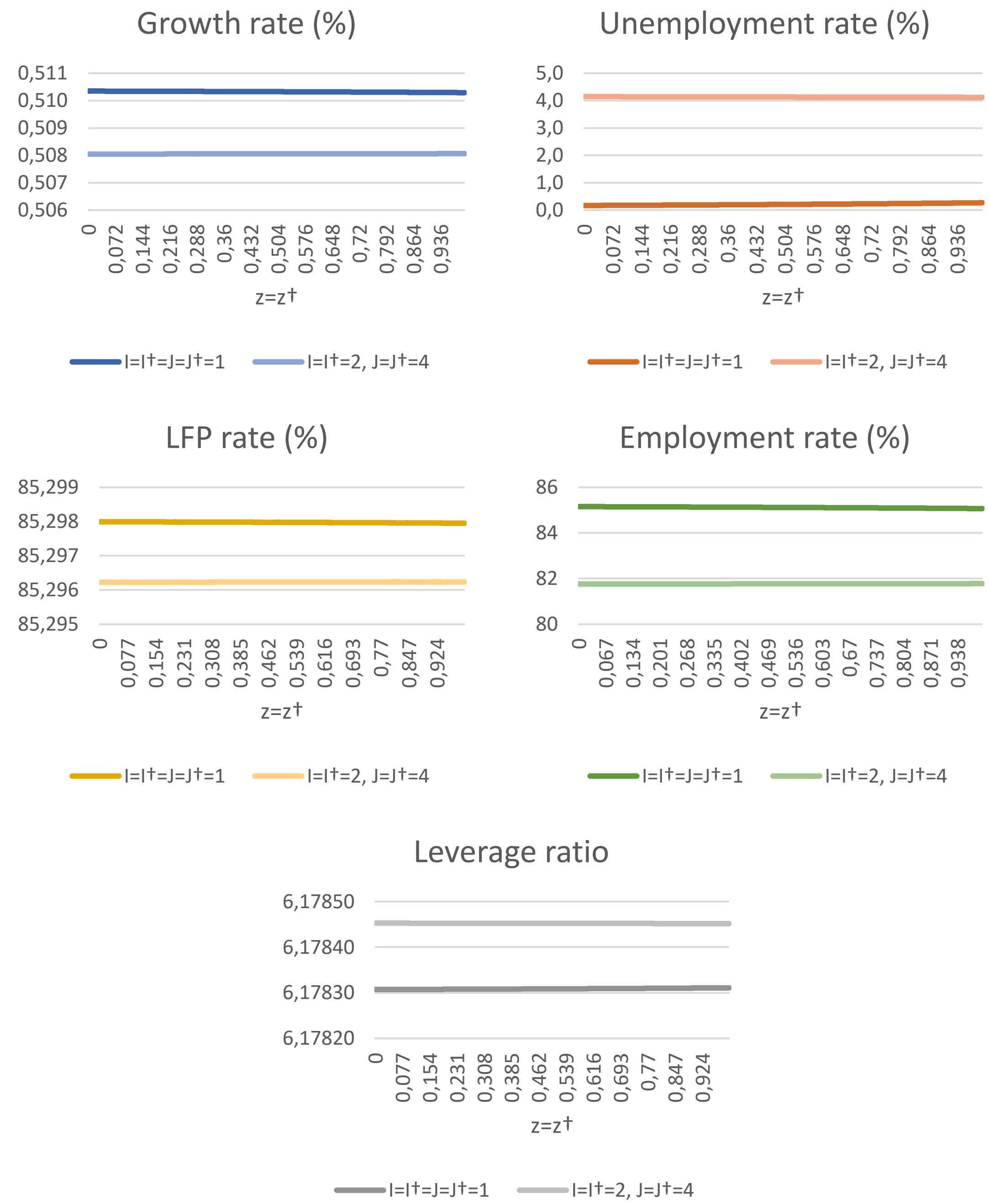
Figure 3.5.2. Sensitivity analysis to changes in $\boldsymbol{q = q^{\dagger}}$, Schumpeterian model



Sensitivity analysis for parameter z

For the parameter that represents unemployment benefits, the flexibility growth rate, LFP rate and employment rate are always higher than the sticky ones, as in the previous chapter. The opposite results are obtained for unemployment and leverage, meaning that the findings of Section 3.3 and 3.4 are always correct whatever the value for the utility of leisure time and unemployment benefits.

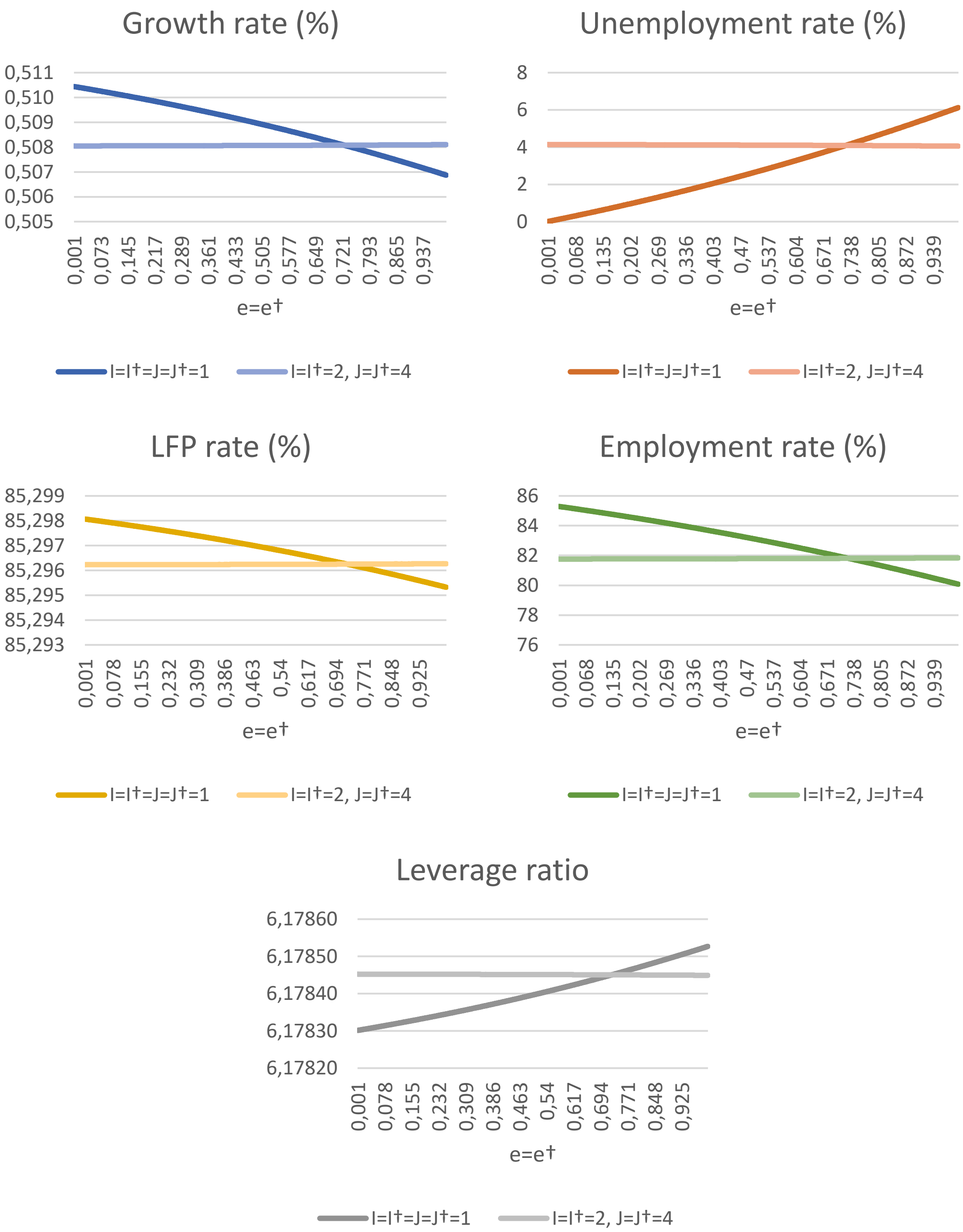
Figure 3.5.3. Sensitivity analysis to changes in $z = z^{\dagger}$, Schumpeterian model



Sensitivity analysis for parameter e

For parameter e, representing the cost of making effort, the sensitivity analysis confirms that our findings are correct until these parameters reach $e = e^{\dagger} = 0{,}725$ for growth, unemployment, LFP, employment and leverage ratio. Most likely values are always lower than this value.

Figure 3.5.4. Sensitivity analysis to changes in $e = e^{\dagger}$, Schumpeterian model

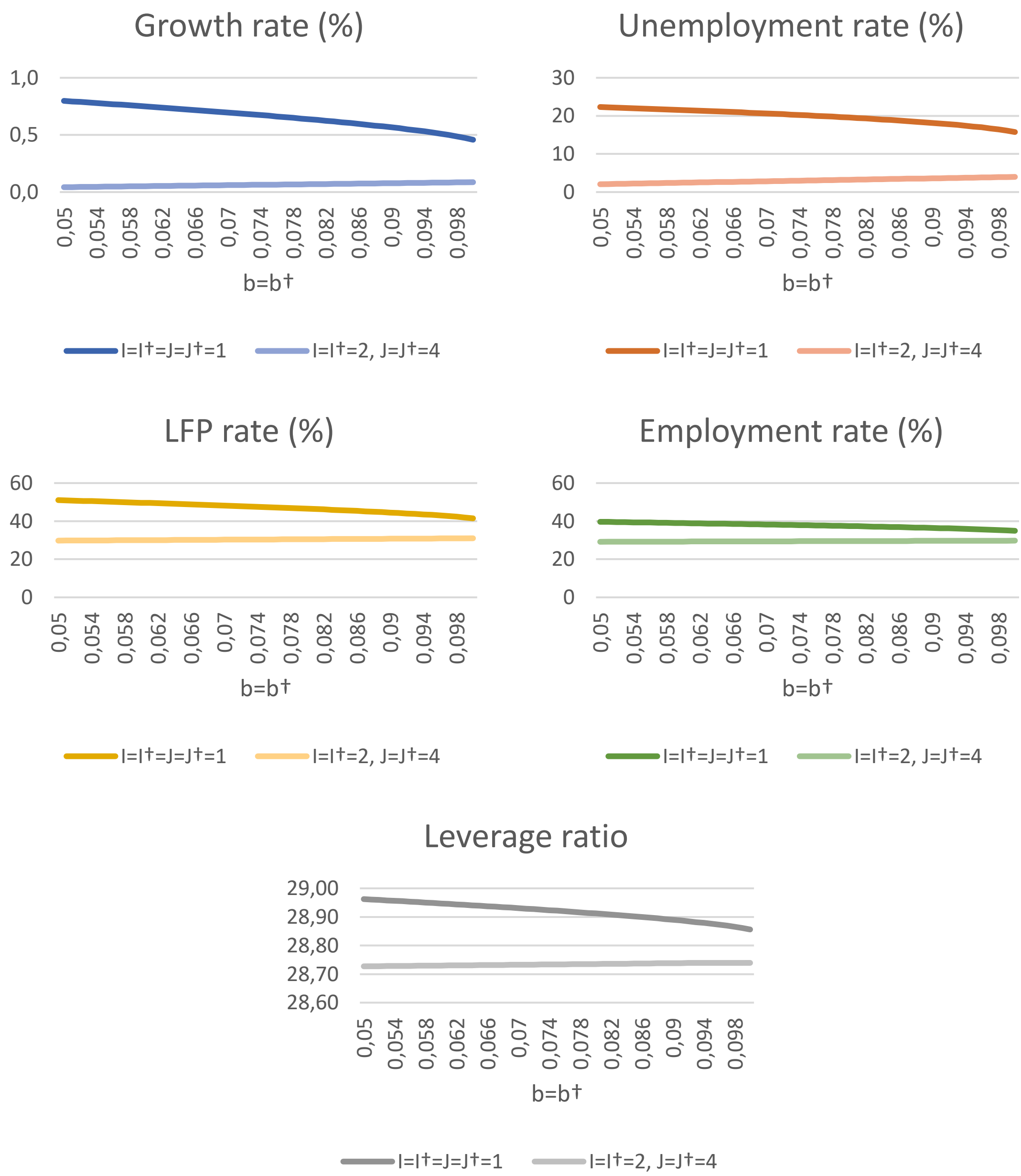


3.5.2. Human capital model

Sensitivity analysis for parameter b

The sensitivity analysis for parameter b shows that all five variables have higher values when wage flexibility is considered than when there are nominal rigidities within a range of reasonable values for this parameter. With wage rigidities, the variables experience a slight increase as we raise the value of the parameter, while with flexibility they show a more noticeable decrease.

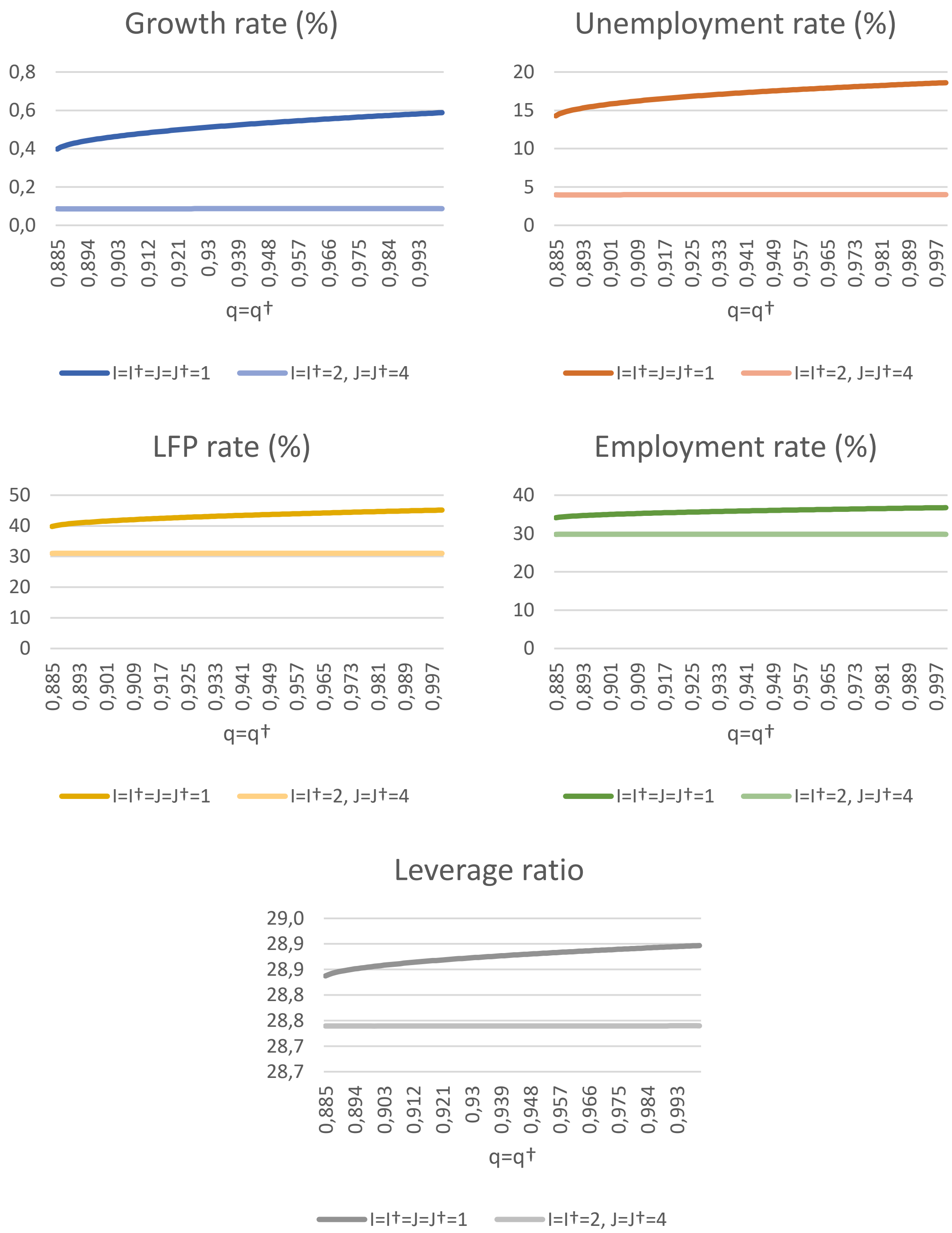
Figure 3.5.6. Sensitivity analysis to changes in $b = b^{\dagger}$, human capital model



Sensitivity analysis for parameter q

For a range of reasonable values of the parameters q, the main macroeconomic variables are always higher considering flexible wages than nominal rigidities in both countries. In this case, as we increase the parameters value, the variables experience a remarkable growth with flexibility and a much more moderate growth under nominal rigidities.

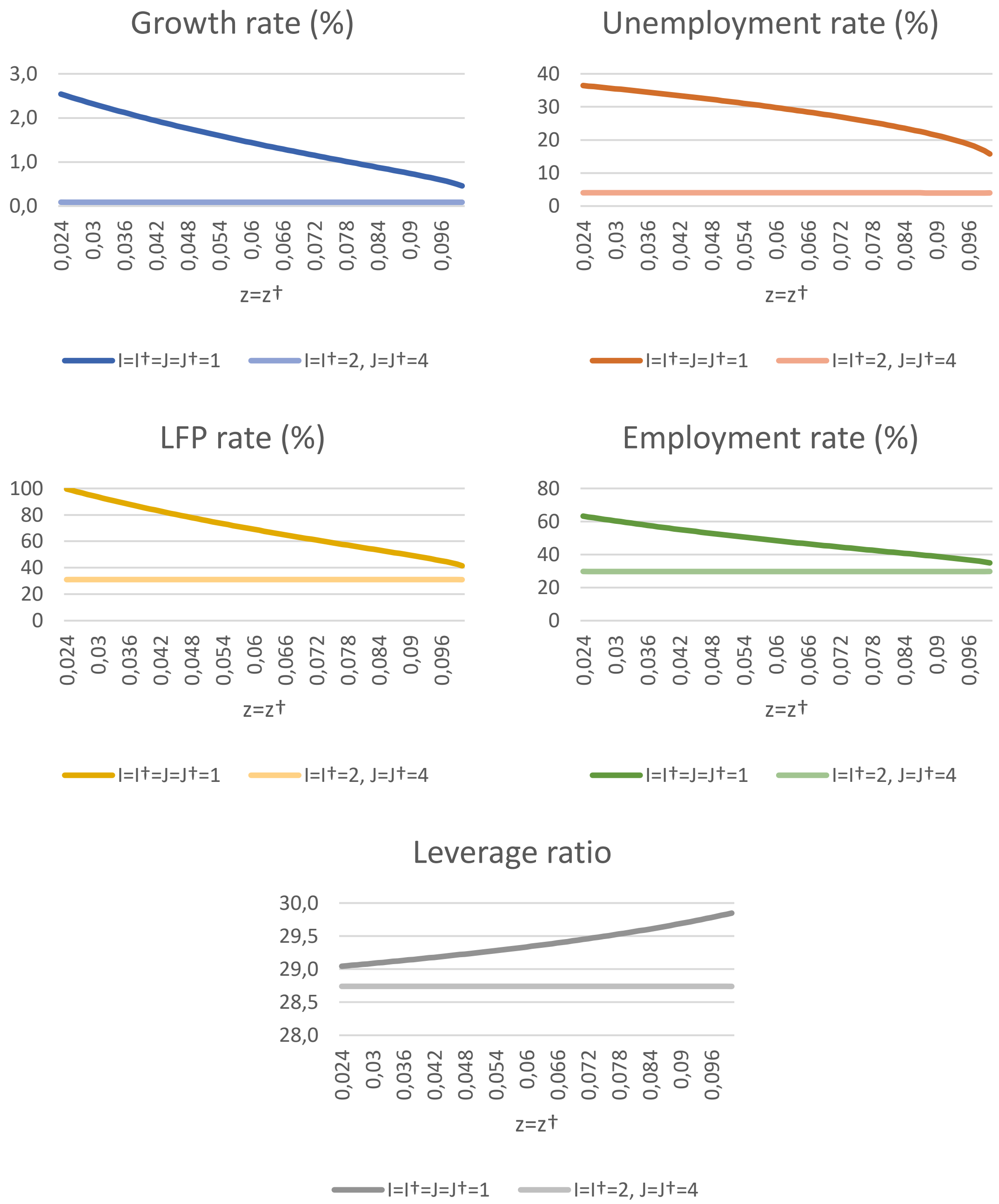
Figure 3.5.7. Sensitivity analysis to changes in $q = q^{\dagger}$, human capital model



Sensitivity analysis for parameter z

The analysis for the parameter z reveals a strong sensitivity under wage flexibility while, as it happens with the other parameters, the main macroeconomic variables remain practically constant with nominal rigidities. As we increase the value of the parameter, growth, LFP, employment and unemployment sharply decrease with wage flexibility, while leverage quickly increases. The results presented in the previous sections are valid for z values below 0.1.

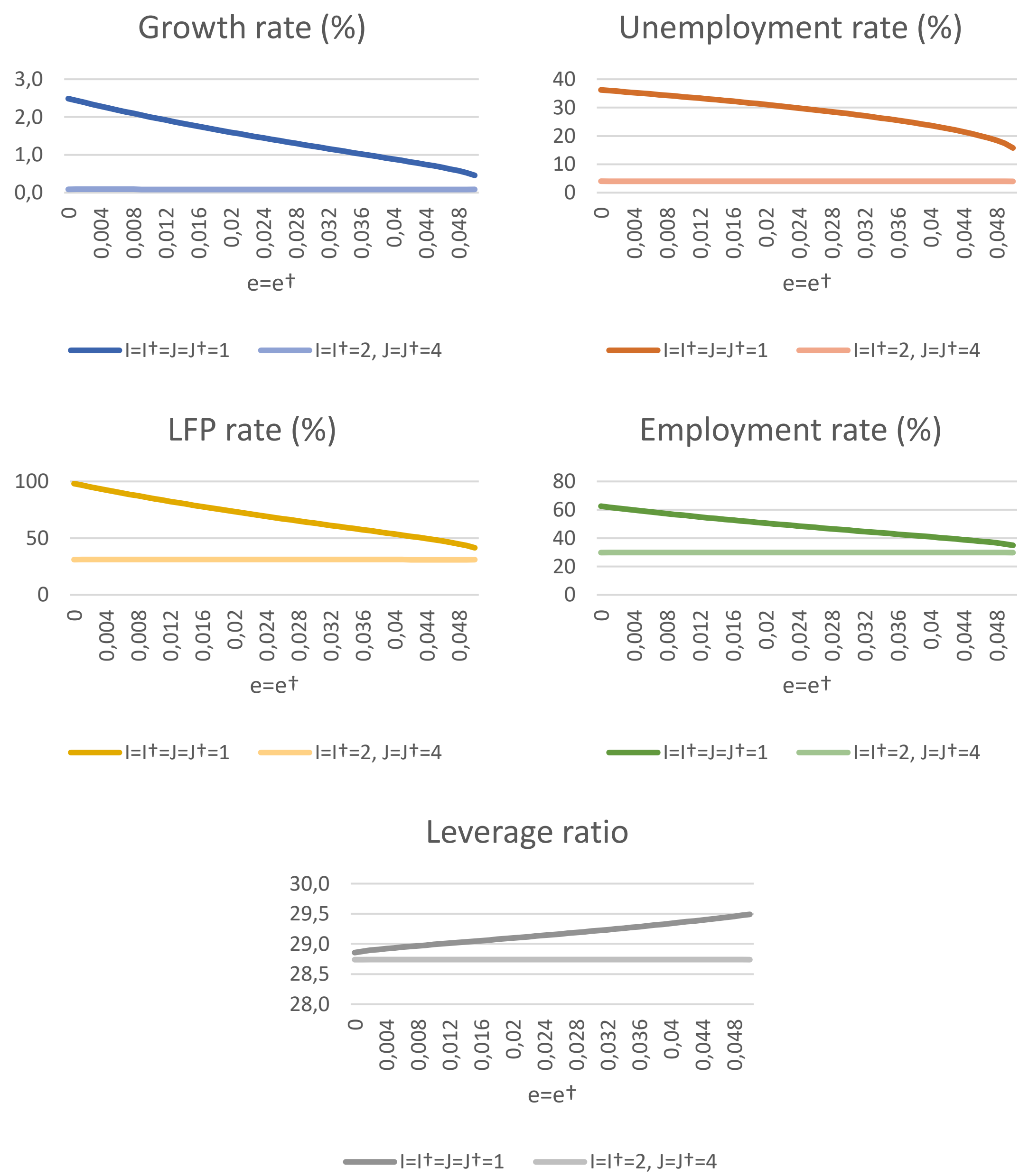
Figure 3.5.8. Sensitivity analysis to changes in $z = z^{\dagger}$, human capital model



Sensitivity analysis for parameter e

The sensitivity analysis for parameter e shows a profile very similar to those obtained for parameter z. Growth, LFP, employment and unemployment rate with flexibility show a notable and sharp decrease as we reduce the value of the parameters, while the value with nominal rigidities is virtually constant. Leverage, on the other hand, decreases as we reduce the value of the parameters. The results obtained in the previous sections are valid for parameters values below 0.05.

Figure 3.5.9. Sensitivity analysis to changes in $e = e^{\dagger}$, human capital model

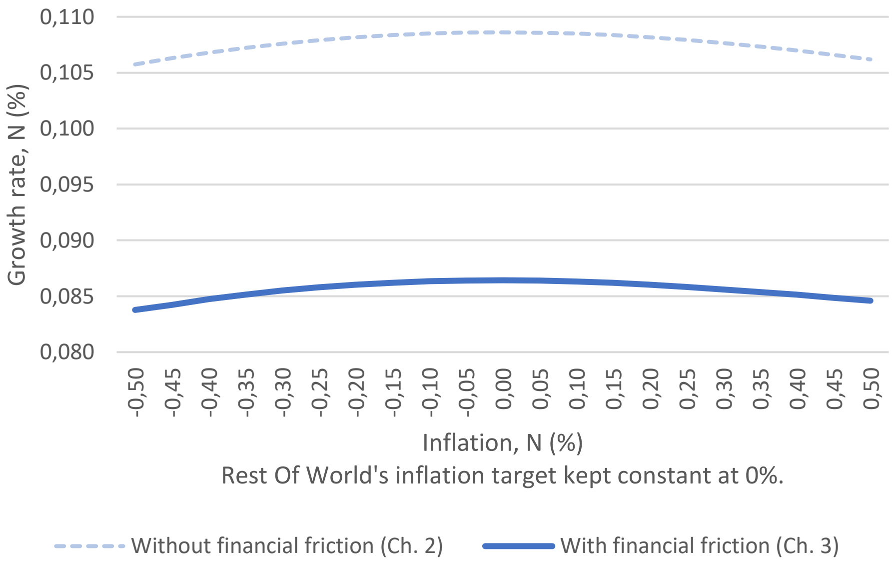


3.6. A wider set of results as consequence of greater structural differences between economies

As in Chapter 2, this section analyzes the results provided by the simulations carried out for the case in which the nation presents greater structural differences than the considered in the previous sections with respect to the rest of the world. We are going to do this analysis, once more, comparing the results obtained until now for the model of Lucas with the new results that we obtain calibrating the model with the parameter values of Spain (N) and OECD average (ROW). In addition, we will also compare these simulations with those in Section 2.7 from the previous chapter, verifying whether the introduction of the financial sector modifies the magnitude of the effect of greater structural differences.

We adopt a different point of view in the results of the previous sections by fixing the inflation target of the rest of the world at zero while varying the inflation target of the nation. Then we compare this perspective of the previous results with those provided by carrying out the same exercise in the model once calibrated with the parameters of Spain, obtaining two-dimensional representations of the resulting simulations.

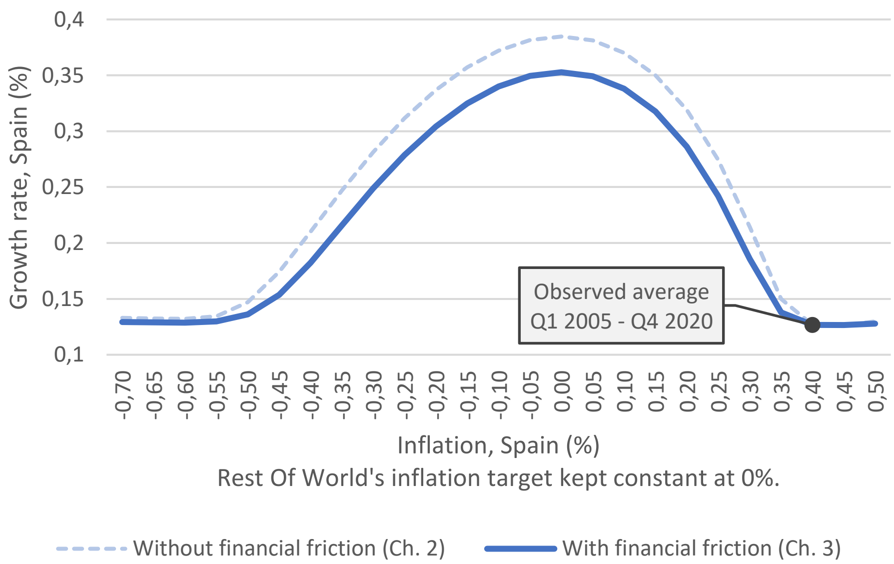
Figure 3.6.1a. Quarterly growth rate, Ch. 3 calibration (%)



Let us begin analyzing the results for the growth rate. Figure 3.6.1a shows the particular case of the results presented so far in this chapter and compares them to those obtained for the model without financial frictions in Chapter 2, evidencing again the loss in the maximum achievable growth rate after the introduction of the financial intermediaries in the model. Notice that this loss in growth rate is nearly the same for any inflation point, around 0.02 percentage points.

Figure 3.6.1b illustrates the same exercise after calibrating the model for the parameter values of Spain. Once more, we find that the gain in the growth rate when Spain moves to the point of null inflation is substantially larger than the increase registered in Figure 3.6.1a, as a consequence of greater structural differentiation between the economies.

Figure 3.6.1b. Quarterly growth rate with financial friction, Spain (%)

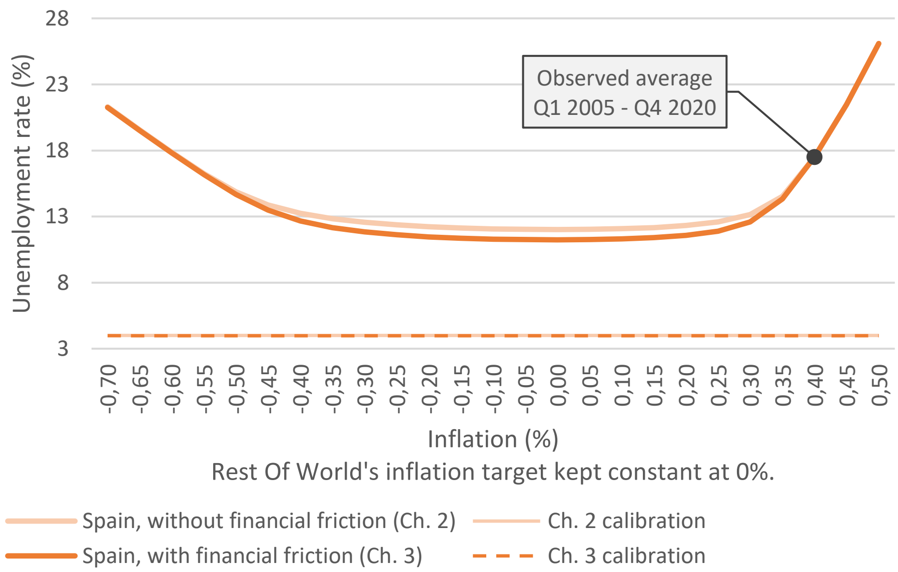


In addition, we also observe a loss in the maximum growth rate after the introduction of the financial intermediaries: while in Chapter 2 we found that Spain could increase its quarterly growth 0.26 percentage points with the adoption of a zero-inflation policy, this gain falls slightly to 0.23 percentage points after the introduction of the new agents. Nevertheless, notice also how this loss becomes smaller as we move further away from the point of null inflation, as the point that represents the observed average of Spain from Q1 2005 to Q4

2020 where the drop almost inexistent. Therefore, it can be concluded that substantial structural differentiation between the economies can also lead to a shift in the magnitude of the effect that the introduction of the financial sector has in the human capital model: now this loss becomes significantly smaller the further away from the point of null inflation.

Now, let us analyze the behavior of the unemployment rate represented in Figure 3.6.2. This figure compares the results obtained in Figure 2.7.2 with those obtained after the introduction of the financial friction. The unemployment rate, once again, no longer remains constant for any value of inflation for the model with the parameter values of Spain, due to greater structural differences with the rest of the world. The relationship inflation-unemployment does not fulfill Friedman's long run Phillips curve, thus evidencing a clear long-run relationship between inflation and unemployment. It is near constant in the surroundings of null inflation but it becomes increasing for values far enough from this value.

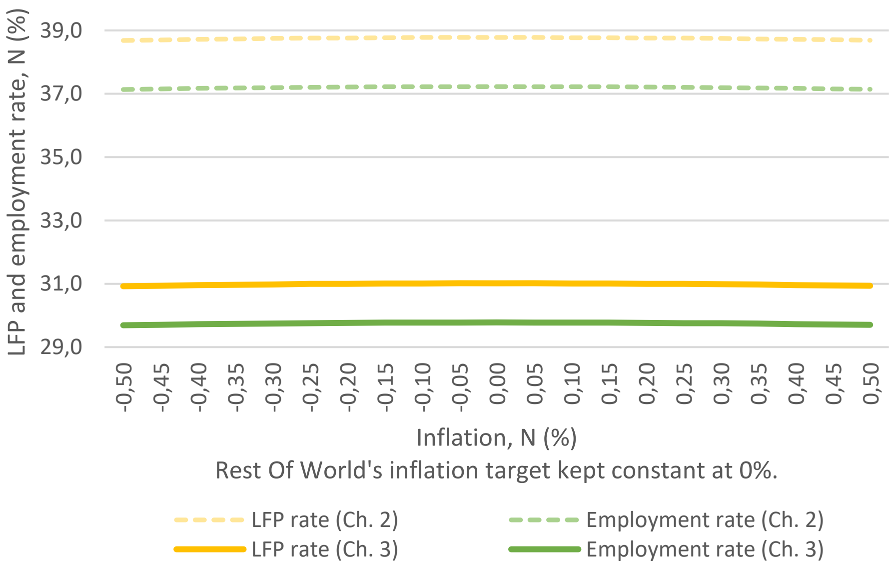
Figure 3.6.2. Unemployment rate, Ch. 3 vs Spain with financial friction (%)



The introduction of financial intermediaries has a greater lowering effect on the unemployment rate: if we found in the previous chapter that Spain could reduce the unemployment rate to 12% by adopting a zero-inflation policy, it would be now 11.2%. This decrease of 0.8 percentage points is significantly larger than the reduction registered with the calibration of the previous sections, where the reduction is limited to 0.02 percentage points, from 4% without financial friction in Chapter 2 to 3.98% with its introduction in this chapter. Moreover, just as we observed for the growth rate, the reduction in the unemployment rate in the case of Spain is not of the same magnitude for any value of the inflation rate since it is greater the more the country approaches the point of null inflation. These results evidence that the introduction of the financial friction affects the magnitude of the effects of greater structural differences.

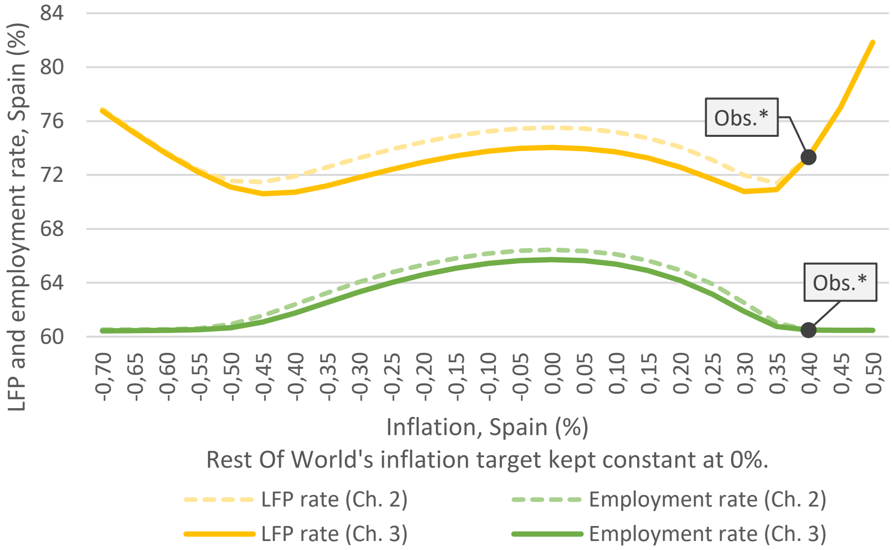
Figure 3.6.3a shows LFP and employment rates behavior for the model with financial intermediaries, compared to those we obtained in Chapter 2 without financial friction in Section 2.7.

Figure 3.6.3a. LFP and employment rate, Ch. 3 calibration (%)



Notice how the reductions in both rates after the introduction of the financial sector for each level of inflation are homogeneous, in contrast to those obtained in Figure 3.6.3b for the parameters of Spain.

Figure 3.6.3b. LFP and employment rate with financial friction, Spain (%)



*Observed average, Q1 2005 - Q4 2020.

In this case, once again, a local maximum appears at the point of null inflation, precisely the point where the decrease reaches its highest magnitude in both rates after the introduction of the new agents. As inflation moves away from that point, this effect falls, becoming substantially smaller as we approach the threshold of inflation or deflation in which both the LFP and the employment rate begin to rise. As explained in the previous chapter, these results represent the particular case of Spain, given that LFP and employment can present a wide diversity of behaviors because of deep structural differences between the two economies.

Overall, the fundamental results obtained in Section 2.7 of the previous chapter after calibrating the human capital model for the parameters of Spain and OECD average still holds after the introduction of the financial intermediaries. A higher variability of the growth rate in response to changes in the long-run inflation target and the nonfulfillment of Friedman's long run Phillips curve remains because of deep structural differences. We must add in this case that the magnitude of the lowering effect that the financial friction has on growth rates, unemployment, LFP and employment in the human capital model is not homogeneous for any given inflation rate. This effect increases as the economy approaches the point of null inflation.

3.7. An empirical application

This last section provides an application looking for the empirical implications of the previous results to six developed countries —Australia, France, Germany, Japan, Spain and the United States— in order to explore the potential gains derived from the policy aimed to achieve their potential long-term economic growth and employment. We consider that the rest of the world is the average of the OCDE countries.

In order to carry out this empirical analysis, we exclusively consider the human capital model developed in this chapter, with nominal rigidities, unemployment and financial friction. We have decided to use the model whose simulations have shown wealthier results.

The observed data have been obtained from the OECD database[7]. Firstly, we estimated the parameters with observations of a selected set of variables[8] with Dynare. Then the models have been additionally calibrated in such a way that steady state growth, employment, labor supply, unemployment and inflation rates reach the average value of the observed series for the period Q1 2005 to Q4 2020 for each country. Table 3.6.1 shows the values of the parameters making possible the steady state of the model to coincide with the average observed data of each economy for the period considered.

Once these parameters have been obtained, the model is re-simulated again but assuming that the economies choose trend null inflation. This way, we are able to compare the observed values with the potential values and conclude whether the economies have any

[7] OECD Statistics [https://stats.oecd.org/]
[8] The variables used in the estimation were growth rate, labor supply, employment, unemployment rate, inflation rate, consumption, foreign trade flows and interest rate.

room for improvement. Table 3.6.2 shows the movements in economic growth and labor market variables for each country when everyone moves to null inflation.

Table 3.6.1. Calibration for each country after estimation, human capital model

Param.	AUS	DEU	ESP	FRA	JPN	USA
β	0,9015	0,929	0,9756	0,96	0,9791	0,929
α	0,6285	0,6502	0,7	0,6502	0,648	0,6502
σ	9,9911	9,9999	10	10,0017	10	10,0230
ν	0,01	0,5348	0,1824	0,3526	0,0501	0,2858
ρ	0	0	0	0	0	0
Ω	0,5	0,5	0,5	0,5	0,5	0,5
ω	0,5	0,5	0,5	0,5	0,5	0,5
ε	1,48	1,48	1,4638	1,483	1,45	1,48
δ	0,008	0,0096	0,064	0,0419	0,054	0,0084
ξ	0,1431	0,1015	0,0424	0,0644	0,0299	0,1078
A	0,9997	0,9997	1,1090	1	1	0,9997
b	0,08	0,1	0,25	0,1	0,1	0,1
q	0,798	0,9	1	0,9	0,9	0,9
z	0,1124	0	0	0	0,1	0
e	0,05	0,0845	0,15795	0,0845	0,0254	0,0845
Γ	0,3988	0,4041	0,5	0,4018	0,4046	0,4081
ψ	0,061	0,0576	0,081	0,0557	0,0578	0,0579
λ	0,273	0,2702	0,168	0,27	0,2699	0,2733

Table 3.6.2. Empirical results Q1 2005 – Q4 2020, human capital model

	AUS	DEU	ESP	FRA	JPN	USA
Quarterly inflation rate, observed	0.57%	0.34%	0.39%	0.3%	0.07%	0.49%
Quarterly inflation rate, target 0%	0%	0%	0%	0%	0%	0%
Difference (1)	**-0.57%**	**-0.34%**	**-0.39%**	**-0.3%**	**-0.07%**	**-0.49%**
Quarterly growth rate, observed	0.59%	0.27%	0.12%	0.15%	0.08%	0.39%
Quarterly growth rate, target 0%	0.60%	0.41%	0.35%	0.24%	0.08%	0.45%
Difference (2)	**0.01%**	**0.14%**	**0.23%**	**0.09%**	**0%**	**0.06%**
LFP rate, observed	76.7%	77.1%	73.1%	70.6%	75.6%	73.8%
LFP rate, target 0%	76.8%	77.8%	74%	69.6%	75.6%	73.9%
Difference (3)	**0.1%**	**0,7%**	**0.9%**	**-1%**	**0%**	**0.1%**
Employment rate, observed	72.5%	72.4%	60.5%	64.3%	72.7%	69.2%
Employment rate, target 0%	72.6%	73.7%	65.7%	65.7%	72.7%	69.7%
Difference (4)	**0.1%**	**1.3%**	**5.2%**	**1.4%**	**0%**	**0.5%**
Unemployment rate, observed	5.47%	6.13%	17.3%	9.01%	3.87%	6.31%
Unemployment rate, target 0%	5.47%	5.24%	11.2%	5.51%	3.87%	5.66%
Difference (5)	**0%**	**-0.89%**	**-6.1%**	**-3.5%**	**0%**	**-0.65%**

We can clearly divide the countries into two groups. The first group of countries shows more substantial increases in economic growth —difference (2)— and higher unemployment drops —difference (5)—. The countries of this group are the members of the Economic and Monetary Union —Spain, France and Germany—. Spain is the country that would show the greatest overall improvement, with an increase in economic growth of 0.23 percentage points per quarter and a strong reduction in unemployment of 6.1 percentage points, down to 11.2% from the average observed rate 17.3%. In economic growth, Germany follows with a gain of 0.14 quarterly percentage points and then France with 0.09. Regarding the unemployment rate, in France it would fall 3.5 percentage points while in Germany just 0.89.

The second group of countries includes the United States, Australia and Japan, with the first leading the moderate improvements after adopting null inflation, both in economic growth, with a quarterly increase of 0.06 percentage points, and in unemployment, with a reduction of 0.65 percentage points. However, the other two countries, Australia and Japan, just show infinitesimal or null improvements after adopting null inflation.

The particular aspect of these results is that we have seen, in the simulations carried out in this chapter and in the previous one, how movements in trend inflation had effects on economic growth but not on the unemployment rate when the structural difference between the two economies is as much in one parameter (discount rate). However, when more structural differences are present, we have also seen in the simulations of both chapters that considerable reductions in the unemployment rate are possible when one of the economies move to null inflation. Obviously, the changes in the unemployment rate for the three EMU countries included of the first group correspond to this second situation of a high degree of structural difference between them and the rest of the world.

This effect takes place because, depending on the degree of structural difference between each economy and the rest of the world, the relationships between the inflation rate and the growth and unemployment rates change their features. The more the difference, the more can be the improvement in both variables because the relationship with the growth rate is steepest and with de unemployment tends to increase when the inflation rate moves away from zero. These features take place in these three countries because their parameters are farther away from those of the rest of the world (OECD average), especially if we compare them with the values of the United States, Australia or Japan, closer to the OECD average. These differences between the nation and the rest of the world, which go further than just the discount rate considered in the simulations, generate the possibilities of improvement. The more the differences between the parameters of a country and those of the rest of the world, the greater the possibility of a higher positive impact on economic growth and a higher decrease in unemployment when the economy shifts to null inflation.

3.8. Conclusions

We have extended in this third chapter the two open-economy New Keynesian models of Chapter 2 with a new agent and implemented an empirical application. We introduce financial intermediaries in order to study how a financial distortion affects the results obtained in the previous chapter on the relationship between long-term growth, employment, LFP and unemployment and trend inflation, seeking in particular to determine the link between those variables and the leverage ratio. Additionally, we have also explored the empirical implication of the extended human capital model because it shows higher long-run sensibility than the Schumpeterian model to changes in the inflation rate, as have been shown throughout the current and previous chapters.

Our main results can be summarized as follows:

(i) The introduction of the financial sector does not change qualitatively the main findings of Chapter 2 regarding the inflation-growth, inflation-unemployment, inflation-LFP and inflation-employment relationships for economies that are identical or different only in the discount rate, but add specific interesting results on the effects of the financial distortion.

(ii) When the structural difference of the two economies is generalized to many of the parameters in the human capital model with sticky wages we find that the relationship between trend inflation and unemployment rate change in a very interesting way because has its minimum for zero inflation and the relationship trend inflation-growth is significantly steeper. This two results point to potential important long run gains from moving to null inflation.

(iii) The leverage ratio is always constant regardless the value of the trend inflation rate when price and wage flexibility prevail in both economies. When wages are sticky, the leverage ratio adopts in the Schumpeterian model a low-sloped paraboloid shape with a minimum at the point of null inflation, while in the human capital model we obtain a bell-shaped average international leverage rate with a maximum at that point, precisely the combination of inflation rates that also maximize the growth rate of both economies. The contagion of non-neutrality is also present when we consider a flexible nation and a rigid rest of the world. Therefore, no clear effect of leverage ratio on economic growth can be inferred, even introducing the same type of financial friction in both models.

(iv) The introduction of the financial intermediaries generates a decrease in growth, LFP and employment rates for any wage settings in both models, with the exception of the human capital model with flexibility in the nation and rigidities in the rest of the world. In this case it generates an increase in the flexible nation and a slightly fall in the rigid rest of the world due to an increase in its exchange rate that stimulates nation's exports.

(v) The effect on the unemployment rate is not clear. In the human capital model, we always observe a decrease when both economies have the same type of wage setting process, but in the Schumpeterian model there is a slight decrease with wage flexibility and a null effect with wage rigidity. When the nation is flexible and the rest of the world is rigid, unemployment falls very slightly in the flexible nation and does not vary in the rigid rest of the world in the Schumpeterian model but, in contrast, these results are the opposite in the human capital model.

(vi) This chapter ends with an exploration of the empirical implications of the model initially simulated that has shown wealthier results, by estimating and calibrating the human capital model for six developed economies: Australia, France, Germany, Japan, Spain and the United States. Once estimated the parameters for each country, we calibrated them to reach its long-run average main variables (growth, LFP, employment and unemployment) in order to re-simulate the resulting models and compare the observed values to the potential with null inflation. The results divide the countries into two groups: the "EMU group", with substantial increases in growth and reductions in unemployment after moving to null inflation, and a second group that includes the United States, Australia and Japan, with the first leading the very moderate potential benefits of adopting a null inflation target, while Australia and Japan present just infinitesimal improvements. The reason of this division is that the more differences a country has compared to the rest of the world (OECD average), the greater the impact on growth and unemployment when moving to null inflation.

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Appendices

Appendix 1.1. Physical capital externality model

Relationships of the steady state for Nation (and the equivalent for Rest of World)

Households
$$\beta \frac{R^{st}}{\mathrm{g}\Pi} = 1 \qquad (1.1.5')$$
Intermediate good producers
$$L = \frac{(1-\alpha)Y^{i^K}}{\Delta_W} \qquad (1.1.9a')$$
$$\Delta_W = \left[\frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{W_{-\mathrm{T}}^*}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad (1.1.9b')$$
$$Y^{i^K} = L^{1-\alpha} \qquad (1.1.10')$$
$$\frac{W_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{-\mathrm{T}}}\left(\frac{\sigma}{\sigma-1}\frac{(1-\alpha)^{\nu}\left(Y^{i^K}\right)^{\nu} C^K}{\Delta_W^{\nu}}\frac{\sum_{\tau=0}^{J-1}\beta^{\tau}\Pi^{\sigma(1+\nu)\tau}}{\sum_{\tau=0}^{J-1}\beta^{\tau}\Pi^{(\sigma-1)\tau}}\right)^{\frac{1}{1+\nu\sigma}} \qquad (1.1.12')$$ for $\mathrm{T} = [0, \ldots, J-1]$
$$r^q = \alpha P^i Y^{i^K} + 1 - \delta \qquad (1.1.13')$$
$$r^q = R \qquad (1.1.14')$$
Capital producers
$$I^K = I^{n^K} + \delta \qquad (1.1.16')$$
$$g = 1 + I^{n^K} \qquad (1.1.17')$$
Final good producers
$$1 = \left(\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \qquad (1.1.22b')$$
$$\frac{P_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{\mathrm{T}}}\frac{\varepsilon}{\varepsilon-1}\frac{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon})^{\tau}P^i}{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon-1})^{\tau}} \qquad (1.1.24')$$ for $\mathrm{T} = [0, \ldots, I-1]$
$$Y^{i^K} = \Delta_P Y^K \qquad (1.1.25a')$$
$$\Delta_P = \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\varepsilon} \qquad (1.1.25b')$$
Central bank
$$R^{st} = R\Pi \qquad (1.1.27')$$

Equilibrium conditions and external sector

Nation

$$Y^K = C^K + I^K + X^K - lX^{\mathrm{K}\dagger} \tag{1.1.28a'}$$

$$X^K = (\rho + \Omega e^{\omega} Y^{\mathrm{K}\dagger})l \tag{1.1.29a'}$$

Rest of World

$$Y^{\mathrm{K}\dagger} = C^{\mathrm{K}\dagger} + I^{\mathrm{K}\dagger} + X^{\mathrm{K}\dagger} - \frac{1}{l}X^K \tag{1.1.28b'}$$

$$X^{\mathrm{K}\dagger} = \left[\rho^\dagger + \Omega^\dagger \left(\frac{1}{e}\right)^{\omega^\dagger} Y^K\right]\frac{1}{l} \tag{1.1.29b'}$$

International relationships

$$b^K = R^i \frac{b_{-1}^K}{g} - X^K + elX^{\dagger^K} \tag{1.1.30a'}$$

$$b^{\dagger^K} = R^i \frac{{b_{-1}^\dagger}^K}{g^\dagger} - \mathrm{X}^{\dagger^K} + \frac{1}{el}X^K \tag{1.1.30b'}$$

$$R^i = \frac{1}{2}[(R^\dagger - R)\, sgn(b^K) + (R^\dagger + R)] \tag{1.1.31'}$$

$$b^{\dagger^K} - \left(\frac{{b_{-1}^\dagger}^K}{\Pi^\dagger g^\dagger}\right) = -\left[\left(\frac{b^K}{el} - \frac{b_{-1}^K}{\Pi^\dagger e_{-1} lg}\right)\right] \tag{1.1.32'}$$

$$R = R^\dagger + \frac{e_{+1} - e}{e} \tag{1.1.33'}$$

For $I = I^\dagger = 2$ and $J = J^\dagger = 4$, the physical capital externality model's system of equations is composed of forty-five equations and forty-five endogenous variables.

Appendix 1.2. Schumpeterian model

Relationships of the steady state for Nation (and the equivalent for Rest of World)

Households

$$\beta\frac{R}{\mathrm{g}\Pi}=1 \tag{1.1.5$'$}$$

Final good producers

$$L=\frac{(1-\alpha)}{a\Delta_W^Y} \tag{1.2.6$'$}$$

$$\Delta_W=\left[\frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{W_{-\mathrm{T}}^*}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{1.2.7$'$}$$

$$\frac{W_{-\mathrm{T}}^*}{P}=\frac{1}{\Pi^{-\mathrm{T}}}\left(\frac{\sigma}{\sigma-1}\frac{C^Y(1-\alpha)^\nu}{A^{Y\,1+\nu}\Delta_W^{(1-\sigma)\nu}}\frac{\sum_{\tau=0}^{J-1}\beta^\tau\Pi^{\sigma(1+\nu)\tau}}{\sum_{\tau=0}^{J-1}\beta^\tau\Pi^{(\sigma-1)\tau}}\right)^{\frac{1}{1+\nu\sigma}} \tag{1.2.8$'$}$$

$$\text{for } \mathrm{T}=[0,\dots,J-1]$$

$$A=\frac{1}{\left(\alpha^{\frac{1}{1-\alpha}}\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha}L} \tag{1.2.9$'$}$$

Intermediate good producers

$$g=\left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}-1\right)\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}}(\gamma-1)+1 \tag{1.2.16$'$}$$

$$\frac{P_{-\mathrm{T}}^*}{P}=\frac{1}{\Pi^{\mathrm{T}}}\frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{1}{1-\alpha}}\right)^\tau}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{\alpha}{1-\alpha}}\right)^\tau} \tag{1.2.17$'$}$$

$$\text{for } \mathrm{T}=[0,\dots,I-1]$$

Equilibrium conditions and external sector

Nation

$$C=1-\left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}-1\right)\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}}A-$$

$$-\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}A-X+lX^\dagger \tag{1.2.18a'}$$

$$X=(\rho+\Omega e^\omega)l \tag{1.2.19a'}$$

Rest of World

$$C^{\dagger} = 1 - \left[\chi^{\dagger}\alpha^{\dagger\frac{1}{1-\alpha^{\dagger}}}L^{\dagger}\frac{1}{I^{\dagger}}\sum_{\tau=0}^{I^{\dagger}-1}\left(\frac{P_{-\tau}^{\dagger*}}{P^{\dagger}}-1\right)\left(\frac{P_{-\tau}^{\dagger}}{P^{\dagger}}\right)^{-\frac{1}{1-\alpha^{\dagger}}}\right]^{\frac{1}{1-\chi^{\dagger}}} A^{\dagger} -$$

$$-\alpha^{\dagger\frac{1}{1-\alpha^{\dagger}}}L^{\dagger}\frac{1}{I^{\dagger}}\sum_{\tau=0}^{I^{\dagger}-1}\left(\frac{P_{-\tau}^{\dagger*}}{P^{\dagger}}\right)^{-\frac{1}{1-\alpha^{\dagger}}} A^{\dagger} - X^{\dagger} + \frac{1}{l}X \qquad (1.2.18b')$$

$$X^{\dagger} = \left[\rho^{\dagger} + \Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\right]\frac{1}{l} \qquad (1.2.19b')$$

International relationships

$$b^{K} = R^{i}\frac{b_{-1}^{K}}{g} - X^{K} + elX^{\dagger K} \qquad (1.1.30a')$$

$$b^{\dagger K} = R^{i}\frac{b_{-1}^{\dagger\ K}}{g^{\dagger}} - \mathrm{X}^{\dagger K} + \frac{1}{el}X^{K} \qquad (1.1.30b')$$

$$R^{i} = \frac{1}{2}\left[(R^{\dagger} - R)\, sgn(b^{K}) + (R^{\dagger} + R)\right] \qquad (1.1.31')$$

$$b^{\dagger K} - \left(\frac{b_{-1}^{\dagger\ K}}{\Pi^{\dagger}g^{\dagger}}\right) = -\left[\left(\frac{b^{K}}{el} - \frac{b_{-1}^{K}}{\Pi^{\dagger}e_{-1}lg}\right)\right] \qquad (1.1.32')$$

$$R = R^{\dagger} + \frac{e_{+1} - e}{e} \qquad (1.1.33')$$

For $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$, the Schumpeterian model's system of equations is composed of thirty-one equations and thirty-one endogenous variables.

Appendix 1.3a. Human capital model

Relationships of the steady state for Nation (and the equivalent for Rest of World)

Households

$$g = \frac{\beta}{1 + \delta - R(1 - X^{\dagger})} - 1 \tag{1.3.7}$$

$$N^0 = \frac{1}{\xi}\left(1 - \frac{\beta \Pi^{J-1}}{1+g}\right) \tag{1.3.8a}$$

$$N^1 = \frac{1}{\xi}\left(1 - \frac{\beta \Pi^{-1}}{1+g}\right) \tag{1.3.8b}$$

Intermediate good producers

$$\Delta_W = \left[\frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{W_{-\tau}^*}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{1.3.12'}$$

$$\frac{W_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{-\mathrm{T}}}\left[\left(\frac{\sigma}{\sigma-1}\right)\left(\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\Delta_W{}^{1-\alpha\sigma}}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} C^K \frac{\sum_{\tau=0}^{J-1}\beta^{\tau} N_{\tau}^{1+\upsilon}}{\sum_{\tau=0}^{J-1}\beta^{\tau}\Pi^{(\sigma-1)\tau}}\right]^{\frac{1}{1-\sigma}} \tag{1.3.13'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$R = \alpha\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left[\frac{1-\alpha}{\Delta_{Wt}}\right]^{\frac{1-\alpha}{\alpha}} \tag{1.3.14'}$$

Final good producers

$$\frac{P_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{\mathrm{T}}}\frac{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon})^{\tau}}{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon-1})^{\tau}} \tag{1.1.24'}$$

for $\mathrm{T} = [0, \dots, I-1]$

$$\Delta_P = \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\varepsilon} \tag{1.1.25b'}$$

Equilibrium conditions and external sector

Nation

$$C^K = \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} - g - \delta + X^K - lX^{\dagger K^{\dagger}} \tag{1.3.15a'}$$

$$X^K = \Omega e^{\omega} l\, \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} \tag{1.3.16a'}$$

Rest of World

$$C^{\dagger K^{\dagger}} = \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} - g^{\dagger} - \delta^{\dagger} + X^{\dagger K^{\dagger}} - \frac{1}{l}X^{K} \qquad (1.3.15b')$$

$$X^{\dagger K^{\dagger}} = \Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\frac{1}{l}\frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} \qquad (1.3.16b')$$

International relationships

$$b^{K} = R^{i}\frac{b_{-1}^{K}}{g} - X^{K} + elX^{\dagger K} \qquad (1.1.30a')$$

$$b^{\dagger K} = R^{i}\frac{b_{-1}^{\dagger\ K}}{g^{\dagger}} - \mathrm{X}^{\dagger K} + \frac{1}{el}X^{K} \qquad (1.1.30b')$$

$$R^{i} = \frac{1}{2}\left[(R^{\dagger}-R)\,sgn(b^{K}) + (R^{\dagger}+R)\right] \qquad (1.1.31')$$

$$b^{\dagger K} - \left(\frac{b_{-1}^{\dagger\ K}}{\Pi^{\dagger}g^{\dagger}}\right) = -\left[\left(\frac{b^{K}}{el} - \frac{b_{-1}^{K}}{\Pi^{\dagger}e_{-1}lg}\right)\right] \qquad (1.1.32')$$

$$R = R^{\dagger} + \frac{e_{+1}-e}{e} \qquad (1.1.33')$$

For $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$, the human capital model's system of equations is composed of thirty-three equations and thirty-three endogenous variables.

Appendix 1.3b Optimal control problem and steady state implications in human capital model

Wage flexibility

The wage is the same for all types of labor services. The Hamiltonian for this problem is:

$$H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 (N_{st+\tau})^{1+\nu} ds\right]$$

$$+ \lambda_{1,t+\tau}\left[\mathrm{D}_{t+\tau} - b_{t+\tau} + b_{t+\tau-1}R_t + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{st+\tau}\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right) ds\right.$$

$$\left. + (R_{t+\tau} - 1 - \delta)K_{t+\tau} - C_{t+\tau}\right] + \lambda_{2,t+\tau}\left\{\int_0^1 \xi(1-u_{st+\tau})N_{st+\tau}h_{st+\tau} ds\right\}$$

subject to (1.3.9), (1.3.13), $(1.3.16a)$, $(1.3.16b)$, $(1.1.30a)$, $(1.1.30b)$ and (1.1.32).

$$H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 (N_{st+\tau})^{1+\nu} ds\right]$$

$$+ \lambda_{1,t+\tau}\left[\mathrm{D}_{t+\tau} + X_t - e_{t+\tau}X^{\dagger}_{t+\tau} + \int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{st+\tau}\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right) ds\right.$$

$$\left. + (R_{t+\tau} - 1 - \delta)K_{t+\tau} - C_{t+\tau}\right] + \lambda_{2,t+\tau}\left\{\int_0^1 \xi(1-u_{st+\tau})N_{st+\tau}h_{st+\tau} ds\right\}$$

The first order conditions are the followings:

$$(A1.3b.1) \quad \frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(A1.3b.2) \quad \beta^{\tau} {N_{st+\tau}}^{\nu}$$

$$= \lambda_{1,t+\tau}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) u_{st+\tau}h_{st+\tau}$$

$$+ \lambda_{2,t+\tau}\xi(1-u_{st+\tau})h_{st+\tau} \quad \forall s \in [0,1]$$

$$(A1.3b.3) \quad \lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) \qquad \forall s \in [0,1]$$

$(A1.3b.4)\quad \lambda_{1,t+\tau+1}-\lambda_{1,t+\tau}=-\lambda_{1,t+\tau}(R_{t+\tau}-1-\delta)-$

$$\lambda_{1,t+\tau}\int_0^1\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right)^{-\sigma}\left[\frac{(1-\alpha)A}{\left(\Delta^i_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}}ds+\lambda_{1,t+\tau}a\frac{e_{t+\tau}X^\dagger_{t+\tau}}{K_{t+\tau}}$$

$(A1.3b.5)\quad \lambda_{2,t+\tau+1}-\lambda_{2,t+\tau}$

$$=-\lambda_{1,t+\tau}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}-(1-a)\frac{e_{t+\tau}X^\dagger_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u_{st+\tau}N_{st+\tau}$$

$$-\lambda_{2,t+\tau}\,\xi(1-u_{st+\tau})N_{st+\tau}\quad \forall s\in[0,1]$$

$$(A1.3b.6)\quad K_{t+\tau+1}=\mathrm{D}_{t+\tau}+\int_0^1\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)L_{i,t+\tau}\left(W^*_{i,t+\tau}\right)ds+(R_{t+\tau}-\delta)K_{t+\tau}-C_{t+\tau}$$

$$(A1.3b.7)\quad h_{t+\tau+1}=\left\{\int_0^1[1+\xi(1-u_{st+\tau})N_{st+\tau}]\frac{h_{st+\tau}}{h_{t+\tau}}ds\right\}h_{t+\tau}$$

In the steady state, from $(A1.3b.1)$:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}}=1+g(\lambda_1)=\frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}}=\frac{\beta}{1+g}$$

From $(A1.3b.3)$:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}}=\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}}=1+g(\lambda_2)=\frac{\beta}{1+g}$$

From $(A1.3b.4)$ and $(A1.3b.1)$:

$$1+g=\frac{\beta}{1+\delta-a\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}\left(1-e\left[\rho^\dagger+\Omega^\dagger\left(\frac{1}{e}\right)^{\omega^\dagger}\right]\right)}$$

From $(A1.3b.5)$ and $(A1.3b.3)$:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}}=1-\zeta N_{st+\tau}=\frac{\beta}{1+g}$$

The steady state supply of labor is the same for all s and is constant over time. From this expression, we obtain the constant value N_{ss} in the steady state:

$$N_{ss} = \frac{1}{\zeta}\left(1 - \frac{\beta}{1+g}\right)$$

From $(A1.3b.2)$ and $(A1.3b.3)$:

$$\frac{\beta^{\tau+1} {N_{st+\tau+1}}^{\upsilon}}{\beta^{\tau} {N_{st+\tau}}^{\upsilon}} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} \frac{h_{st+\tau+1}}{h_{st+\tau}}$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}} \frac{\beta}{1+g} => g(h_s) = g$$

The growth rate of human capital is the same as the output growth rate and the same for all s. We obtain the steady state value of u from the accumulation process of human capital:

$$h_{st+\tau+1} = h_{st+\tau} + \xi(1 - u_{st+\tau})N_{st+\tau}h_{st+\tau}$$

The steady state growth rate of human capital is:

$$g(h_s) = g = \xi(1 - u_{st+\tau})N_{st+\tau} = \xi(1 - u_{ss})N_{ss}$$

where u_{ss} is the steady state value for any s. From this expression, we can deduce that the value of *u* is also the same for all types of labor services and is constant over time:

$$u_{ss} = 1 - \frac{g}{\xi N_{ss}}$$

We close the system of equations in steady state with the expressions obtained in this subsection.

Sticky wages

Note that the first order condition for $u_{st+\tau}$ in $(A1.3b.3)$ implies that the real wage at time $t+\tau$ must be the same for all individuals. However, since the nominal wage correspond to the effective labor, the re-optimized real wage with rigidity should be constant in the steady state and, therefore, the nominal re-optimized wage grows at the same rate as the aggregate price. This implies that, when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, this contradicts $(A1.3b.3)$. Therefore, the previous problem is not valid with wage rigidity.

The Hamiltonian for this situation is:

$$
\begin{aligned}
H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 (N_{st+\tau})^{1+\nu} ds\right] \\
+ \lambda_{1,t+\tau}\Bigg[\mathrm{D}_{t+\tau} + X_t - e_{t+\tau}X_{t+\tau}^{\dagger} + \int_0^1 \left(\frac{W_{st+\tau}^{*}}{P_{t+\tau}}\right) L_{st+\tau}\left(\frac{W_{st+\tau}^{*}}{P_{t+\tau}^{i}}\right) ds \\
+ (R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}\Bigg] \\
+ \sum_{q=0}^{J-1} \lambda_{2,t+\tau}^{1q}\left\{\int_{\frac{q}{J}}^{\frac{q+1}{J}} \xi\left(1 - u_{i,t+\tau}^{1q}\right) N_{st+\tau}^{1q} h_{st+\tau}^{1q}\, ds\right\} \\
\mathrm{q} = 1, 2, \ldots, \mathrm{J} - 1
\end{aligned}
$$

subject to $(1.3.5)$, $(1.3.9)$, $(1.3.10)$, $(1.3.11)$, $(1.3.12)$, $(1.3.14)$, $(1.1.22a)$, $(1.1.22b)$, $(1.3.13)$, $(1.1.24')$, $(1.3.16a)$, $(1.3.16b)$, $(1.1.30a)$, $(1.1.30b)$ and $(1.1.32)$.

The first order conditions are the followings:

$$(A1.3b.8) \quad \frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(A1.3b.9) \qquad \beta^{\tau} {N_{st+\tau}}^{v}$$

$$= \lambda_{1,t+\tau}\left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u_{st+\tau}h_{st+\tau}$$

$$+ \lambda_{2,t+\tau}\xi(1-u_{st+\tau})h_{st+\tau} \quad \forall s \in [0,1]$$

$$(A1.3b.10.1), (A1.3b.10.2), \dots, (A1.3b.10.\mathrm{J}) \qquad \lambda_{2,t+\tau}$$

$$= \frac{\lambda_{1,t+\tau}}{\xi}\left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right]$$

$$q = 0, 1, 2, \dots, J-1$$

$$(A1.3b.11) \quad \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau} = -\lambda_{1,t+\tau}(R_{t+\tau} - 1 - \delta) -$$

$$\lambda_{1,t+\tau}\int_0^1 \left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}}\right)\left(\frac{W^{*}_{st+\tau}}{P^{i}_{t+\tau}}\right)^{-\sigma}\left[\frac{(1-\alpha)A}{\left(\Delta^{i}_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}} ds + \lambda_{1,t+\tau}a\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{K_{t+\tau}} \qquad \forall s \in [0,1]$$

$$(A1.3b.12.1), (A1.3b.12.3), \dots, (A1.3b.12.\mathrm{J}) \quad \lambda^{q}_{2,t+\tau+1} - \lambda^{q}_{2,t+\tau}$$

$$= -\lambda_{1,t+\tau}\left(\frac{W^{q}_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u^{q}_{st+\tau}N^{q}_{st+\tau}$$

$$- \lambda^{q}_{2,t+\tau}\,\xi\left(1-u^{q}_{st+\tau}\right)N^{q}_{st+\tau} \quad \forall s \quad \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \qquad q = 0, 1, 2, \dots, J-1$$

$$(A1.3b.13) \quad K_{t+\tau+1}$$

$$= \mathrm{D}_{t+\tau} + X_{t+\tau} - e_{t+\tau}X^{\dagger}_{t+\tau} + \int_0^1 \left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}}\right)L_{st+\tau}\left(\frac{W^{*}_{st+\tau}}{P^{i}_{t+\tau}}\right)di$$

$$+ (R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}$$

$$(A1.3b.14) \quad h_{t+\tau+1} = \left\{\int_0^1 [1 + \xi(1-u_{st+\tau})N_{st+\tau}]\frac{h_{st+\tau}}{h_{t+\tau}}ds\right\}h_{t+\tau}$$

In the steady state, from $(A1.3b.8)$:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g} \text{ (*)}$$

From $(A1.3b.11)$ and (*):

$$1+g=\frac{\beta}{1+\delta-a\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}\left(1-e\Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\right)}$$

From $(A1.3b.10.11)$ - $(A1.3b.10.J)$ (which represents labor services, which change or do not change wages in $t+\tau+1$):

$$\frac{\lambda^{q}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}}=\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}}=1+g(\lambda^{q}_{2})=\frac{\beta}{1+g} \quad q=0,1,2,\dots,J-1 \quad \forall s\in\left[\frac{q}{J},\frac{q+1}{J}\right] \text{in t}+\tau$$

Consequently, there will be two values of *N*. From A1.12 and (*):

$$\frac{\lambda^{q}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}}=1-\xi N^{q}_{st+\tau}=1-\xi N^{1q}_{st+\tau}=\frac{\beta}{1+g}$$

$$N^{q}=N=\frac{1}{\xi}\left(1-\frac{\beta}{1+g}\right) \quad q = 0, 1, 2, \dots, \text{J-1 in t+t} \quad \forall s\in\left[\frac{q}{J},\frac{q+1}{J}\right]$$

From $(A1.3b.9)$:

$$\frac{\beta^{\tau+1}N_{st+\tau+1}{}^{v}}{\beta^{\tau}N_{st+\tau}{}^{v}}=\frac{\lambda^{q}_{2,t+\tau+1}}{\lambda^{q}_{2,t+\tau}}\frac{h_{st+\tau+1}}{h_{st+\tau}} \qquad q = 0, 1, 2, \dots, \text{J-1} \qquad \forall s\in\left[\frac{q}{J},\frac{q+1}{J}\right] \text{ in t+t}$$

$$\beta=\frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta}{1+g} => g=1+g(h)-1 \qquad q=0,1,2,3,\dots,J-1 \quad \forall s\in\left[\frac{q}{J},\frac{q+1}{J}\right] \text{ in t+t}$$

From the last expression, there will also be three expressions of *u* in the steady state.

Appendix 2.1. Schumpeterian model

Relationships that must be considered for both Nation and Rest of World

Households

$$N_{-\mathrm{T}} = \left(\frac{1}{C^Y}(1-d)w^Y_{-\mathrm{T}}\right)^{\frac{1}{\nu}} \qquad N = \frac{1}{J}\sum_{\tau=0}^{J-1} N_{-\tau} \tag{2.1.3'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$\beta\frac{R}{\mathrm{g}\Pi} = 1 \tag{2.1.4'}$$

Final good producers

$$L_{-\mathrm{T}} = \left(\frac{(1-\alpha)L^{\frac{1-\sigma}{\sigma}}}{w^Y_{-\mathrm{T}}}\right)^{\sigma} \qquad LL = \frac{1}{J}\sum_{\tau=0}^{J-1} L_{-\tau} \tag{2.1.6'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$L = \frac{(1-\alpha)}{\Delta^Y_W} \tag{2.1.7'}$$

$$\Delta^Y_W = \left[\frac{1}{J}\sum\nolimits_{\tau=0}^{J-1}(w^Y_{-\mathrm{T}})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{2.1.8'}$$

$$d = \frac{N-L}{N} \tag{2.1.11'}$$

$$w^Y_{-\mathrm{T}} = \frac{1}{\Pi^{-\mathrm{T}}}\frac{\frac{e}{A^Y}\Delta_b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R + b\Delta_b)}{4R + b\Delta_b + d\Delta_d}\frac{z}{A^Y}\Delta_d}{\Delta^b_w\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] - \Delta^{bq}_w(4R + b\Delta_b)} \tag{2.1.15'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$A^Y = \frac{1}{\left(\alpha^{\frac{1}{1-\alpha}}\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha} L} \tag{2.1.16'}$$

Intermediate good producers

$$g = \left[\chi\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P} - 1\right)\left(\frac{P^*_{-\tau}}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}}(\gamma - 1) + 1 \tag{2.1.19'}$$

$$\frac{P^*_{-\mathrm{T}}}{P} = \frac{1}{\Pi^{\mathrm{T}}}\frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{1}{1-\alpha}}\right)^{\tau}}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{\alpha}{1-\alpha}}\right)^{\tau}} \tag{2.1.20'}$$

for $\mathrm{T} = [0, \dots, I-1]$

Equilibrium conditions and external sector

For the Nation:

$$C^Y = 1 - \left[\chi\alpha^{\frac{1}{1-\alpha}} L \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P}-1\right)\left(\frac{P^*_{-\tau}}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}} A^Y -$$

$$-\alpha^{\frac{1}{1-\alpha}} L \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P}\right)^{-\frac{1}{1-\alpha}} A^Y - X^Y + lX^{\dagger^{Y^\dagger}} \qquad (2.1.21a')$$

$$X^Y = (\rho + \Omega e^{\omega})l \qquad (2.1.22a')$$

For the Rest of World:

$$C^{\dagger^{Y^\dagger}} = 1 - \left[\chi^\dagger\alpha^{\dagger\frac{1}{1-\alpha^\dagger}} L^\dagger \frac{1}{I^\dagger}\sum_{\tau=0}^{I^\dagger-1}\left(\frac{P^{\dagger*}_{-\tau}}{P^\dagger}-1\right)\left(\frac{P^\dagger_{-\tau}}{P^\dagger}\right)^{-\frac{1}{1-\alpha^\dagger}}\right]^{\frac{1}{1-\chi^\dagger}} A^{\dagger^{Y^\dagger}} -$$

$$-\alpha^{\dagger\frac{1}{1-\alpha^\dagger}} L^\dagger \frac{1}{I^\dagger}\sum_{\tau=0}^{I^\dagger-1}\left(\frac{P^{\dagger*}_{-\tau}}{P^\dagger}\right)^{-\frac{1}{1-\alpha^\dagger}} A^{\dagger^{Y^\dagger}} - X^{\dagger^{Y^\dagger}} + \frac{1}{l}X^Y \qquad (2.1.21b')$$

$$X^{\dagger^{Y^\dagger}} = \left[\rho^\dagger + \Omega^\dagger\left(\frac{1}{e}\right)^{\omega^\dagger}\right]\frac{1}{l} \qquad (2.1.22b')$$

International relationships

$$b = R^i\frac{b_{-1}}{g} - X^Y + elX^{\dagger^{Y^\dagger}} \qquad (2.1.24a')$$

$$b^\dagger = R^i\frac{b^\dagger_{-1}}{g^\dagger} - X^{\dagger^{Y^\dagger}} + \frac{1}{el}X^Y \qquad (2.1.24b')$$

$$R^i = \frac{1}{2}[(R^\dagger - R)\,sgn(b) + (R^\dagger + R)] \qquad (2.1.25')$$

$$b^\dagger - \left(\frac{b^\dagger_{-1}}{\Pi^\dagger}\right) = -\left[\left(\frac{b}{e} - \frac{b_{-1}}{\Pi^\dagger e_{-1}}\right)\right] \qquad (2.1.26')$$

$$R = R^\dagger + \frac{e_{+1} - e}{e} \qquad (2.1.27')$$

For $I = I^\dagger = 2$ and $J = J^\dagger = 4$, the Schumpeterian model's system of equations is composed of sixty-one equations and sixty-one endogenous variables.

Appendix 2.2a. Human capital model

Relationships that must be considered for both Nation and Rest of World

Households

$$N^0 = \frac{1}{\xi(1-d)}\left(1 - \frac{\beta\Pi^{J-1}}{1+g}\right) \qquad (2.2.2a')$$

$$N^1 = \frac{1}{\xi(1-d)}\left(1 - \frac{\beta\Pi^{-1}}{1+g}\right) \qquad (2.2.2b')$$

$$N = \frac{1}{J}\left(N^0 + (J-1)N^1\right) \qquad (2.2.2c')$$

$$g = \frac{\beta}{1+\delta - R\left(1 - X^{\dagger K^{\dagger}}\right)} - 1 \qquad (2.2.3')$$

$$u^0 = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^0}\left[(1+g)\frac{1}{\Pi^3}\left(\frac{N^1}{N^0}\right)^{\nu} - 1\right]\right\} \qquad (2.2.5a')$$

$$u^{01} = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\Pi\left(\frac{N^0}{N^1}\right)^{\nu} - 1\right]\right\} \qquad (2.2.5b')$$

$$u^1 = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\frac{1}{\Pi} - 1\right]\right\} \qquad (2.2.5c')$$

Intermediate good producers

$$\Delta_W = \left[\frac{1}{J}\sum_{\tau=0}^{J-1} w_{-\tau}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad (2.2.8')$$

$$R = \alpha\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left[\frac{1-\alpha}{\Delta_W}\right]^{\frac{1-\alpha}{\alpha}} \qquad (2.2.9')$$

$$L^0 = (1-d)u^0N^0 \qquad (2.2.10a')$$

$$L^{01} = (1-d)u^{01}N^{01} \qquad (2.2.10b')$$

$$L^1 = (1-d)u^1N^1 \qquad (2.2.10c')$$

$$L = \frac{1}{J}\left(L^0 + L^{01} + (J-2)L^1\right) \qquad (2.2.10d')$$

$$d = \frac{N-L}{N} \qquad (2.2.12')$$

$$w_{-\mathrm{T}} = \frac{1}{\Pi^{-\mathrm{T}}}\frac{\frac{e}{A}\Delta_b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R + b\Delta_b)}{4R + b\Delta_b + d\Delta_d}\frac{z}{A}\Delta_d}{\Delta_w^b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] - \Delta_w^{bq}(4R + b\Delta_b)} \qquad (2.2.13')$$

$$\text{for } \mathrm{T} = [0, \dots, J-1]$$

Final good producers	
$$\frac{P^*_{-\mathrm{T}}}{P}=\frac{1}{\Pi^{\mathrm{T}}}\frac{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon})^{\tau}}{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon-1})^{\tau}}$$ for $\mathrm{T}=[0,\dots,I-1]$	(2.2.15′)
$$\Delta_P=\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P}\right)^{-\varepsilon}$$	(2.2.16′)
Equilibrium conditions and external sector	
For the Nation:	
$$C^K=\frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}}-g-\delta+X^K-lX^{\dagger K^{\dagger}}$$	(2.2.17a′)
$$X^K=\Omega e^{\omega}l\,\frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}}$$	(2.2.18a′)
For the Rest of World:	
$$C^{\dagger K^{\dagger}}=\frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}}-g^{\dagger}-\delta^{\dagger}+X^{\dagger K^{\dagger}}-\frac{1}{l}X^K$$	(2.2.17b′)
$$X^{\dagger K^{\dagger}}=\Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\frac{1}{l}\frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}}$$	(2.2.18b′)

International relationships

$$b=R^i\frac{b_{-1}}{g}-X^K+elX^{\dagger K^{\dagger}}\qquad(2.1.24a')$$

$$b^{\dagger}=R^i\frac{b^{\dagger}_{-1}}{g^{\dagger}}-\mathrm{X}^{\dagger K^{\dagger}}+\frac{1}{el}X^K\qquad(2.1.24b')$$

$$R^i=\frac{1}{2}[(R^{\dagger}-R)\,sgn(b)+(R^{\dagger}+R)]\qquad(2.1.25')$$

$$b^{\dagger}-\left(\frac{b^{\dagger}_{-1}}{\Pi^{\dagger}}\right)=-\left[\left(\frac{b}{e}-\frac{b_{-1}}{\Pi^{\dagger}e_{-1}}\right)\right]\qquad(2.1.26')$$

$$R=R^{\dagger}+\frac{e_{+1}-e}{e}\qquad(2.1.27')$$

For $I=I^{\dagger}=2$ and $J=J^{\dagger}=4$, the human capital model's system of equations is composed of forty-nine equations and forty-nine endogenous variables.

Appendix 2.2b. Optimal control problem in the human capital model and steady state implications

Wage flexibility

The wage is the same for all types of labor services.

The Hamiltonian for this problem is:

$$H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 (N_{st+\tau})^{1+\nu} ds\right]$$

$$+ \lambda_{1,t+\tau}\left[\mathrm{D}_{t+\tau} + X_t - e_{t+\tau}X^{\dagger}_{t+\tau} + \int_0^1 (1-d_{t+\tau})\left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}}\right) L_{st+\tau} ds\right.$$

$$\left. + (R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}\right]$$

$$+ \lambda_{2,t+\tau}\left\{\int_0^1 \xi\big(1 - u_{st+\tau}(1-d_{t+\tau})\big)N_{st+\tau}h_{st+\tau}\right\}$$

subject to $(2.2.4)$, $(2.2.13)$, $(2.2.18)$, $(2.1.24a)$, $(2.1.24b)$ and $(2.1.26)$.

The first order conditions are the followings:

$$(\mathrm{A2.2b.1}) \quad \frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(\mathrm{A2.2b.2}) \quad \beta^{\tau} {N_{st+\tau}}^{\upsilon}$$

$$= \lambda_{1,t+\tau}(1-d_{t+\tau})\left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u_{st+\tau}h_{st+\tau}$$

$$+ \lambda_{2,t+\tau}\xi\big(1 - u_{st+\tau}(1-d_{t+\tau})\big)h_{st+\tau} \quad \forall i \in [0,1]$$

$$(\mathrm{A2.2b.3}) \quad \lambda_{2,t+\tau} = \frac{\lambda_{1,t+\tau}}{\xi}\left(\frac{W^{*}_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) \quad \forall s \in [0,1]$$

$$(\text{A2.2b.4})\quad \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau}$$

$$= -\lambda_{1,t+\tau}(R_{t+\tau} - \delta)$$

$$- \lambda_{1,t+\tau} \int_0^1 (1 - d_t) \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) \left(\frac{W^*_{st+\tau}}{P^m_{t+\tau}}\right)^{-\sigma} \left[\frac{(1-\alpha)A}{\left(\Delta^i_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}} ds$$

$$+ \lambda_{1,t+\tau} a \frac{e_{t+\tau} X^{\dagger}_{t+\tau}}{K_{t+\tau}}$$

$$(\text{A2.2b.5})\quad \lambda_{2,t+\tau+1} - \lambda_{2,t+\tau}$$

$$= -\lambda_{1,t+\tau}(1 - d_{t+\tau}) \left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) u_{st+\tau}N_{st+\tau}$$

$$- \lambda_{2,t+\tau}\, \xi\left(1 - u_{st+\tau}(1 - d_{t+\tau})\right)N_{st+\tau} \qquad \forall s \in [0,1]$$

$$(\text{A2.2b.6})\quad K_{t+\tau+1}$$

$$= \mathrm{D}_{t+\tau} + X_t - e_{t+\tau}X^{\dagger}_{t+\tau} + \int_0^1 (1 - d_{t+\tau}) \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) N_{i,t+\tau}\left(W^*_{i,t+\tau}\right) ds$$

$$+ (1 + R_{t+\tau} - \delta)K_{t+\tau}$$

$$(\text{A2.2b.7})\quad h_{t+\tau+1} = \left\{\int_0^1 \left[1 + \xi\left(1 - u_{st+\tau}(1 - d_{t+\tau})\right)N_{st+\tau}\right] \frac{h_{st+\tau}}{h_{t+\tau}} ds\right\} h_{t+\tau}$$

In steady state, from (A2.2b.1):

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g(C)} \quad (*)$$

From (A2.2b.3) and (*):

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_2) = \frac{\beta}{1+g(C)}$$

From (A2.2b.4) and (*):

$$1+g=\frac{\beta}{1+\delta-a\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}\left(1-e\Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\right)}$$

From $(\mathrm{A2.2b.5})$:

$$\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}}=1-\xi(1-d_{st+\tau})N_{st+\tau}=\frac{\beta}{1+g(C)}$$

The supply of labor is the same for all *i* and is constant over time. From this expression, the constant value of N_{ss} in steady state can be obtained:

$$N_{ss}=\frac{1}{\xi(1-d_{ss})}\left(1-\frac{\beta}{1+g(C)}\right)$$

From $(\mathrm{A2.2b.2})$:

$$\frac{\beta^{\tau+1}{N_{st+\tau+1}}^{\upsilon}}{\beta^{\tau}{N_{st+\tau}}^{\upsilon}}=\frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}}\frac{h_{st+\tau+1}}{h_{st+\tau}}$$

$$\beta=\frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta}{1+g(C)}=>g(h)=g(C)$$

The growth rate of human capital is the same as the consumption growth rate and the same for all *s*. We can see that from the accumulation process of human capital

$$h_{st+\tau+1}=h_{st+\tau+}+\xi(1-(1-d_{t+\tau})u_{st+\tau})N_{st+\tau}h_{st+\tau}$$

its growth rate is:

$$g(h_s)=g(C)=\ \xi(1-(1-d_{t+\tau})u_{st+\tau})N_{st+\tau}$$

where u_{ss} is the steady-state value for any s. From this expression, we can deduce that the value of *u* is also the same for all types of labor services and is constant over time:

$$u_{ss}=\frac{1}{1-d_{ss}}\left(1-\frac{g(C)}{\xi N_{ss}}\right)$$

With those expressions, the system of equations in steady state is closed.

Sticky wages

Note that the first-order condition for $u_{st+\tau}$ in $(\text{A2.2b}.3)$ implies that the real wage at time $t+\tau$ has to be the same across all individuals. However, since the nominal wage is expressed in terms of effective labor, the re-optimized real wage should be constant at the steady state, and therefore the nominal re-optimized wage grows at the same rate as the aggregate price. This implies that when the trend inflation is different from zero, there will be variations in the real wage across individuals. Obviously, this contradicts $(\text{A2.2b}.3)$. Then the previous problem is not valid with wage stickiness.

The Hamiltonian for this situation is:

$$H_{t+\tau} = \beta^{\tau}\left[\log(C_{t+\tau}) - \frac{1}{1+\nu}\int_0^1 (N_{st+\tau})^{1+\nu} ds\right]$$

$$+\lambda_{1,t+\tau}\left[\mathrm{D}_{t+\tau}+X_t - e_{t+\tau}X^{\dagger}_{t+\tau} + \int_0^1 (1-d_{t+\tau})\left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right) L_{i,t+\tau}\left(\frac{W^*_{st+\tau}}{P^m_{t+\tau}}\right) ds\right.$$

$$\left. + (R_{t+\tau}-\delta)K_{t+\tau} - C_{t+\tau}\right]$$

$$+\sum_{q=0}^{J-1}\lambda^q_{2,t+\tau}\left\{\int_{\frac{q}{J}}^{\frac{q+1}{J}} \xi(1-u^q_{s,t+\tau}(1-d_{t+\tau}))N^q_{st+\tau}h^q_{st+\tau}\, ds\right\}$$

subject to $(2.2.4)$, $(2.2.6)$, $(2.2.7)$, $(2.2.8)$, $(2.2.9)$, $(1.1.22a)$, $(1.1.22b)$, $(2.2.13)$, $(2.2.15)$, $(2.2.18)$, $(2.1.24a)$, $(2.1.24b)$ and $(2.1.26)$.

The first order conditions are the followings:

$$(\text{A2.2b}.8) \quad \frac{\beta^{\tau}}{C_{t+\tau}} = \lambda_{1,t+\tau}$$

$$(\text{A2.2b.9})\quad \beta^{\tau} {N_{st+\tau}}^{\upsilon}$$

$$= \lambda_{1,t+\tau}(1-d_t)\left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u_{st+\tau}h_{st+\tau}$$

$$+ \lambda_{2,t+\tau}\xi(1-u_{st+\tau}(1-d_{t+\tau}))h_{st+\tau} \quad \forall s \in [0,1]$$

$$(\text{A2.2b.10.1}), (\text{A2.2b.10.3}), \ldots, (\text{A2.2b.10.J}) \qquad \lambda^q_{2,t+\tau}$$

$$= \frac{\lambda_{1,t+\tau}}{\xi}\left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right) \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right]$$

$q = 0, 1, 2, \ldots, J-1$

$$(\text{A2.2b.11})\quad \lambda_{1,t+\tau+1} - \lambda_{1,t+\tau}$$

$$= -\lambda_{1,t+\tau}(R_{t+\tau} - \delta)$$

$$-\lambda_{1,t+\tau}(1-d_t)\int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)\left(\frac{W^*_{st+\tau}}{P^i_{t+\tau}}\right)^{-\sigma}\left[\frac{(1-\alpha)A}{\left(\Delta^i_{w,t+\tau}\right)^{1-\sigma\alpha}}\right]^{\frac{1}{\alpha}} ds$$

$$+ \lambda_{1,t+\tau}a\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{K_{t+\tau}}$$

$$(\text{A2.2b.12.1}), (\text{A2.2b.12.3}), \ldots, (\text{A2.2b.12.J})\quad \lambda^q_{2,t+\tau+1} - \lambda^q_{2,t+\tau}$$

$$= -\lambda_{1,t+\tau}(1-d_{t+\tau})\left(\frac{W^*_{st+\tau}}{P_{t+\tau}} - (1-a)\frac{e_{t+\tau}X^{\dagger}_{t+\tau}}{u_{st+\tau}N_{st+\tau}h_{st+\tau}}\right)u^q_{st+\tau}N^q_{st+\tau}$$

$$-\lambda^q_{2,t+\tau}\,\xi(1-u^q_{st+\tau}(1-d_{t+\tau}))N^q_{st+\tau} \quad \forall s$$

$$\in \left[\frac{q}{J}, \frac{q+1}{J}\right] \qquad q = 0, 1, 2, \ldots, J-1$$

$$(\text{A2.2b.13})\quad K_{t+\tau+1}$$

$$= \mathrm{D}_{t+\tau} + X_t - e_{t+\tau}X^{\dagger}_{t+\tau} + (1-d_t)\int_0^1 \left(\frac{W^*_{st+\tau}}{P_{t+\tau}}\right)L_{st+\tau}(W^*_{st+\tau})di$$

$$+ (1 + R_{t+\tau} - \delta)K_{t+\tau} - C_{t+\tau}$$

$$(\text{A2.2b.14})\quad h_{t+\tau+1} = \left\{\int_0^1 [1 + \xi(1-u_{st+\tau}(1-d_{t+\tau}))N_{st+\tau}]\frac{h_{st+\tau}}{h_{t+\tau}}di\right\}h_{t+\tau}$$

In steady state, from $(\text{A2.2b.}8)$:

$$\frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g(\lambda_1) = \frac{\beta^{\tau+1}/C_{t+\tau+1}}{\beta^{\tau}/C_{t+\tau}} = \frac{\beta}{1+g(C)}$$

From $(\text{A2.2b.}10.1)$–$(\text{A2.2b.}10.\text{J})$ (representing labor services that do not change wages):

$$\frac{\lambda^q_{2,t+\tau+1}}{\lambda^q_{2,t+\tau}} = \frac{\lambda_{1,t+\tau+1}}{\lambda_{1,t+\tau}} = 1 + g\left(\lambda^q_2\right) = 1 + g(\lambda_2) = \frac{\beta}{1+g(C)} \qquad q = 0, 1, 2, \dots, J-1$$

Therefore, there will two values of *N*. From $(\text{A2.2b.}12)$:

$$\frac{\lambda^q_{2,t+\tau+1}}{\lambda^q_{2,t+\tau}} = 1 - \xi(1-d_{t+\tau})N^q_{st+\tau} = 1 - \xi(1-d_{t+\tau})N^q_{st+\tau} = \frac{\beta}{1+g(C)} \Rightarrow N^1 = \frac{1}{\xi(1-d)}\left(1 - \frac{\beta\Pi^{-1}}{1+g}\right) \quad q = 0, 1, 2, \dots, J\text{-}2$$

$$\frac{\lambda^0_{2,t+\tau+1}}{\lambda^{J-1}_{2,t+\tau}} = 1 - \xi(1-d_{t+\tau})N^0_{st+\tau} = \frac{\beta\Pi^{J-1}}{1+g} \Rightarrow N^0 = \frac{1}{\xi(1-d)}\left(1 - \frac{\beta\Pi^{J-1}}{1+g}\right)$$

From $(\text{A2.2b.}11)$ and (*):

$$1 + g = \frac{\beta}{1 + \delta - a\left[A\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\Delta_W}\right)^{\frac{1-\alpha}{\alpha}}\left(1 - e\Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\right)}$$

From $(\text{A2.2b.}9)$:

$$\frac{\beta^{\tau+1}{N_{st+\tau+1}}^{\upsilon}}{\beta^{\tau}{N_{st+\tau}}^{\upsilon}} = \frac{\lambda_{2,t+\tau+1}}{\lambda_{2,t+\tau}}\frac{h_{st+\tau+1}}{h_{st+\tau}}$$

$$\beta = \frac{h_{st+\tau+1}}{h_{st+\tau}}\frac{\beta}{1+g(C)} \Rightarrow g(C) = 1 + g(h) - 1 \;\Rightarrow\; g(C) = g(h)$$

$$\forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \qquad q = 0, 1, 2, \dots, \text{J} - 1$$

Consequently, there will three expressions of *u* in steady state:

$$u^0 = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^0}\left[(1+g)\frac{1}{\Pi^3}\left(\frac{N^1}{N^0}\right)^{\nu} - 1\right]\right\} \forall s \in \left[\frac{J-1}{J}, 1\right] \text{ in t+τ}$$

$$u^{01} = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\Pi\left(\frac{N^0}{N^1}\right)^{\nu} - 1\right]\right\} \forall s \in \left[\frac{J-2}{J}, \frac{J-1}{J}\right] \text{ in t+τ}$$

$$u^1 = \frac{1}{1-d}\left\{1 - \frac{1}{\xi N^1}\left[(1+g)\frac{1}{\Pi} - 1\right]\right\} \; q = 0, 1, 2, 3, \dots, J-3 \quad \forall s \in \left[\frac{q}{J}, \frac{q+1}{J}\right] \text{ in t + t in t+τ}$$

Appendix 3.1. Schumpeterian model

Relationships that must be considered for both Nation and Rest of World

Households

$$N_{-\mathrm{T}} = \left(\frac{1}{C^Y}(1-d)w^Y_{-\mathrm{T}}\right)^{\frac{1}{\nu}} \qquad N = \frac{1}{J}\sum_{\tau=0}^{J-1} N_{-\tau} \tag{2.1.3'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$\beta \frac{R}{\mathrm{g}\Pi} = 1 \tag{2.1.4'}$$

Financial intermediaries

$$\nu = (1-\gamma)(R^k - R) + \gamma\beta G(S)\nu_{+1} \tag{3.1.6'}$$

$$\eta = (1-\gamma)R + \gamma\beta G(T)\eta_{+1} \tag{3.1.7'}$$

$$\phi = \frac{\eta}{\lambda - \nu} \tag{3.1.8'}$$

$$G(T) = (R^k - R)\phi + R \tag{3.1.10'}$$

$$G(S) = G(F) \tag{3.1.11'}$$

$$G(S) = \gamma[(R^k - R)\phi + R] + \psi R\phi \tag{3.1.13'}$$

Final good producers

$$L_{-\mathrm{T}} = \left(\frac{(1-\alpha)L^{\frac{1-\sigma}{\sigma}}}{R^k w^Y_{-\mathrm{T}}}\right)^{\sigma} \qquad LL = \frac{1}{J}\sum_{\tau=0}^{J-1} L_{-\tau} \tag{3.1.15'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$L = \frac{(1-\alpha)}{R^k \Delta^Y_W} \tag{3.1.17'}$$

$$\Delta^Y_W = \left[\frac{1}{J}\sum_{\tau=0}^{J-1} (w^Y_{-\mathrm{T}})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{2.1.8'}$$

$$d = \frac{N-L}{N} \tag{2.1.11'}$$

$$w^Y_{-\mathrm{T}} = \frac{1}{\Pi^{-\mathrm{T}}} \frac{\frac{e}{A^Y}\Delta_b\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R + b\Delta_b)}{4R + b\Delta_b + d\Delta_d}\frac{z}{A^Y}\Delta_d}{\Delta^b_w\left[4R + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R + b\Delta_b + d\Delta_d}\right] - \Delta^{bq}_w(4R + b\Delta_b)} \tag{2.1.15'}$$

for $\mathrm{T} = [0, \dots, J-1]$

$$A^Y = \frac{1}{\left(\left(\frac{\alpha}{R^k}\right)^{\frac{1}{1-\alpha}} \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P^*_{-\tau}}{P}\right)^{-\frac{1}{1-\alpha}}\right)^{\alpha} L} \tag{3.1.18'}$$

Intermediate good producers

$$g = \left[\frac{\chi}{R^k}\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}-1\right)\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{\chi}{1-\chi}}(\gamma-1)+1 \qquad (3.1.23')$$

$$\frac{P_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{\mathrm{T}}}\frac{1}{\alpha}\frac{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{1}{1-\alpha}}\right)^{\tau}}{\sum_{\tau=0}^{I-1}\left(\beta\Pi^{\frac{\alpha}{1-\alpha}}\right)^{\tau}} \qquad (2.1.20')$$

for $\mathrm{T} = [0, \dots, I-1]$

Equilibrium conditions and external sector

For the Nation:

$$C^Y = 1 - \left[\frac{\chi}{R^k}\alpha^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}-1\right)\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\chi}}A^Y -$$

$$-\left(\frac{\alpha}{R^k}\right)^{\frac{1}{1-\alpha}}L\frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\frac{1}{1-\alpha}}A^Y - X^Y + lX^{\dagger^{Y^\dagger}} \qquad (3.1.26a')$$

$$X^Y = (\rho + \Omega e^{\omega})l \qquad (2.1.22a')$$

For the Rest of World:

$$C^{\dagger^{Y^\dagger}} = 1 - \left[\frac{\chi^\dagger}{R^{k^\dagger}}\alpha^{\dagger\frac{1}{1-\alpha^\dagger}}L^\dagger\frac{1}{I^\dagger}\sum_{\tau=0}^{I^\dagger-1}\left(\frac{P_{-\tau}^{\dagger*}}{P^\dagger}-1\right)\left(\frac{P_{-\tau}^{\dagger}}{P^\dagger}\right)^{-\frac{1}{1-\alpha^\dagger}}\right]^{\frac{1}{1-\chi^\dagger}}A^{\dagger^{Y^\dagger}} -$$

$$-\left(\frac{\alpha^\dagger}{R^{k^\dagger}}\right)^{\frac{1}{1-\alpha^\dagger}}L^\dagger\frac{1}{I^\dagger}\sum_{\tau=0}^{I^\dagger-1}\left(\frac{P_{-\tau}^{\dagger*}}{P^\dagger}\right)^{-\frac{1}{1-\alpha^\dagger}}A^{\dagger^{Y^\dagger}} - X^{\dagger^{Y^\dagger}} + \frac{1}{l}X^Y \qquad (3.1.26b')$$

$$X^{\dagger^{Y^\dagger}} = \left[\rho^\dagger + \Omega^\dagger\left(\frac{1}{e}\right)^{\omega^\dagger}\right]\frac{1}{l} \qquad (2.1.22b')$$

International relationships

$$b = R^i\frac{b_{-1}}{g} - X^Y + elX^{\dagger^{Y^\dagger}} \qquad (2.1.24a')$$

$$b^\dagger = R^i\frac{b_{-1}^\dagger}{g^\dagger} - \mathrm{X}^{\dagger^{Y^\dagger}} + \frac{1}{el}X^Y \qquad (2.1.24b')$$

$$R^i = \frac{1}{2}\left[\left(R^\dagger - R\right)sgn(b) + \left(R^\dagger + R\right)\right] \qquad (2.1.25')$$

$$b^\dagger - \left(\frac{b_{-1}^\dagger}{\Pi^\dagger}\right) = -\left[\left(\frac{b}{e} - \frac{b_{-1}}{\Pi^\dagger e_{-1}}\right)\right] \qquad (2.1.26')$$

$$R = R^\dagger + \frac{e_{+1} - e}{e} \qquad (2.1.27')$$

For $I = I^\dagger = 2$ and $J = J^\dagger = 4$, the Schumpeterian model's system of equations is composed of seventy-five equations and seventy-five endogenous variables.

Appendix 3.2. Human capital model

Relationships that must be considered for both Nation and Rest of World

Households	
$$N^0 = \frac{1}{\xi(1-d)}\left(1-\frac{\beta\Pi^{J-1}}{1+g}\right)$$	$(2.2.2a')$
$$N^1 = \frac{1}{\xi(1-d)}\left(1-\frac{\beta\Pi^{-1}}{1+g}\right)$$	$(2.2.2b')$
$$N = \frac{1}{J}(N^0 + (J-1)N^1)$$	$(2.2.2c')$
$$g = \frac{\beta}{1+\delta-R^k\left(1-X^{\dagger K^{\dagger}}\right)} - 1$$	$(2.2.3')$
$$u^0 = \frac{1}{1-d}\left\{1-\frac{1}{\xi N^0}\left[(1+g)\frac{1}{\Pi^3}\left(\frac{N^1}{N^0}\right)^{\nu} - 1\right]\right\}$$	$(2.2.5a')$
$$u^{01} = \frac{1}{1-d}\left\{1-\frac{1}{\xi N^1}\left[(1+g)\Pi\left(\frac{N^0}{N^1}\right)^{\nu} - 1\right]\right\}$$	$(2.2.5b')$
$$u^1 = \frac{1}{1-d}\left\{1-\frac{1}{\xi N^1}\left[(1+g)\frac{1}{\Pi} - 1\right]\right\}$$	$(2.2.5c')$
Financial intermediaries	
$$\nu = (1-\gamma)(R^k - R) + \gamma\beta G(S)\nu_{+1}$$	$(3.1.6')$
$$\eta = (1-\gamma)(R+1) + \gamma\beta G(T)\eta_{+1}$$	$(3.1.7')$
$$\phi = \frac{\eta}{\lambda - \nu}$$	$(3.1.8')$
$$G(T) = (R^k - R)\phi + (R+1)$$	$(3.1.10')$
$$G(S) = G(F)$$	$(3.1.11')$
$$G(S) = \gamma[(R^k - R)\phi + (R+1)] + \psi R\phi$$	$(3.1.13')$
Intermediate good producers	
$$\Delta_W = \left[\frac{1}{J}\sum_{\tau=0}^{J-1} w_{-\tau}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$	$(2.2.8')$
$$R = \alpha\left[\frac{A}{1+R^k}\left(\frac{\varepsilon-1}{\varepsilon}\right)\right]^{\frac{1}{\alpha}}\left[\frac{1-\alpha}{(1+R^k)\Delta_W}\right]^{\frac{1-\alpha}{\alpha}}$$	$(3.2.3')$
$$L^0 = (1-d)u^0N^0$$	$(2.2.10a')$
$$L^{01} = (1-d)u^{01}N^{01}$$	$(2.2.10b')$
$$L^1 = (1-d)u^1N^1$$	$(2.2.10c')$
$$L = \frac{1}{J}(L^0 + L^{01} + (J-2)L^1)$$	$(2.2.10d')$

$$d = \frac{N-L}{N} \qquad (2.2.12')$$

$$w_{-\mathrm{T}} = \frac{1}{\Pi^{-\mathrm{T}}} \frac{\frac{e}{A}\Delta_b\left[4R^k + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R^k + b\Delta_b + d\Delta_d}\right] + \frac{q\Delta_q(4R^k + b\Delta_b)}{4R^k + b\Delta_b + d\Delta_d}\frac{z}{A}\Delta_d}{\Delta_w^b\left[4R^k + b\Delta_b + q\Delta_q - \frac{q\Delta_q d\Delta_d}{4R^k + b\Delta_b + d\Delta_d}\right] - \Delta_w^{bq}(4R^k + b\Delta_b)} \qquad (2.2.13')$$

$$\text{for } \mathrm{T} = [0, \dots, J-1]$$

Final good producers

$$\frac{P_{-\mathrm{T}}^*}{P} = \frac{1}{\Pi^{\mathrm{T}}} \frac{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon})^{\tau}}{\sum_{\tau=0}^{I-1}(\beta\Pi^{\varepsilon-1})^{\tau}} \qquad (2.2.15')$$

$$\text{for } \mathrm{T} = [0, \dots, I-1]$$

$$\Delta_P = \frac{1}{I}\sum_{\tau=0}^{I-1}\left(\frac{P_{-\tau}^*}{P}\right)^{-\varepsilon} \qquad (2.2.16')$$

Equilibrium conditions and external sector

For the Nation:

$$C^K = \frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{(1+R^k)\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} - g - \delta + X^K - lX^{\dagger K^{\dagger}} \qquad (3.2.7a')$$

$$X^K = \Omega e^{\omega} l\, \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{(1+R^{k\dagger})\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} \qquad (3.2.8a')$$

For the Rest of World:

$$C^{\dagger K^{\dagger}} = \frac{A^{\dagger\frac{1}{\alpha^{\dagger}}}}{\Delta_P^{\dagger}}\left[\left(\frac{\varepsilon^{\dagger}-1}{\varepsilon^{\dagger}}\right)\frac{1-\alpha^{\dagger}}{(1+R^{k\dagger})\Delta_{\mathrm{W}}^{\dagger}}\right]^{\frac{1-\alpha^{\dagger}}{\alpha^{\dagger}}} - g^{\dagger} - \delta^{\dagger} + X^{\dagger K^{\dagger}} - \frac{1}{l}X^K \qquad (3.2.7b')$$

$$X^{\dagger K^{\dagger}} = \Omega^{\dagger}\left(\frac{1}{e}\right)^{\omega^{\dagger}}\frac{1}{l}\frac{A^{\frac{1}{\alpha}}}{\Delta_{\mathrm{P}}}\left[\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1-\alpha}{(1+R^k)\Delta_{\mathrm{W}}}\right]^{\frac{1-\alpha}{\alpha}} \qquad (3.2.8b')$$

International relationships

$$b = R^i\frac{b_{-1}}{g} - X^K + elX^{\dagger K^{\dagger}} \qquad (2.1.24a')$$

$$b^{\dagger} = R^i\frac{b_{-1}^{\dagger}}{g^{\dagger}} - \mathrm{X}^{\dagger K^{\dagger}} + \frac{1}{el}X^K \qquad (2.1.24b')$$

$$R^i = \frac{1}{2}[(R^{\dagger} - R)\, sgn(b) + (R^{\dagger} + R)] \qquad (2.1.25')$$

$$b^{\dagger} - \left(\frac{b_{-1}^{\dagger}}{\Pi^{\dagger}}\right) = -\left[\left(\frac{b}{e} - \frac{b_{-1}}{\Pi^{\dagger}e_{-1}}\right)\right] \qquad (2.1.26')$$

$$R = R^{\dagger} + \frac{e_{+1} - e}{e} \qquad (2.1.27')$$

For $I = I^{\dagger} = 2$ and $J = J^{\dagger} = 4$, the human capital model's system of equations is composed of sixty-one equations and sixty-one endogenous variables.