Directed ratchet transport of cold atoms and fluxons driven by biharmonic fields: A unified view

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This Letter discusses two retrodictions of the theory of ratchet universality which explain previous experimental results concerning directed ratchet transport of cold atoms in dissipative optical lattices in one case and of fluxons in uniform annular Josephson junctions in the other, both driven by biharmonic fields. It has to be emphasized that these retrodictions are in sharp contrast with the current standard explanation of such experimental results, and they offer optimal control of the ratchet-like motion of such entities. New experimental proposals with cold atoms and fluxons are discussed, providing additional tests for novel predictions from ratchet universality.

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I. INTRODUCTION

Symmetry principles constrain the possible forms of the laws of nature by constituting a synthesis of those regularities which are independent of the specific dynamics. They usually play a deep and subtle role in the sense that many physical phenomena ultimately come to be explained in terms of mechanisms of symmetry breaking. A notable instance is the so-called ratchet effect [1-3], i.e., the possibility of generating directed transport from a fluctuating environment without any net external force. Indeed, it has been a fundamental research topic in diverse areas of science and technology since the end of the last century partly because of its potential applications for manipulating such systems as coupled Josephson junctions [4] and molecular motors [5], as well as for designing micro- and nano-devices suitable for on-chip implementation. Directed ratchet transport (DRT) is now qualitatively understood to be a result of the interplay of nonlinearity, symmetry breaking [6], and non-equilibrium fluctuations including temporal noise [2], spatial disorder [7], and quenched temporal disorder [8].

The symmetry analysis *alone*, however, is insufficient to predict the direction and strength of the DRT. Recently, some of such fundamental aspects, including current reversals [9] and the quantitative dependence of DRT strength on the system's parameters [10], have begun to be elucidated. In this regard, the theory of ratchet universality (RU) [11-13] predicts that there exists a universal force waveform which optimally enhances directed transport by symmetry breaking. The theory of RU refers to the criticality scenario that emerges when the generalized parity symmetry and the generalized time-reversal symmetry are broken, *regardless* of the nature of the dynamic equation in which the breaking of such symmetries results in DRT. For noiseless ratchets, the effectiveness of this theory has been demonstrated in diverse physical contexts in which the driving forces are chosen to be biharmonic, such as in the cases of topological solitons [8], Bose-Einstein condensates exposed to a sawtooth-like optical lattice potential [14], matter-wave solitons [10], one-dimensional granular chains [15], and Bose-Einstein condensates under an unbiased periodic driving potential [16]. Thus, the effectiveness of RU in quantum systems has been previously demonstrated, including the cases of directed transport of atoms in a Hamiltonian quantum ratchet (the values of the parameters used, which were chosen to maximize the directed transport, correspond to those of the universal biharmonic waveform, cf. Ref. [14]) and driven Bose-Einstein condensates (the authors show that the ratchet current is maximum for the values of the parameters that correspond to those of the ratchet potential associated with the universal biharmonic waveform, cf. Ref. [16]). There have also been quantitative explanations in coherence with the degree-of-symmetry-breaking mechanism, as predicted by the theory of RU [11,12], of the interplay between thermal noise and symmetry breaking in the DRT of a Brownian particle moving on a periodic substrate subjected to a homogeneous temporal biharmonic force [17-19], and of a driven Brownian particle subjected to a vibrating periodic potential [20]. Numerical analyses of a driven Brownian particle in the presence of non-Gaussian noise [21] and coupled Brownian motors with stochastic interactions in a crowded environment [22] have confirmed the RU predictions. Additionally, RU has recently been demonstrated in the bidirectional escape from a symmetric potential well [23].

This present paper discusses two *retrodictions* of the theory of RU which explain previous experimental results concerning DRT of cold atoms in dissipative optical lattices [24] in one case (Sec. II), and magnetic flux quanta (fluxons) in uniform annular Josephson junctions [25] in the other (Sec. III), both driven by biharmonic fields. It has to be emphasized that these retrodictions are in sharp contrast with the current standard explanation of such experimental results, and they suggest new aspects for experimental testing. Finally, we conclude with Sec. IV by summarizing our conclusions and discussing future work.

II. COLD ATOMS

Motivated by investigation into the mechanisms that yield directed diffusion in a symmetric periodic potential, Schiavoni et al. [24] made an experimental and numerical study of cold atoms in a one-dimensional dissipative optical lattice where directed motion appears as a result of the breaking of the system's temporal symmetry after applying a biharmonic phase modulation to one of the lattice beams. In the accelerated reference frame, the atoms experience a stationary optical potential together with an inertial force

$$F(t) = \gamma\left[A\cos(\omega t) + B\cos(2\omega t - \phi)\right], \qquad (1)$$

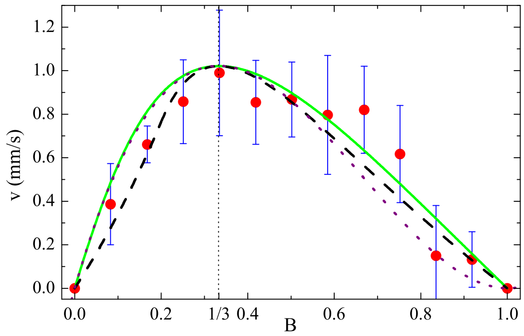
where γ is an amplitude factor, $A = 1 - B$, and the parameters $B \in [0,1]$ and $\phi \in [0, 2\pi]$ are the relative amplitude and initial phase difference of the two harmonics, respectively. Commenting on their experimental results, the authors claimed that: "By increasing B from the zero value the atoms are set into directed motion, and a maximum for the c.m. velocity is reached for $B \simeq 0.5$, i.e., for about equal amplitudes of the even and odd harmonics." This statement has had the *unfortunate consequence* that most subsequent publications citing Ref. [24] have considered $B = 1/2$ to be the condition that maximizes ratchet transport in systems subjected to a biharmonic temporal force.

It will be shown below that the maximum c.m. velocity is reached for $B = 1/3$, as predicted by the theory of RU [11-13]. Indeed, it has been demonstrated for temporal and spatial biharmonic forces that optimal enhancement of DRT is achieved when maximally effective (i.e., critical) symmetry breaking occurs, which implies the existence of a particular universal waveform [11-13]. Specifically, the optimal value of the relative amplitude B comes from the condition that the amplitude of the odd harmonic must be twice that of the even harmonic in Eq. (1), i.e., $1 - B_{opt} = 2B_{opt} \Longrightarrow B_{opt} = 1/3$. Notice that this means that the contributions of the amplitudes of the two harmonics to the directed motion of the atoms are *not* independent, which in Ref. [24] is solely taken into account in the estimate of the optical pumping rate (escape rate) $\Gamma' \sim \sin^2 k\Delta z$, with $\Delta z \sim A^2 B$ being the displacement of the centre of oscillation of the atoms in a potential well from the well centre, *after* the substitution $A = 1 - B$. In such a case, one has that $\Gamma' = \Gamma'(B)$ presents a single maximum at $B_{opt} = 1/3$ for which the asymmetry between the escape rates towards the left and right wells is maximal, and hence one again expects a maximal nonzero current of atoms for $B = 1/3$, as is indeed confirmed by the experimental results [24] (see Fig. 1). RU predicts that the strength of the nonzero mean current, $\langle v \rangle$, has the functional dependence

$$\langle v \rangle \sim S(B)\, p(\phi) \qquad (2)$$

[12], where $S(B)$ accounts for the degree of breakage of the shift symmetry $F(t + T/2) = -F(t)$, while the 2π-periodic function $p(\phi)$ accounts for the degree of breakage of the time-reversal symmetry $F(-t) = F(t)$ and has two extrema at the optimal values $\phi_{opt} = \{\pi/2, 3\pi/2\}$. This dependence on ϕ is indeed confirmed by the experimental results shown in Fig. 2 of Ref. [24]. Also, $S(B)$ presents features similar to those of the normalized version of the biharmonic force [Eq. (1)], $F^*(t)$, for *any* value of ϕ (see Ref. [12] for additional details). Figure 1 shows plots of the optical pumping rate $\Gamma'(B)$ and the function $S(B)$ for two limiting cases of the initial phase difference: one of the optimal values ($\phi_{opt} = \pi/2$) and one of the least favourable values ($\phi_{opt} = 0$) [26]. These curves fit the experimental data reasonably well, and present a single maximum at $B = 1/3$, as expected [11,12]. This retrodiction therefore indicates that the results of Ref. [24] provide a first experimental proof of RU in the context of cold atoms in optical lattices.

FIG. 1: Velocity of the centre of mass of the atomic cloud for $\phi = \pi/2$ (dots are the experimental data from Fig. 3 in Ref. [1]), and the curves $S_{\phi=\pi/2} = C_1\mathcal{F}_{\phi=\pi/2}(B)$ (solid line), $S_{\phi=0} = C_2\mathcal{F}_{\phi=0}(B)$ (dashed line), and $C_3(1-B)^2 B$ (dotted line) as functions of the relative amplitude B [see the text; Eq. (1)]. Fixed parameters: $C_1 = 6.6, C_2 = 1.95, C_3 = 6.9$ [26].



Further experiments with cold atoms could readily discriminate between the RU and the harmonic-mixing perturbation theory predictions by considering the inertial force $\gamma[\alpha(1-B)\cos(\omega t) + B\cos(2\omega t - \phi)]$, with $\alpha > 0$, instead of that given by Eq. (1). In such a case, one obtains that $\Gamma'(B) \sim \alpha^2(1-B)^2 B$ presents *again* a single maximum at $B_{opt} = 1/3$, irrespective of the particular value of parameter α, while RU predicts an α-dependent maximal current of atoms for

$$B_{opt} = \alpha/(2+\alpha). \qquad (3)$$

III. MAGNETIC FLUX QUANTA

Motivated by investigation into the mechanisms that yield DRT of solitons in long Josephson junctions, Ustinov et al. [25] studied experimentally the rectified dc voltage induced by the ratchet-like motion of a fluxon driven by a biharmonic microwave force of zero mean, applied to a spatially uniform long Josephson junction. The dynamics of the superconducting phase difference across the junction is described by the perturbed sine-Gordon equation

$$\varphi_{tt} - \varphi_{xx} + \sin\varphi = -\alpha\varphi_t + \gamma + \widetilde{\gamma}(t),$$
$$\widetilde{\gamma}(t) \equiv \widetilde{\gamma}_1 \sin(\Omega t) + \widetilde{\gamma}_2 \sin(2\Omega t + \theta), \quad (4)$$

with the boundary conditions $\varphi(l) = \varphi(0) + 2\pi$, $\varphi_x(l) = \varphi_x(0)$, where α is the dissipation constant due to quasiparticle tunneling current, γ and $\widetilde{\gamma}_1, \widetilde{\gamma}_2$ are the dc and ac normalized amplitudes (bias current densities), respectively, θ is the initial phase difference of the two microwave harmonics, and the ac power levels P_1 and P_2 satisfy the scaling $\widetilde{\gamma}_i \sim \sqrt{P_i}, i = 1,2$. The breakage of the shift symmetry of the ac field $\widetilde{\gamma}(t)$ leads to the ratchet-like motion of a fluxon, which is manifested in the nonzero rectified voltage across the junction at zero bias current. Commenting on their experimental and numerical results, the authors claimed that: "The results of the experimental measurements and numerical simulations [of Eq. (4)] are in good correspondence with the results of the first order (point-particle approximation) soliton perturbation theory... . Using this approach, the mean fluxon velocity (in the absence of dc bias) can be computed...as follows":

$$\langle v\rangle \sim \widetilde{\gamma}_1^2 \widetilde{\gamma}_2 \sin(\theta + \theta_0) \sim P_1\sqrt{P_2}\sin(\theta + \theta_0), \quad (5)$$

where $\theta_0 = \arctan\left\{2(\Omega/\alpha)/\left[3 + (\alpha/\Omega)^2\right]\right\}$. The authors assumed a sufficiently high dissipation constant α such that $\theta_0 \approx 0$, and found that the rectified voltage presents a 2π-periodic dependence on θ with two extrema at the optimal values $\theta_{opt} = \{\pi/2, 3\pi/2\}$ (cf. Fig. 3 in Ref. [25]). This dependence on θ is indeed predicted from RU for the biharmonic field $\widetilde{\gamma}_1 \sin(\Omega t) + \widetilde{\gamma}_2 \sin(2\Omega t + \theta)$ when the dissipation phase $\theta_0 + \pi/2$ reaches its highest value, i.e., when $\theta_0 \to 0$ [11]. Remarkably, their experimental results (see Fig. 2) indicate that the maximum amplitude of the rectified voltage, $V_{\max}$, presents, as a function of the relative power $dB = dB_{1(2)} \equiv 10\log\left[P_{1(2)}/P_{2(1)}\right]$, a *maximum* at some optimum value of the power level P_1 (P_2) while keeping the other power level P_2 (P_1) constant. The following remarks would now seem to be in order. First, the theoretical prediction given by Eq. (5) indicates that $V_{\max}$ should present a *monotonous* behaviour as a function of dB for both data series, thus failing to explain the experimental results shown in Fig. 2. Second, RU predicts again that optimal enhancement of DRT of fluxons requires that the amplitude of the odd harmonic must be twice that of the even harmonic in Eq. (4), i.e., $\widetilde{\gamma}_1 = 2\widetilde{\gamma}_2 \Longrightarrow P_1 \sim 4P_2$, and hence that $V_{\max}$ should present an absolute *maximum* at $dB_{\max} \sim 6$ (-6) when the power level P_2 (P_1) is kept constant (see Fig. 2), thereby explaining the overall behaviour of the experimental data. Notice that the values $dB \sim \pm 6$ are independent of the particular values of the power levels which are kept constant (not stated in Ref. [25]) in each of the two data series. This retrodiction therefore indicates that the results of Ref. [25] provide a first experimental proof of RU in the context of fluxons in annular Josephson junctions driven by biharmonic fields.

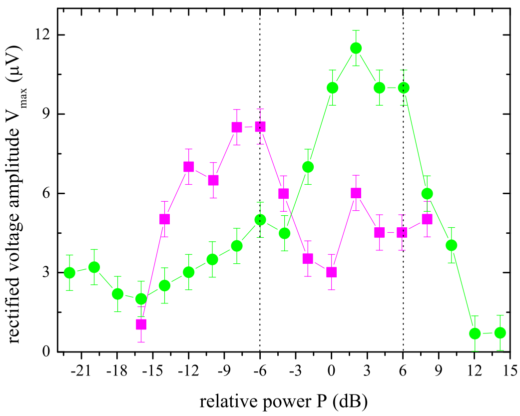


FIG. 2: Maximum amplitude of the rectified dc voltage versus relative power $dB \equiv 10\log\left[P_{1(2)}/P_{2(1)}\right]$ of the two harmonics (experimental data from Fig. 4 in Ref. [25]; $P_1 = const$ (squares), $P_2 = const$ (circles)). Vertical dotted lines indicate the values $dB = \pm 6.02$ (see the text).

Finally, we propose additional experimental tests with fluxons by considering the ac field

$$\widetilde{\gamma}(t) \equiv \gamma\left[\eta\sin(\Omega t) + \alpha(1-\eta)\sin(2\Omega t + \theta)\right] \quad (6)$$

instead of that given by Eq. (4), where γ is an amplitude factor, $\alpha > 0$, and $\eta \in [0,1]$. This means that $P_1 \sim \gamma^2\eta^2, P_2 \sim \alpha^2\gamma^2(1-\eta)^2$, and hence $dB_1 \equiv 20\log[\eta/(\alpha(1-\eta))] = -dB_2$. Now, the theoretical prediction given by Eq. (5) reads $\langle v\rangle \sim \alpha\gamma^3\eta^2(1-\eta)\sin(\theta+\theta_0)$, which presents a single maximum at $\eta_{opt} = 2/3$, irrespective of the particular value of parameter α, and hence the amplitude of the rectified voltage should present single maxima at $dB_{\max} = \pm 20\log(2/\alpha)$, thus indicating an explicit dependence on parameter α. In contrast, RU predicts once again that optimal enhancement of DRT of fluxons requires that the amplitude of the odd harmonic must be twice that of the even harmonic in Eq. (6), i.e., $\eta_{opt} = 2\alpha/(1+2\alpha) \Longrightarrow P_1 \sim 4P_2$, and hence that the amplitude of the rectified voltage should present again absolute maxima at $dB_{\max} \sim \pm 6$, *irrespective* of the particular value of parameter α. Also, RU predicts that the mean fluxon ve-

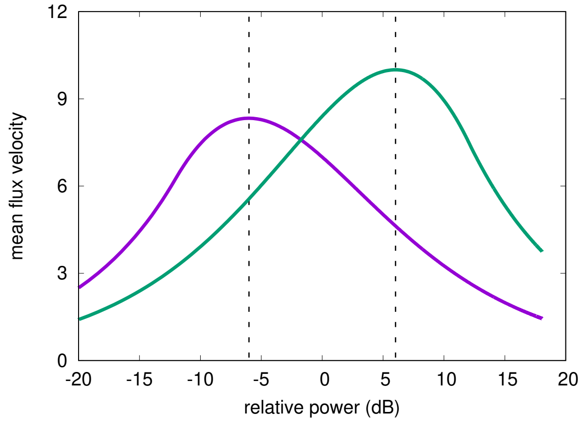


FIG. 3: Mean fluxon velocity as a function of the relative power $dB = \pm 20 \log \{\eta / [\alpha (1-\eta)]\}$ [see the text; Eq. (7)] for $\theta = \theta_{opt} \equiv \pi/2$ and $\alpha = \{1, 2, 3\}$ [curves $S(dB_1) = C_1 \mathcal{L}(dB_1)$ and $S(dB_2) = C_2 \mathcal{L}(dB_2)$ with maxima at $dB_{\max} = -6.02$ and $dB_{\max} = 6.02$, respectively]. Vertical dashed lines indicate the values $dB = \pm 6.02$. Fixed parameters: $C_1 = 50, C_2 = 60$ [27].

locity has the functional dependence

$$\langle v \rangle \sim S(dB)\, p(\theta), \tag{7}$$

[12] where $p(\theta)$ is a 2π-periodic function, while $S(dB)$ presents features similar to those of the normalized version of the biharmonic field [Eq. (6)], $\widetilde{\gamma}^*(t)$, for any value of θ (see Ref. [12] for additional details). Figure 3 shows plots of the functions $S(dB_{1,2})$ for one of the optimal values of the initial phase difference ($\theta_{opt} = \pi/2$) and three values of α [27]. One finds indeed that the respective curves are identical for *any* value of α and present maxima at $dB_{\max} \sim \pm 6$, as predicted by RU, while resembling the corresponding experimental data series shown in Fig. 2.

IV. SUMMARY AND OUTLOOK

In conclusion, we have discussed two retrodictions of the theory of RU which explain previous experimental results concerning DRT of cold atoms in dissipative optical lattices in one case, and fluxons in uniform annular Josephson junctions in the other, both driven by biharmonic fields (rocking ratchets). We expect that the theory of RU will explain other different implementations of the ratchet effect, such as gating ratchets [28], directed transport in coupled systems without external bias [29], and ratchets without any periodic substrate potential [30]. Our current work is aimed at exploring these cases [31].

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[27] The corresponding analytical expressions of $\mathcal{L}(dB_{1,2})$ for $\alpha=2$ can be obtained straightforwardly from the expression of $W(\eta)$ (cf. Eq. (4) in Ref. [17]) after the substitutions $\eta \rightarrow \left[1+(1/\alpha)10^{\mp dB/20}\right]^{-1}$.

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