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## **Turbidity Currents, Revisited**

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# TURBIDITY CURRENTS, REVISITED

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## ABSTRACT

Reviewed is the hydrodynamics of turbidity currents, which constitute a special case of density (or gravity) currents. Caused essentially by density differences resulting from the presence of sediments in the current, turbidity currents generally plunge in ambient water and flow over the bottom topography. The entrainment of sediments from the bed, and the entrainment of ambient water from above the current both play an important role. Certain turbidity currents can achieve high velocities and become auto-accelerative, if sediment entrainment from the bed continues.

The current is described as being made up of a front followed by a body. The hydrodynamic equations are derived taking into account the entrainment of sediment and ambient water. The equation of the interface is developed and explained for various special cases. The empirical equations required for closing the system of governing equations are presented. The shape and the velocity of the front of the current is discussed, and the distribution of velocity and concentration in the body are briefly presented.

*Keywords:* Turbidity Current, Front Velocity, Water Entrainment, Sediment Entrainment.

## 1. DESCRIPTION OF TURBIDITY CURRENT

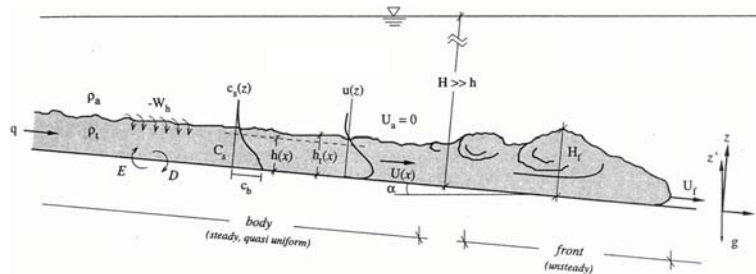


Figure 1 Sketch of a turbidity current (Graf and Altinakar, 1998).

A density or gravity current is a (two-phase) flow of a fluid of density  $\rho_t$ , which is caused essentially by the influence of a density difference,  $\Delta\rho$ , on the gravity,  $g$ . It is as if the gravity were reduced by the ratio of  $\Delta\rho/\rho_a$ , where  $\rho_a$  is the density of the ambient fluid. The reduced gravity, which is the driving force of a density current, is expressed as:

$$g' = g (\rho_t - \rho_a) / \rho_a = g \Delta\rho / \rho_a \quad (1)$$

The density difference may be caused by the difference in temperature,  $\Delta\rho \cong 2\text{kg}/\text{m}^3$ , salinity,  $\Delta\rho \cong 20\text{kg}/\text{m}^3$ , or turbidity,  $\Delta\rho \cong 20$  to  $200\text{kg}/\text{m}^3$ .

A *turbidity current* is thus a density current where the heavy (turbulent) fluid is a

mixture of light ambient fluid of density  $\rho_a$ , and of granular (non-cohesive) material of density  $\rho_s$ , in suspension. Such a current must generate enough turbulence to carry the granular material in suspension. The average density of a turbidity current is given by:

$$\rho_t = C_s \rho_s + (1 - C_s) \rho_a = \rho_a + (\rho_s - \rho_a) C_s \quad (2)$$

where  $C_s$  is the volume concentration of the granular solid material averaged over the height of the current; if  $\rho_t > \rho_a$ , the turbidity current is a bottom current. The depth-averaged reduced gravity, Eq. 1, can also be written as:

$$g' = g [(\rho_t - \rho_a) / \rho_a] C_s = g R C_s \quad (3)$$

where  $R$  is the specific density of suspended granular material. Considering that  $c_s$  is the local concentration within the current at the height  $z$  from the bottom, the local reduced gravity can be written as:

$$g_z' = g R c_s \quad (4)$$

If the suspension of sediments in the current is sufficiently diluted,  $\Delta\rho / \rho_a \ll 0.1$ , one can simplify equations using Boussinesq approximation (see Turner, 1973), which implies that  $\Delta\rho / \rho_a \cong 0$  when it multiplies the inertia terms, and  $\Delta\rho / \rho_a \neq 0$  when it multiplies the gravitational acceleration  $g$ .

The *densimetric Froude number*, which expresses the ratio of inertia forces to reduced-gravity forces, is an important dimensionless number:

$$Fr_D = U / \sqrt{g' h \cos \alpha} \quad (5)$$

where  $\alpha$  is the bottom slope angle and  $U$  the depth-averaged velocity. However, the following form, which is called *global Richardson number*, is more commonly used:

$$Ri = 1 / Fr_D^2 = g' h \cos \alpha / U^2 \quad (6)$$

Letting  $u$  and  $\rho$  represent the velocity and density at the height  $z$ , respectively, One can also define a *local or gradient Richardson number* at the height  $z$  from the bottom as follows (Turner, 1973):.

$$Ri_z = -g (\partial\rho / \partial z) / [\rho (\partial u / \partial z)^2] \quad (7)$$

As it will be discussed later in detail, the movement of the turbidity current depends on the entrainment of the ambient fluid from its upper boundary, which is parameterized by the *entrainment coefficient*,  $E_w$ . This coefficient is dependent on the global Richardson number:

$$E_w = f(Ri) \quad (8)$$

Another important parameter that needs to be defined is the *buoyancy flux*, or *reduced sediment flux*, per unit width, defined as:

$$B = g' h U = g R (C_s U h) = g' q \quad (9)$$

where  $q$  is the discharge per unit width. One distinguishes now between *conservative turbidity* currents with  $dB / dx = 0$ , and *non-conservative turbidity* currents with  $dB / dx \neq 0$ . In the latter ones the change in the buoyancy flux occurs due to entrainment of the eroded bed material into the current and/or deposition of the transported sediment on the bed. Turbidity currents are often non-conservative ones.

Turbidity current (see Figure 1) is made up of a front or head advancing into the ambient fluid, being followed by the body. The driving force for the front is the pressure gradient due to the density difference between the current and the ambient fluid. The flow in the front is three dimensional and unsteady. The driving force for the body is the gravitational force of the heavier fluid. The flow is often considered to be a steady. The flow duration of a turbidity current, thus its length, will depend upon the incoming reduced sediment flux,  $B$ .

The interface between the turbidity current and the ambient fluid (Figure 1) is usually not easy to distinguish. For this reason, the average current height,  $h_t$ , and average current velocity,  $\bar{U}$ , are defined as integral scales (see Turner, 1973, p. 179):

$$Uh = \int_0^{\infty} u dz = \int_0^{h_t} u dz = \bar{U}h_t = q \quad \text{and} \quad U^2 h = \int_0^{\infty} u^2 dz = \int_0^{h_t} u^2 dz = \beta_u \bar{U}^2 h_t \quad (10, 11)$$

where  $u(z)$  is the point velocity, which becomes zero at  $z = h_t$ , and  $\beta_u$  is a shape coefficient (Boussinesq) depending on the velocity distribution. Once  $\bar{U}$  and  $h_t$  are known, the average concentration,  $C_s$ , is determined from:

$$C_s Uh = \int_0^{\infty} (uc_s) dz = \int_0^{h_t} (uc_s) dz = C_s \bar{U}h_t \quad (12)$$

## 2. PLUNGE POINT

When a stream transporting large quantities of sediments enters a stagnant reservoir of reasonably clear water, the larger particles deposit immediately at the beginning of the reservoir by forming a delta. The flow with fine sediments advances further into the reservoir, and at some point plunges to the bottom and establishes a bottom turbidity current (Figure 2).

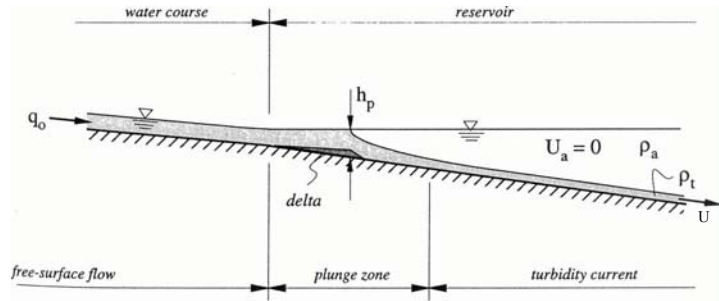


Figure 2 Plunge zone of a sediment laden river entering a reservoir (Graf and Altinakar, 1998)

The plunge zone is often clearly visible. The water depth at the plunge point,  $h_p$ , can be calculated by considering that the momentum of the flow (in channel and in reservoir) is conserved. By taking  $q_o$  as the unit discharge entering a reservoir, Akiyama and Stefan, 1985, provide an approximate relation for the densimetric Froude number at the plunge point:

$$Fr_p = (q_o / h_p) / \sqrt{g' h_p} \cong 0.68 \quad (13)$$

which is valid for a wide range of bed slopes:  $0.017 < S_f < 0.123$ .

A turbidity current, when it passes along a reservoir, lake or ocean, has a tendency to deposit its granular material, causing rather important sedimentation (see Graf, 1983). Spectacular sedimentation was observed behind Hoover Dam and Elephant Butte Dam, but also in the delta formed by the Rhone River entering Lake Geneva or in the submarine Scripps Canyon of the Pacific Ocean. However, turbidity currents can also be beneficial for the life of artificial reservoirs. During floods, the deposited sediments can possibly create turbidity currents, which in turn transport the sediments downstream towards the dam. By clever manipulation of the bottom outlets, turbidity currents can be used to evacuate the sediments accumulated in the reservoir.

## 3. HYDRODYNAMIC EQUATIONS

Consider the body of a turbidity current, two-dimensional and plane, with the flow,  $(u, 0, w)$ , being turbulent and incompressible (Figures 1 and 3). The height,  $h$ , the velocity,  $U$ , and the concentration,  $C_s$ , are average values, defined by the integral scales (Eqs. 10, 11, and 12). The current moves in the longitudinal direction,  $x$ , over a bottom slope,  $S_f$ , with an angle,  $\alpha$ , under a deep layer,  $H \gg h$ , of ambient stagnant fluid of density,  $\rho_a$ , being slightly smaller than the density of the turbidity current,  $\rho_t > \rho_a$ .

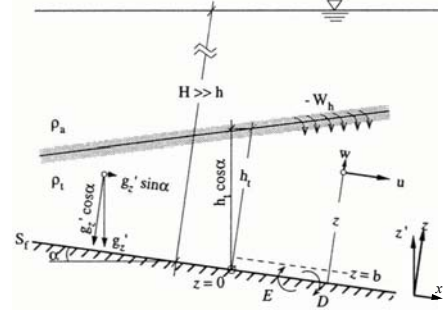


Figure 3 Definition sketch of the body of a turbidity current

The flow is well established, continuous, steady and gradually varied. The equation of continuity and of motion for the fluid phase (mixture of water/sediment) and for the solid phase, may be established (Figure 3). This current, being relatively thin,  $h \ll H$ , is taken to be a boundary-layer flow, where the conditions of  $u \gg w$  and  $\partial/\partial z \gg \partial/\partial x$  are valid. The *continuity for the fluid phase* is written as:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (14)$$

Upon integration of Eq. 14 over the depth,  $0 < h < h_t$ , one obtains:

$$\int_0^{h_t} \frac{\partial u}{\partial x} dz + \int_0^{h_t} \frac{\partial w}{\partial z} dz = \left( \frac{\partial}{\partial x} \int_0^{h_t} \frac{\partial u}{\partial x} dz - u_{h_t} \frac{\partial h_t}{\partial x} \right) + w_{h_t} - w_b = 0 \quad (15)$$

By definition, the horizontal velocity at the interface and the vertical velocity at the bed are zero :  $u_{h_t} = 0$  and  $w_b = 0$ . The vertical velocity at the interface,  $w_{h_t} = W_h = E_w U$ , is defined as the velocity of entrainment of the ambient fluid into the current. It is assumed to be proportional to the velocity of the current through an entrainment coefficient,  $E_w$ , which depends on the global Richardson number, Ri.

The *equation of continuity for the solid phase* is given by the equation of diffusion of granular material (see Graf, 1971, chap. 8.3) or :

$$\frac{\partial(uc_s)}{\partial x} + \frac{\partial(wc_s)}{\partial z} = v_{ss} \frac{\partial c_s}{\partial z} + \varepsilon_s \frac{\partial^2 c_s}{\partial z^2} \quad (16)$$

where  $v_{ss} (\cong v_{ss} \cos \alpha)$  is the settling velocity and  $\varepsilon_s$  is the diffusion coefficient of granular material. Elder's relation  $\varepsilon_s (\partial^2 c_s / \partial z^2) \cong -\partial/\partial z (\overline{c_s'w'})$ , where  $(\overline{c_s'w'})$  is the Reynolds flux of the solid phase (sediments), can be used to replace the diffusion term in Eq. 16. Integrating Eq. 16 over the depth,  $0 < h < h_t$ , one obtains:

$$\frac{\partial}{\partial x} \int_0^{h_t} (uc_s) dz = -v_{ss} c_s \Big|_{z=b} + (\overline{c_s'w'}) \Big|_{z=b} \quad (17)$$

The terms  $(\overline{c_s'w'}) \Big|_{z=b} = v_{ss} E_s = E(b)$  and  $-v_{ss} c_s \Big|_{z=b} = v_{ss} c_b = D(b)$  represent the erosion of the sediments from the bed and deposition of the sediment on the bed, respectively (Parker et al., 1987). In these relationships  $E_s$  is the sediment entrainment coefficient and  $c_b$  the local concentration near the bed. Inserting these relations in Eq. 17, yields the final form of the equation of continuity for the solid phase:

$$\frac{\partial(C_s U h)}{\partial x} = v_{ss} (E_s - c_b) = E(b) - D(b) \quad (18)$$

One can identify the following cases: 1) when  $E(b) > D(b)$  the turbidity current is non-

conservative and erosive; 2) when  $E(b) < D(b)$  the current is non-conservative and positive; 3) when  $E(b) = D(b)$  the current is *conservative* and/or in *equilibrium*.

The *equation of motion of the turbidity current* can be written by combining equation boundary layer flow with the equation of continuity:

$$\frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho_t} \frac{\partial}{\partial x} (p_t + \rho_t g z') + \frac{1}{\rho_t} \frac{\partial \tau_{zx}}{\partial z} \quad (19)$$

The shear stress for turbulent flow is expressed as  $\tau_{zx} = -\rho_t (\overline{u'w'})$  and Boussinesq approximation is used. The pressure in the turbidity current can be decomposed into the pressure due to ambient fluid,  $p_a$ , and the pressure due to excess density,  $\rho_s = (p_t - p_a)$ , resulting from the presence of suspended fine sediments. Assuming also that the pressure in the ambient fluid to be hydrostatic,  $(p_a + \rho_a g z') = \text{const}$ , one can write:

$$\begin{aligned} \frac{\partial (p_t + \rho_t g z')}{\partial x} &= \frac{\partial (p_a + \rho_a g z')}{\partial x} + \frac{\partial (p_s + \Delta \rho g z')}{\partial x} = \\ \frac{\partial p_s}{\partial x} + \frac{\partial (\rho_t g'_z z')}{\partial x} &= -\rho_t g R \cos \alpha \frac{\partial}{\partial x} \int_z^H c_s dz - \rho_t g R c_s \sin \alpha \end{aligned} \quad (20)$$

where  $g R c_s = g'_z$ ,  $z' = z \cos \alpha$ ,  $-dz'/dx = \sin \alpha$ , and  $\Delta \rho g = \rho_a g' \cong \rho_t g'$ . Integration of Eq. 19 over the depth,  $0 < h < h_t$ , gives:

$$\frac{\partial}{\partial x} \int_0^{h_t} u^2 dz - u_{*b}^2 \frac{\partial h_t}{\partial x} + (u_{*h} w_{*h} - u_{*b} w_{*b}) = -g R \cos \alpha \frac{\partial}{\partial x} \int_0^{h_t} c_s (H - z) dz - g R \sin \alpha \int_0^{h_t} c_s dz - u_{*b}^2 \quad (21)$$

in which  $-u_{*b}^2 / \rho_t = \int_0^{h_t} (\partial \tau_{zx} / \partial z) dz$ . Introducing integral scales Eqs. 10, 11, and 12, into Eq.

21, one obtains:

$$\frac{\partial U^2 h}{\partial x} = -\frac{1}{2} \cos \alpha \frac{\partial (S_1 g' h^2)}{\partial x} + S_2 g' h \sin \alpha - u_{*b}^2 \quad (22)$$

where the shape coefficients,  $S_1$  and  $S_2$ , are defined as:

$$S_1 = \frac{2}{g' h^2} \int_0^{h_t} (g R c_s) (H - z) dz \quad \text{and} \quad S_2 = \frac{2}{g' h} \int_0^{h_t} (g R c_s) dz \quad (23)$$

One can reasonably well assume that  $S_1 \cong 1$  and  $S_2 \cong 1$  (Parker et al., 1987, and Altinakar et al, 1993). When pressure and resistance forces are negligible, a simple form of the equation of motion, Eq. 22, can be written as:

$$\frac{\partial U^2 h}{\partial x} = g' h \sin \alpha \quad (24)$$

#### 4. INTERFACE PROFILE CURVES

By combining equations of continuity, Eqs. 15 and 18, with the equation of motion, Eq. 22, one obtains an equation of the interface between the current and the ambient fluid:

$$\frac{\partial h}{\partial x} = \frac{1}{(1 - Ri)} \left\{ \frac{(4 - Ri)}{2} E_w + \frac{Ri}{2} \frac{v_{ss}}{UC_s} (E_s - c_b) - Ri \tan \alpha + \left( \frac{u_{*b}}{U} \right)^2 \right\} \quad (25)$$

The global Richardson number, Eq. 6, is taken as  $Ri = g' h \cos \alpha / U^2 = g R c_s h \cos \alpha / U^2$ . By

using the global Richardson number to parameterize the reduced sediment flux (i.e. buoyancy flux),  $Ri = g' h U \cos \alpha / U^3 = B \cos \alpha / U^3$ , one can also obtain an equation for the variation of Richardson number:

$$\frac{h}{3Ri} \frac{dRi}{dx} = \frac{1}{(1-Ri)} \left\{ \left[ E_w + \frac{1}{3} \frac{v_{ss}}{UC_s} (E_s - c_b) \right] \frac{1}{2} (2 + Ri) - Ri \operatorname{tg} \alpha + \left( \frac{u_{*b}}{U} \right)^2 \right\} \quad (26)$$

For conservative currents, the terms underlined in Eqs. 25 and 26 become zero. A conservative current for which  $dRi/dx = 0$  is in equilibrium. Solving Eqs 25 and 26 together, one obtains  $dh/dx = E_w$ , which expresses a linear growth for the current height, i.e.  $h = E_w (x - x_o) + h_o$ , where  $h_o$  is the initial depth at the origin  $x_o$ . In this case the velocity of the current is given by  $U = (B \cos \alpha / Ri)^{1/3}$ . For a conservative turbidity current over a weak slope, thus with negligible entrainment of ambient fluid,  $E_w \cong 0$ , Eq. 25 becomes:

$$\frac{\partial h}{\partial x} = \frac{(u_{*b}/U)^2 - Ri S_f}{(1-Ri)} \quad (27)$$

This equation is similar to the equation of gradually varied flow in open channels, indicating the analogy between a conservative turbidity current and free-surface flow in a channel. Similar to the open-channel flow, one can also make a distinction between subcritical flow,  $Ri > 1$  (or  $Fr_D < 1$ ), and supercritical flow,  $Ri < 1$  (or  $Fr_D > 1$ ).

Under certain conditions, the flow of a conservative turbidity current can be expressed as a uniform flow. Imposing  $dh/dx = 0$  in Eq. 27, making use of the Eq. 6 and by taking  $S_f \cong \sin \alpha$ , one obtains  $g' h S_f \cong u_{*b}^2$ . Defining a friction coefficient as  $f_{CT} \cong 8(u_{*b}/U)^2$ , one can obtain a Chezy-like uniform flow equation:

$$U = \sqrt{8/f_{CT}} \sqrt{g' h S_f} \quad (28)$$

Turbidity currents are subject to friction both at the bed and at the interface with the ambient fluid. Harleman suggests to use  $f_{CT} = f(1 + \alpha_H)$ , in which  $f$  is the friction coefficient due to the bed, for free surface flow and  $\alpha_H \cong 0.43$  for turbulent flow.

## 5. ENTRAINMENT COEFFICIENTS

The system of equations describing the dynamics of turbidity currents, i.e., Eqs. 15, 18 and 22, can only be closed by specifying empirical relationships for the entrainment coefficients for water,  $E_w$ , and sediment,  $E_s$ , the sediment concentration near the bed,  $c_b$ , the fall velocity of sediments,  $v_{ss}$ , and the bed shear velocity,  $u_{*b}$ .

The entrainment coefficient,  $E_w$  relating the entrainment velocity at the interface,  $W_h$ , to the average current velocity,  $U$ , is a function of the Richardson number,  $Ri$ . The experimental data from different types of density and turbidity currents are plotted in Figure 4a. Despite the obvious experimental dispersion, the following empirical relationship given by Parker et al., 1987, represents the entire data range quite well:

$$E_w = 0.075 \left( 1 + 718 Ri^{2.4} \right)^{-0.5} \quad (29)$$

It is interesting to note the rapid decrease of  $E_w$  with increasing  $Ri$ . In fact,  $Ri$ , thus  $E_w$ , depends on bed slope. For large slopes ( $\alpha > 12^\circ$ ), one can write  $E_w = Ri \tan \alpha$ . Referring to Figure 4a, for  $Ri \rightarrow 0$  we have  $E_w \rightarrow 0.075$ , which is the entrainment coefficient of a fluid

jet in an ambient fluid of same density; for  $Ri \rightarrow 1$  we have  $E_w \cong 0.003$ , which corresponds to negligible entrainment. Across a stable interface with  $Ri > 1$ , there is almost no mixing, but internal gravity waves can form. These internal waves break and contribute to mixing, i.e. entrainment if the criterion of Keulegan is fulfilled:

$$Ri / Re = \nu g' / U^3 < 0.18^3$$

where  $Re$  is the Reynolds number and  $\nu$  is the kinematic viscosity of the current.

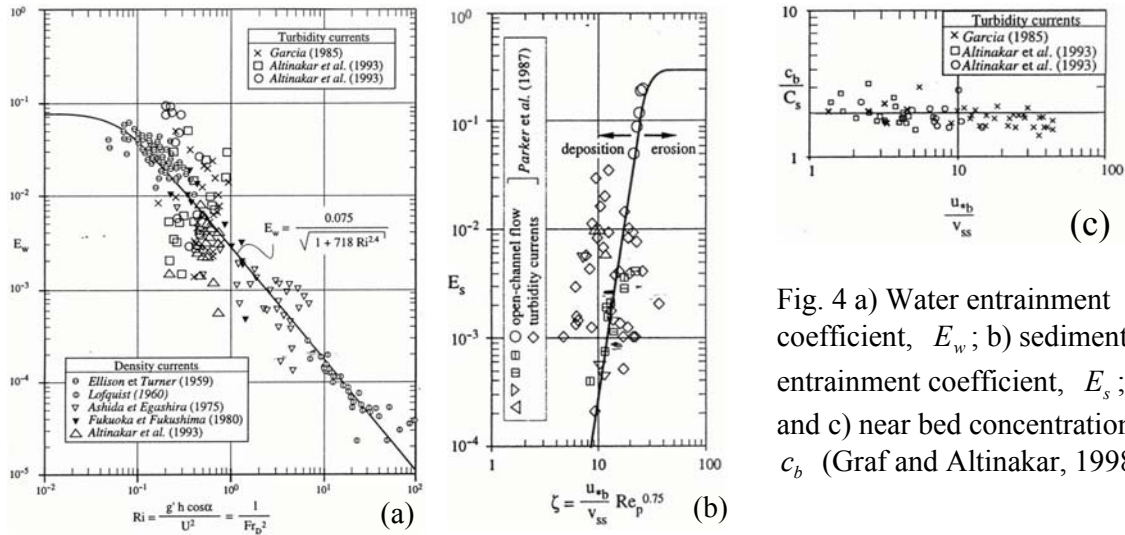


Fig. 4 a) Water entrainment coefficient,  $E_w$ ; b) sediment entrainment coefficient,  $E_s$ ; and c) near bed concentration,  $c_b$  (Graf and Altinakar, 1998)

Sediment entrainment coefficient data obtained by Akiyama and Stefan, 1985, from open channel experiments, and by Altinakar et al., 1993, from turbidity current experiments are plotted in Figure 4b. The following empirical relationship proposed by Parker et al, 1987, can be used to calculate the sediment entrainment coefficient:

$$E_s = (3 \times 10^{-11} \zeta^7) / (1 + 10^{10} \zeta^7) \quad (30)$$

In which the parameter  $\zeta$  depends on the particle Reynolds number,  $Re_p$ :

$$\zeta = \frac{u_{*b}}{v_{ss}} Re_p^{0.75} \quad \text{with} \quad Re_p = \frac{d_{50} \sqrt{gRd_{50}}}{\nu}$$

In the above equations  $d_{50}$  is the median sediment size. It is important to note that Eq. 30 describes a very steep curve which converges to a maximum value of  $E_s \cong 0.3$ .

The near bed concentration is generally evaluated at the height  $b \cong 0.05h_i$  and has a functional form given as  $c_b / C_s = f(u_{*b} / v_{ss})$ . The data from turbidity current experiments by Parker et al, 1987, and Altinakar et al., 1993, are plotted in Figure 4c. This plot shows that, for the range  $1 < u_{*b} / v_{ss} < 50$  one can take  $c_b / C_s \cong 2$ .

The settling velocity of the sediment particles can be calculated using different methods found in the literature (Graf, 1971). For very fine particles one can use Stokes' law:

$$v_{ss} = d^2 g R / (18 \nu) \quad (31)$$

As discussed in the previous section, the bed shear velocity can be calculated from:

$$u_{*b} = (f/8) U^2 \quad (32)$$



## 6. FRONT OF THE CURRENT

The front of the current preceding the quasi-uniform body flow is a three-dimensional unsteady flow region with intense mixing with the ambient fluid. The driving force is the gradient of pressure resulting from the density difference between the front and the ambient fluid. The shape of the front (Figure 5) is characterized by its height,  $H_f$ , and the nose which is situated at the height,  $h_f$ , slightly above the bed.

The velocity of the front can be calculated using simple hydraulic considerations (see Turner, 1973, p.73). Assuming a quasi-uniform flow moving on a frictionless surface with zero slope and neglecting the mixing, one obtains  $U_f = \sqrt{2g'h}$ . Using a large number of experiments of density and turbidity currents on a large range of slopes, Altinakar et al., 1990, proposed the following relationship:

$$U_f = 0.75 \sqrt{g'H_f} \quad (33)$$

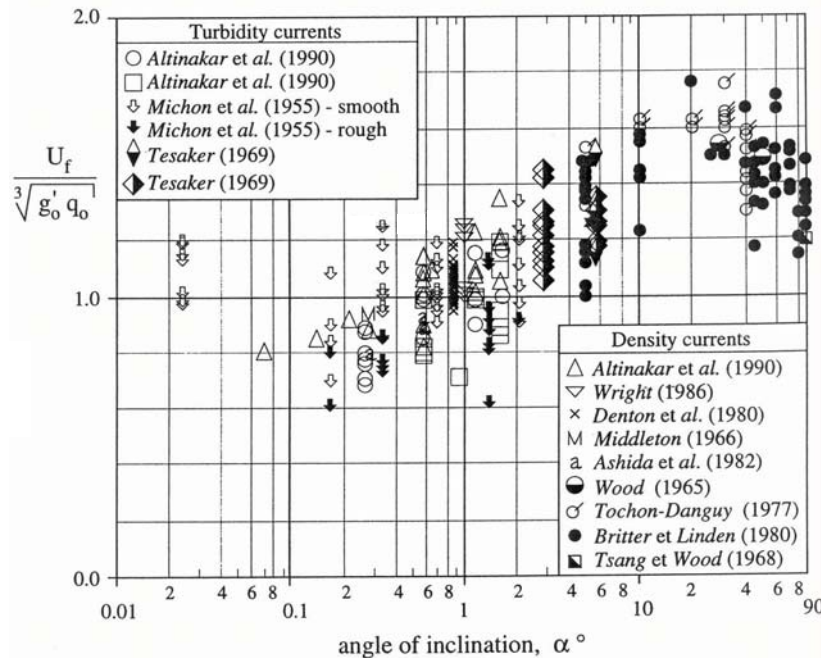


Figure 6 Dimensionless velocity of the front,  $U_f / (g'_o q_o)^{1/3}$ , as a function of the bottom slope,  $\alpha$  (Graf and Altinakar, 1998).

Britter and Linden, 1980, and Altinakar et al, 1990, have observed that, for a continuously fed conservative current, the velocity of the front remains more or less constant independent of the distance covered. This implies that the reduced gravity compensates the

frictional force. However, for turbidity currents over small slopes,  $\alpha < 0.5^\circ$ , and for depositing currents, the front decelerates slightly and the height of the front increases with distance, which can be attributed to entrainment of ambient fluid. It should also be noted that the front velocity is always smaller than the velocity of the trailing body. This is necessary in order to compensate the fluid lost into the ambient environment through intensive mixing.

The velocity of the front,  $U_f$ , can also be related to the reduced sediment flux of the entering current,  $B_o = g'_o q_o$ , and to the bed slope,  $\alpha$ , or:

$$U_f = (g'_o q_o)^{1/3} f(\alpha) \quad (34)$$

The roughness of the bed,  $f$ , the entrainment coefficient,  $E_w$ , and the Reynolds number,  $Re = Uh/\nu$ , could also play a role. In Figure 6, the dimensionless head velocity  $U_f / (g'_o q_o)^{1/3}$  is plotted as a function of the angle of inclination  $\alpha$ . The dimensionless velocity increases with increasing slope up to  $\alpha \cong 35^\circ$ , then decreases due to increased friction and mixing at the interface. For practical purposes one can assume  $U_f / (g'_o q_o)^{1/3} = 1.5 \pm 0.2$  for large slopes  $5^\circ < \alpha < 90^\circ$ . For small slopes,  $\alpha < 5^\circ$ , the dimensionless head velocity varies in the range  $0.7 < (U_f / (g'_o q_o)^{1/3}) < 1.5$ .

## 7. DISTRIBUTION OF VELOCITY AND CONCENTRATION

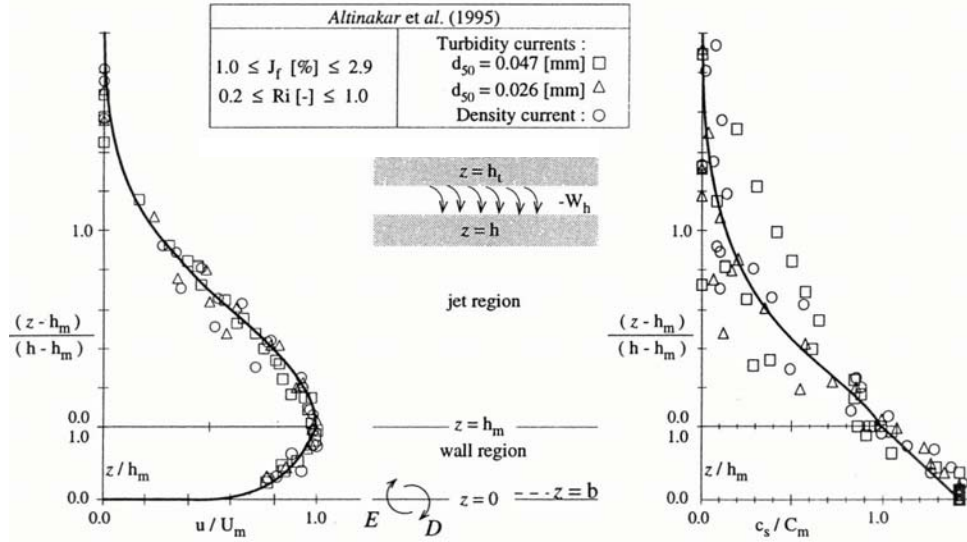


Figure 7 Non-dimensional distribution of velocity,  $u(z)$ , and concentration,  $c_s(z)$ , measured by Altinakar (1988) for different types of gravity currents - turbidity current and saline density currents (Graf and Altinakar, 1998).

Referring to Figure 7, the flow in the body of the turbidity current may be assimilated to a turbulent wall jet composed of two regions separated at the height  $h_m$ , where the local time averaged velocity is maximum,  $u = U_m$ . In the wall region,  $z < h_m$ , the turbulence is created at the wall, and the entrainment or deposition of sediments takes place. The velocity distribution in this region is logarithmic and given by:

$$u(z)/u_{*b} = (1/\kappa) \ln z + Const \quad (35)$$

In the jet region,  $z > h_m$ , the turbulence is created by friction and by entrainment. The velocity profile is Gaussian. Altinakar et al., 1996, proposes the following relationship:

$$u(z)/U_m = \exp\left\{-\alpha_c \left[\frac{(z-h_m)}{(h-h_m)}\right]^2\right\} \quad \text{with} \quad \alpha_c = 1.4 \quad (36)$$

Based on laboratory experiments, they also found that:

$$h_m/h \approx 0.3 \quad U_m/U \approx 1.3 \quad h_t/h \approx 1.3 \quad U/\bar{U} \approx 1.3 \quad (37)$$

The concentration distribution in the wall region,  $z < h_m$ , is similar to the concentration profile of suspended sediments in open channel flow. It is given by:

$$c_s(z)/c_b = \left[\frac{(h_m-z)}{z} \frac{b}{(h_m-b)}\right]^{\mathcal{Z}} \quad (38)$$

where  $c_b$  is the near bed concentration and the Rouse exponent is defined as  $\mathcal{Z} = v_{ss}/\kappa u_{*b}$

In the jet region,  $z > h_m$ , the distribution of concentration is Gaussian. Altinakar et al., 1996, proposes the expression:

$$c_s(z)/C_m = \exp\left\{-\beta_c \left[\frac{(z-h_m)}{(h-h_m)}\right]^{4/3}\right\} \quad (39)$$

in which one takes  $1.7 < \beta_c < 4.1$ . The value of  $\beta_c$  increases with the distance  $x$ . The concentration evaluated at  $z = h_m$  is denoted as  $C_m$ . Altinakar et al., 1996, experimentally found that:

$$c_b/C_s \approx 2 \quad \text{and} \quad c_b/C_m \approx 1.4 \quad (40)$$

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