Conference Paper, Published Version

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Proceedings of ninth International Conference on Hydro-Science and Engineering (ICHE 2010), IIT Madras, Chennai, India.
2 - 5 August 2010

NONLINEAR EFFECTS UPON WAVE PROPAGATION IN EXPERIMENTS OF WAVE BLOCKING

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***Abstract**: Nonlinear dispersion effects are investigated in modeling of blocking of periodic waves. It is found that inclusion of these in the model results in a larger wave group velocity, as expected, and gives a better fit with experimental data as compared to the use of linear dispersion relation.*

***Keywords**: wave blocking; periodic waves; modeling; nonlinear dispersion effects.*

1. INTRODUCTION

Water waves meeting an adverse current shorten and steepen as the current velocity increases in the upstream direction and wave blocking may occur, which can be accompanied with wave breaking (Chawla and Kirby, 1998; Suastika *et al*., 2000). In such a situation, nonlinear effects are expected to play an important role. The phenomenon of wave blocking has been studied extensively in recent years, particularly in connection with modeling of the wave evolution in coastal regions (Chawla and Kirby 2002; Suastika and Battjes, 2005; Suastika, 2009). Suastika and Battjes (2009) developed a linear model for modeling of the wave amplitude evolution in the case of blocking of periodic waves. Their model is able to reproduce the observed pattern of the wave amplitude variation well, although the predicted blocking point location lies about one wave length downstream from the observation, ascribed to the exclusion of nonlinear effects in the model. In the present study, nonlinear dispersion effects are investigated by applying respectively linear and Stokes third-order dispersion relations in the model of Suastika and Battjes (2009). Model results are compared with the experimental data of Suastika *et al.* (2000).

2. DISPERSION RELATION

The kinematics of wave-current interactions and of wave blocking are best illustrated by an investigation of the dispersion relation. A discussion of this for gravity waves propagating on an ambient current has been given in e.g. Peregrine (1976), Peregrine and Jonsson (1983) and Jonsson (1990). For the case of blocking of gravity-capillary waves, a discussion of the dispersion relation is given in Shyu and Phillips (1990) and Trulsen and Mei (1993).

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Although a discussion of the kinematics of wave-current interactions and of wave blocking has been given in the above mentioned studies, here a discussion of these is presented for the sake of easy reference, in which particular attention is paid to nonlinear effects. For the general case of 2-dimensional wave propagation, the wave frequency is Doppler-shifted due to the current, which is represented as

$$\omega = \sigma + \underline{k} \cdot \underline{U} \tag{1}$$

where ω is the wave frequency relative to the fixed bed, σ is the wave frequency relative to the water mass (the intrinsic wave frequency), $\underline{k}$ is the wave number vector and $\underline{U}$ is the current velocity relative to the bottom. The intrinsic linear dispersion relation of gravity waves in water of finite depth is represented as

$$\sigma^2 = gk \tanh(kh) \tag{2}$$

where g is the gravitational acceleration, $k = |\underline{k}|$ is the magnitude of the wave number vector and h is the mean water depth. Equations (1) and (2) can be combined to give

$$(\omega - \underline{k} \cdot \underline{U})^2 = gk \tanh(kh) = \sigma^2 \tag{3}$$

We consider the case where periodic gravity waves (with frequency ω relative to the fixed bed) generated in still water, collinearly meet an adverse current with velocity increasing in the upstream direction. For the purpose of a discussion of the phenomenon of wave blocking, we define $\underline{k} = k\, \underline{e}_x$ and $\underline{U} = U\, \underline{e}_x$, where $\underline{e}_x$ is the unit vector in the (incident) wave propagation direction. With these definitions, the solutions to Eq. (3) are given as

$$\omega - kU = \pm [gk \tanh(kh)]^{1/2} \tag{4}$$

In the present case, $U \leq 0$ so that $k\,U \leq 0$ and only the positive branch of the r.h.s. of Eq. (4) can be taken. Figure 1 shows a schematic diagram of graphical solutions of Eq. (4) for this particular case. The intrinsic phase velocity c is given as $c = \sigma/k$ and the intrinsic wave group velocity c_g (or intrinsic wave energy transport velocity) as $c_g = \partial\sigma/\partial k$.

In still water ($U = 0$), $\sigma = \omega$ and the wave number k is given by Eq. (2) (solution r_0 of Eq. (4) in Fig. 1). In the region with an adverse current ($U < 0$), in which $|U| < |U_B|$, where U_B is the blocking velocity ($U_B < 0$), there are two solutions for a wave train with frequency ω relative to the fixed bed in water of depth h, which are denoted by r_1 and r_2 in Fig. 1. Solution r_1 represents waves with net phase and net energy transport velocities directed upstream ($c + U > 0$, $c_g + U > 0$)

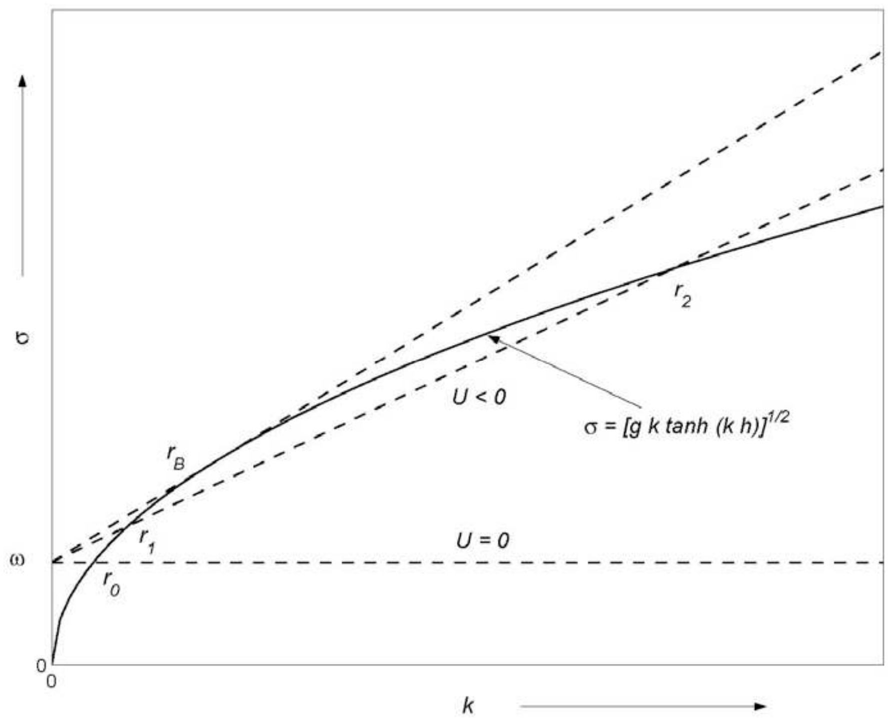

Fig. 1. Solutions of the dispersion relation for given wave frequency ω relative to the fixed bottom, current velocity *U* and water depth *h*.

and solution r_2 represents waves with net phase velocity directed upstream ($c + U > 0$) but with net energy transport velocity directed downstream ($c_g + U < 0$). In the case that $|U| > |U_B|$, there is no (real) solution to the dispersion relation Eq. (4).

In the case considered here, where gravity waves generated in still water meet an adverse current with velocity increasing in the upstream direction, the frequency ω has the same value at all positions. As the adverse current becomes stronger (the slope of the line $\sigma = \omega - k\,U$ in Fig. 1 becomes larger) the incident waves become shorter (the wave number k corresponding to r_1 is larger than that corresponding to r_0). The maximum adverse current velocity that the incident waves can face is given by the condition that the line $\sigma = \omega - k\,U$ is tangent to the curve $\sigma = [gk\tanh(kh)]^{1/2}$. This is the blocking condition where the intrinsic wave group velocity is equal to the local adverse current velocity: $\partial\sigma/\partial k + U = 0$ or $c_g + U = 0$. At the blocking point, the roots r_1 and r_2 coincide ($r_1 = r_2 = r_B$). Since the energy flux for waves r_2 is directed downstream, these waves must be generated upstream from where they are present. They represent waves reflected at the blocking point, where they have the same wave length as the incident waves ($r_2 = r_1 = r_B$). Going downstream the current weakens, so that r_2 shifts to larger wave number (the reflected waves shorten).

Having discussed the kinematics of blocking of linear gravity waves due to a counter current, we discuss now some implications of nonlinearity in the dispersion relation. For that purpose, we consider the Stokes third-order dispersion relation (the waves are in relatively deep water in the region with blocking) as also considered by Chawla and Kirby (2002) as follows:

$$\sigma^2 = gk\tanh(kh)\left[1 + (ak)^2\left(\frac{8 + \cosh(4kh) - 2\tanh^2(kh)}{8\sinh^4(kh)}\right)\right] \tag{5}$$

where σ is the intrinsic wave frequency, g is the gravitational acceleration, k is the wave number, h is the water depth and a is the wave amplitude. The higher-order correction to the linear dispersion relation is represented by the factor in the squared brackets in Eq. (5).

In laboratory experiments of wave blocking performed by Suastika *et al*. (2000), the water depth along the measurement section is about 0.5 m and the wave amplitude near the blocking point is about 1.5 cm. Figure 2 shows a comparison between the linear and Stokes third-order intrinsic dispersion relations for waves in water of depth h = 0.5 m and with wave amplitude a = 1.5 cm. Figure 2 shows that in this particular case the Stokes third-order dispersion relation diverts from the linear one for wave number k larger than about 10 rad/m. That is, nonlinear effects becomes to play a role for wave steepnes ak larger than about 0.15. Furthermore, as has been pointed out, the wave shorten and steepen as they propagate on a counter current with velocity increasing in the upstream direction. So, nonlinear effects are expected to play a role even in the region relatively far from the blocking point. We note that there is a limiting wave steepness ak of about 0.3 where waves in a deep water start to break.

As stated above, the intrinsic wave group velocity $c_g = \partial\sigma/\partial k$, that is the gradient of the $\sigma(k)$-curve shown in Figs. 1 and 2. Figure 2 shows that $\partial\sigma/\partial k$ becomes larger with the inclusion of nonlinear effects, that is nonlinear effects increase the wave group velocity. This implies that the (model) blocking point will shift further upstream with the inclusion of nonlinear effects (as compared to the use of linear dispersion relation). This effect has been pointed out by Suastika and Battjes (2009). A quantitative comparison between experimental data and model results using the linear and Stokes third-order dispersion relations is presented in the following section.

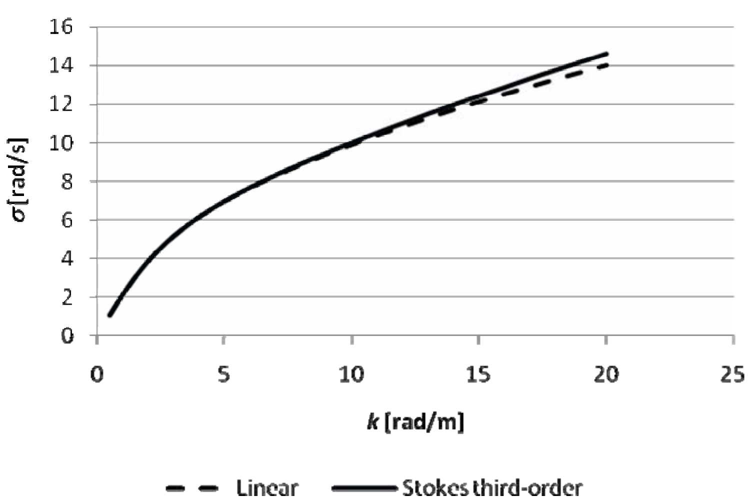

Fig. 2. Comparison between the linear and Stokes third-order intrinsic dispersion relations for waves in water of depth *h* = 0.5 m and wave amplitude *a* = 1.5 cm.

3. COMPARISON BETWEEN MODEL RESULTS AND EXPERIMENTAL DATA

In this section, we present results of modeling of the wave amplitude evolution in the case of blocking of periodic waves for which the model of Suastika and Battjes (2009) has been used. The reader is referred to Suastika and Battjes (2009) for the details of the model. Model results are compared with the experimental results of Suastika *et al*. (2000). In the results presented below, periodic waves with period T = 1.1 s and target wave amplitude in still water of 1.0 cm are considered. The still water depth along the measurement section is h = 0.55 m and the maximum adverse current discharge is Q_{max} = 0.12 m^3/s. For a description of the experimental arrangement, see Suastika *et al*. (2000) or Suastika and Battjes (2009).

Figure 3 shows a comparison between the observed and modeled (fundamental) wave amplitudes in which linear (dash-dot line) and Stokes third-order (solid line) dispersion relations are utilized in the model, respectively. Figure 3 shows that utilizing either the linear or Stokes third-order dispersion relation, the model is able to reproduce the observed pattern of the wave amplitude variation along the flume well. However, the observed blocking-point position lies about 0.5 m (about one wave length) further upstream than the modeled one when using the linear dispersion relation (as reported by Suastika and Battjes, 2009). Using the Stokes third-order dispersion relation, the modeled blocking-point position shifts about one wave length further upstream, resulting in a better fit between the observed and modeled amplitude variations along the flume as compared to the use of the linear dispersion relation.

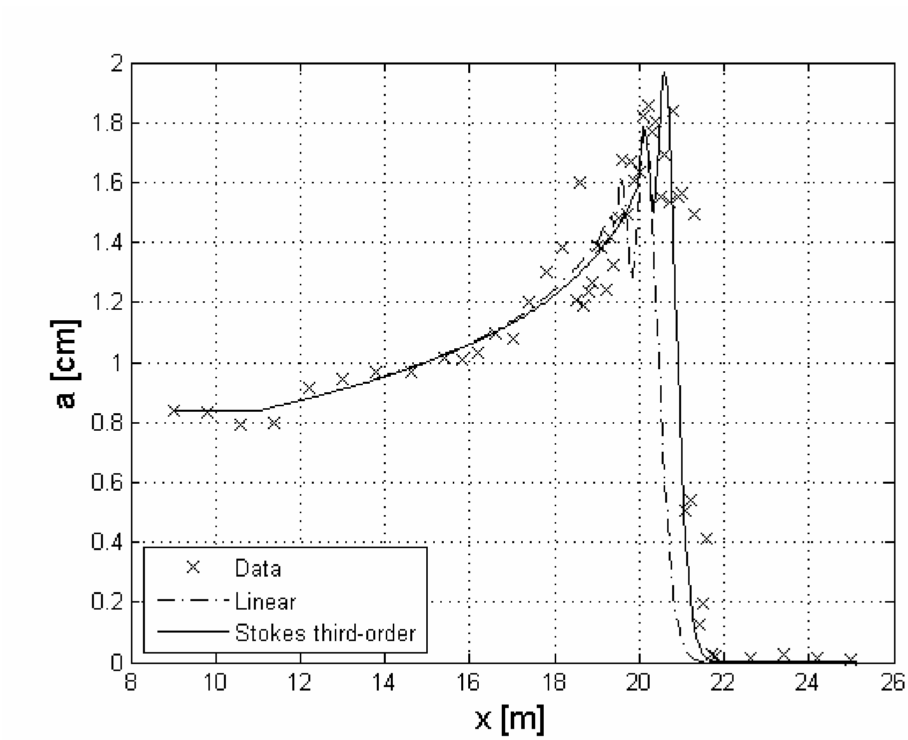

Fig. 3. Comparison between observed and modeled wave amplitudes, utilizing linear and Stokes third-order dispersion relations in the model.

CONCLUSIONS

Water waves meeting an adverse current and in situation of wave blocking tend to be very steep and may break on the current. In such a situation, nonlinear effects are expected to play an important role. In the present study, nonlinear dispersion effects are investigated in modeling of the wave amplitude evolution in the case of blocking of periodic waves. Model results are compared with experimental data. It is found that, the observed blocking-point position lies about one wave length further upstream than the modeled one when using the linear dispersion relation. Using the Stokes third-order dispersion relation, the modeled blocking-point position shifts about one wave length further upstream, resulting in a better fit between the observed and modeled amplitude variations along the flume as compared to the use of the linear dispersion relation.

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