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## **Investigations on the Establishment of Uniform Flow in Compound Channel Flumes**

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# INVESTIGATIONS ON THE ESTABLISHMENT OF UNIFORM FLOW IN COMPOUND CHANNEL FLUMES

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## ABSTRACT

There has been recently increasing concern about the uniformity of flow at measuring sections in straight compound channel experiments. Experiments have been usually carried out with the same total head at the inlet of the main channel and floodplain and as a result the velocities entering into the main channel and onto the floodplain are also the same. As flow enters onto the floodplain, the floodplain discharge exceeds the discharge for a uniform flow condition, hence mass transfer towards the main channel progressively occurs along the flume until flow becomes uniform for both main channel and floodplain.

This paper investigates the influence of relative depth on the length required for a uniform flow condition to be achieved in compound channel flumes. The quasi one-dimensional model, the Independent Sub-Sections Method (ISM), the two-dimensional model Telemac 2D and a three-dimensional numerical model have been used to simulate the experiments conducted at LMFA (France) and at UCL (Belgium). The ISM has been subsequently used to investigate more upstream conditions at the LMFA and their resulting flow developments in relation to the flume length. The results show that as the relative depth increases, the length required for uniform flow condition increases and can even exceed the actual length of some of the experimental flumes studied in literature.

*Keywords:* uniform flow, flow development, straight compound channel and numerical modelling

## 1. INTRODUCTION

Experiments on flow in compound channels have been usually carried out with the same total head at the inlet of the main channel and floodplain. As a result the velocities entering into the main channel and onto the floodplain are also the same (Figure 1) or, in practice, almost identical. As flow enters onto the floodplain, the floodplain discharge exceeds the discharge for a uniform flow condition, hence mass transfer towards the main channel progressively occurs along the flume until flow becomes uniform for both main channel and floodplain. The distance  $Lu$  for such mass transfer to decay and the flow distribution along the channel to remain stable can be significant.

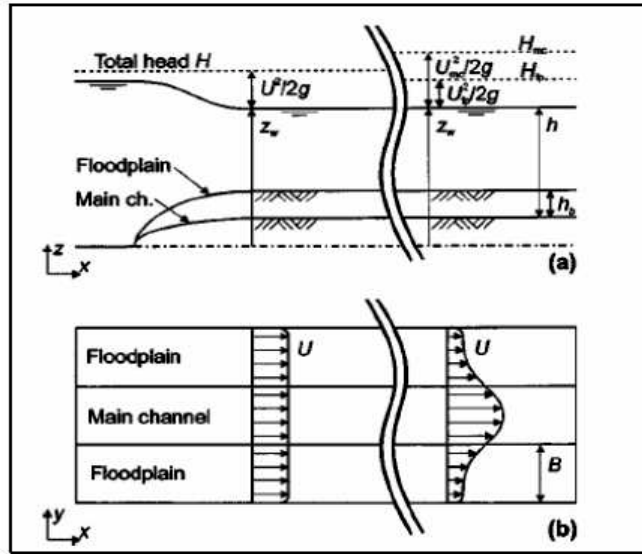


Figure 1: Water surface, head and velocity profiles in a compound channel flume with classical inlet, near inlet and at distance  $d/s$ : a) side view b) plan view (Bousmar et al. 2005)

In order to investigate this issue further and in more details, numerical simulations have been carried out. The results are presented in this paper.

## 2. METHODOLOGY

The experiments performed at the Catholic University of Louvain (UCL, in Belgium) and at the Laboratory of Acoustic and Fluid Mechanics (LMFA, France) in straight compound channels presented in Bousmar et al. (2005) were modelled numerically. The quasi one-dimensional model, the Independent Sub-Sections Method (ISM) (Proust et al., 2006), the two-dimensional commercial software Telemac 2D (Hervouet, 2007) and a three-dimensional numerical model (Vyas, 2007) have been used to simulate the experiments. The first objective of the numerical modelling was to obtain base models calibrated by the experimental results.

The characteristics of the flumes used for the experiments are presented in table 1.

Table 1: Characteristics of the LMFA and UCL flume

Flume	LMFA	UCL
Type	Asymmetrical	Symmetrical
Length (m)	8	10
Total width (m)	1.2	1.2
Slope	$1.8 \cdot 10^{-3}$	$0.99 \cdot 10^{-3}$
Floodplain width (m)	0.8	0.4
Bankfull depth (m)	0.051	0.050
Roughness	Smooth	Smooth

Experiments in UCL were staged in order to analyse the evolution of flow distribution over a longer distance than the physical flume length. This was achieved by using variable opening

screens that enabled upstream control of each subsection supply. The first experiment was carried out without controlling flow at the inlet. Flow distribution was measured at the outlet. This was subsequently set at the inlet by upstream flow control, so that the experiment could be carried over one more flume length. This process was repeated until a uniform flow distribution could be achieved.

In LMFA, separate inlets were used so that the upstream floodplain and main channel flows could be supplied separately at the inlet of the flume. In order to investigate the impact of an unbalanced upstream flow distribution, the upstream floodplain flow was increased compared to its uniform flow value for a given total flow.

The flow conditions investigated are presented in Table 2. Telemac 2D was used for the simulation of the experiments in UCL while the ISM and the three-dimensional model were used to simulate all the experiments presented in Table 2.

Table 2: Flow conditions investigated at LMFA and UCL flume

	<b>Total flow (L/s)</b>	<b>Percentage increase in upstream floodplain flow</b>	<b>Water depth (mm)</b>	<b>Relative depth ratio</b>
<b>LMFA</b>	17.3	56%	62	0.09
	24.7	53%	72	0.18
	24.7	38%	72	0.18
	36.3	32%	85	0.27
<b>UCL</b>	10.0	67%	61.1	0.18
	14.0	48%	68.6	0.29
	24.0	32%	85.3	0.40

Uniform flow was considered as achieved when the relative difference between the computed flow distribution and the experimental uniform flow distribution presented discrepancies lower than 1%. A brief description of the different numerical models used in this study is presented below.

### The Independent Subsections Method

The Independent Subsections Method (ISM), solves a system of ordinary differential equations (ODE) composed of three momentum equations and one equation of mass conservation. This system of ODE calculates the water level and the mean velocity in the main channel and on the floodplain (Proust et al., 2006). The ISM models the shear stress at the interface and the momentum exchange explicitly. The subscripts *mc*, *lfp* and *rfp* are used to describe the mean value of hydraulic parameters and refer to the main channel, the left and right floodplain respectively.

The one-dimensional momentum equations for the sub-sections are expressed as:

$$\left(1 - \frac{U_{lfp}^2}{gh_{lfp}}\right) \frac{\partial h_{lfp}}{\partial x} = S_0 - S_{f,lfp} + \frac{U_{lfp}^2}{gB_{lfp}} \frac{dB_{lfp}}{dx} + \frac{\tau_{lfp} h_{lfp}}{\rho g A_{lfp}} + q_{lfp} \frac{(2U_{lfp} - U_{int,lfp})}{gA_{lfp}} \quad (1)$$

$$\left(1 - \frac{U_{rfp}^2}{gh_{rfp}}\right) \frac{\partial h_{rfp}}{\partial x} = S_0 - S_{f,rfp} + \frac{U_{rfp}^2}{gB_{rfp}} \frac{dB_{rfp}}{dx} + \frac{\tau_{rfp} h_{rfp}}{\rho g A_{rfp}} + q_{rfp} \frac{(2U_{rfp} - U_{int,rfp})}{gA_{rfp}} \quad (2)$$

$$\left(1 - \frac{U_{mc}^2}{gh_{mc}}\right) \frac{\partial h_{mc}}{\partial x} = S_0 - S_{fmc} + \frac{U_{mc}^2}{gB_{mc}} \frac{dB_{mc}}{dx} - \frac{(\tau_{lfp} h_{lfp} + \tau_{rfp} h_{rfp})}{\rho g A_{mc}} - \frac{q_{lfm} (2U_{mc} - U_{int,lfp})}{gA_{mc}} \dots \quad (3)$$

$$\dots - \frac{q_{rfm} (2U_{mc} - U_{int,rfp})}{gA_{mc}}$$

Where  $S_0$  and  $S_{f,i}$  are the bed slope and the friction slope respectively,  $A_i$  and  $B_i$  refer to the wetted area and the width respectively,  $Q_i$  and  $U_i$  are the flow and the velocity in each subsection, with  $i=mc, lfp$  or  $rfp$ . The friction slope  $S_{f,i}$  is calculated using the Manning's formula. The lateral mass discharge between the right floodplain (resp. the left floodplain) and the main channel are expressed as  $q_{lfm}$  and  $q_{rfm}$  (algebraic values).

Mass conservation over the total cross-section is expressed as:

$$\frac{\partial Q_{mc}}{\partial x} + \frac{\partial Q_{l.fp}}{\partial x} + \frac{\partial Q_{r.fp}}{\partial x} = 0 \quad (4)$$

$\tau_i$  and  $U_{i,int}$  are the shear stress and the velocity at the interface respectively.  $\tau_i$  is modelled following the mixing length concept in a horizontal plane. The velocity at the interface between subsections  $U_{i,int}$  is modelled with empirical formulae calibrated in both prismatic and non-prismatic geometries, that account for the direction of mass transfer between subsections (Proust et al., 2006).

## Telemac 2D

TELEMAC 2D, a finite-element-based model of free surface flow solving the 2-D Saint-Venant equations was used in this study. The model was developed by the National Hydraulics Laboratory of Electricité de France (EDF) and has been successfully applied to engineering research and practice.

In Telemac 2D, the mass conservation is expressed as (5):

$$\frac{\partial HU_d}{\partial x} + \frac{\partial HV_d}{\partial y} = 0 \quad (5)$$

Where  $H$  is the water depth and  $U_d$  and  $V_d$  are the longitudinal and lateral depth-averaged velocities respectively. The momentum equations are expressed as (6) and (7):

$$\frac{\partial HU_d U_d}{\partial x} + \frac{\partial HU_d V_d}{\partial y} = -Hg \frac{\partial z}{\partial x} + HF_x + \text{div}(H\mu_e \overrightarrow{\text{grad}}(U_d)) \quad (6)$$

$$\frac{\partial HU_d V_d}{\partial x} + \frac{\partial HV_d V_d}{\partial y} = -Hg \frac{\partial z}{\partial x} + HF_y + \text{div}(H\mu_e \overrightarrow{\text{grad}}(V_d)) \quad (7)$$

Where  $\mu_e$  is the depth-averaged eddy viscosity and  $F_x$  and  $F_y$  are the forces per unit volume in the x and y directions. An Elder formulation of the turbulence was chosen for the simulations.

### Three-dimensional numerical model

The finite volume three dimensional model used in this analysis was developed by Vyas (Vyas, K., 2007). The code is based on Lilek, Z. (1995) and is using a collocated finite volume discretization. The code solves the Reynolds Averaged Navier-Stokes equations coupled with the continuity equation. The pressure-velocity coupling is achieved using the SIMPLE algorithm of Patankar and Spalding (1972). The governing equations were solved with the linear  $k-\varepsilon$  model of Rodi (1993) to calculate the Reynolds stresses and thus simulate the turbulent flows.

Mass conservation is expressed as:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \quad (8)$$

The momentum equations are expressed as:

$$\begin{aligned} U \frac{\partial \rho U}{\partial x} + U \frac{\partial \rho V}{\partial y} + U \frac{\partial \rho W}{\partial z} &= -\frac{\partial P}{\partial x} + \mu_T \frac{\partial^2 U}{\partial x^2} + \rho g_x \\ V \frac{\partial \rho U}{\partial x} + V \frac{\partial \rho V}{\partial y} + V \frac{\partial \rho W}{\partial z} &= -\frac{\partial P}{\partial y} + \mu_T \frac{\partial^2 V}{\partial x^2} - \frac{\partial \overline{v v}}{\partial y} - \frac{\partial \overline{u v}}{\partial z} + \rho g_y \\ W \frac{\partial \rho U}{\partial x} + W \frac{\partial \rho V}{\partial y} + W \frac{\partial \rho W}{\partial z} &= -\frac{\partial P}{\partial z} + \mu_T \frac{\partial^2 W}{\partial x^2} - \frac{\partial \overline{v w}}{\partial y} - \frac{\partial \overline{w w}}{\partial z} + \rho g_z \end{aligned} \quad (9)$$

Where  $U$ ,  $V$ ,  $W$  and  $u$ ,  $v$ ,  $w$  are the mean and fluctuating velocities in the longitudinal, lateral and vertical directions,  $\mu_T$  is the turbulent or eddy viscosity,  $\rho$  is the fluid density,  $P$  is the mean static pressure,  $g_x$ ,  $g_y$  and  $g_z$  are the components of the gravitational acceleration in the  $x$ ,  $y$  and  $z$  directions. The two transport equations for turbulent kinetic energy  $k$  and kinetic energy dissipation  $\varepsilon$  given by Rodi (1993) were used to calculate the distribution of turbulent eddy viscosity  $\mu_T$ . The model constants ( $C_{\varepsilon 1}$ ,  $C_{\varepsilon 2}$ ,  $\sigma_k$ ,  $\sigma_\varepsilon$  and  $C_\mu$ ) proposed by Rodi were adopted without modifications. The law of the wall function as proposed by Launder and Spalding (1974) was used at the channel boundary. The model operated with a rigid lid approximation. The turbulent energy dissipation rate at the free surface was following the expression of Naot and Rodi (1982). The model is capable of predicting deviation from the imposed free surface and changes in water depths are derived from the computed pressure field. This model has been validated against various experimental results (Vyas, K., 2007) in straight open compound channels.

While the linear  $k-\varepsilon$  model is not capable of accurately predicting the normal Reynolds stresses and cannot reproduce the secondary currents generated by the anisotropy of turbulence (Speziale, 1987), it has been regarded as sufficient for the objectives of the current analysis. Indeed, one of the primary objectives of this work is to quantify the length necessary to obtain a constant flow distribution along the flume. The focus is on the flow distribution along the flume and the water profiles. Rameshwaran (2003) highlighted that a three-dimensional model with a linear  $k-\varepsilon$  model could reproduce reasonably well the flow field yet noticing that discrepancies existed at the interface between the main channel and the

floodplain. It is however reasonable to tackle the issue of mass transfer in flow development with the linear  $k-\epsilon$  model as a first step. Non-isotropic turbulence models will be used to investigate a length of uniform flow in future research.

### 3. NUMERICAL RESULTS AND DISCUSSIONS

The computed evolution of floodplain discharge (as percentage of total discharge  $Q$ ) for the LMFA and the UCL flumes are presented in Figures 2 and 3 respectively.

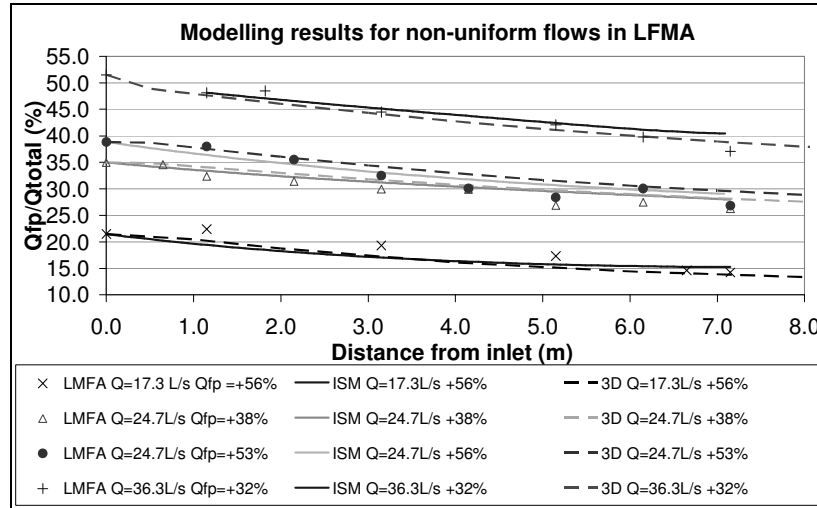


Figure 2: Computed evolution of discharge distribution in the floodplain in LMFA

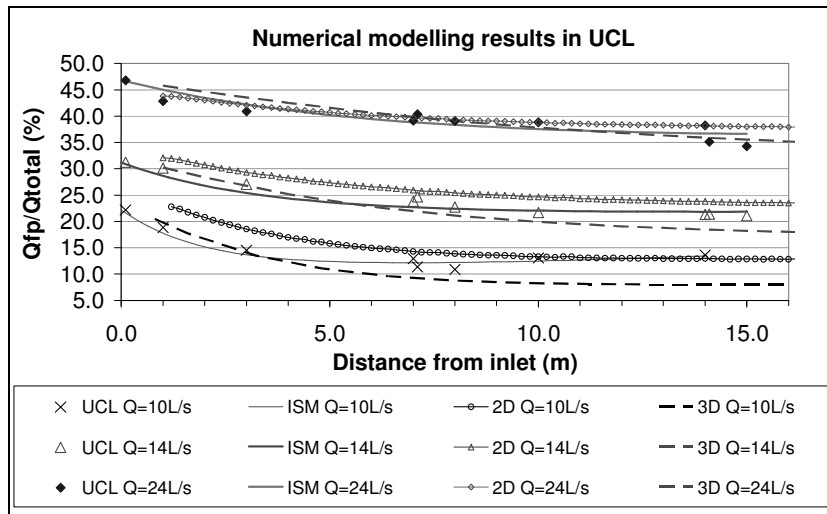


Figure 3: Computed evolution of discharge distribution in the floodplain in UCL

The computed water profiles for the LMFA and the UCL flumes are presented in Figures 4 and 5 respectively.

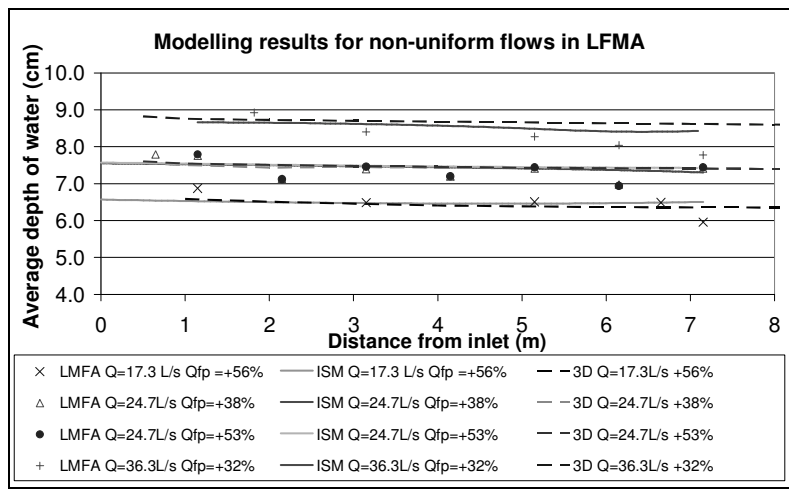


Figure 4: Computed water profiles compared to the experiments at LMFA

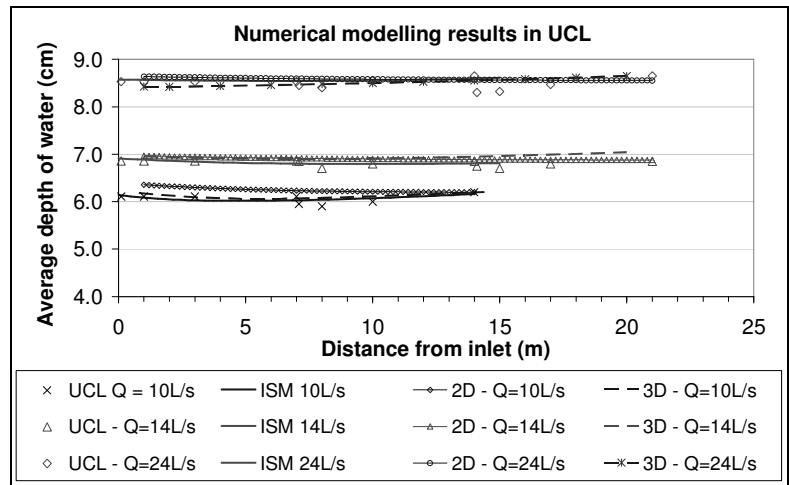


Figure 5: Computed water profiles compared to the experiments at UCL

During the experiments, a uniform flow distribution was achieved in UCL within the physical length of the flume for  $Q=10\text{L/s}$ . The upstream flow control enabled the determination of a uniform flow establishment length  $L_u$  for  $Q=14\text{L/s}$  and  $Q=24\text{L/s}$  as the downstream flow of the first experiment was used as an upstream condition in second and third experiments. This clearly illustrates the impact of a unique upstream reservoir on the establishment of a uniform flow. In LMFA, where separate inlets were used to increase the floodplain flow compared to its uniform flow value, mass transfer was observed over the whole length of the flume.

The results obtained from the ISM and the three-dimensional model were in good agreement with the experimental results at LMFA where flow distributions within a relative error of 5% were obtained for all simulations. The ISM, the two and three dimensional models also computed flow distribution within 5% of the experimental results for the higher discharge  $Q=24\text{L/s}$  in the UCL flume. However, while the ISM computed flow distributions close to the experimental results for  $Q=10\text{L/s}$  and  $Q=14\text{L/s}$ , the three-dimensional model computed a lower floodplain flow distribution for both results, with relative errors of 40% and 17% respectively.

Mass transfer decreasing from the floodplain to the main channel are observed for all the numerical simulations for the given upstream conditions as was observed during the



experiments.

It is interesting to note that the ISM and the three-dimensional model computed similar water depth profiles with small variations (typically less than 2 mm) in the water depth. These small variations imply that mass transfer can occur with very small changes in the water depth. Such changes are notably difficult to measure in large flumes, particularly during experiments for which the water surface is not perfectly smooth.

As observed experimentally, the computed flows in the flume require some distance to reach their uniform flow distributions. Depending on the stringency of the criteria used to define a uniform flow and the level of destabilisation of the upstream flow distribution, the flumes in both LMFA and UCL can appear to be too short for a uniform flow to be reached when using a unique reservoir. However, when using separate inlets in the LMFA, a uniform flow distribution within the floodplain and the main channel is reached within the first two meters of the flume.

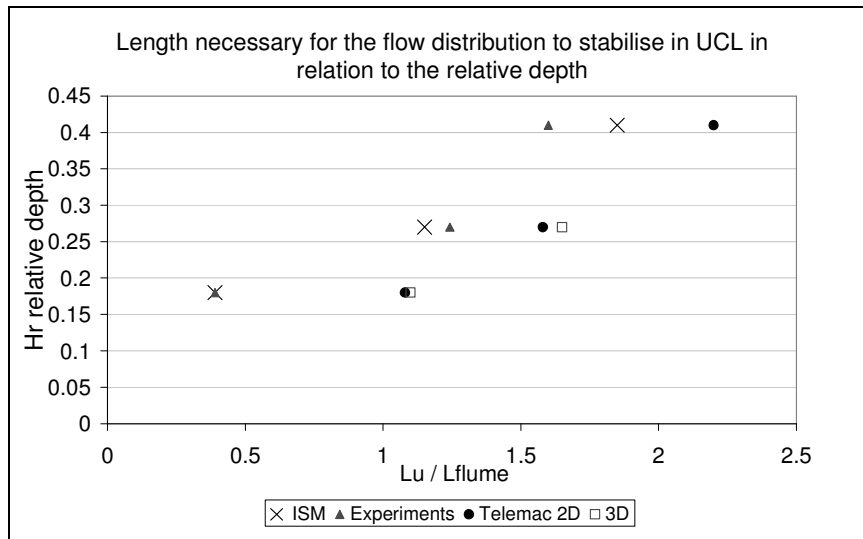


Figure 6: Length necessary in UCL flume for the flow distribution to stabilise in function of the relative depth Hr

Figure 6 also shows that the length required for the flow distribution to become stable increases with the relative depth ratio Hr. The experimental results suggest that this length is about 3.9m for Hr=0.18, 12.4m for Hr=0.29 and 16.0m for Hr=0.40 in the UCL flume. The ISM and the three-dimensional numerical models both reveal similar trends. For the three dimensional model, the flow distribution could not meet the criteria of flow uniformity defined above for the highest relative depth ratio within the modelled flume length of 20m.

In order to characterise the length required for the mass transfer along the channel to have fully stabilised, additional simulations were carried out with different floodplain flows given upstream, corresponding to different levels of destabilisation for Q=24.7L/s at LMFA. The results are presented in Figure 7. It must be noted that the ISM numerical simulations for which floodplain flows are given upstream with a value below that of a uniform floodplain flow do not have corresponding experiments.

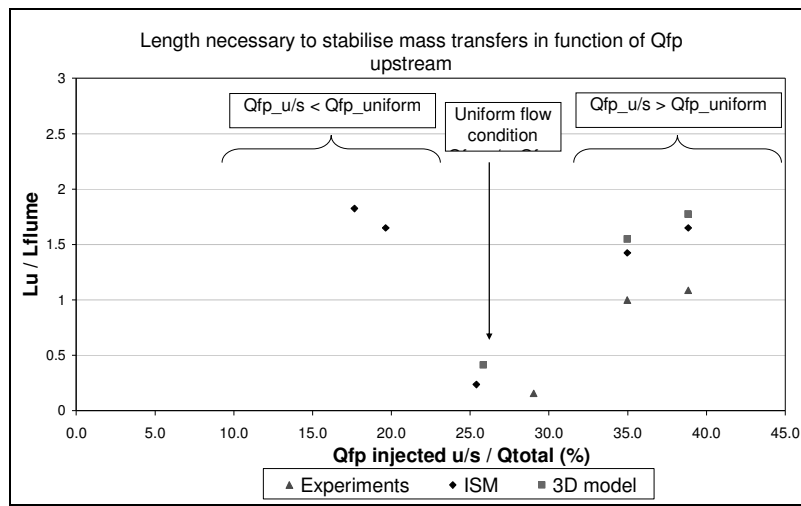


Figure 7: Length necessary in LMFA flume for the flow distribution to stabilise for  $Q=24.7\text{L/s}$  in function of the floodplain flow  $Q_{fp}$  injected upstream

Figure 7 highlights how significant the upstream conditions can be to stabilise the flow distribution in the flume. The use of separate inlets drastically shortens the distance  $L_u$  as the uniform flow distribution can be directly given upstream. On the other hand,  $L_u$  increases drastically if the upstream floodplain flow differs significantly from its uniform flow value. The flume appears to be too short to meet the uniform flow criteria if the upstream floodplain inflow is more than 35% of its uniform flow value. In the case of the example presented above, the uniform flow distribution is achieved within the first few meters of the flume (1.15m) if separate inlets are used. The LMFA experimental flow  $Q=24.7\text{L/s}$  for which  $Q_{fp}$  has been increased to 53% with the separate inlets has an upstream flow distribution similar to that of a unique reservoir situation. The upstream floodplain and main channel velocities are close to each other ( $U_{mc}=0.49\text{m/s}$  and  $U_{fp}=0.43\text{m/s}$ ). For this experiment,  $L_u$  is equal to 8.5m as opposed to 1.15m when separate inlets are used.

#### 4. CONCLUSION

Numerical modelling was carried out using the Independent Sub-sections Method (ISM), the two-dimensional commercial software Telemac 2D and a three-dimensional model in order to investigate the establishment of uniform flow in experimental flumes. The experiments carried out at LMFA and UCL flumes were simulated by the three different numerical models. The ISM computed flow distributions along the channel which were within 5% of relative error with all experimental flow distributions. The three-dimensional model also computed flow distributions within 5% of all experimental flow distributions at the LMFA. Telemac 2D, only used to model the UCL experiments, simulated flow distribution similar to the experimental flow distribution, the largest discrepancy being 9% for  $Q=14\text{L/s}$ . The most significant discrepancy in flow distribution was obtained between the three-dimensional model and the flow  $Q=10\text{L/s}$  at UCL (40%).

Clearly, flow depth measurements alone are not enough to verify the uniformity of a flow. As was demonstrated, excess flow from the floodplain can be transferred towards the main channel with water depths quasi identical to that of uniform flow conditions. The use of separate inlet conditions as an upstream control has proved to significantly shorten the length  $L_u$  required for no mass transfer. This type of upstream control enables the establishment of a

uniform flow within the first few meters of a flume. If an upstream flow distribution is too different from the uniform flow distribution,  $Lu$  might exceed the physical length of a flume. The length required for the flow distribution along the channel to become stable increases with the relative depth ratio  $Hr$ . The experimental results suggest that this length is about 3.9m for  $Hr=0.18$ , 12.4m for  $Hr=0.29$  and 16.0m for  $Hr=0.40$  in the UCL flume. The ISM and the three-dimensional numerical models both confirm the same trends.

The hypothetical case of an upstream floodplain flow given below its uniform flow value was also simulated. Mass transfer from the main channel to the floodplain was observed and some length comparable to the length observed for increased floodplain flows was also necessary to achieve a uniform flow distribution.

## ACKNOWLEDGMENTS

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## REFERENCES

- Bousmar D., Rivière N., Proust S., Paquier A., Morel R. and Zech Y. (2005), Upstream discharge distribution in compound-channel flumes, *Journal of Hydraulic Engineering*, 131(5), pp408-412.
- Hervouet JM. (2007), Hydrodynamics of Free Surface Flows - Modelling with the Finite Element Method, edited by *John Wiley & Sons*.
- Lauder, B.E. and Spalding, D.B. (1974). The Numerical computation of turbulent flows. *Computer Methods in Applied Mechanics and Engineering*, 3, 269–289.
- Lilek Z., Mzaferij S. and Peric M. (1995). Efficiency and accuracy aspects of a full-multigrid SIMPLE algorithm for three-dimensional flows. *Numerical Heat Transfer, Part B*, 31, 23–42.
- Naot D. and Rodi W. (1982). Calculation of secondary currents in channel flow. *Journal of Hydraulic Division, ASCE*, 111, 116–125.
- Patankar S.V. and Spalding D.B. (1972). A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. *International Journal of Heat and Mass Transfer*, 15, 1787–1806.
- Proust S., Bousmar D., Rivière N., Paquier A, and Zech Y. (2006) A methodology for computing non-uniform flows in compound channels." Proceedings of *River Flow 2006 conference*, pp405-413.
- Proust S. (2005). Ecoulements non-uniformes en lits composés : effets de variations de largeur du lit majeur," *PhD thesis*, INSA de Lyon, n°2005-ISAL-0083, 362 p, (in French). [www.lyon.cemagref.fr/doc/these/proust/index.shtml](http://www.lyon.cemagref.fr/doc/these/proust/index.shtml).
- Rodi W. (1993), Turbulence models and their application in hydraulics: a state-of-the-art review, *IAHR Monograph*.
- Speziale C.G. (1987). "On Nonlinear  $k - l$  and  $k - \epsilon$ " models of turbulence. *Journal of Fluid Mechanics*, 178, 459–475.
- Vyas K. (2007) "Multigrid Modelling of 3D Anisotropic Turbulent Flow in Compound Meandering Channels", *PhD. Thesis*, Civil and Building Engineering Department of Loughborough University.