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TWO-PHASE DEPTH-INTEGRATED MODEL FOR UNSTEADY RIVER FLOW

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ABSTRACT

Relevant geomorphic changes in water bodies often occur in relatively short time intervals. For instance, river bed evolution is mainly driven by the few flood events occurring in a season, during which most of the annual sediment volume is eroded somewhere and deposited elsewhere. Moreover, some catastrophic events like dam-break or levee breaching evolve as fast transients both for water and sediment transport, usually with strong interaction between flow and bottom evolution. Under the above circumstances, the classical textbooks approach for solid transport modelling, which invokes the immediate adaptation of solid transport to hydrodynamics and adopts therefore equilibrium (i.e. uniform flow) formulas, may furnish unsatisfactory (and unrealistic) results. A fully unsteady transport model with strong coupling between water and sediment is then required.

Mainly in the last two decades several models matching the above requirements have been presented and reported in the literature, making use of additional differential equations to represent sediment dynamics.

Within the above framework, a novel fully conservative model for sediment transport and bed evolution in river flow has been derived by a two-phase approach, and it is illustrated in the present paper. Referring to a two-dimensional depth-averaged framework, two scalar mass conservation equations and two vectorial momentum conservation equations are written for water and sediment separately, and coupled with bottom evolution equation due to erosion/deposition. Sediment inertia, momentum exchange between phases by drag forces and tangential stresses opposing motion are taken into account in a fully coupled formulation for solid transport under non-stationary conditions.

Keywords: numerical modelling, river hydraulics, sediment transport.

1. INTRODUCTION

Prediction of river evolution in alluvial channels by numerical models requires the estimation of bed load transport. According to classical textbooks' approach, solid discharge is assumed to depend on flow variables as in corresponding uniform flows. Yet for non-uniform and unsteady flows the instantaneous adaptation of sediment transport to hydrodynamics is therefore postulated. However such hypotheses may lead to unrealistic results, especially when dealing with fast transients.

In past literature several models overcoming these limitations (Nakagawa & Tsujimoto, 1980, Armanini & Di Silvio, 1988, Phillips & Sutherland, 1989, Di Cristo et. Al., 2002, Fraccarollo & Capart, 2002) have been presented. A common feature for most of them is the introduction of additional equations which aim to simulate the sediment adaptation to flow conditions. Armanini and Di Silvio (1988) proposed a conceptual model built up introducing multiple adaptation lengths. More recently, models based either on sediment dynamics descritpion (Di Cristo et al., 2002), or on two-layer schematisation (Fraccarollo & Capart, 2002) have been proposed.

In the present work a two-phase model for unsteady river flows is presented. The hyperbolic nature of the model is demonstrated, independently on the flow condition. The above feature, which is thought to be relevant for numerical simulation, may be lost, at least in certain flow conditions, for models based on two-layer flow. (e.g. Savary and Zech, 2007; Greco et al., 2008b). At the end of the paper, results of numerical simulation of one- and two-dimensional dam-break are shown.

2. MATHEMATICAL MODEL

2.1 Modelling hypotheses

The flow is regarded as a two-phase (solid/liquid) mixture: in the lower part of the flow depth the two phases share the same space, so that in each point both water or solid particles simultaneously exist. Model equations reflect mass and momentum conservation principle for water and sediments separately. The following hypotheses are assumed:

1) solid transport occurs as bed-load with constant sediment volume concentration (C); interaction between phases is represented by momentum exchanged through the drag forces;
2) both phases exchange mass with the standing bed;
3) cross-sectional surface concentration is equal to the volume concentration.
4) hydrostatic pressure distribution is postulated for the water/sediment mixture, while Terzaghi's principle is invoked to evaluate the pressure acting on each phase.

2.2 Model equations

Control volume is a rectangular prism of base area *dx* x *dy* and height equal to the total flow depth *h*. Under classical shallow-water assumption, conservation of mass and momentum leads to:

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(Q_x+Qs_x)+\frac{\partial}{\partial y}(Q_y+Qs_y)+\frac{\partial Z}{\partial t}=0 \tag{1}$$

$$\frac{\partial \delta}{\partial t}+\frac{\partial Qs_x}{\partial x}+\frac{\partial Qs_y}{\partial y}+(1-p)\frac{\partial Z}{\partial t}=0 \tag{2}$$

$$\frac{\partial Q_x}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_x^{\,2}}{h-\delta}+g\frac{h^2}{2}\right)+\frac{\partial}{\partial y}\left(\frac{Q_xQ_y}{h-\delta}\right)+gh\left(\frac{\partial Z}{\partial x}+Sf_x\right)=0 \tag{3}$$

$$\frac{\partial Q_y}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q_xQ_y}{h-\delta}\right)+\frac{\partial}{\partial y}\left(\frac{Q_y^{\,2}}{h-\delta}+g\frac{h^2}{2}\right)+gh\left(\frac{\partial Z}{\partial y}+Sf_y\right)=0 \tag{4}$$

$$\frac{\partial Qs_x}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Qs_x^{\,2}}{\delta}+\frac{g\Delta}{C(\Delta+1)}\frac{\delta^2}{2}\right)+\frac{\partial}{\partial y}\left(\frac{Qs_xQs_y}{\delta}\right)+g\delta\frac{\Delta}{\Delta+1}\frac{\partial Z}{\partial x}+Ss_x=0 \tag{5}$$

$$\frac{\partial Qs_y}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Qs_xQs_y}{\delta}\right)+\frac{\partial}{\partial y}\left(\frac{Qs_y^{\ 2}}{\delta}+\frac{g\Delta}{C(\Delta+1)}\frac{\delta^2}{2}\right)+g\delta\frac{\Delta}{\Delta+1}\frac{\partial Z}{\partial y}+Ss_y=0 \tag{6}$$

$$\frac{\partial Z}{\partial t}=-e_b \tag{7}$$

in which t is the time, x and y the two spatial coordinates, h is the total flow depth, δ the sediment volume for unit base area, Q_x and Q_y (Qs_x and Qs_y) are the two components of liquid (solid) discharge along x and y and Z denotes the bottom elevation. p is bottom porosity, g the gravity and Δ+1 the ratio of sediment to water density. It is worth of note in the proposed model conservation of momentum of both water and sediments is enforced, while in the classical textbook approach the latter is not considered. Source terms for water (solid) momentum are denoted as Sf_x and Sf_y (Ss_x and Ss_y), while e_b is the bottom erosion/deposition velocity.

The source term for water is the total resistance force acting per unit weight $\vec{Sf}$. It is computed as the sum of the contribute of bottom friction (computed using a Chezy-like formula with constant friction factor) and of the drag exchanged between the two phases:

$$\vec{Sf}=\frac{\vec{U}_w\left|\vec{U}_w\right|}{ghCh^2}+\frac{C_D\delta}{ghd}\left(\eta\vec{U}_w-\vec{U}_p\right)\left|\eta\vec{U}_w-\vec{U}_p\right| \tag{8}$$

where d is the sediment diameter, $\vec{U}_w$ and $\vec{U}_p$ the average water and sediment velocities, and C_h is the non-dimensional Chezy coefficient. Finally, C_D denotes the drag coefficient, with form factors and constants lumped in. In evaluating drag term, a reduced local fluid velocity is considered trough the coefficient η<1.

The corresponding solid source term $\vec{Ss}$ accounts for drag exchange and the collisional shear stress computed, after Bagnold, as a coefficient a multiplied by the square of the particle velocity:

$$\vec{Ss}=\alpha\vec{U}_s\left|\vec{U}_s\right|+\frac{\Delta}{\Delta+1}\frac{C_D\delta}{ghd}\left(\eta\vec{U}_w-\vec{U}_p\right)\left|\eta\vec{U}_w-\vec{U}_p\right| \tag{9}$$

A closure relation for the entrainment/deposition term e_b is then required. A novel formulation for this term is proposed herein, which is based on the interpretation of the river bottom as a shock surface. Across the bottom, in fact, both phases experiment a jump in mass and momentum flux. A fluid-like stress model is assumed also for solid phase, so that the stress tensor reduces to a pressure and a tangential stress. For both liquid and solid phase local flow conditions are computed by Reynolds equations. Again, Terzaghi's principle is invoked to express the pressure on both phases below the bottom surface (at rest), while the tangential stress is assumed in the form of a Mohr-Coulomb law.

By the above hypotheses, Rankine-Hugoniot conditions for the local flow equations may be written. Such conditions express the congruence between the values of mass and momentum flux of each phase on both sides of the bed surface. The latter is assumed to move with velocity e_b. Similarly to Fraccarollo and Capart (2004), this allows to relate the velocity of the bottom surface to the shear stresses exerted on the two sides of the bottom surface. On the upper side of bottom surface, these are represented by water friction and solid collisional shear and Mohr-Coulomb friction, while on the lower one (augmented with the value needed to dislocate the first particle of the bed, as in Savary and Zech, 2007). The following relation is obtained for e_b:

$$e_b = -\frac{\dfrac{\vec{U}_w\left|\vec{U}_w\right|}{ghCh^2} + (\Delta+1)\alpha\left|\vec{U}_s\right|^2 - g\Delta\left[\delta\tan\varphi + 0.047\left(1-\dfrac{\tan\vartheta}{\tan\varphi}\right)\right]}{\eta\left|\vec{U}_w\right|p + (\Delta+1)\left|\vec{U}_s\right|(1-p)} \tag{10}$$

In (10), φ denotes the friction angle of bed material and ϑ is the local bottom slope. It is worth of remark that while Terzaghi's principle is a common assumption in Soil Mechanics for pressure repartition within the quiescent bed, its application to bed load transport constitutes is here simply postulated, being its verification well beyond the scope of present note. We just note that due to this assumption, no discontinuity in the phase pressure distribution exists across the bottom surface.

Finally, it has to be observed that pressure repartition mainly affects the expression of the normal surface forces in the dynamical equation of both phases, and in turn the model eigenstructure.

2.3 Eigenstructure analysis

Analysis of mathematical nature of the PDEs system is discussed herein, owing to its relevance from both theoretical and practical point of view. For hyperbolic-type models, the analysis of characteristic celerities allows to fully define the boundary conditions to define a well-posed problem.

For sake of simplicity, the 1-D version of the model is considered in the following. The resulting five-equation model has four real and non-null eigenvalues. Assuming the following peculiar definitions for Froude number, namely:

$$Fr = \frac{U_w}{\sqrt{gh}} \tag{11}$$

and

$$Fr_s = \frac{U_s}{\sqrt{g\dfrac{\delta}{C}\dfrac{\Delta}{\Delta+1}}} \tag{12}$$

the celerities of the system are :

$$\begin{aligned} c_{1,2} &= U_w\left(1 \pm \frac{1}{Fr}\right) \\ c_{3,4} &= U_s\left(1 \pm \frac{1}{Fr_s}\right) \end{aligned} \tag{13}$$

It is interesting to note that (11) defines an unusual group number given by the ratio between water average velocity and celerity corresponding to the total flow depth. The Froude number (12) on the other hand represents the ratio of solid-phase average velocity to the celerity of a mass of liquid of height δ/C and density ($\Delta/\Delta+1$) times smaller than water.

Despite model hyperbolicity would appear on intuitive basis as a obvious property for a model aimed to simulate processes with strictly time-marching propagation of information starting from initial condition, it has recently been observed that in the context of morphological models for fast transients, the above properties is not always matched. (Savary and Zech, 2007; Greco et al., 2008b)

3. NUMERICAL IMPLEMENTATION

Numerical solution of the equations deduced in the previous section is achieved trough the following numerical scheme. The numerical method is based on the finite-volume predictor-corrector explicit scheme proposed in Leopardi (2001), with second order accuracy both in space and time. Spurious oscillations are damped trough the addition, after each temporal predictor-corrector step, of a diffusive term, implemented according to Jameson et al. (1981).

A special treatment is applied to the slope terms appearing in water momentum equations. The bed slope source term is discretised according to the procedure outlined by Greco et al. (2008a), therefore including an original wet/dry correction.

4. SAMPLE APPLICATIONS

4.1 Dam-break over horizontal and stepped erodible bed

Propagation of dam-break induced waves over an erodible bed is characterised by fast and severe erosion both upstream and downstream of the original position of the dam, and by some special features without any corresponding counterpart in the fixed-bed case.

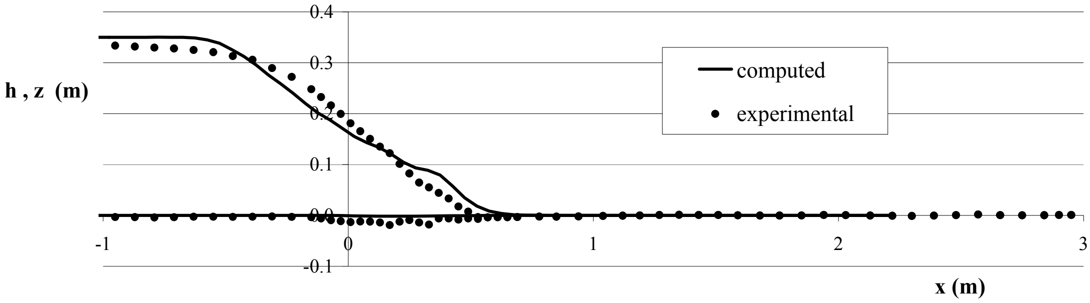
Spinewine & Zech (2007) have published a considerable set of experimental data on small scale dam break over erodible bed. Publication of these data in electronic format allows accurate comparisons with the results from numerical models.

Among the investigated experimental conditions, only the runs with sand particles in the configurations named (a) and (b) will be considered herein. The former configuration is characterised by flat erodible bed with 35 cm water height upstream the dam; the latter one presents a downward -facing bottom step of height 5 cm, and water height upstream the dam is 35 cm. The sand has the following mechanical properties: particle sizes ranging from 1.2 to 2.4 mm, with mean diameter d_{50} = 1.82 mm, density ρ_s = 2683 kg/m^3, friction angle φ=30° and negligible cohesion. Bottom solid packing concentration is c_b = 53%.

Tests (a) and (b) with sand were reproduced using the following values of model parameter values were assumed based on best-fitting of experimental data:

- α=0.25
- η = 0.7;
- Ch = 20;
- C_d = 0.03.

Figures 1 and 2 report the comparison between simulation and experiments test at different times, for the flat bed and for the upward step test, respectively. Figures 1 show that flat-bed configuration experiment is reasonably reproduced by the model. Wavefront celerity is significantly affected by erosion and deposition, with time-depending values. Bed scour and friction both result in a notable deceleration of the wavefront respect to the rigid bed case, as discussed in detail by Spinewine and Zech (2007). Wavefront positions for experiment (a) are plotted in figure 3: it is easy to observe that the computed wavefront is initially faster than the experimental one but after a little it becomes slightly slower.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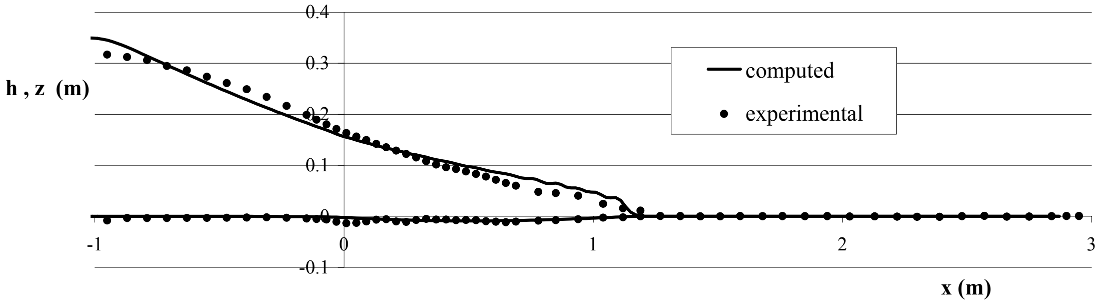
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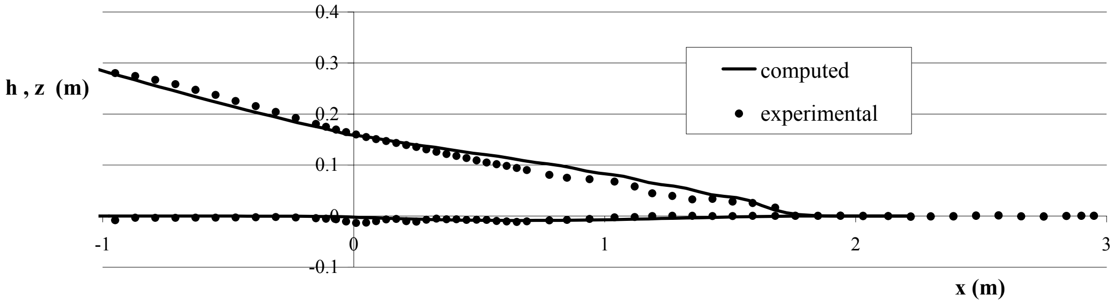
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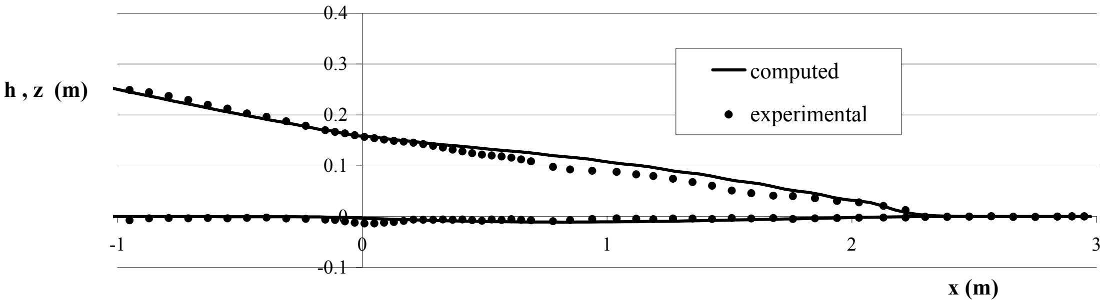
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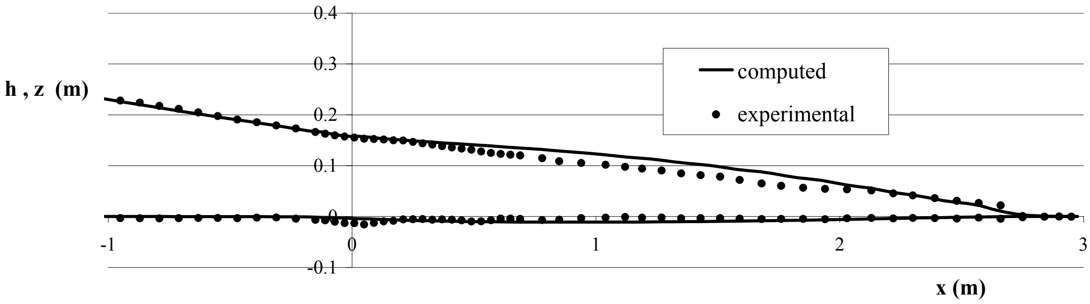
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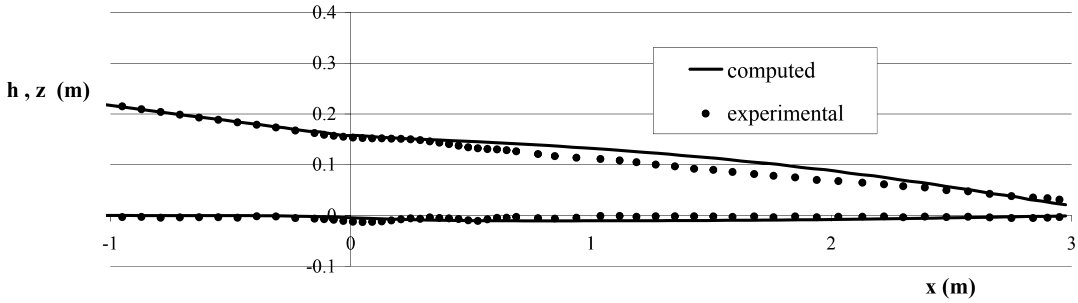
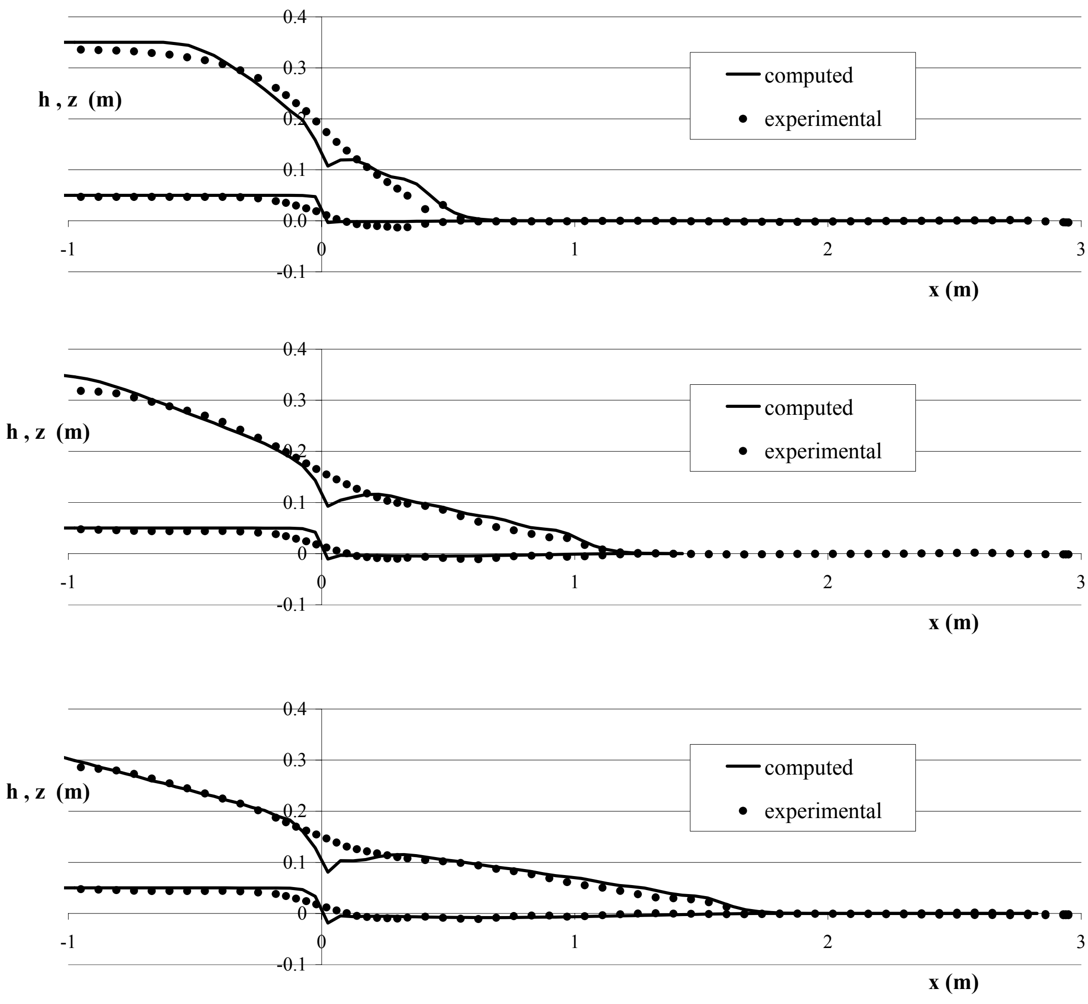


Figure 1 – Dam break over initially flat bed (t=0.25, 0.5, 0.75, 1, 1.25, 1.5 s)

Configuration (b) is also reproduced in a globally reasonable manner, despite simulated profiles significantly differ from experimental ones in the region close to the original dam position: indeed, erosion of the bottom step is less intense than observed. Nevertheless, while in the absence of any *ad-hoc* treatment aimed to represent the collapse of the step based on some friction angle evaluation, the initial slope gradually reduces with a qualitatively reasonable shape, due to the special expression assumed for the erosion term in (10) acting as a triggering mechanism for solid transport.



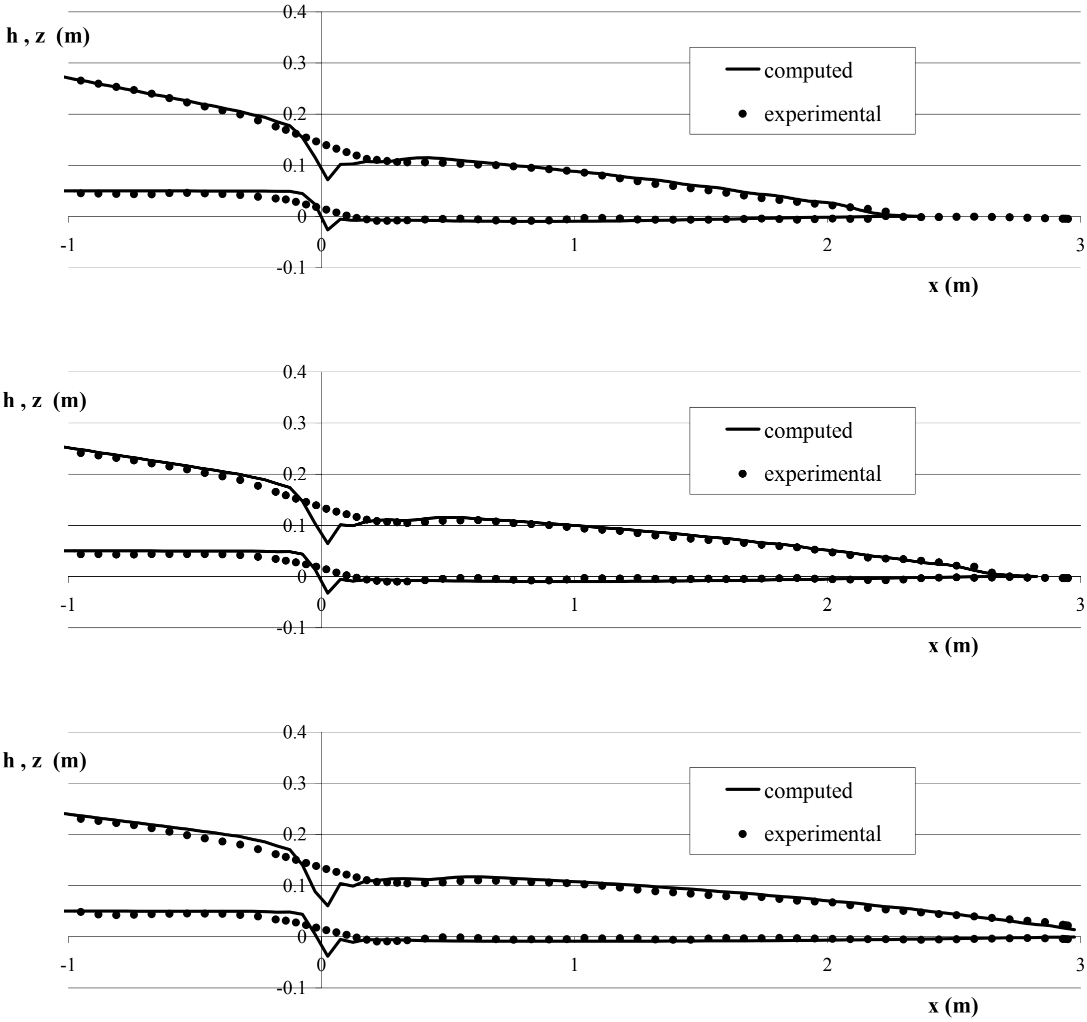


Figure 2 – Dam break over initially stepped bed (t=0.25, 0.5, 0.75, 1, 1.25, 1.5 s)

Also in simulating these tests the computed wavefront is initially slightly faster than the experimental one and it becomes slower after a little.

The Authors of the experiments observed that the classical pivot point of the shallow water theory, at which the flow is supposed to undergo a transition from subcritical to supercritical regime, is preserved in the experiments. It stands almost exactly at the predicted depth of 4/9 of the initial height, but is located slightly downstream of the initial gate position, at $x \approx 9$ cm. The above feature is reasonably reproduced in the numerical simulations

4.2 2-D Dam break

The scheme considered for two dimensional dam break over mobile bed is the one recently discussed by Spinewine (2005). The flow domain consists of a narrow channel followed by an idealized floodplain generated by the sudden increase of the width on one side. The same scheme was reproduced by the authors (Greco et al., 2006) using a previous version of the present model.

The same values of model parameter of the previous 1-dimensional examples have been used.:

Figure 3 reports some top view of bottom surface 1, 2, 3 and 5 s after dam removal. The model reproduces qualitatively the principal features of the phenomenon, with a maximum scour location lose to the floodplain enlargement and an oblique deposit just downstream.

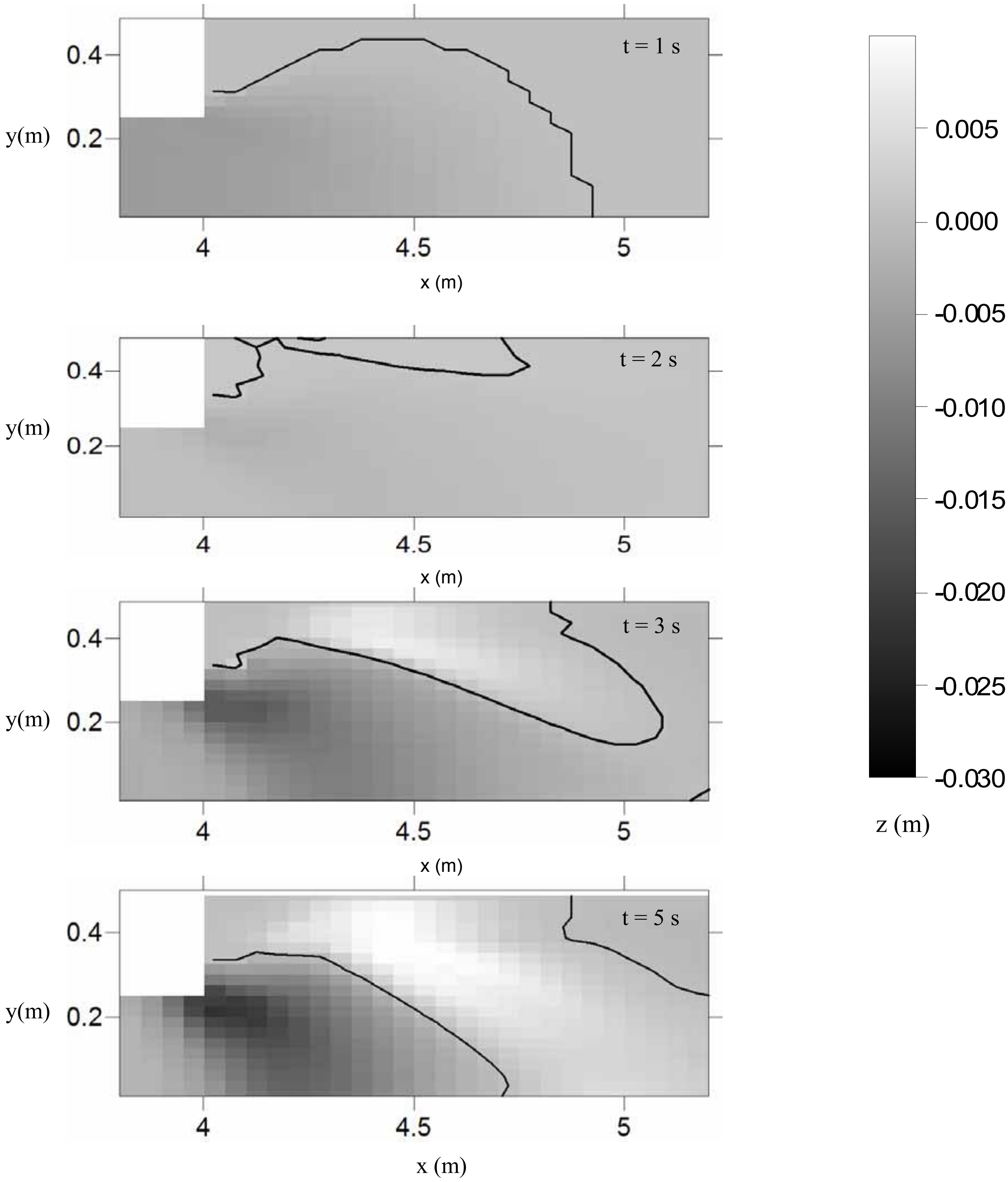


Figure 3. Top view of bottom surface close to the floodplain widening (t=1,2,3,5 s). The iso-contour relative to initial bed level is superposed as a solid line.

5 CONCLUSION

In present paper a two-phase morphodynamical model is presented. Model equations are presented in their two-dimensional dept-averaged formulation, based on mass and momentum conservation for each of the two phases. An argument based on Rankine-Hugoniot conditions is used to express the erosion/deposition rate e_b. The analysis of the characteristic celerities of the one-dimensional version of the model is performed, and the differential problem is shown to be unconditionately hyperbolic.

Preliminary application of the model is performed by simulation of some of the experiments reported in Spinewine & Zech (2007). Computed results reasonably agree with observed data. In particular, wavefront celerity is reproduced with an enough accuracy degree and the pivot point in water profiles, observed by the Authors, appears also in simulated results.

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