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An Asynchronous Solver for Differential Equations Arising from River Basin Models

AN ASYNCHRONOUS SOLVER FOR DIFFERENTIAL EQUATIONS ARISING FROM RIVER BASIN MODELS

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This presentation documents our development of a numerical solver for systems of ordinary differential equations (ODEs) determined by a tree structure. These types of systems arise in distributed hydrological models. Our main motivation is the application of this solver to hillslope-link river basin models for real-time flood forecasting over large basins (> 16,000 km^2). We therefore consider several aspects of this problem to develop an efficient numerical solver. Details of this work can be found in Small et al. (Submitted).

The topology of a river network can be described as a directed tree under the assumption that no loops are present in the network. We consider the hillslope-link decomposition of a river basin described by Mantilla and Gupta (2005). Equations for each link (i.e., river segment between two junctions) follow the general form described by Mantilla (2007) and Menabde and Sivapalan (2001). For each link, a differential equation can be written for water discharge by describing the link as a non-linear reservoir. A simple model is given by

$$\frac{dq}{dt}(i,t) = \frac{1}{\tau} q^{\lambda_1} \left(\sum_{j \to i} q(j,t) - q(i,t) + c_1 s_p^{5/3} \right) \tag{1}$$

$$\frac{ds_p}{dt}(i,t) = c_2 p(i,t) - c_3 s_p^{5/3} \tag{2}$$

Here the dynamics of the discharge $q(i,t)$ and depth of ponded water $s(i,t)$ on the hillslope are rep- resented. The function $p(i,t)$ is the rainfall rate at link i and the sum in equation (1) is taken over all upstream links. More detailed models that involve descriptions of soil dynamics, evaporation, and other dynamics may also be considered.

To develop an efficient numerical solver for this problem, several aspects of the problem must be addressed. For the integration, we apply dense output Runge-Kutta methods. These methods allow for approximate solutions to the ODEs at each link between time steps by means of polynomial interpolation. This interpolation allows for asynchronous integrators at each link in the basin, i.e. the use of different step sizes and different Runge-Kutta methods at each link.

The precipitation function $p(i,t)$, provided from Hydro-NEXRAD (see Krajewski et al. (2011)), is given as hourly rates in mm/hr and can be viewed as a step function. This introduces discontinuities into the system ODEs which should be tracked downstream through the basin to insure efficiency and accuracy of the numerical integrators. The introduction of dams to form reservoirs in the basin will also create further discontinuities. A discontinuity, whether from rainfall or reservoirs, will appear only in higher order derivatives as it is propagated downstream, and does not warrant tracking if it will not interfere with the error control for the integrators.

Although the order in which the ODEs at each link are solved is important, much room for par- allelization of the numerical solver exists. We apply a graph partitioning algorithm to distribute each link amongst different processes. With this distributed memory architecture, the

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ODEs correspond- ing to each link may be solved in parallel, greatly reducing runtime. The processes communicate asynchronously to improve efficiency.

With these considerations, we apply our implementation to the 16,000 km^2 Cedar River Basin located in Eastern Iowa. This basin can be decomposed into around 300,000 hillslope-link pairs determined from a 30 m digital elevation model. The system of ODEs with 20 days of rainfall data can be integrated in 5 minutes using our computing resources.

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