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Integrated Modelling of Beach Water Table Fluctuations

MODELLING BEACH WATER TABLE FLUCTUATIONS USING AN INTEGRATED GROUND-SURFACE WATER MODEL

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ABSTRACT

A two-layer integrated model is used to predict simultaneously free surface and groundwater flows, with the surface water layer being located on the top of the ground water layer. The depth integrated Reynolds equations are used to represent the surface water flows and the extended Darcy's equation is used to represent the groundwater flows. A hydrostatic pressure distribution is assumed to be applicable for both these two types of flows, thus the surface water elevation gradient can be use to drive the groundwater flows. The governing equations for the two types of flows are discretized in a similar manner and they are combined to give one set of tri-diagonal matrix equations. Two test cases are used to validate the model, with results showing that the model is capable of simulating the combined ground and surface water flows. In particular, the model is capable of predicting water table fluctuations near sloping beaches, where the water table in the aquifer sometimes decouples from the free water surface. Detailed analysis is undertaken to investigate the capability of the model for predicting the formation of seepage face and the location of the exit point.

Keywords: water table fluctuation, coastal aquifer, seepage face and exit point

1. INTRODUCTION

In natural coastal systems, the interface between a coastal aquifer and an oscillating clear water body is generally non-vertical. A prime example is a sloping beach face. Due to the existence of the slope, the periodic boundary condition can no longer be applied at a fixed x-coordinate, thus it becomes a moving boundary problem. The sloping boundary acts as a highly nonlinear filter and is responsible for the generation of higher harmonics in the water table oscillations (Horn, 2006; Cartwright et al, 2004). If the tide drops quickly, and the groundwater flow in the adjacent coastal aquifer does not respond in time, the water table in the aquifer decouples from the free water surface in the ocean, resulting in the formation of a seepage face on the sloping beach (Nielsen, 1990). An accurate prediction of the location of the intersecting point between the groundwater table and beach surface, i.e. the exit point, is important since it affects the behaviour of wave run-up on the beach and hence the sediment transport process (Horn, 2006; Cartwright et al., 2006). Thus, it is desirable for a numerical model to be capable to simulate the formation of the exit point and seepage face on a sloping beach. In simulating a moving boundary problem, it is generally difficult to conserve the mass and momentum if the status of a computational cell changes from dry to wet or vice versa. It is particularly challenging to deal with the mass conservation in a partly wet cell.

Details are given herein of the refinement and application of a coupled ground-surface water numerical model by Yuan et al (2008) to predict the water table fluctuations in an isotropic, homogeneous aquifer across a sloping beach, with the ground- and surface water flows being simultaneously modelled. The model has been applied to two test cases to simulate the combined ground- and surface water flows in coastal areas. The emphasis is placed on investigating the model accuracy in predicting the formation of the seepage face and the location of the exit point.

2. GOVERNING EQUATION AND SOLUTION METHODS

The depth-integrated continuity equation for unconfined groundwater flow in a Cartesian coordinate system can be written in the following form (Yuan et al, 2008):-

$$n_e^{\zeta}\frac{\partial \zeta^g}{\partial t}+\frac{\partial p^g}{\partial x}+\frac{\partial q^g}{\partial y}=q_m^g \tag{1}$$

where ζ^g = elevation of groundwater free, or upper, surface; n_e^{ζ} = porosity of the aquifer at the free surface; q_m^g = source or sink discharge per unit horizontal area; p^g and q^g = discharges per unit width in the x- and y-directions, respectively, defined as:-

$$p^g=\int_{-h_b}^{\zeta^g}(n_e u)\,dz\,,\; q^g=\int_{-h_b}^{\zeta^g}(n_e v)\,dz \tag{2}$$

where $-h_b$ = elevation of groundwater lower surface (or bottom of aquifer).

The Dupuit-Forchheimer assumption of hydrostatic pressure is often applied to study the tide-induced unconfined groundwater table fluctuations (Horn, 2006; Cartwright et al., 2004; Nielsen, 1990). Under this assumption, the governing equation for unconfined groundwater flows for an isotropic and homogeneous medium can be expressed as:-

$$p^g=-KH^g\frac{\partial \zeta^g}{\partial x} \tag{3}$$

$$q^g=-KH^g\frac{\partial \zeta^g}{\partial y} \tag{4}$$

where H^g = aquifer thickness, and K = hydraulic conductivity.

For many practical surface flow problems in coastal and estuarine waters it is appropriate to assume a hydrostatic pressure distribution in the vertical direction. Thus, a depth integrated two-dimensional equations can be used to describe the surface water flows. The continuity equation is given as:-

$$\frac{\partial \zeta^s}{\partial t}+\frac{\partial p^s}{\partial x}+\frac{\partial q^s}{\partial y}=q^s{}_m \tag{5}$$

The momentum equations in the x- and y-directions are given as:

$$\frac{\partial p^s}{\partial t}+\frac{\partial}{\partial x}\left(\beta U p^s\right)+\frac{\partial}{\partial y}\left(\beta p^s V\right)=fq^s-gH\frac{\partial \zeta^s}{\partial x}+\frac{\gamma\rho_a W_x W_s}{\rho}-\frac{gp^s\sqrt{\left(p^s\right)^2+\left(q^s\right)^2}}{H^2C^2}+E\left(\frac{\partial^2 p^s}{\partial x^2}+\frac{\partial^2 q^s}{\partial y^2}\right) \tag{6}$$

$$\frac{\partial q^s}{\partial t}+\frac{\partial}{\partial x}\left(\beta U q^s\right)+\frac{\partial}{\partial y}\left(\beta V q^s\right)=fp^s-gH\frac{\partial \zeta^s}{\partial y}+\frac{\gamma\rho_a W_y W_s}{\rho}-\frac{gq^s\sqrt{\left(p^s\right)^2+\left(q^2\right)^2}}{H^2C^2}+E\left(\frac{\partial^2 q^s}{\partial x^2}+\frac{\partial^2 q^s}{\partial y^2}\right) \tag{7}$$

where ζ^s = water surface elevation above datum, p^s and q^s = discharges per unit width in the x- and y-directions, respectively, defined as:-

$$p^s=\int_{-h}^{\zeta^s} u dz = \mathrm{UH}, \qquad q^s=\int_{-h}^{\zeta^s} v dz = VH \tag{8}$$

in which H = total water depth; β = momentum correction factor; h = water depth below datum; U and V = depth average velocity components in the x- and y-directions, respectively; f = Coriolis parameter given by $f=2\omega\sin\varphi$, with ω being the rotating speed of the earth and φ being the latitude at the studying point; g = acceleration due to gravitation, C = Chezy's bed roughness coefficient, E = depth mean eddy viscosity, γ = air-water resistance coefficient, ρ = water density, ρ_a = air density, W_x and W_y = wind velocity components in the x- and y-directions respectively, $W_s=\sqrt{W_x^2+W_y^2}$.

Equation (5) is a special case of Equation (1), with the porosity value being equated to unity. In this study Equations (3) and (4) are used as the equivalent of Equations (6) and (7) for groundwater flow.

In developing the linked groundwater and surface water flow model a hydrostatic pressure distribution is assumed to apply over the whole water depth and the horizontal pressure gradient is therefore proportional to the water elevation gradient. Therefore the gradient of the free surface water elevation can be directly used in Equations (3) and (4) as the driving force for the groundwater flow. The total water discharge is then defined as the sum of the surface water (above beach surface) and groundwater flow (below beach surface) discharges, giving:-

$$p=p^s+p^g \tag{9a}$$

$$q=q^s+q^g \tag{9b}$$

The shallow water surface flow equations (5)-(7) and the groundwater flow equations (1)-(3) are both discretized based the ADI method. The resulting equations are rearranged and integrated according to equations (9a) and (9b) to form a set of tri-diagonal matrix equations, which are then solved efficiently (see Yuan et al, 2008).

3. MODEL RESULTS

3.1 Tide-induced water table fluctuations

In this test case tide-induced water table fluctuations in a coastal aquifer with a sloping beach are studied (see Figure 1). Three scenarios have been simulated, corresponding to three different beach slopes, namely 0.05, 0.1 and 0.2. The following parameters are used in the numerical simulation: the tidal amplitude $A=1m$, the tidal cycle $T=3600s$, the effective porosity $n_e=0.3$ and the hydraulic conductivity $K=0.01ms^{-1}$. The space steps in both the x- and y-directions are set to 1m, and the time step is set to 0.5s.

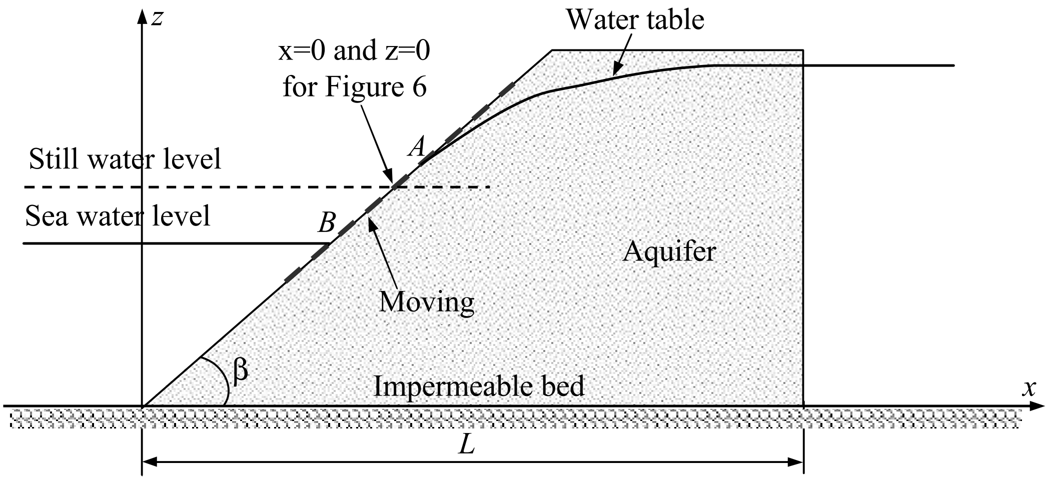

Figure 1 Sketch of tidal water table fluctuation in a coastal aquifer

Based on the linearised Boussinesq equation Li et al (2000) deduced a perturbation solution for predicting beach water table fluctuations, in which the effect of a moving boundary is taken into account. For monochromatic tidal waves, their solution can be expressed as follows:-

$$\zeta = Ae^{-kx}\cos(\omega t - kx) + \frac{A\varepsilon}{2}\left[1 + \sqrt{2}e^{-\sqrt{2}kx}\cos\left(2\omega t - \sqrt{2}kx + \frac{\pi}{4}\right)\right] \qquad (10a)$$

with

$$\varepsilon = Ak\cot(\beta) \qquad (10b)$$

$$k = \sqrt{\frac{n_e\omega}{2kh}} \qquad (10c)$$

where A = tide wave amplitude, ω = angular frequency of the tide, k = wave number and β = beach angle.

The numerical model predicted results and analytical solutions of the upper and lower limits of water levels by Li et al (2000) are shown in Figure 2. In this figure the origin of the coordinate is set as the point of intersection between the beach face and the still water surface (see Figure 1). It can be seen that the numerical results are in good agreement with the analytical solutions for cases (a) and (b) ($\beta = 0.2$ and $\beta = 0.1$). However, the predictions for case (c) ($\beta = 0.05$) does not agree with the corresponding analytical solutions. The main reasons for this discrepancy are thought to be: (i) the analytical solution is not suitable for a beach with a very mild slope, because the perturbation parameter ε may not be a valid small number when β is very small (in case (c), $\varepsilon = 1.618$), and (ii) the seepage effect is neglected in the analytical solution. It can be seen from Figure 2 that the predicted asymptote of the groundwater table becomes higher as the beach slope decreases. This is consistent with the filed observations and the existed theories (Turner et al., 1997).

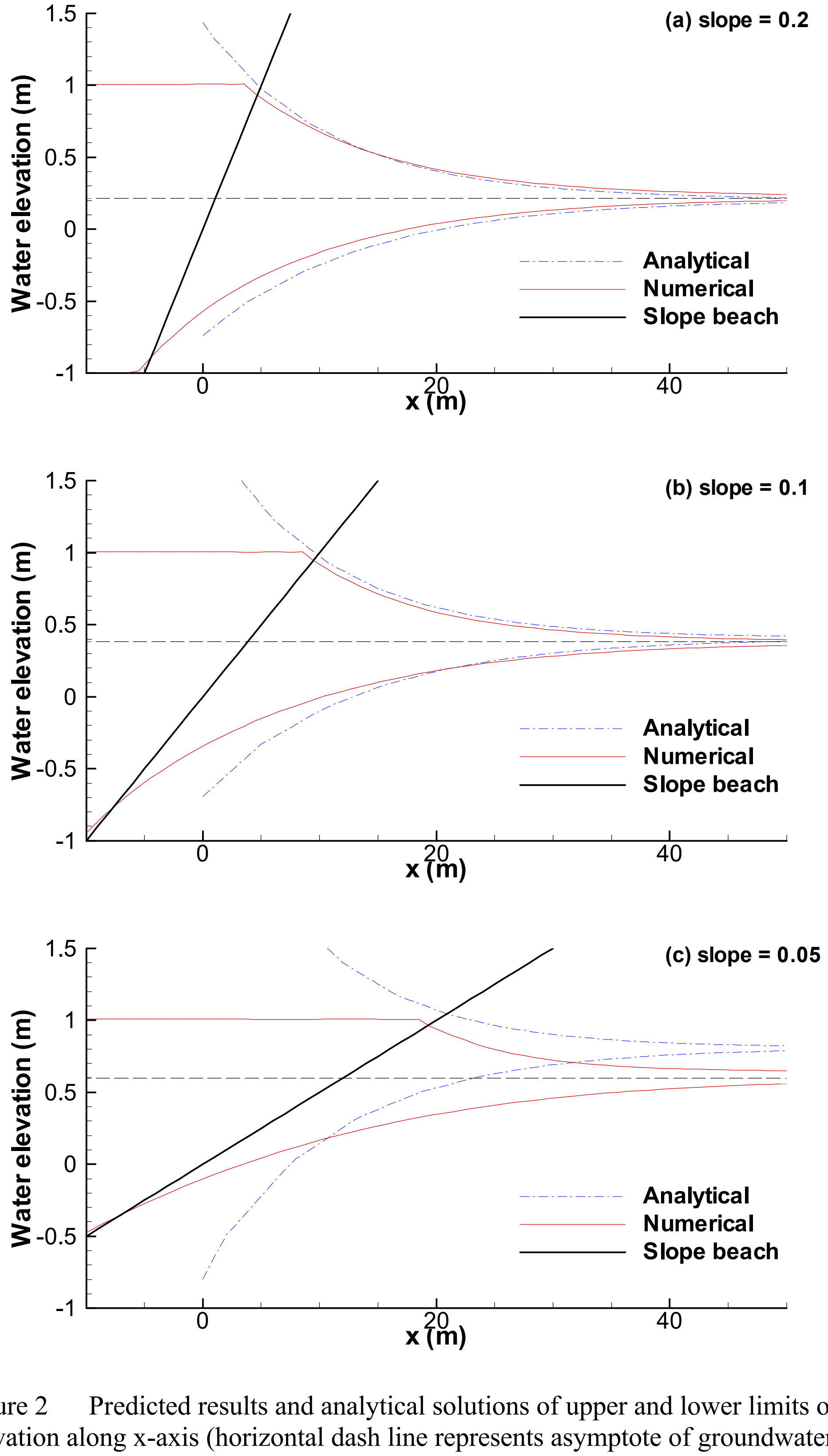

Figure 2 Predicted results and analytical solutions of upper and lower limits of water elevation along x-axis (horizontal dash line represents asymptote of groundwater table)

3.2 Exit Point and Seepage Face

In order to check the capability of the model for simulating the formation of the exit point and seepage face, tide-induced water table fluctuation in a coastal aquifer with a sloping beach is studied. The configuration of the problem is illustrated in Figure 1, which is the same as that given by Li et al (1997). In this study point A is the exit point and Point B is the point of intersection of the surface water level and the beach slope. During a tidal cycle, Point A may be coupled with or decoupled from Point B.

The parameters used in the numerical simulation are the same as those used in Li et al (1997): the beach slope $\beta = 45^o$, the mean water level $d = 5m$, the tidal amplitude $A = 3m$, the tidal cycle $T = 24hour$, the effective porosity $n_e = 0.4$, the hydraulic conductivity $K = 0.38\,m/hour$, the river water level is fixed to be 10m and the length of the base of the aquifer $L = 50m$.

Figure 3 shows the present model predicted water levels at four points in the aquifer. The space steps in both the x- and y-directions are 0.125m and the time step is 0.05s. The corresponding numerical results obtained by Li et al (1997) are also illustrated in the figure. It can be seen that due to the effect of the sloping beach the water surface becomes asymmetric with steep rising and flat falling water levels, with this trend being consistent with the field observations (Nielsen, 1990). In general, the predicted water levels agree with the corresponding results obtained by Li et al (1997). However, the present results are slightly more asymmetric than those obtained by Li et al.

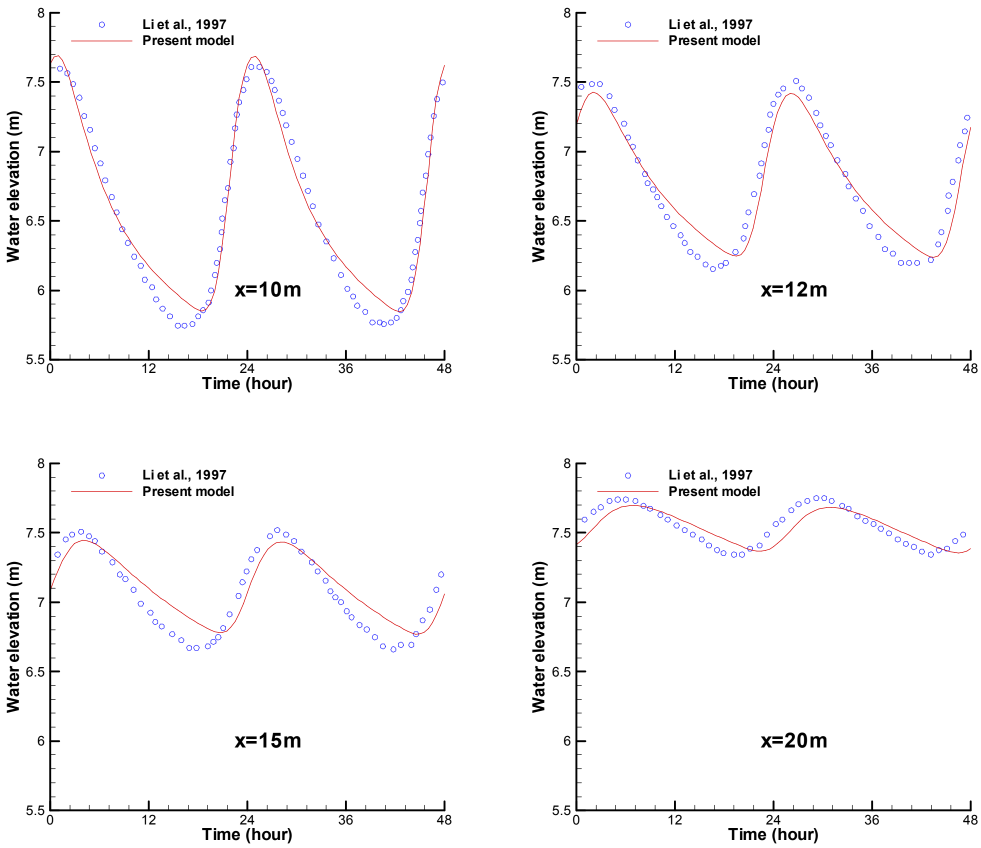

Figure 3 Comparison of predicted water levels at four points in the aquifer against results obtained by Li et al. (1997)

Figure 4 shows the formation of the exit point and seepage face on the slope beach during a tidal cycle. It is clear that during the ebbing phase, the groundwater flow becomes decoupled from the surface water flow, and an exit point and seepage face formed at about t = 2 hr. The exit point moves downwards along the beach face during the ebbing phase. During the flooding phase the groundwater flow becomes coupled with the surface water flow after the surface water level becomes close to the exit point, and the exit point vanishes at about t = 17hr. The predicted elevation of the exit point are compared against the results obtained by using the models by Li et al (1997) and Turner (1993) in Figure 5. It can be seen that the present results are in good agreement with the existed results.

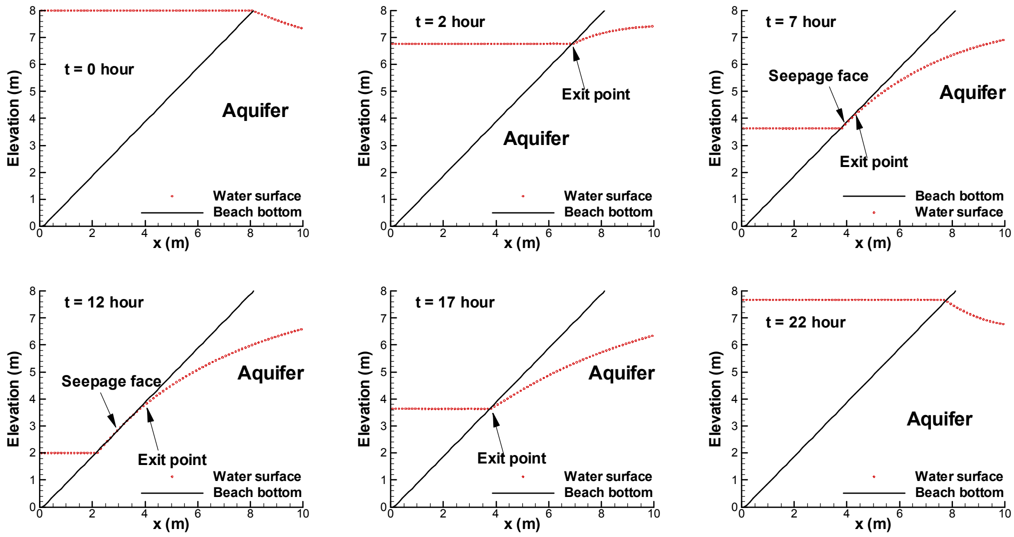

Figure 4 Predicted water levels, showing formation of exit point and seepage face

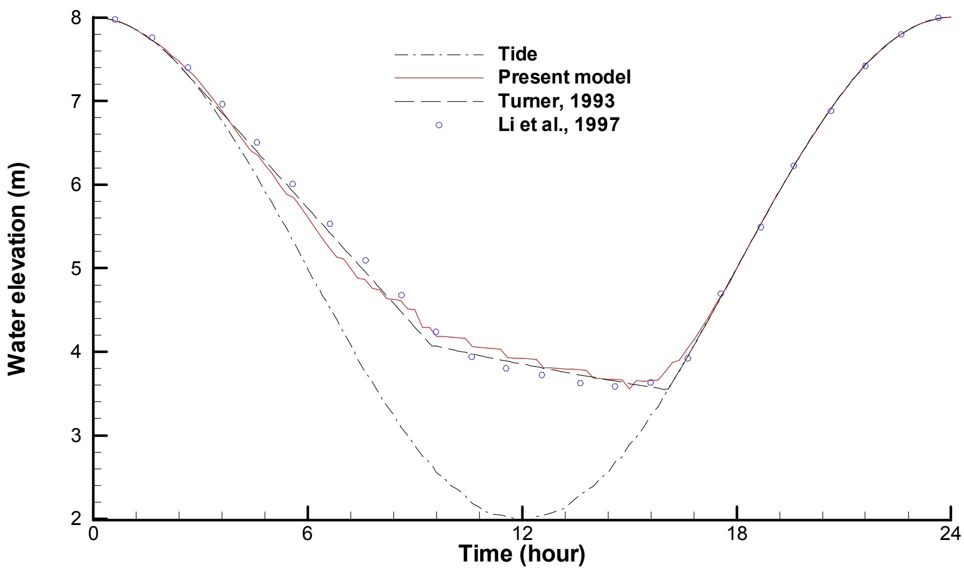

Figure 5 Comparison between the current model predicted water elevation distribution and those by and those by Turner (1993) and Li et al (1997)

4. CONCLUSIONS

An important phenomenon in beach hydraulics is that the exit point may decouple from the driving head, resulting in the formation of a seepage face on a sloping beach during the falling tide period. The seepage face and the exit point are formed naturally when water flows across the interface of two different media. Using the present model the groundwater and surface water flows are implicitly linked, thus no additional numerical effort is required to treating the moving boundary explicitly at every time step. Continuous water surface can be obtained between surface and groundwater flows. The seepage face and exit point are automatically formed.

Two test cases are used to test the model, with emphasis being given to prediction of the formation of the seepage face and exit point. The integrated model is shown to be capable of predicting the tidal induced water table fluctuations in the coastal aquifer with both vertical and slope beaches. Comparison against existing analytical solutions shows that the effect of moving boundary can be simulated correctly in the present model. The formation of the seepage face and exit point on a slope beach can be predicted accurately, with the predicted exit point locations matching closely existing laboratory measurements and analytical and numerical model predictions.

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