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# VALIDATION OF INSTANTANEOUS VELOCITIES MEASUREMENTS WITH ADV EQUIPMENT IN TURBULENT HIGH TWO-PHASE FLOWS 

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#### Abstract

Measurement instruments by Doppler effects are very reliable to characterize the instantaneous velocities in laminar and turbulent flows without the air presence, because the water constitutes the fundamental element of sound transmission.

However, in two-phase-flows (water-air), for example, inside of a hydraulic jump, the register measurements can be wrong when air bubbles pass through the measurement volume, because in this instant the sound echo is not correctly transmitted.

In these circumstances it is necessary to verify the registers and to carry out a digital filtering of the information, in order to eliminate and/or correct the anomalous data but maintaining the continuity of the register.

In this paper an easy and effective procedure is presented to carry out the filtering and is based on a register progressive cut of the lower and upper limits, in function of the $5 \%$ and $95 \%$ statistical.

The kindness of the method is verified with instantaneous velocities measurements in a laboratory channel and inside of some hydraulic jumps configurations. The obtained results are compared and contrasted with the already supported experimental and theoretical results.

Finally, a parametric study was carried out about the characteristics of jump length and the relation among the conjugate depths, the analysis of the energy dissipation and the distribution of velocities in different positions of the free and submerged hydraulic jumps.


Keywords: acoustic doppler velocimetry, turbulent high two-phase flow, digital filtering

## 1. INTRODUCTION

A hydraulic jump is a sudden transition from a high-velocity, supercritical open channel flow into a slow-moving, subcritical-flow. It is characterized by a sudden rise of the free-surface, with strong energy dissipation and mixing, large-scale turbulence, air entrainment, waves and spray.

Measurement instruments by Doppler effects are very reliable to characterize the instantaneous velocities in laminar and turbulent flows without the air presence, because the water constitutes the fundamental element of sound transmission. However, in two-phaseflows (water-air), the register measurements can be wrong when air bubbles pass through the measurement volume, because in this instant, the sound echo is not correctly transmitted.

In these circumstances, it is necessary to verify the registers and to carry out a digital filtering of the information, in order to eliminate and/or correct the anomalous data but maintaining the continuity of the register. The kindness of the method is verified with instantaneous velocities measurements in a laboratory channel and inside of some hydraulic jumps configurations, free and submerged.

The series of experiments were carried out in a horizontal channel, 5 m long and 0,081 m wide and a flows range between 0.019 and $0.048 \mathrm{~m}^{2} / \mathrm{s}$. At the upstream end, the channel
flow was controlled by ogee spillway and a vertical sluice gate. At the downstream end, the depth control was established by means of a tail-gate. The Froude number range was between 1.7 and 5.5 and the Reynolds number of $1 \times 10^{4}$ to $3 \times 10^{4}$. Table 1 shows a summary of the principal characteristics of experimental flow conditions.

Table 1 Summary of experimental flow conditions.

| $y_{l}$ |  | $V_{1}$ |  | $F_{1}$ |  | $R e_{1} \times 10^{4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cm |  | $\mathrm{m} / \mathrm{s}$ |  |  |  |  |  |
| $\begin{gathered} \text { Ogee } \\ \text { spillway } \end{gathered}$ | Sluice gate | $\begin{gathered} \text { Ogee } \\ \text { spillway } \end{gathered}$ | Sluice gate | $\begin{gathered} \text { Ogee } \\ \text { spillway } \end{gathered}$ | Sluice gate | $\begin{gathered} \text { Ogee } \\ \text { spillway } \end{gathered}$ | Sluice gate |
| Jump initial position from contracted depth - $x=0 \mathrm{~cm}$. Ogee: 20 depths - Gate: 9 depths |  |  |  |  |  |  |  |
| 0.8-2.8 | 1.1-4.0 | 1.4-1.8 | 1.5-1.8 | 2.9-5.8 | 1.7-5.5 | 1.1-2.9 | 1.5-3.0 |
| Jump initial position from contracted depth - $x=14 \mathrm{~cm}$. Ogee: 20 depths - Gate: 9 depths |  |  |  |  |  |  |  |
| 0.9-2.9 | 1.2-3.7 | 1.5-1.8 | 1.2-1.8 | 3.2-5.2 | 2.0-4.4 | 1.1-2.9 | 1.5-3.0 |
| Jump initial position from contracted depth - $x=28 \mathrm{~cm}$. Ogee: 20 depths - Gate: 8 depths |  |  |  |  |  |  |  |
| 0.9-2.9 | 1.2-3.0 | 1.5-1.8 | 1.5-1.7 | 3.2-5.2 | 2.7-4.6 | 1.1-2.9 | 1.5-3.0 |
| Jump initial position from contracted depth $-x=56 \mathrm{~cm}$. Ogee: 20 depths - Gate: 8 depths |  |  |  |  |  |  |  |
| 0.9-2.8 | 1.4-3.3 | 1.4-1.8 | 1.3-1.6 | 3.2-4.8 | 2.6-3.5 | 1.0-2.9 | 1.5-2.9 |
| Jump initial position from contracted depth - x $=70 \mathrm{~cm}$. Gate: 4 depths |  |  |  |  |  |  |  |
|  | 2.3-3.3 |  | 1.5-1.6 |  | 2.6-3.2 |  | 2.3-2.8 |
| Jump initial position from contracted depth - $x=90 \mathrm{~cm}$. Gate: 4 depths |  |  |  |  |  |  |  |
|  | 2.4-3.4 |  | 1.4-1.6 |  | 2.5-3.1 |  | 2.2-2.8 |

With these conditions it was possible to obtain developing and developed flows and data with and without scale effects. The developed flow was reached approximately to a distance of 30 times the contracted depth. It will be seen later, the jumps downstream of ogee spillway are exempt from scale effect for Reynolds number greater than 18000, being this value slightly inferior to the obtaining by Chanson (2005). However, in the case of jumps downstream of the vertical gate, the scale effects could be negligible for $R e>25000$.

## 2. DATA FILTERING AND VERIFICATION

The data filtering is based on a register progressive cut of the lower and upper limits, in function of the $5 \%$ and $95 \%$ statistical. Next the filtering process is shown.

From the mean, $\bar{u}$ and maximum, $u_{\max }$ values registered in the data series, the first relative amplitude is determined, $A_{1}=u_{\text {max }}-\bar{u}$.

Next is found the value $u_{\text {min }}=\bar{u}-A_{1}$ and the general amplitude, $A=u_{\text {max }}-u_{\text {min }}$.
Finally they are obtained the superior cut value, $X_{\text {max.c. }}$ and the lower cut value, $X_{\text {min.c }}$ from the initial series, so that

$$
\begin{align*}
& X_{\text {max.c. }}=u_{\max }-(0.05 A)  \tag{1}\\
& X_{\text {min.c. }}=u_{\text {min }}+(0.05 A) \tag{2}
\end{align*}
$$

This process can be repeated if the data series need it. However it is recommended not to do more than three filtering data, so that the initial series will be little altered.

In Figure 1 are shown the velocity registers with no filter and first filter. The registers
have 4096 data and were obtained with a rate of data acquisition of 5 points per second. It can be observed that in this particular case, it was necessary filtering the register, once. The mean value of the register is maintained and the maximum values are slightly altered.


Figure 1 Velocity register of 4096 data. No filter and first filter, respectively.
In Figure 2 are shown the velocities in function of the relative depth registered inside of the free and submerged hydraulic jumps. The measurement section corresponding to 0.75 of the hydraulic jump length and Froude Number in the contracted depth, $F_{l}=2.87$.


Figure 2 Velocities inside of the free and submerged hydraulic jumps. Froude number in the contracted depth, $F_{l}=2.87$. Gate opening $a=y_{0}$.

## 3. FLOW CONDITIONS AND CHARACTERISTICS OF HYDRAULIC JUMP

The length of the hydraulic jump is defined like the length of the zone where the energy dissipation is carried out, so the following general relations can be derived:

$$
\begin{equation*}
f\left(L / H_{L}, H_{L} / H_{*}, L / y_{2}, y_{2} / y_{1}, y_{4} / y_{2}, y_{4} / y_{3}, y_{3} / y_{0}, u / u_{m}\right)=0 \tag{3}
\end{equation*}
$$

Where $H_{L}$ and $H_{*}$, are the energy loss and the total specific energy at the upstream end of the jump, respectively. The principals geometric characteristics are $\left(L, y_{0}, y_{1}, y_{2}, y_{3}, y_{4}\right)$, and $\left(u, u_{m}\right)$, the mean and maximum velocities inside of the hydraulic jump. Based on this relation, it is proposed some experimental relations for free and submerged hydraulic jumps.

## Free hydraulic jumps

In Figure 3 are indicated the results obtained by Bradley and Peterka (1957) and Ohtsu et al. (1990). The values $L_{r j}<L_{j}<L_{t}$, are the roller length (end of the separation zone), jump length and the length where the velocity distribution change is neglected, respectively.

We can observe that all experimental data, although with some spreading grade, fall in the corresponding location.


Figure 3 Characteristics of the free hydraulic jump length $L / y_{2}=f\left(F_{1}\right)$.

Figure 4 shows the experimental data with some of the conjugate depth relations, $\left(y_{2} / y_{1}\right)$, so:
Bélanguer (1838):

$$
\begin{equation*}
y_{2} / y_{1}=1 / 2\left\lfloor 1+8 F_{1}^{2}-1\right\rfloor \tag{4}
\end{equation*}
$$

Harleman (1958) and Rajaratnam (1965):

$$
\begin{equation*}
\left(y_{2} / y_{1}\right)^{3}-\left[2 F_{1}^{2}+1-S_{f}\right]\left(y_{2} / y_{1}\right)+2 F_{1}^{2}=0 \tag{5}
\end{equation*}
$$

Leutheusser and Kartha (1972):

$$
\begin{equation*}
F_{1}^{2}=\frac{\left(y_{2} / y_{1}\right)\left[\left(y_{2} / y_{1}\right)^{2}-1\right]}{2.06\left[\left(y_{2} / y_{1}\right)-1\right]-0.0244\left(y_{2} / y_{1}\right)^{2}} \tag{6}
\end{equation*}
$$

where $F_{1}=u / \sqrt{g y_{1}}$, Froude number in contracted depth, $S_{f}$, integral of bed shear stress or boundary force and $g$, gravity acceleration.

The last two relations present similar results for fully developed flow and, Bélanguer relation, for developing flow. In the figure are represented only the data that are exempt from the scale effect, so, flow controlled by ogee spillway, $R e>18000$ and vertical gate, $R e>25000$.


Figure 4 Comparison of experimental data with conjugate depths relations.

## Submerged hydraulic jumps

We consider the contracted depth, $y_{0}=a$, and $y_{3}$ and $y_{4}$ the depths at the upstream end and downstream end of the submerged jump.

Figure 5 shows the relation, $L_{s j}=f\left(y_{4} / y_{2}\right)$, where $y_{2}$ is calculated with the Belanger's equation of the conjugate depths, $y_{2} / y_{0}=1 / 2\left\lfloor\sqrt{1+8 F_{0}^{2}}-1\right\rfloor$.

The best fit proposed by Ohtsu et al. (1990) for submerged jump is:

$$
\begin{equation*}
L_{s j} / y_{2}=\left[5\left(y_{4} / y_{2}\right)+0,9\right] \tag{7}
\end{equation*}
$$

Where, in the case of $y_{4} y_{2}=1$, it is obtained the relation for free hydraulic jump:

$$
\begin{equation*}
L_{s j}=5,9 y_{2} \tag{8}
\end{equation*}
$$

However, we must note that the coefficient 5.9 would be only valid in the range of

Froude numbers [ $6 \leq F_{l} \leq 11$ ]. This coefficient varies approximately between 4 and 6 in the range $\left[2 \leq F_{1} \leq 5\right.$ ].


Figure 5 Characteristics of free and submerged hydraulic jump length.
The best fit to the experimental data is like follows:

$$
\begin{equation*}
L_{s j c} / y_{2}=\left[7,44\left(y_{4} / y_{2}\right)-3,04\right] \tag{9}
\end{equation*}
$$

So, in the case of free hydraulic jump, $y_{4} / y_{2}=1$, the relation is reduced to:

$$
\begin{equation*}
L_{s j c}=4,4 y_{2} \tag{10}
\end{equation*}
$$

The experimental data corresponds to the range $\left[2.87 \leq F_{0} \leq 4.96\right]$. The coefficient 4.4 represents an intermediate value of the free jump coefficient in the range [ $2 \leq F_{1} \leq 5$ ].

In accordance with equation (10) and the criteria to define the length of hydraulic jump, $L_{j c}$, we can observe that this length is inferior to the length obtained by Bradley and Peterka (1957) and other authors. This difference is in accordance with the criteria to define the jump length in our data, $L_{j c}$, an intermediate length between the roller length, $L_{r j}$, and the measurement defined by Bradley and Peterka, $L_{j}$.

We can observe in Figure 5 that until a submergence approximately of $y_{4} / y_{2}=1.25$, the submerged hydraulic jump length is lower than the corresponding classical free hydraulic jump in the range [ $6<F_{1}<11$ ]. However, for larger submergence, the lengths are bigger than the free hydraulic jump lengths and, it tends to coincide with the law proposed by Ohtsu et al. (1990) on the value, $y_{4} / y_{2}=1.60$.

In Figure 6 are shown the experimental results and the theoretical laws obtained by application of the continuity and momentum criteria to the control volume of a submerged hydraulic jumps in a horizontal and rectangular channel. The theoretical law is:

$$
\begin{equation*}
y_{3} / y_{0}=\sqrt{\frac{2 F_{0}^{2}\left[1-\left(y_{4} / y_{0}\right)\right]}{\left(y_{4} / y_{0}\right)}+\left(y_{4} / y_{0}\right)^{2}} \tag{11}
\end{equation*}
$$

Where the index 0 designates to hydraulic parameters in the outlet plane of the gate, the index 3 and 4, at the upstream end and at the downstream end of the submerged hydraulic jump, respectively. As it can be seen, the obtained fit degree is quite suitable.


Figure 6 Submerged hydraulic jumps for different Froude's numbers.
Figure 7 shows the relation $\left(y_{4} / y_{3}\right) / F_{0}=f\left(y_{3} / y_{0}\right)$. It can be observed that the data come together in the following experimental law with a regression coefficient $\mathrm{R}^{2}=0,984$ :

$$
\begin{equation*}
\left(y_{4} / y_{3}\right) / F_{0}=\left[1,173\left(y_{3} / y_{0}\right)^{-0,766}\right] \tag{12}
\end{equation*}
$$



Figure 7 Experimental law of the relation $\left[\left(y_{4} / y_{3}\right) / F_{0}\right]=f\left(y_{3} / y_{0}\right)$.

## Energy dissipation in hydraulic jumps

The difference between the total energy at the beginning of the jump and the sum of four items (mean momentum flux $M$, turbulent momentum flux $T$, pressure $P$ and integrated bed shear stress) represents the loss through viscous dissipation. In spite of large effect that the turbulence has on the flow, the turbulent momentum flux is surprisingly small (Vischer and Hager, 1995 and Ohtsu et al., 1990).

So, $L_{j}$ can be interpreted like the necessary length for that the energy dissipation in the hydraulic jump will be completed. The energy loss, $H_{L}$, between the beginning $(x=0)$ and at end of the jump ( $x=L_{j}$ ), can be expressed with the following one-dimensional equation:

$$
\begin{equation*}
H_{L}=H_{1}-H_{2}=\left(v_{1}^{2} / 2 g+y_{1}\right)-\left(v_{2}^{2} / 2 g+y_{2}\right) \tag{12}
\end{equation*}
$$

Using (12) and the continuity equation, the relative energy loss in a free hydraulic jump can be expressed like:

$$
\begin{equation*}
H_{L} / H_{1}=\frac{2\left[1-\left(y_{2} / y_{1}\right)\right]+\left[1-1 /\left(y_{2} / y_{1}\right)^{2}\right] F_{1}^{2}}{2+F_{1}^{2}} \tag{13}
\end{equation*}
$$

In similar way, the relative energy loss in a submerged hydraulic jump can be too expressed like:

$$
\begin{equation*}
H_{L} / H_{0}=\frac{2\left[\left(y_{3} / y_{0}\right)-\left(y_{4} / y_{0}\right)+\left(1-1 /\left(y_{4} / y_{0}\right)^{2}\right) F_{0}^{2}\right.}{2\left(y_{3} / y_{0}\right)+F_{0}^{2}} \tag{14}
\end{equation*}
$$

If $y_{3} / y_{0}=1$ and $F_{0}=F_{1}$, the equation (14) is reduced to the equation (13).
Figure 8 shows the experimental results and the theoretical laws, equations (13) and (14). It can be seen that the data obtained are satisfactory. It is possible to do an effective use of submerged hydraulic jump, in the ranges $\left[1.1<y_{3} / y_{0}<8\right]$ and $F_{0}>1.5$.


Figure 8 Relative energy losses in free and submerged hydraulic jumps.

## Velocity distribution in free and submerged hydraulic jumps

From the analysis of the mean velocity experimental distribution in different sections of the free and submerged hydraulic jump, it was obtained a similar velocity distribution, in the range $\left[0.2 \leq x / L_{j} \leq 0.7\right]$. In figure 9 we can see the experimental data, the fitting curve and the principal parameters.

The scalar length, $Y$, is the depth where the velocity is equal to the half of the registered maximum velocity, $\bar{u}=u_{m} / 2$, and, $y_{\max }$, is the depth where $u=u_{m}$.


Figure 9 Velocity distribution law inside of free and submerged hydraulic jumps. Validity range: $2.5 \leq F_{r l} \leq 5 ; 0.25 \leq \mathrm{x} / L_{s j c} \leq 0.75$ y $4 \leq y_{4} / y_{0} \leq 10$.

The best fit of the velocity distribution law inside of free and submerged hydraulic jumps are:

$$
\begin{gather*}
\frac{\bar{u}}{u_{m}}=\left\{\frac{1}{k}\left(\frac{y}{Y}\right)\right\}^{1 / n} ; \quad 0 \leq \frac{y}{Y} \leq k  \tag{15}\\
\frac{\bar{u}}{u_{m}}=\exp \left[-\frac{1}{2}\left\{\frac{1.177}{1-k}\left(\frac{y}{Y}-k\right)\right\}^{2}\right] ; \quad k \leq \frac{y}{Y} \leq 1.5 \tag{16}
\end{gather*}
$$

Where, $k=y_{\text {max }} / Y$.
Table 2 shows the coefficient, $k$, exponent, $n$, and the corresponding validity ranges of the velocity distribution law in free and submerged hydraulic jumps.

It is interesting to note that the difference in the characteristics of developing and developed flow, are completely diffused inside of hydraulic jump. This last phenomenon is produced because the turbulence diffuses all flow characteristics such as momentum, energy or even turbulence itself (Rouse et al., 1959).

The present results constitute complementary laws of the proposed by Ohtsu et al. (1990). They generalised the equation for the classical jump and demonstrated a similarity for jumps that are intermediate between the classical jump and the classical wall jet.

Table 2 Coefficient, $k$, and exponent, $n$, of the velocity distribution law in hydraulic jumps.

| Velocity distribution law | Range of application | $k$ | $n$ |
| :--- | :---: | :---: | :---: |
| Free hydraulic jump. Undeveloped flow | $2.5 \leq F_{l} \leq 5$ <br> $0.25 \leq \mathrm{x} / L_{j i} \leq 0.75$ | 0.393 | 9.9 |
| Submerged hydraulic jump. Undeveloped flow | $2.5 \leq F_{l} \leq 5$ <br> $0.25 \leq \mathrm{x} / L_{j j} \leq 0.75$ | 0.302 | 9.15 |
| $2.5 \leq F_{I} \leq 5$ <br> Free and submerged hydraulic jump <br> Undeveloped flow | $0.25 \leq \mathrm{x} / L_{j c} \leq 0.75$ <br> $4 \leq \mathrm{x} / L_{j c} \leq 10$ | 0.342 | 9.5 |
| Free hydraulic jump. Undeveloped flow <br> Ohtsu et al. (2000) | $5 \leq F_{I} \leq 7.3$ <br> $0.2 \leq \mathrm{x} / L_{j c} \leq 0.7$ | 0.333 | 12 |
| Free hydraulic jump. Developed flow <br> Ohtsu et al. (2000) | $5.3 \leq F_{I} \leq 7.3$ <br> $0.2 \leq \mathrm{x} / L_{j c} \leq 0.7$ | 0.351 | 7 |

## REFERENCES

Bélanguer, J.B. (1838), Résumé de leçons, Mémoire, Ecole Nationale des Ponts et Chaussées, París.
Chanson, H. (2005), Air bubble entrainment in hydraulic jumps. Similitude and scale effects. The University of Queensland, Department of Civil Engineering, Report CH57/05.
García, S. (2008), Caracterización de resaltos hidráulicos libres a partir de velocidades instantáneas con equipo Doppler. Flujo aguas abajo de un aliviadero. Proyecto Fin de Carrera. EU de Ingeniería Civil. Universidad Politécnica de Cartagena. Spain
Harleman, D.R.F. (1958), Discussion of turbulence characteristics of the hydraulic jump. Proc. ASCE, Journal of Hydraulic Division., Nov.
Leutheusser, H.J. and Kartha, V.C. (1972), Effect of inflow condition on hydraulic jump. Proc. ASCE, Journal of Hydraulic Division., Aug.
Márquez, C. (2006), Caracterización paramétrica de resaltos hidráulicos libres y sumergidos a partir de medidas de velocidades instantáneas con equipo Doppler. Proyecto Fin de Carrera. EU de Ingeniería Civil. Universidad Politécnica de Cartagena. Spain
Ohtsu, F., Yasuda, Y. and Awazu, S. (1990), Free and submerged hydraulic jumps in rectangular channels. Report of the Research Institute of Science and Technology, Nihon University, No 35.
Rajaratnam, N. (1965), The hydraulic jump as wall jet. Proc. ASCE, Journal of Hydraulic Division., 91(HY5),pp. 107-132.
Rao, G. and Rajaratnam, N. (1963), The submerged hydraulic jump. Proc. ASCE, Journal of Hydraulic Division., January.
Rouse, H., Siao, T.T. and Nagaratnam, S. (1959). Turbulence characteristics of the hydraulic jump. Trans. ASCE 124: pp. 926-966.
Silvester, R. (1964), Hydraulic jump in all shapes on horizontal channels. Proc. ASCE, Journal of Hydraulic Division, Jan., pp. 23-55.
Vicente, J. (2008), Caracterización de resaltos hidráulicos libres a partir de velocidades instantáneas con equipo Doppler. Flujo aguas abajo de una compuerta. Proyecto Fin de Carrera. EU de Ingeniería Civil. Universidad Politécnica de Cartagena. Spain
Vischer, D.L. and Hager, W.H. (1995), Energy Dissipators. No 9 Hydraulic Structures Design Manual, International Association for Hydraulic Research, IAHR. Taylor \& Francis Group.

