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APPLICATIONS OF MLPG_R & SALE/QALE-FEM FOR WAVE - STRUCTURE INTERACTIONS

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***Abstract**: Over the past years, we are developing numerical codes to tackle wave-structure interaction problems based on the Fully Nonlinear Potential Theory (FNPT) or general viscous flow model. In the case of FNPT, we solve the Laplace's equation of velocity potential using the finite element method (FEM) that are free from singularities and 10 times faster than BEM in handling complex 3D overturning waves, floating body interactions and so on. Whereas, to solve the general viscous flow model, i.e. Navier-Stokes (NS) equations with proper boundary conditions, a meshfree method, namely Meshless Local Petrov Galerkin Method using Rankine source function (MLPG_R), has been recently developed. The MLPG_R is relatively new algorithm to tackle the violent wave structure interactions, which may have higher computational efficiency and accuracy in terms of pressure calculation than other meshfree methods (like SPH) for the same purpose. In the present paper, we will summarize the applications of the developed algorithms to various wave structure interaction problems namely, submerged structure, 6 DOF, breaking wave interactions over slope, sloshing and so on. Finally, the paper ends with preliminary results on the novel coupling based on MLPG_R and FNPT methods.*

***Keywords**: nonlinear potential flow,FEM, viscous flow model, MLPG_R, couple, breaking wave, 6-DOF.*

INTRODUCTION

In the recent years, due to the advances in computational techniques and computational power, modelling very complex practical problem like full nonlinear wave interaction with 3D offshore structures, floating bodies with 6-Degree of freedom, studying the kinematics of breaking wave interactions in time domain simulations are possible. The computation of very steep and/or overturning waves, their loads on offshore structures as well as responses of the floating structures is still of a very challenge and time-consuming task. Because of the strong nonlinearity involved sometimes, solutions based on linear or other simplified theories may be insufficient

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and so fully nonlinear theory is necessary. Two types of fully nonlinear models, i.e. NS model (governed by the Navier-Stokes and the continuity equations together with proper boundary conditions) and FNPT models (fully nonlinear potential model), may be used. The latter are much easier and needs less computational resource than the former with satisfactory accuracy if waves are not breaking and/or structures involved are large, whereas, one need to use the NS model for modelling violent waves and its interaction with structures.

The problems formulated by FNPT model are usually solved by a time marching procedure. In this procedure, the key task is to solve the boundary value problem by using an numerical method, such as the boundary element or desingularized boundary integral methods (both are shortened as BEM in this paper) and the finite element method (FEM). The BEM has been attempted by many researchers, complete review can be found in Kim et al.(1999). The FEM has been developed by Wu & Eatock Taylor (1994) for 2D cases and by Ma et al. (2001) for 3D case, and it is further extended to handle complex objects and floating bodies simulation by using QALE-FEM (Quasi-Arbitrary Lagrangian and Eulerain) and SALE-FEM (Semi-Arbitrary Lagrangian and Eulerain) by Ma and Yan(2006) and Sriram (2008), even though the basic formulations are similar, the algorithm and implementation of these two methods are carried out at City University London and IITMadras, respectively.

The problems formulated by NS model are carried out by using well known time-split algorithm. The numerical approach for modelling can be based on mesh and meshfree methods. The mesh free methods are the promising approach in recent years, for the simulation of violent waves. The notable works on this topic for the wave structure interactions using smoothed particle hydrodynamics (Monaghan ,1994), Moving particle semi-implicit method (MPS by Kosizhuka et al., 1995), and more recently, meshless local petrov galerkin with ranking source (MLPG_R by Zhou and Ma, 2009). This paper presents the application of the state-of-the art models (SALE-FEM/QALE-FEM/MLPG_R) developed by the authors over the past many years and the preliminary results of the new coupled algorithm by taking the advantage and disadvantage of the available methods.

MATHEMATICAL BACKGROUND

Fully nonlinear potential flow theory (FNPT) using FEM

As pointed out above, the QALE-FEM/SALE-FEM are based on fully nonlinear potential theory to model the waves and on the fully nonlinear dynamics equation to model the floating bodies, if any. Motions of bodies and waves are fully coupled. The solutions are found by using a time marching procedure. At each time step, boundary value problems (BVPs) about the velocity potential and/or about the time derivative of the potential are solved by the finite element method (FEM) in which the fluid domain is discretised into tetrahedral elements.

Similar to the usual formulation for the FNPT Model, the velocity potential (ϕ) satisfies Laplace's equation,

$$\nabla^2\phi = 0 \qquad (1)$$

in fluid domain. On the free surface $z = \zeta(x, y, t)$, it satisfies the kinematic and dynamic conditions in the following Lagrangian form,

$$\frac{Dx}{Dt} = \frac{\partial\phi}{\partial x}, \frac{Dy}{Dt} = \frac{\partial\phi}{\partial y}, \frac{Dz}{Dt} = \frac{\partial\phi}{\partial z} \qquad (2)$$

$$\frac{D\phi}{Dt} = -gz + \frac{1}{2}|\nabla\phi|^2 \qquad (3)$$

where $\frac{D}{Dt}$ is the substantial (or total) time derivative following fluid particles and g is the gravitational acceleration. In Eq. (3), the atmospheric pressure has been taken as zero. On all rigid boundaries, such as the wavemaker and the floating body, the velocity potential satisfies

$$\frac{\partial\phi}{\partial n} = \vec{n}\cdot\vec{U}(t) \qquad (4)$$

where $\vec{U}(t)$ and $\vec{n}$ are the velocity and the outward unit normal vector of the rigid boundaries, respectively.

The problem described by Eqs. (1) to (4) is solved by using a time step marching procedure, more detail about the FEM formulation can be cited in Ma *et al.*, (2001). In the case of 2D, the discretisation is carried out by using triangular element (SALE-FEM) and for 3D it is based on tetra-hedron element (QALE-FEM). Compared to the conventional FEM, the QALE/SALE-FEM has three distinctive features generally. (1) The complex unstructured mesh is generated only once at the beginning and is moved by using a robust spring/vertex analogy method specially developed for this kind of problem. (2) A robust method is adopted to compute the fluid velocities on body and free surfaces which is suitable for moving unstructured meshes. (3) The semi-implicit iterative procedure is utilised to deal with full coupling between floating bodies and water waves. In addition, the numerical testes show that the quality of mesh during long time simulation are well retained and the high accuracy and efficiency can be achieved by using the QALE-FEM. The detail about the algorithm for QALE-FEM can be cited in Ma and Yan (2008), which is completely based on improved spring analogy method for moving the mesh, that are specifically tuned for handling water wave problems. Whereas, in SALE-FEM, the mesh motion is carried out by using an improved spring analogy method like in QALE-FEM as well as it could use Vertex method for moving the mesh, the details are reported in Sriram *et al.* (2009).

Navier stokes equation using MLPG_R

The above FNPT, does not consider viscous effects and may not handle breaking waves, that are deemed necessary for the design of any offshore or coastal structure.Hence, for doing so, one need to consider the full Navier stokes equations. The Navier-Stokes equation (referred as NS equation) and continuity equations together with proper boundary conditions are considered to simulate the water waves, which are given as follows:

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \vec{g} + \nu\nabla^2\vec{u} \qquad (5)$$

$$\nabla.\vec{u} = 0 \qquad (6)$$

Where, ρ is water density, g is gravitational acceleration, $\vec{u}$ is the fluid velocity vector, P is the pressure.

The Lagrangian forms of the kinematic and dynamic conditions on the free surfave are given by,

$$\frac{D\vec{r}}{Dt} = \vec{u} \quad \text{and} \quad P = 0 \qquad (7)$$

where $\vec{r}$ is the position vector. On the rigid boundary surface, the following boundary condition should be satisfied.

$\vec{u}.\vec{n} = \vec{U}.\vec{n}$ and $\vec{n}.\nabla p = \rho(\vec{n}.\vec{g} - \vec{n}.\dot{\vec{U}})$ (8)

The numerical approach for solving the above equations are based on well known time-split algorithm, in which the evaluation of pressure is based on semi-implicit equation give by

$$\nabla^2 p^{n+1} = \alpha \frac{\rho^{n+1} - \rho^*}{dt^2} + (1-\alpha)\frac{\rho}{dt}\nabla.\vec{u}^*, \quad (9)$$

Where * represents the intermediate velocity, more details can be cited in Ma (2005). In the governing equation, ∝=0 corresponds to the equation used by Ma(2005), whereas, ∝=1 corresponds to the equation used in MPS and PFEM. Zhou and Ma(2009) suggested ∝=0.1 to 0.2 for breaking wave simulation when using MLPG_R. As can be seen, the key task of this procedure is to solve Eq. (9) in order to evaluate the pressure. Many numerical methods, for example, finite element and finite different methods, may be adopted to solve this equations. In this paper, the collocated formulation of the MLPG_R method is used, which leads to the solve Eq. (9) directly, similar to the MPS method. We will give a brief overview of the numerical method that deals with the above fractional time split algorithm in the following sections. The details about the formulation are given in the above said paper and not repeated herein. For identifying the free surface particles a novel method based on the Mixed Particle Number Density and Auxiliary Function Method is used (Zhou and Ma, 2009).

APPLICATIONS

In this section, we will provide a brief overview of the developed model for the different purpose. SALE/QALE-FEM as well as MLPG_R has been applied to model various cases including those for waves only and those for interaction between waves and fixed/floating bodies. We have not enough space in this paper to present all these results. Only some selected examples as well as the results from coupling algorithm are provided. More could be found in our other papers.

Modeling 2-D or 3-D Steep Waves using SALE-FEM or QALE-FEM

Even though, SALE-FEM is developed at IITMadras, India and QALE-FEM at City University London, UK., the efficiency and the accuracy of the method are tested separately by validating with experimental and available literature data. In this section, we will report the comparison between the methods for a freak wave simulation, before reporting their applications. The length of the tank is 60m and the simulation is carried out for an input freak wave condition that could generate a near overturning. The simulation results are reported in Fig 1 for the location at 10m and 26.25m from the wave paddle. The figure shows that the results from these codes are almost identical. It should be noted that the QALE-FEM is 3D code and the SALE-FEM is a 2D code. Hence based on the purpose, one could use the respective code, i.e., for handling 3D freak waves, using QALE-FEM is much easier as reported in Fig.2 which illustrate the kinematics of 3D freak waves.

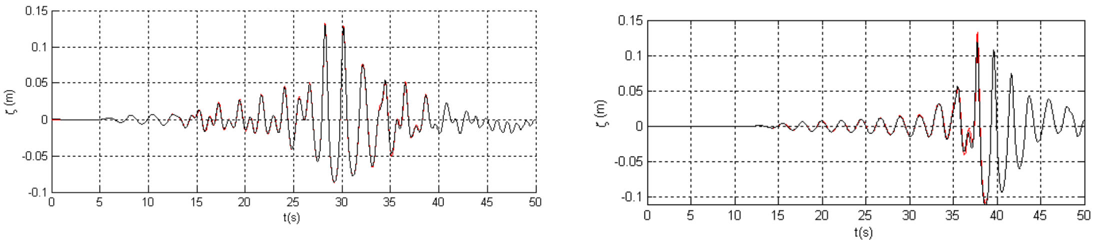


Fig.1. Comparison between SALE-FEM and QALE-FEM for the freak wave simulation (black –SALE-FEM, red – QALE-FEM, Left fig: at 10m, Right Fig: 26.25m)

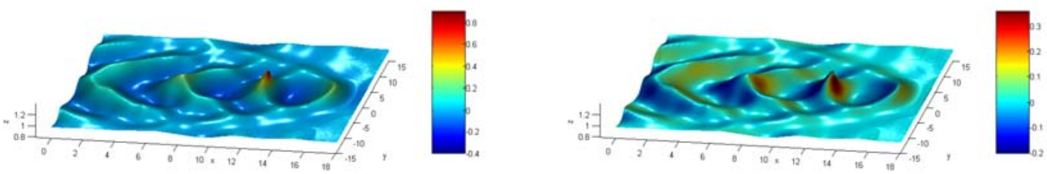


Fig. 2 Distribution of the fluid velocity on the free surface at $\tau \approx 29.68$ (N_θ =30, θ_{max} =45^0, ω_i =1.0 and a_i =0.01, slope = 1:20; a: x-component (u); b: y-component (v); c: z-component (w))

SALE-FEM

While a wave propagates over a submerged bar, the transmitted wave will disintegrate into shorter waves. The accurate description of the wave nonlinearity and wave dispersive characteristics is essential since the dispersive character of free higher harmonics is a major factor i.e, the amount of energy transferred between the harmonics. When the wave form propagates over submerged obstacles, the lee side of which is in deep water region, nonlinearity would tend to become weak leading to the non-existence of bound waves (nonlinear distortion of the long waves). A drastic change in the wave form will take place due to the higher harmonics that travel with different phases. Thus, predicting the above said phenomena proves to be classical case study to test the applicability of the numerical model for wave – propagation models. The experimental setup consisting of a tank of length 65m and water depth of 0.5m with a submerged bar as in Ohyama *et al.* (1995). Case C of the above paper is used for the present simulation using unstructured mesh (based on Vertex method) and compared with experimental measurements. The results are shown in Fig. 3, which shows a good comparison between the simulated and experimental measurements for the station at the top of the bar, whereas, a reasonable comparison at the deeper region, owing to the turbulence effect in the experiments. The snapshot of the mesh before and after the bar using the dynamic moving mesh algorithm based on vertex method is projected in Fig. 4. The SALE-FEM is applied to wide variety of problem mainly for studying without any structure and understanding the kinematics like splitting of solitary waves, sloshing, numerical and physical speed of waves and so on.

QALE-FEM

QALE-FEM is effectively used for dealing with overturning waves. A typical example is shown in Fig.4 in which a 3D solitary wave propagating over a sloping seabed with y-directional variation. The same case has been simulated using BEM method by many authors. Some have been used for comparison (Fig.5), which demonstrated that the FEM and BEM lead to same accuracy level. However, when comparing the computational efficiency, the QALE-FEM requires 1.80 hrs for modelling this problem in 2.3Ghz processor with 1GB RAM. For the same case, Higher order BEM requires 52.8hrs in CRAY-C90 (Grilli et al., 2001), whereas, fast BEM (Fochesato & Dias, 2006) requires 19hrs in 2GHz processor with 1GB RAM. Thus, it clearly indicates, the QALE-FEM is much faster than the fastest available BEM model.

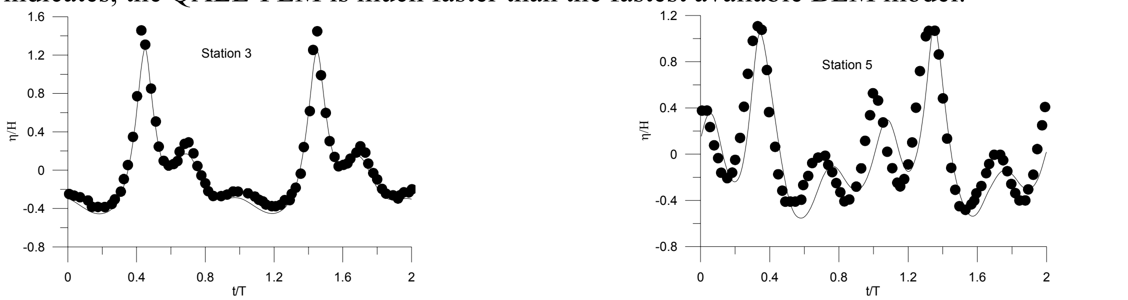


Fig.3. Comparison between numerical (———) and experimental (●●●●●●) measurements (Ohyama *et al.*, 1995).

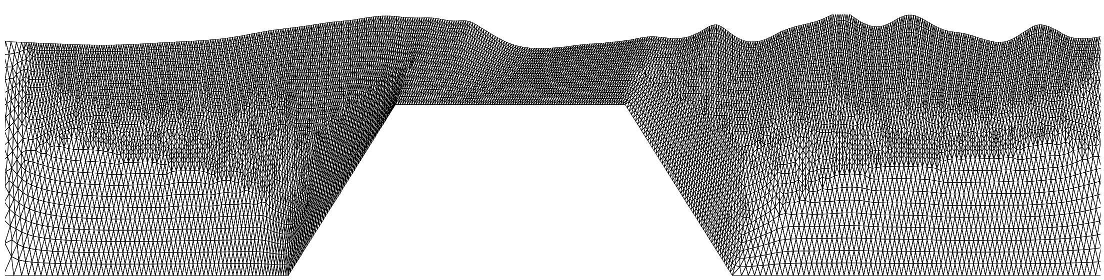


Fig.4. Moved mesh during simulation using vertex method for submerged bar.

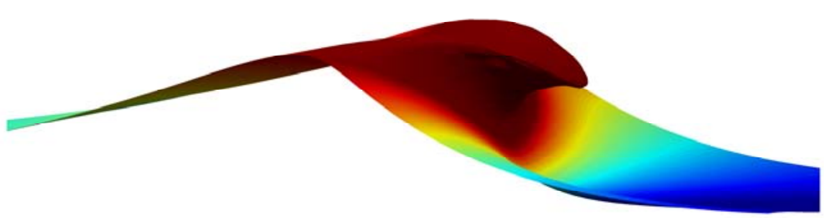


Fig.4 Solitary waves propagating over sloping seabed ($\tau \approx 8.827$, H=0.6, sc=1/15; $k_c^+=0.25$; the colour bar represents the wave elevation)

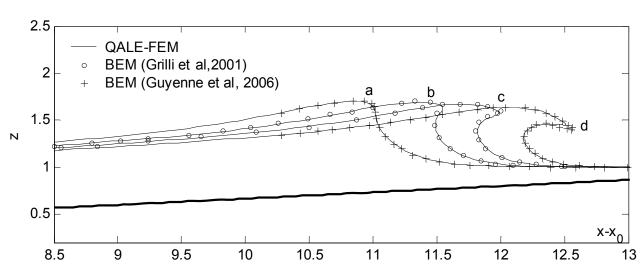
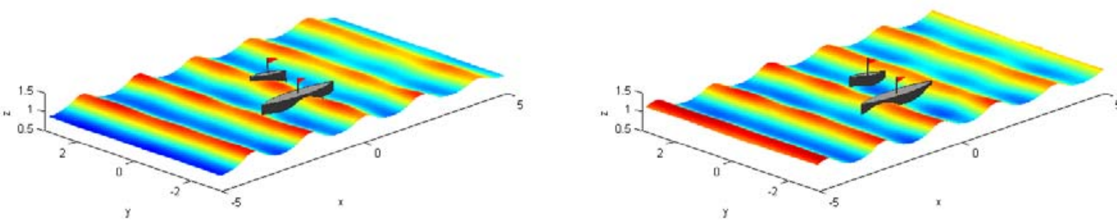


Fig.5 Free surface profile at (a) $y=\pm 4$ and (b) $y=0$ (H=0.6; s_c=1/15; k_c=0.25; Curve a: $\tau \approx 7.89$; b: $\tau \approx 8.25$; c: $\tau \approx 8.57$; d: $\tau \approx 8.83$; thick solid line represents the seabed geometry; BEM results are taken from Grilli et al., 2001 and Guyenne and Grilli, 2006)

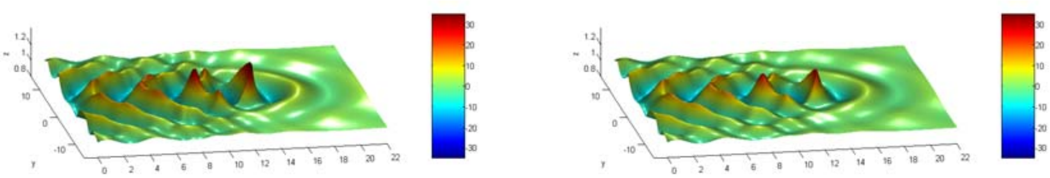
Except the problem regarding wave interacting with submerged structure, e.g. Fig.4, the QALE-FEM has also been applied to model the interaction between floating structures (single or multiple) with steep waves. The structures may be fixed, in forced motion or freely responses to the waves. They may be in head sea or with any incident angles and undergo a motion of 6 degrees of freedom (DOFs). Different applications may be found in our previous paper. Here only one case is presented for demonstration. Full nonlinear analysis on this kind of problems is extremely challenging and has been found to be very rare so far in public domain. A typical result for the two wiggly hull in the close proximity undergoing 6-Dof motion is reported in Fig. 6. For modelling floating bodies, an efficient semi-iterative scheme is devised (Ma and Yan, 2009)



(a) $\tau \approx 53.7$ (b) $\tau \approx 59.3$

Fig.6. Snapshots of the part of free surface for two different Wigley hulls (No.1: L_b=2.5,B_b=0.4 and D_r=0.3, $\theta_i = 0$; No.2: L_b=1.0,B_b=0.2 and D_r=0.15, $\theta_i = 20^o$; $\omega = 1.7691$, a=0.03; L=15, B=6)

The recent development of the QALE_FEM has enabled it to model wind effects and current effects on 2-D or 3D waves, including extreme waves. Some examples are given in Fig.7 and Fig. 8 for demonstration.



(a) wind speed= $4\sqrt{gd}$ (b) without wind

Fig.7. directional focusing freak waves in cases with or without wind at $\tau \approx 29\sqrt{d/g}$ (N_θ=30, $\theta_{\max}$=45^0, ω_i=1.0 and a_i=0.01, colorbar represents ς / a_i)

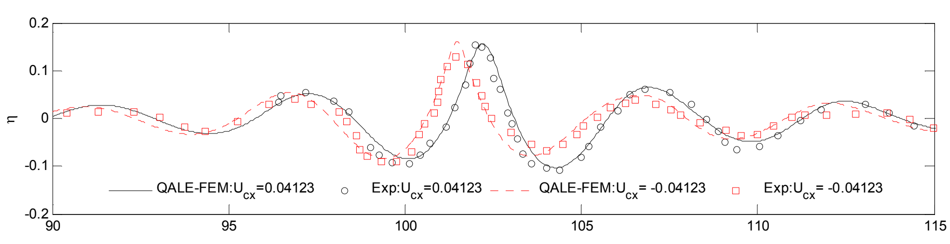


Fig 8 Wave histories recorded at focusing point having current effect (N=32, x_f = 12.5, τ_f = 102.5, ω_{min} = 1.0724; ω_{max} = 2.2846, the wave spectrum and the experimental data)

Violent wave simulation using the MLPG_R

The simulations of violent waves are the challenging task and are difficult to model based on mesh based methods. Due the recent advances in the computational technique, the meshfree methods are quite popular. One such meshfree method is the MLPG_R developed at City University. This method was under constant development in improving the pressure calculation that poses a major problem in meshfree methods. To show that the numerical model can tackle wide variety of problems like flood inundation with the structure, tsunami propagation to the coastline, long wave interacting with offshore wind energy structures, slamming effect on coastal structures, the dam breaking upon a rigid object is studied, which creates sudden impact force and flow separation, which are vital in understanding the wave structure interactions and it has been compared with the experimental results as in Fig. 6. Similar to SALE/QALE-FEM, the MLPG_R is applied for wide variety of water wave applications like 3D sloshing undergoing 6-Dof motions, interaction with fixed cylinder, breaking wave interaction over slope, freak wave simulations and currently being tested for wave - elastic structure interactions. More details can be found in Ma and Zhou (2009) and Zhou and Ma (2010).

Coupling QALE-FEM with MLPG_R for 2-D breaking waves (QALE-MLPG)

Thus, from the above discussion, it is clearly evident that the developed method at City University are applied to real complex practical problems, that are difficult to simulate using other methods. But nevertheless, each of these methods is having their own advantages and disadvantages. When one solves the FNPT, then we couldn't model breaking waves or consider viscous effects, but conserve the energy for long time simulations and efficient in computational aspects. Whereas, when one solves the NS, then loss of energy for long time simulation is inevitable and computational time will be tremendous, but it could handle violent waves and consider viscous effects. Thus, coupling these two theories will be very interesting, challenging and the resulting method could be free from any such disadvantages. But, the numerical coupling is not that much easier since these two methods are based on different theory, for strong interactions, one should use iterative schemes, whereas, for weak interactions (one way information to any one of the model) is much easier. In this paper, we will report this weak coupling interactions, where in the result from the FNPT is feed into the NS model for analyzing breaking wave over slope. For, this novel pressure based coupling algorithm is used, i.e., the pressure information from the QALE-FEM will be used at the point of interface for the MLPG_R method. The nondimensional length of the tank and slope are 15 and 1:15 respectively, the input solitary wave steepness is 0.45. Further, details about the experimental setup can be referred in Li and Raichlan (1998).

Thus, in this coupling the QALE-FEM is used to run the full simulations and the results are extracted before the breaking, i.e. over the sloping region and the MLPG_R is used to run only near to the breaking location. Thus, there is no need to run for the full computational domain, wherein, there will be loss in energy and higher computational time using MLPG_R. The result from this algorithm in comparison with the experimental results is depicted in Fig. 7a and b. The total CPU time for running this model using MLPG_R in the entire domain was 7.27 hrs (i.e,. from the wave paddle end) using 8GB RAM and 3.33GHz processor., whereas, for the coupled algorithm it was 0.64 hrs.(roughly, half the time for QALE-FEM and MLPG_R). The ratio of the CPU time is 11.34. Thus, one can get the higher resolution breaking details as well as avoid the excessive computational time by using the new coupled algorithm.

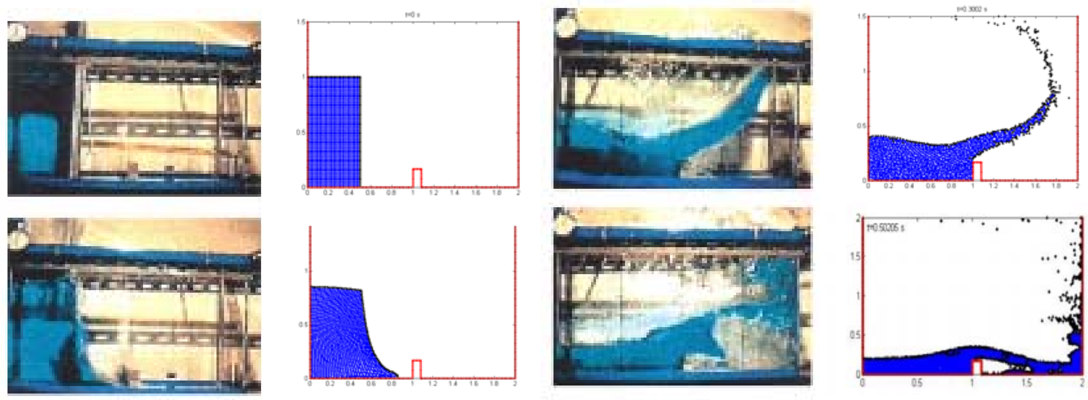


Fig. 6. Comparison between experiments (Koshizuka *et al.*1995) and MLPG_R results for dam breaking problem, showing violent wave interactions

CONCLUSIONS

The paper presents our recent effort in the state-of- art simulation for fully nonlinear free surface and violent wave interactions for fixed and floating structures that are complex to deal with and of practical importance. Further, the coupling of the model based on FNPT and NS are proved to be effective in computational time.

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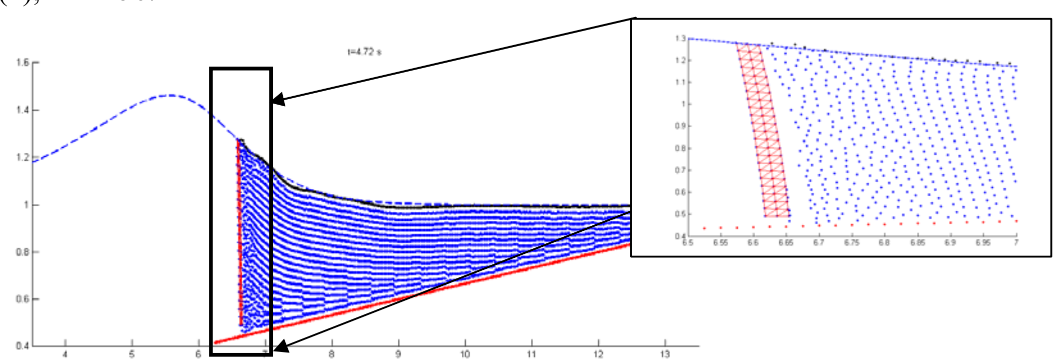

Fig. 7a. Pictorial representation of Coupling methodology, QALE-FEM solitary profile (blue dashed), MLPG_R (water particle, blue dot, free surface particle : black dot,Solid particle : red dot). Figure also shows the zoomed in view at the coupling location.

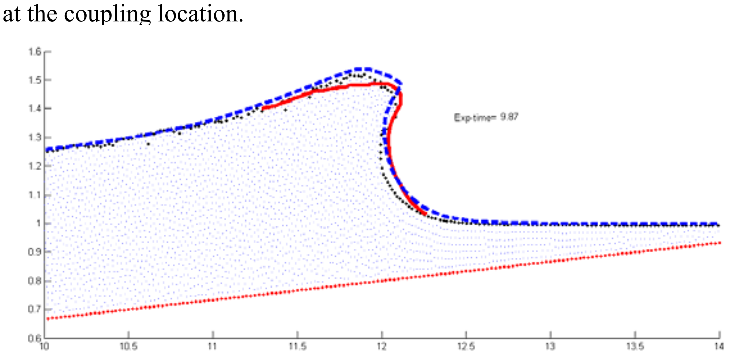

Fig. 7b. Comparison between experiments (red line), QALE-FEM (blue dashed) and QALE-MLPG (black dots) showing the over turning profile.