Mirajkar, A. B.; Patel, P. L.

Multiobjective Fuzzy Linear Programming Under Uncertain Resource Parameters

MULTIOBJECTIVE FUZZY LINEAR PROGRAMMING UNDER UNCERTAIN RESOURCE PARAMETERS

A B Mirajkar[1] and P L Patel[2]

ABSTRACT

Multi-objective fuzzy linear programming (MOFLP) problem has been formulated with three objectives, viz. maximization of net benefit(NB), employment generation(EG) and minimization of cost of cultivation(CC) to obtain optimal cropping pattern for the Kakrapar right bank main canal (KRBMC) command area, under Ukai-Kakrapar irrigation project, Gujarat, India. All said objectives are conflicting with one another; and uncertain due to resources associated with them. Uncertainty in the objectives and resources has been taken into consideration by fuzzification of both the objectives as well as resource parameters with three cases. In the first case, the objectives are treated as fuzzy, i.e. the membership function has been developed on the tolerance range for each objective. In second case, the right hand side of resource constraints are considered to be fuzzy, over the tolerance range; and in the third case the technological coefficients as well as resources have been treated as fuzzy. The level of satisfaction (λ) for the first, second and third cases have been obtained as, 0.503, 0.49 and 0.17 respectively. Study reveals that all possible uncertainties are necessary to be considered in obtaining the optimal solution for optimal usage of resources.

Keywords: Uncertainty, multi-objective fuzzy linear programming, level of satisfaction, optimal irrigation pattern, Kakrapar right bank canal command area.

1. INTRODUCTION

The irrigation efficiency improvement up to 20% is the main focus of Twelth five year plan as far as water resources development of India is concerned. The irrigation planning for water resources system is a complex phenomenon as it involves uncertainty in various resource parameters. Uncertainty associated with various resources can be considered by using fuzzy linear programming. Raju and Nagesh Kumar (2000) formulated the multi-objective fuzzy linear programming (MOFLP) with three conflicting objectives, viz. maximization of net benefits, crop production and labor employment, for Sri Ram sagar project, Andhra Pradesh, India. Gasimov and Yenilmez (2002) outlined the methodology for fuzzy linear programming with two cases, including fuzziness in technological coefficients and, in combination of resources and technological coefficients to tackle the uncertainty. Regulwar and Gurav (2010) proposed the irrigation planning with uncertainty by employing multi-objective fuzzy linear programming for four cases, viz. objectives, resources, technological coefficients and, combination of technological coefficient and resources as fuzzy for Jayakwadi irrigation project, Maharashtra, India and found the most realistic situation for the real

[1]Research Scholar, Department of Civil Engg., Sardar Vallabhbhai National Institute of Technology, Surat-395007,Guajarat, India (ashmirajkar@gmail.com, abm@ced.svnit.ac.in)
[2]Professor, Department of Civil Engg., Sardar Vallabhbhai National Institute of Technology, Surat-395007,Guajarat, India (premlalpatel1966@gmail.com, plpatel@ced.svnit.ac.in)

world problem amongst them. Mirajkar and Patel (2011) have studied fuzzy based optimal irrigation planning for Kakarapar Right Bank Canal Command Area, Gujarat, India, for two objectives and found the level of satisfaction (λ) as 0.501. Gurav and Regulwar (2012) presented the MOFLP considering fuzziness in resources and decision variables, with four conflicting objectives. The presented study was aimed to consider a situation closer to real world problem for sustainable development of an irrigation project by giving due considerations to both fuzziness in resources as well as decision variables in a MOFLP model. The present study deals with, multi-objective fuzzy linear programming problem, with three objectives, viz. maximization of net benefit(NB), employment generation(EG) and minimization of cost of cultivation(CC) to obtain optimal cropping pattern for the Kakrapar right bank canal command area, under Ukai-Kakrapar irrigation project, Gujarat, India. The Sugarcane, wheat and paddy along with nine other crops are grown in the command area in kharif & rabi seasons. All aforesaid objectives are conflicting with one another and uncertain due to resources associated with them. Uncertainty in the objectives and resources has been taken into consideration by fuzzification of both, objectives as well as resource parameters. Three cases, viz. I. Objectives fuzzy II. Resources fuzzy and III. Technological coefficients as well as resources fuzzy have been analyzed. In the first case, the membership function has been developed on the tolerance range of each objective. In second case, the right hand sides of resource constraints are considered to be fuzzy, over the tolerance range and, in third case, the technological coefficients as well as resources are treated as fuzzy.

2. METHODOLOGY AND MODEL DEVELOPMENT

2.1 Description of Study Area

The Ukai-Kakrapar project on river Tapi is the second largest multipurpose project in Gujarat, India. The Kakrapar weir is located at 29 kms, in the downstream of the dam for meeting the irrigation demand through Kakrapar left and right bank main canals, see Figure 1.

Kakrapar right bank main canal originate from Kakrapar weir and irrigate the land to the tune 113123 ha. The major crops being grown in the command area are sugarcane, paddy, wheat and juwar along with other crops such as vegetables, banana, groundnut, maize, bajri etc. The hot, kharif and rabi seasons, in the study area, are considered from March 15-June 14, June 15-Nov. 14 and Nov. 15-March 14 respectively. Under existing scenario, the water is supplied to the farmers on fortnightly basis and charged in terms of per watering per hectare of the crop area.

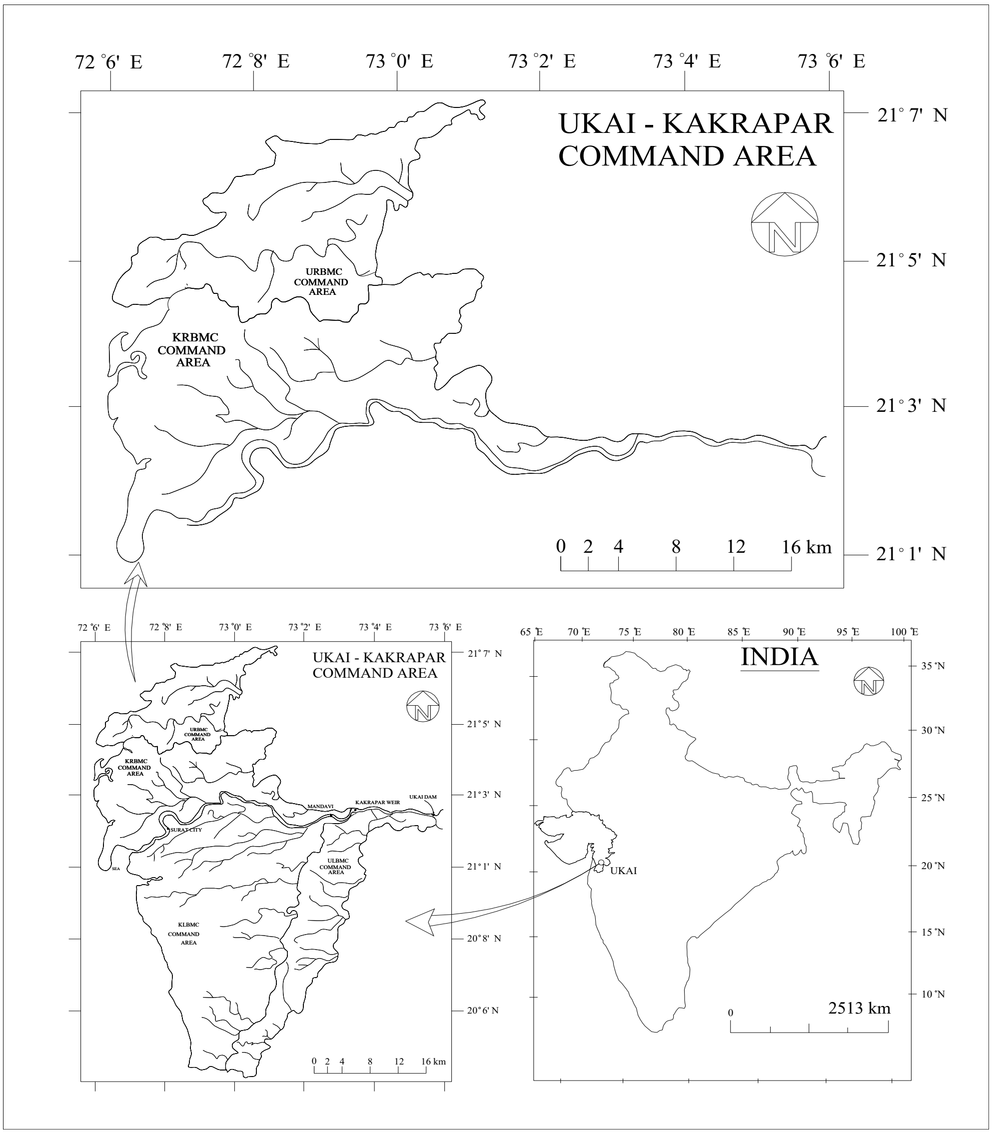

Figure 1 Index map of Ukai-Kakrapar irrigation project, Gujarat, India (Mirajkar and Patel, 2011)

Three objective functions, viz. maximization of net benefits, maximization of employment generation and minimization of cost of cultivation, have been considered in the present study to obtain optimal cropping pattern in KRBMC command area for given set of available water and land as resource constraints. Keeping in view availability of surface water vis-a vis ground water in the study area for irrigation use, surface water has only been considered in foregoing model formulation. The soil properties have been assumed to be homogeneous in nature in the command area and relationships between the parameters within the model are assumed to be linear in nature. Also, water available at the head regulator has been assumed to be sufficiently available (considering full irrigation) to meet the demand in the command area.

2.2 Objective Function

Three objective functions, in the present study are described in the succeeding paragraphs:

Maximization of Net Benefit (Z_1)

The objective function for maximization of net benefit can be expressed as,

$$\text{Max NB} = \sum_{i=1}^{n} B_i A_i \tag{1}$$

where, n = number of crops in a command area (12 in present case), B_i = net benefit coefficient for i^{th} crop in Rs./ha, which has been calculated after due deduction for cost of cultivation (includes watering charges, and labor, seeds, fertilizers, and field equipments cost is assumed 20% of the gross benefit) from gross benefit, A_i = area of i^{th} crop in ha, 1 = paddy(k) , 2 = juwar(k), 3 = vegetables(k), 4 = wheat(r), 5 = vegetables(r) , 6 = juwar(r), 7 = paddy(hw), 8 = groundnut(hw), 9 = cotton(ts),10 = vegetables(ts), 11 = sugarcane(p), 12 = banana(p). Here, k, r, hw, ts and p, represent kharif, rabi, hot weather, two seasonal, and perennial crops respectively

Maximization of Employment Generation (Z_2)

For the motivation and prosperity of the labor, the employment in the farming sector can be maximized and relevant objective function for the command area can be expressed as,

$$\text{Max EG} = \sum_{i=1}^{n} L_i A_i \tag{2}$$

L_i= labor requirement for i^{th} crop in man days during the season.
(The number of labor man days for each crop has been obtained from the data, collected from Navsari Agricultural University, Gujarat)

Minimization of Cost of Cultivation (Z_3)

For obtaining maximum benefit, the cost incurred in the cultivation of a crop is required to be minimized. The objective function for minimization of cost of cultivation can be expressed as,

$$\text{Min CC} = \sum_{i=1}^{n} C_i A_i \tag{3}$$

C_i = total cost of cultivation for i^{th} crop in Rs./ha., (which includes cost of watering, and cost of labor, seeds, fertilizers, and field equipments assumed to be 20% of gross benefit)

3. CONSTRAINTS

The objective functions Eqs. (1), (2) and (3) have been solved using crisp linear programming model subject to the following constraints:

3.1 Maximum Sowing Area Constraint

The sum of the total area to be allocated for various crops for different seasons in a command should not be more than the total Cultivable Command Area (CCA),

$$\sum_{i} A_i \leq CCA, \quad \text{for kharif, two seasonal and perennial crops} \tag{4a}$$

i = 1, 2, 3, 9,10,11,12

$$\sum_i A_i \leq CCA, \quad \text{for rabi, two seasonal and perennial crops} \tag{4b}$$

$$i = 4,5,6,9,10,11,12$$

$$\sum_i A_i \leq CCA, \quad \text{for hot weather and perennial crops} \tag{4c}$$

$$i = 7, 8, 11, 12$$

where, CCA = total cultivable command area.

3.2 Affinity Constraint

The area allocated to a crop in the model should be equal or less than maximum area, and greater or equal to minimum area to be sown from socio-economic considerations.
The maximum and minimum area requirements of various crops were obtained by comparing three cropping patterns, namely, planning stage (1966), development stage (1978) and existing cropping scenarios (2010) in the command area. The upper and lower bounds of crop area can be expressed as,

$$A_i \leq A_{i\,\max} \tag{5a}$$

$$A_i \geq A_{i\,\min} \tag{5b}$$

Here, $A_{i\ max}$ and $A_{i\ min}$, respectively, denote the maximum and minimum area requirement for i^{th} crop in the command area.

3.3 Water Availability Constraint

The water required for the crop growth in a command area has to be met through releases from the source, i.e., Kakrapar weir through its right bank canal. The water allocation constraint, mathematically can be expressed as,

$$\sum_{i=1}^{n} NIR_i A_i \leq \sum_{t=1}^{12} R_t \eta_s \tag{6}$$

Here, NIR_i = total net irrigation requirement of i^{th} crop during the base period, (calculated using Modified Penman's Method) with due deduction to effective rainfall, R_t = release, available for the crops at the head regulator of the weir for irrigation in a year (t=1-12). η_s = surface water conveyance efficiency of canal, and has been taken as 0.54 for the study area. The data related to availability of releases, surface water conveyance efficiency and NIR (Computed using modified Penman method) for KRBMC was obtained from Surat Irrigation Circle, Surat, Gujarat, India.

3.4 Non Negativity Constraints

All the variables in the model should be non-negative.

$$L_i >= 0,\ C_i >=0,\ A_i>=0,\ C_{sw} >=0,\ R_z>=0 \tag{7}$$

4. MULTI-OBJECTIVE FUZZY LINEAR PROGRAMMING (MOFLP)

4.1 Case I: Fuzzy Linear Programming with Fuzzy Objective Functions ($\tilde{Z}$)

The objective function for Multi-Objective Fuzzy Linear Programming (MOFLP), mathematically, can be expressed as (Li *et al.,* 2006),

$$\text{Max } Z= [c_1x,\ldots,c_Nx]^T=[Z_1(x),\ldots,Z_N(x)]^T \tag{8}$$

$$\text{s.t. } x \in X, \qquad X = \{x \in R^n : Ax \le b, x \ge 0\} \tag{9}$$

where $A = (a_{ij})_{m\times n}$, $c_i \in R^n$ $(0 \le i \le N), b \in R^m$. Here, $Z_1, Z_2 \ldots Z_N$ represent the objective functions involved in a particular problem (N=3 for present study), (a_{ij}) is the constraint coefficient for i^{th} constraint and j^{th} variable in the matrix of size $m\times n$. On solving the aforesaid equation (Eq.8), all the objectives may hardly reach to their optimal solutions, subjected to the given set of constraints (Eq.9). Accordingly, for all the objective functions, the decision maker have to choose some efficient solution as the final one depending upon overall level of satisfaction for all the objective functions together. The methodology for solving Fuzzy Linear Programming (FLP) was evolved by Zimmermann (1996) which had given an effective way of measuring level of satisfaction in MOFLP. The minimum ideal solution of the objective function can be considered as the initial solution of the objective functions, and is given by,

$$Z^- = [Z_1^-,\ldots,Z_N^-] = [\min(Z_1(x)),\ldots,\min(Z_N(X))] \tag{10}$$

The membership function for the degree of satisfaction of each objective function can be defined as,
For maximization type of objective function:

$$u_k(x) = \begin{cases} 0 & Z_k(x) > Z^-{}_k \\ \dfrac{Z_k - Z_k^-}{Z_k^+ - Z_k^-} & Z^-{}_k < Z_k(x) \le Z^+{}_k, \quad k = 1,\ldots,N \\ 1 & Z_k(x) \ge Z_k^+ \end{cases} \tag{11a}$$

For minimization type of objective function:

$$u_k(x) = \begin{cases} 1 & Z_k(x) > Z^*{}_k \\ \dfrac{Z_k(x) - Z_k^-}{Z^*{}_k - Z_k^-} & Z^-{}_k < Z_k(x) \le Z^*{}_k \quad k=1,\ldots,N \\ 0 & Z_k(x) \le Z^-{}_k \end{cases} \tag{11b}$$

Here, Z_k^+ and Z_k^-, respectively, represent maximum and minimum values of k^{th} objective function while considering the solutions of all objective functions using crisp linear programming approach and (Zimmermann, 1996) can be expressed as,

$$\max \lambda \tag{12}$$

$$\text{s. t. } \lambda \le u_k(x), \qquad k = 1,\ldots N \tag{13}$$

$$\lambda \in [0, \ 1] \qquad x \in X \tag{14}$$

with the existing set of constraints. Here, λ is newly introduced variable, which is to be maximized; converting each objective function into constraints; and its value varies between 0 and 1.

Steps to be followed in Fuzzy Linear Programming with Fuzzy Objective Functions ($\tilde{Z}$):
Solve crisp linear programming problem for each objective.
With results obtained in the step 1, prepare the pay off matrix.
From the step 2, upper and lower bounds of each objective (Z_U and Z_L) are obtained.
Formulate the linear membership function for each objective with (Z_U and Z_L).
Formulate equivalent FLP with newly introduced variable λ and membership functions.
Solve the MOFLP for compromised solution, i.e. level of satisfaction λ.

The algorithms for Case-II and Case-III are given below (for detailed mathematical part Klir and Yuan, 2007 can be refereed).

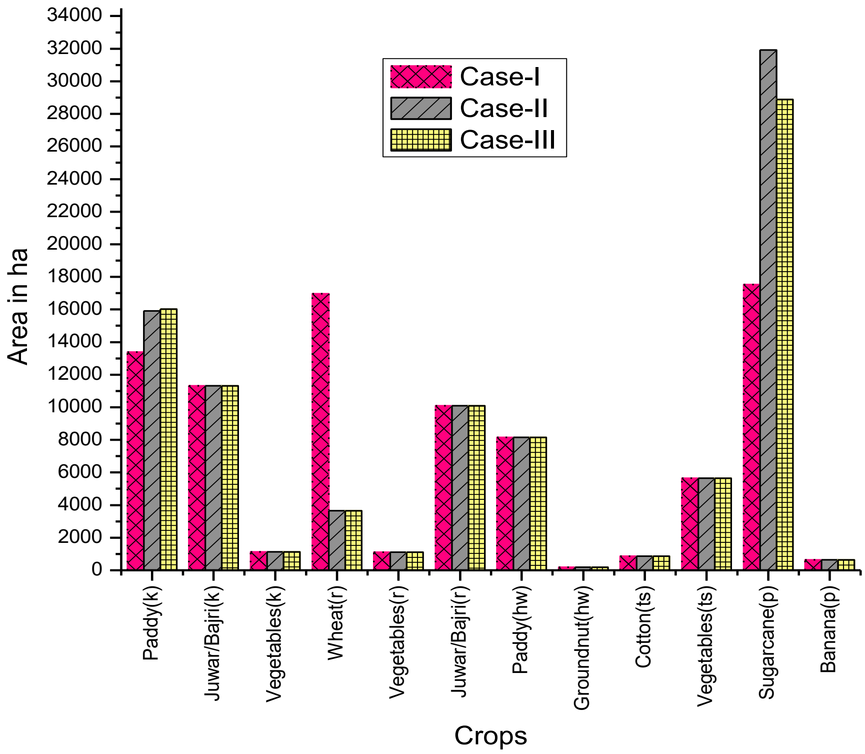

Figure 2 Area allocation by three different methods

4.2 Case II: The Fuzzy Linear Programming Problem, with Fuzzy Right-Hand Side ($\tilde{b}_i$)

Solve crisp linear programming problem for lower limit of *(bi*) resources, for each objective function.
Solve crisp linear programming problem for upper limit of *(bi+pi)* resources, for each objective function. Where p_i is the tolerance range.
From the results obtained in the steps1 and 2, obtain the values of each objective function and prepare the pay off matrix.
Mark the z_u and z_l as upper and lower bounds of each objective function from the pay off matrix obtained in step 3.
Considering the upper and lower bounds for each objective, develop the linear membership function for objectives as well as for resources for the system under study.
Formulate equivalent LP and MOFLP and solve the model.

Formulate equivalent FLP with newly introduced variable λ and membership functions in the form of constraints.
Solve the MOFLP for compromised solution, i.e. level of satisfaction λ.

Table 1 Areas allocated to different crops (in ha) LP and MOFLP models

Crop No. (*i*)	Crops	Solutions obtained by various Models					
		Crisp Linear Programming Individual solutions			MOFLP Models		
		NB	EG	CC	Case-I	Case-II	Case-III
1	Paddy(k)	13100	16965	13100	13386.38	15909.36	16021.41
2	Juwar/Bajri(k)	11310	11310	8100	11310	11310	11310
3	Vegetables(k)	1131	1131	690	1131	1131	1131
4	Wheat(r)	3654	3654	3654	16965	3654	3654
5	Vegetables(r)	1120	1120	1120	1120	1120	1120
6	Juwar/Bajri(r)	10091	10091	10091	10091	10091	10091
7	Paddy(hw)	8145	8145	8145	8145	8145	8145
8	Groundnut(hw)	192	192	192	192	192	192
9	Cotton(ts)	860	860	860	860	860	860
10	Vegetables(ts)	5655	5655	1335	5655	5655	5655
11	Sugarcane(p)	38337.21	37350.34	4998	17529.92	31925.56	28892.81
12	Banana(p)	633	633	633	633	633	633
Total		94228.21	97106.34	52918	87018.3	90625.65	87705.22
Irrigation Intensity in %		83.30	85.84	46.78	76.92	80.11	77.53

Table 2 Pay off matrix with all objective values for various models

		Solutions obtained by various models					
		Crisp Linear Programming Individual solutions			MOFLP Models		
		NB	EG	CC	Case-I	Case-II	Case-III
Max	Net benefit (NB) in million Rs	5593.764524^U	5559.323244	1552.544730^L	3585.053539	4931.188150	4593.843179
Max	Employment generation (EG) in thousand man-days	14631.2498	14824.0187 ^{U}	5499.487^L	10189.2082	13430.52535	12728.72921
Min	Cost of cultivation(CC) in Million Rs	3457.538056^U	3454.374170	1076.746788^L	2260.131574	3084.500848	2888.820051
Case I (Lamda)					0.503 (λ)		
Case II (Lamda)						0.49(λ)	
Case III (Lamda)							0.17(λ)

suffix ^{U} and ^{L} are the upper and lower bounds of the objective functions.

4.3 Case III: Fuzzy Linear Programming Problem with Fuzzy Technological Coefficients as well as Fuzzy Right-Hand-Side Numbers

Solve individual crisp objective function as LP with (a_{ij}+d_{ij}) and resources bi, where a_{ij} is technological coefficients and d_{ij} is the tolerance range for the same.
Solve the LP with a_{ij} and (b_i+p_i) same as that in step 1.
From the results of LP problems in step 1and step 2, prepare the payoff matrix.
Comparing the values of objectives, and mark the upper and lower bounds (z_u and z_l)
Establish the linear membership function over the tolerance range for each objective function, also for fuzzy technological coefficients and fuzzy right hand side for the system under consideration.
Develop the equivalent NLP model for multi-objective fuzzy non linear programming model (MOFNLP)
The above formulation which is nonlinear can be converted into linear by varying λ between 0 and 1.
Solve above to get the level of satisfaction λ as a compromised solution.

5. RESULT ANALYSIS

Three objectives, maximization of net benefits (NB), employement generation (EG) and minimization cost of cultivation (CC) have been considered in the present study. Crisp linear programming models for each objective have been solved. Table 1 shows the area allocated by different models (Figure 2). Table 2 shows the pay off matrix with maximum and minimum calculated values of each objective function. The irrigation intensity, if compared (Table 1) obtained in case I, II and III, it is more in Case II, where the resources are considered to be fuzzy with degree of satisfaction (λ) as 0.49. On otherhand, in case III, where resources and technological coefficients are treated fuzzy, gives very low value of level of satisfaction (λ) as 0.17.

The area to be grown for wheat crops is higher in case I as compared to other cases, i.e. case II and case-III (Figure 2). Similarly from Figure 2 and Table 2, it can be seen that area allocated by the model for sugarcane is higher in case II than other two cases, which is also true for the field conditions as many farmers are orienting themselves for sugarcane, being a cash crop.

Further, in case II, maximum net benefit and maximum employement generation have been obtained out of all three cases considered in the present study.

All the three models, i.e.case I, case II and case III, allocate the same area of crops except for rabi wheet, kharif paddy, and perennial sugarcane, may be due large variation in later crops and higher benefit coefficient. Keeping in view, trend of crop area allocated, maximum net benefit, employment generation derived and level of satisfaction (λ) obtained; it would be appropriate to consider case II for actual application.

6. CONCLUSIONS

Three objective functions, namely, maximization of net benefit, maximization of employment generation and minimization of cost of cultivation; resource parameters, namely, availability of water and land; and technological coefficients and resources parameters have been considered fuzzy as case I, case II and case III respectively. MOFLP, for all the three cases, has been applied and corresponding values level of satisfaction (λ) and have been found to be 0.53, 0.49, 0.17 and 3189.975883, 4931.188150, 4593.843179 respectively. In case II, relatively maximum irrigation intensity, maximum net benefits, maximum employment generation, better degree of satisfaction and realistic cropping pattern have been obtained; and can be recommended for actual application in the field. Intergovernmental Panel on Climate Change (IPCC) suggested recent trends in hydro-meteorological condition over the past 30 years. This study is the part of global efforts to create resilience against uncertainty arising in water sector due to the variation in climate, in developing countries like India.

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REFERENCES

Gasimov, R.N. and Yenilmez, K. (2002) "Solving Fuzzy Linear Programming Problems with Linear Membership Functions", Journal of Math Turk, Vol. 26, pp. 375-396.

Gurav, J.B. and Regulwar, D.G. (2012) "Multi-objective Sustainable Irrigation Planning with Decision Parameters and Decision Variables Fuzzy in Nature", Journal of Water Resources and Management, Springer, Vol. 26, pp. 3005-3021.

Klir, G.J., Yuan, B. (2007) Fuzzy sets and fuzzy logic theory and applications, Prentice Hall of India Private Limited, New Delhi.

Mirajkar, A.B. and Patel, P.L. (2011) "A Fuzzy Based Optimal Irrigation Planning for Kakrapar Right Bank Canal Command Area, Gujarat, India", Journal of Hydraulic Engineering-Special Issue, Indian Society for Hydraulics (ISH), Vol.17, No.3, pp. 43-50.

Regulwar, D.G. and Gurav J.B. (2010) "Irrigation Planning Under Uncertainty - A Multiobjective Fuzzy Linear Programming Approach", Journal of Water Resources Management, Springer, DOI10.1007/s11269-010-9750-5

Srinivasa Raju, K. and Nagesh Kumar, D. (2000) "Irrigation Planning of Sri Ram Sagar Project Using Multi Objective Fuzzy Linear Programming", Journal of Hydraulic Engineering, Indian Society for Hydraulics (ISH), Vol. 6, pp. 55-63.