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ANALYTIC ELEMENT METHOD AND GENETIC ALGORITHM BASED MODELS FOR ESTIMATING STREAM BED RESISTANCE

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***Abstract**: The parameter estimation improves understanding of the interaction between river bed aquifer and stream under human intervention which is needed for effective management of water resources. Analytic Element Method (AEM) has been found to be a good choice for aquifer flow simulation. Genetic Algorithms (GA), which has been successfully applied to groundwater management problems, has been found more efficient for parameter estimation than gradient based methods. In this study, a stream aquifer simulation model based on Analytic Element Method (AEM) has been developed and elements for discharge specified well, head specified well, and resistance specified line sink are implemented. For estimating hydraulic conductivity and stream bed resistance for steady state stream aquifer interaction problem, a GA model has been developed. Further the coupled AEM-GA model has been developed by combining the AEM with GA model. Both the AEM and GA models are verified by using mathematical problems with known solutions. The Sum of Squared Error (SSE) function is computed as fitness function. The GA evaluates these fitness values and performs selection, cross over and mutation to generate a new population. The procedure is executed for specified number of generations. Finally the unknown parameters are estimated by the model with best fitness values. The AEM-GA model has been applied to a field problem for estimation of hydraulic conductivity and bed resistance. The model performance has been found to be satisfactory.*

***Keywords**: Analytical element method; Genetic algorithm; stream-aquifer interactions.*

INTRODUCTION

The growth and development in urban and industrial water use have led to situations in few areas that heavily depend upon riverbed pumping to meet the water demands under low surface water flow conditions. Riverbed pumping can be carried out by series of tube-wells, radial collector wells or infiltration galleries. To design such structures or to operate such structure individually or in a group, an understanding of the interaction between river bed aquifer and stream under

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human intervention is needed to predict the response of flow process under various scenarios. The flow models help in developing strategy to utilize the water resources in an efficient manner. The selection of model depends on the scale of the model and the objectives to be achieved. Accordingly, model studies can be carried out using theoretical models (analytical solution, numerical models), physical models or by field investigations. As very few analytical solutions are available and physical and field investigations are cumbersome, investigators mainly use numerical models to simulate flow and transport in groundwater – surface water interaction problems. Use of numerical models involving finite difference, finite element and boundary element methods has increased in the last few decades due to the growth in computer technologies and development of improved algorithms. A major problem in the numerical modeling of surface water groundwater systems is that of obtaining reliable estimates for the parameters that appear in the various governing equations, as many of these parameters are difficult to measure directly, given that most of the subsurface is inaccessible. Hydraulic conductivity, storage coefficient, stream bed resistance are some of them. The use of simulation models have been extended for estimating such parameters by coupling simulation model with optimization tool.

Finite difference method (FDM) and finite element method (FEM) are the most commonly used numerical methods for the solution of flow problems (Wang and Anderson, 1995). However these methods increases in complexities and computational efforts for certain class of problems. The designer and decision makers prefer the modeling tools that can simulate the field condition with flexibility and convenience. Analytic Element Method is found to have certain capabilities which overcome the difficulties associated with grid based algorithms (Fitts, 2002).

For the determination of aquifer parameters, other than field investigations, optimization methods such as GA and particle swarm optimization (PSO) can be used. These heuristic methods have been increasingly used for global optimization problems and outperform the conventional gradient based methods in many areas. The aquifer parameters can be predicted by using optimization model by minimizing the sum squared error functions. The combined simulation and optimization model accounts for the complex behavior of the groundwater system and identifies the best management strategy under consideration of the management objectives and constraints. Simulation-optimization groundwater management models have been developed for a variety of applications, such as restoration of contaminated groundwater, control of aquifer hydraulics, allocation of ground and surface water resources, and evaluation of groundwater policies (Fitts, 2002). Further AEM can be used as a numerical engine along with recent optimization techniques such as genetic algorithm (GA) to develop a management model of stream aquifer systems with convenience.

In this study, a stream aquifer simulation model based on Analytic Element Method (AEM) has been developed and elements for discharge specified well, head specified well and resistance specified line sink are implemented. Also a binary coded GA model has been developed for optimization. Further both AEM and GA model have been coupled as a simulation optimization model. For estimating hydraulic conductivity and stream bed resistance, the coupled AEM-GA model has been developed by combining the AEM with binary coded Genetic Algorithm

Optimization. The AEM-GA model has been used to analyze the field pumping of Mahi riverbed aquifer (Gujarat, India) system.

ANALYTIC ELEMENT METHOD

The Analytic Element Method (AEM) was developed by Strack(1989) to simulate the aquifer flow problems with an accuracy that was similar to analytic solution and ease of representing hydro-geologic elements similar to the finite elements or finite difference techniques. Strack(1989) and Haitjema(1995) gave detailed mathematical descriptions of the analytic elements and their numerical implementation. The AEM has been applied successfully to many field problems of varying scales, and is growing rapidly in terms of accuracy of simulation, capabilities and range of applicability (Hunt, 2006). It has also been applied for solving problems of pumping of shallow riverbed aquifer (Patel et al., 2010). The method is based on Dupuit–Forchheimer assumption which describes shallow confined flow or unconfined flow as two-dimensional flow. Strack introduced a discharge potential, Φ(Eq. (1)) which can always be expressed in terms of the piezometric head ϕ as,

$$\Phi = \frac{1}{2} k\phi^2 \qquad (\phi \le H) \tag{1}$$

where k=hydraulic conductivity, H=thickness of the aquifer and ϕ=piezometric head.

The discharge potential defined above satisfies the continuity equation and gives Laplace equation (Haitjema, 1995). The hydro-geologic features are represented by analytic functions which satisfy the Laplace equation known as analytic element. Most elements are considered in the form of complex potential, $\Omega = \Phi + i\Psi$, whose real part is the potential function $\Phi = \Re(\Omega)$, and the imaginary part is the stream function, $\Psi = \Im(\Omega)$.
The complex potential of a well located at $z_0 = x_0 + iy_0$ with the pumping rate of Q_w is as follows:

$$\Omega_w = \frac{Q_w}{2\pi} \ln(z - z_0) + C \tag{2}$$

and the potential function is:

$$\Phi_w = \Re(\Omega_w) = \frac{Q_w}{4\pi} \ln[(x - x_0)^2 + (y - y_0)^2] + C \tag{3}$$

The linesink element, to represent edge of river channel, with end points z_1 and z_2 in complex plain and discharge per unit length or intensity, σ is defined by Haitjema(1995) as,

$$\Omega_{ls} = \frac{\sigma}{2\pi} e^{-i\alpha} [(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2) + (z_1 - z_2)] + c \tag{4}$$

where, α is the angle between the line sink and the positive x-axis and z(x+*i*y) is independent variable. The Eq.(3) and Eq.(4) can be conveniently expressed as,

$$\Phi_e = \Re(\Omega_e) = F_e(x, y) P_e + c \tag{5}$$

Here $F_e(x,y)$ is influence function for element 'e' which purely depends on geometric condition and location of an element and P_e is the strength parameter of the element 'e' which is equal to discharge or discharge intensity and *c* is integration constant.

To obtain a full description of hydro-geological setup, these potential functions must be superimposed for each element. Due to the linearity of the *Laplace*-equation it means a simple sum of the individual potentials.

So the comprehensive potential describing the aquifer is:

$$\Phi = \Phi_w + \sum \Phi_{ls} + C \tag{6}$$

Equation-(6) is applied to control points of all line sink elements for which heads are known. It is also applied to one reference point with known head. Thus unknown strength parameters are found. Once the strength parameters are obtained, Equation (6) is used to find discharge potential at location of observation wells in the aquifer. The value of discharge potential is then converted to head.

GENETIC ALGORITHM

Hydrogeologic system comprises of many parameters such as hydraulic conductivity, storage coefficient, recharge rate, pumping rate, leakance, riverbed conductivity etc. Field investigations are sometimes insufficient to estimate their values. Parameter estimation studies are generally carried out using conventional optimization techniques, such as gradient based methods, in conjunction with flow simulation model. Since past few years, evolutionary algorithms such as Genetic Algorithm (GA) have been increasingly applied to achieve optimal solution. GA has the following general steps (Deb,1995):

1) Begin with a population of individuals with alleles generated at random.
2) Determine the fitness of the individuals by decoding their chromosomes and applying the fitness function
3) If an optimization criterion is not met then creation of new generation starts.
4) Select parents for next generation with probability proportional to fitness.
5) Mate selected parents to produce offspring, with possible mutation, to populate the new generation.
6) Repeat 2 - 5 until satisfactory solution is found.

In the present study, a model has been developed using GA for parameter estimation. The model is coupled with AEM model to test the fitness of parameters. If h_o is the observed head in an observation well and if h_c is the computed head using the AEM model for the given model area, then the sum of square errors (SSE) is calculated as

$$SSE = \sum_{i=1}^{n} (h_{o,i} - h_{c,i})^2 \tag{7}$$

where n=total number of observed head data.

The above equation of SSE is used as the objective function in the present study, and the target is to minimize SSE. As the GA is naturally a maximization algorithm with a positive fitness value, the objective function value (SSE) is translated into the fitness function. As it is known that GA requires a fitness value to compare the different solutions and then to go for achieving more suitable or say global optimum solution. This fitness value is calculated using the transformation function. The following fitness function is used for such transformation.

$$F(x) = \frac{1}{1 + SSE} \tag{8}$$

AEM-GA MODEL

To address the problems related to stream aquifer systems, a model AEM has been developed in Matlab environment by using the analytic element method described above to deal with stream aquifer interaction. The AEM possess features to represent streams and some man made features such as wells and horizontal wells in an effective manner so as the stream aquifer processes can be accurately modeled. Also based on the GA procedure mentioned, the GA model has been developed. The developed AEM model has been verified with analytical and other numerical solutions (Patel et al, 2010) and found to be satisfactory. The developed GA model has been verified with the example of Himmelblau function (Deb, 1995) and found to be satisfactory.

AEM-GA model has been developed by combining the AEM with GA. AEM-GA model instantiates population of hydraulic conductivity of riverbed aquifer and stream bed layer. Using these parameters, AEM procedure is followed for computation of heads at observation wells. The Sum of Squared Error (SSE) function is computed followed by its conversion into fitness function value. The GA evaluates these fitness values and performs selection, cross over and mutation to generate a new population. The procedure is executed for specified number of generations. Finally the unknown parameters are estimated by the model with best fitness values. The flow chart of the AEM-GA model is shown in Fig.1.

APPLICATION

A pumping test is carried out near Mahi River (India) having thin sediment deposit underlain by shallow coarse sand and gravel aquifer. The locations of pumping well and observation wells with respect to river water channel are shown in Fig. 2. The well discharge was measured as 150 m^3/h and heads at observation wells were obtained as shown in Fig.2. The AEM model is constructed with discharge specified well and resistance specified linesink elements to investigate the hydraulic conductivity of bed aquifer and hydraulic conductivity of thin relatively less pervious stream bed layer. The river channel was considered as 50 m wide and having 2 cm thick sediment layer at the bed of channel.

The AEM-GA model was run to predict hydraulic conductivity of river aquifer and hydraulic conductivity of stream bed material. The population size was selected as 20. The mutation and survival are set to 0.05 and 1 respectively. After 20th generation, the results are given in Table 1. The hydraulic conductivity values of bed aquifer and stream bed material are found to be 20.39 m/h and 3.817727E-003 m/h respectively. The SSE is 4.328415E-005 at the termination of model simulation. It is observed that the hydraulic conductivity value of river bed aquifer predicted by AEM-GA model is reasonably matching with the available field test values obtained at different nearby locations. It is also interesting to note that the hydraulic conductivity of aquifer and hydraulic conductivity of bed material varies in different ranges. The thin sediment layer offers resistance to induced recharge due to pumping which is adequately investigated by the model.

CONCLUSIONS

In this study, an AEM model has been developed for groundwater flow simulation. Further a GA based model ahs been developed for parameter estimation. Both AEM and GA models are coupled to get a simulation-optimization model. The coupled model has been applied for the parameter estimation of a field problem. The results are verified with available field observations and found to be satisfactory. Analytic Element Method is very simple to construct a model. As no elements or grid generation is involved, the data requirement is minimum. The hydro-geologic elements such as streams, wells, etc can be easily constructed. The solution obtained by AEM is fast and precise. GA on other hand provides optimal solution with flexibility. The parameters are estimated using combination of AEM and GA with reasonable accuracy. Thus coupling of GA code with AEM provides very powerful tool for parameter estimation.

Table 1. Comparison of results of estimation of 'k' using AEM-GA model

Parameter	Value
Max generations	20
Population size	20
Mutation	0.05
Survival	1
Output	
Actual generations	20
Hydraulic conductivity of aquifer (m/h)	2.039054E+001
Hydraulic conductivity of stream-bed (m/h)	3.817727E-003
Objective Function	4.328415E-005

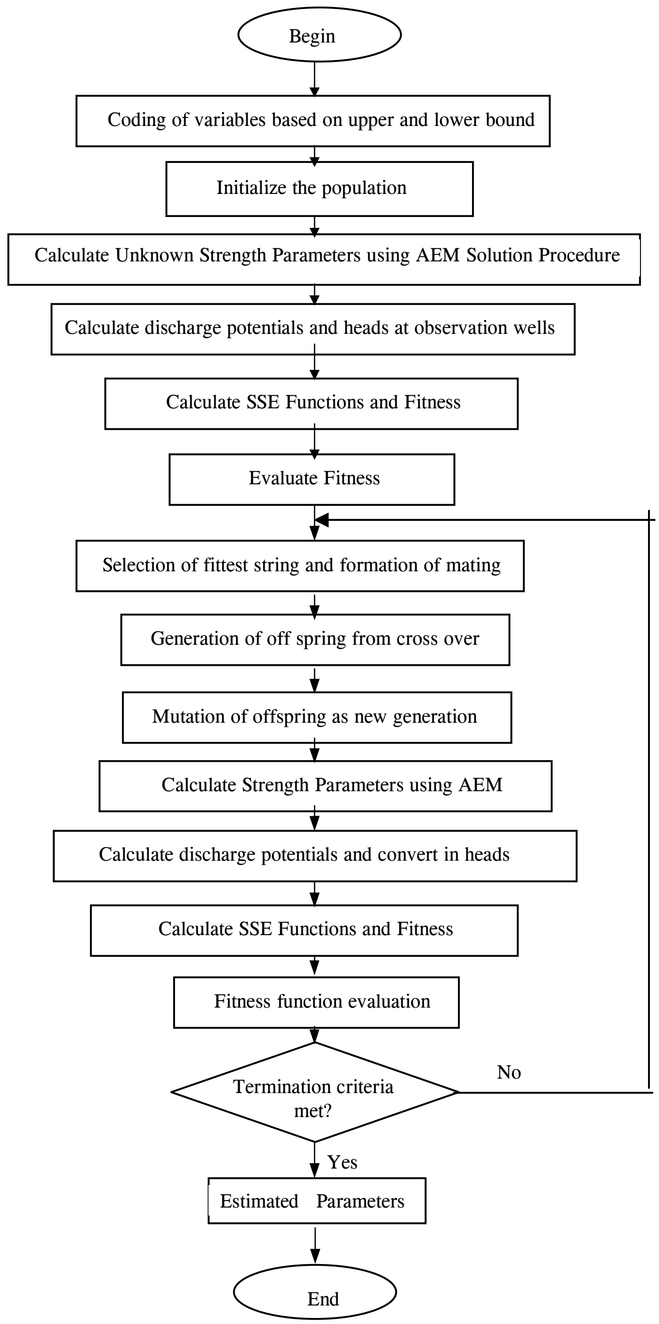

Fig. 1 Flow chart of AEM-GA Model for parameter estimation

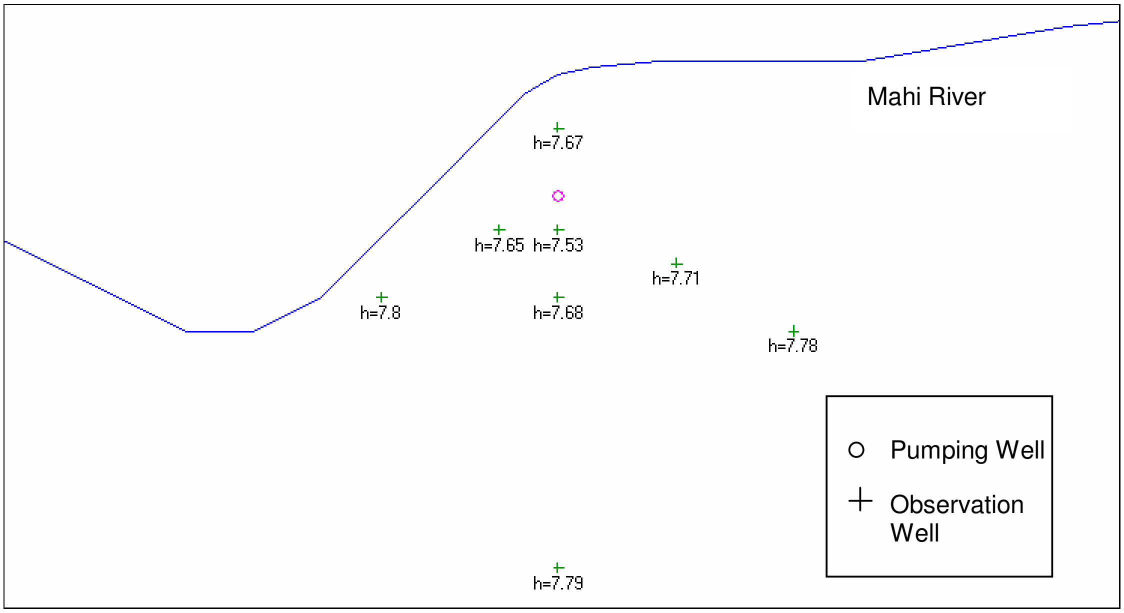

Fig. 2 AEM model for analyzing pumping test data at Mahi River.

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