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MODELING SEDIMENT PARTICLE MOVEMENT IN REGULAR AND EXTREME FLOW EVENTS USING A STOCHASTIC DIFFUSION JUMP PARTICLE TRACKING MODEL

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ABSTRACT

Movement of sediment particles in regular and extreme flow events can be described by a stochastic diffusion jump process. The stochastic differential equation (SDE) in the proposed stochastic diffusion jump model classifies the movement of particles using three major terms including a mean drift motion, a random Brownian type turbulent motion and a Poisson jump term caused by the occurrence of the hydrologic extreme events. The random term is represented by the Wiener process. The jump term is modelled as the Poisson process. The magnitude of particle movement in response to extreme flow events, characterized as the Poisson jump, depends on the hydraulic characteristics of extreme events and the properties of the sediment particles. The frequency of occurrence of the extreme events in the proposed model can be explicitly accounted for in the evaluation of movement of sediment particles. The particle relaxation time is defined by the time needed for a particle to adjust to a change in its ambient environment. Herein it can also be applied to sediment particles as the time needed for a particle to move from the regular state to the extreme flow state. Since sediment particle size is normally larger than the fluid particle size, there is a lag in time in response to the extreme event because of the particle inertia effect. The proposed particle tracking model is able to account for such delayed responses in sediment transport modeling, and to test the validity of a commonly adopted assumption, the Reynolds analogy. The ability to incorporate such phase lags is considered an advance in sediment transport modeling. One examples is presented to illustrate the realizations of sediment transport. The mean and variance of particle trajectory can be obtained from simulations of the proposed model.

Keywords: stochastic diffusion jump processes, hydrologic extreme flow, sediment transport, relaxation time, particle trajectory

1. INTRODUCTION

Flow dynamics and sediment transport bring about morphological changes as well as movements of contaminated materials, which can cause severe problems. Especially, the problems can become serious when extreme events such as floods occur. The extreme events enhance the unsteadiness of flows and magnify the lag effects of sediments in response to extreme flows. Despite the seriousness of problems, the unsteady flow and sediment transport have been traditionally approximated at steady or quasi-steady situations in the existing numerical models. Additionally, in general sediment transport modeling, deterministic differential equations for transport are first established and then solved analytically or numerically (Syvitski et al., 1995; Arnold et al., 1998; Moulin et al., 1998). In recent years, a few stochastic modeling approaches have attempted to demonstrate the essential stochastic characteristics of sediment transport processes. However, these studies mostly focus on the statistical properties of hydrological features such as stream flow or river morphology, not those of the sediment particle transport (Singer et al., 2004; Van Vuren et al., 2005). Although these models are also a stochastic approach related to the flood effect and morphological changes, they concentrated on the Monte Carlo simulation of sediment movements with the morphological response. To our knowledge, the stochastic model of discrete sediment particle transport is unprecedented.

The stochastic approach proposed in this study is similar to the particle tracking model in the sense that it deals with sediment particles as a collection of discrete particles. Moreover, compared to the existing approaches, there are mainly two distinct points in the proposed approach. Firstly, it can model the stochastic movement of sediment particle; thus, it can take parameters such as diffusion coefficients of the probability distribution into consideration so as to effectively model the random term due to turbulence. Secondly, it can consider the impact of extreme flow events such as floods on sediment transport by treating the occurrence of extreme events as a stochastic process. The impact of extreme flow events on particle movement is not negligible; i.e., the particle movement is rapidly altered by the extreme flow events. Therefore, the proposed model that includes the jump term for extreme events may enhance the prediction of sediment transport models substantially.

The extreme events can be considered an external force that can accelerate flow, expressed by a change of particle velocity between before and after the extreme event during the relaxation time. The relaxation time is defined by the time taken by a particle to adjust to a change in environment. Although the lag effects have recently started being considered in sediment transport models (Wu et al., 2006), the relaxation time has not been thoroughly examined to date. Therefore, the main purposes of this paper are to define the relaxation time during extreme events in a river flow and to establish a stochastic diffusion jump particle tracking model for sediment movement when the extreme flow occurs.

2. STOCHASTIC MODELS

2.1 Stochastic Diffusion Model

Particle movement in a flow can be delineated by a stochastic diffusion model. The Langevin equation is a stochastic diffusion equation describing the Brownian motion. The Langevin equation of particle displacement is

$$d\mathbf{X}_t = \underbrace{\bar{\mathbf{u}}(t,\mathbf{X}_t)dt}_{\text{drift term}} + \underbrace{\boldsymbol{\sigma}(t,\mathbf{X}_t)d\mathbf{B}_t}_{\substack{\text{random term}\\ \text{due to turbulence}}} \quad \text{or} \tag{1a}$$

$$\mathbf{U} = \frac{d\mathbf{X}_t}{dt} = \underbrace{\bar{\mathbf{u}}(t,\mathbf{X}_t)}_{\text{drift term}} + \underbrace{\boldsymbol{\sigma}(t,\mathbf{X}_t)\frac{d\mathbf{B}_t}{dt}}_{\substack{\text{random term}\\ \text{due to turbulence}}} \tag{1b}$$

where $\mathbf{X}_t$ is the position or trajectory of a particle = $\left[x(t)\ \ y(t)\ \ z(t)\right]^T$, $\bar{\mathbf{u}}(t,\ X_t)$ is the drift velocity vector, $\boldsymbol{\sigma}(t,\ X_t)$ is the diffusion coefficient tensor (3×3 diagonal matrix in flow), $\mathbf{B}_t$ is the three-dimensional vector of the Wiener process, i.e., $\mathbf{B}_t - \mathbf{B}_s$ has a normal distribution with a zero mean and variance as $(\boldsymbol{\sigma}\boldsymbol{\sigma}^T)(t-s)$ for $s \le t$, which is independent of $\mathbf{X}_t$. $d\mathbf{B}_t/dt$ $(=W(t))$ is the Gaussian White noise (Gardiner, 1985). The first term on the right hand side of equation (1a) or (1b) explains the mean drift motion of particles and the second term for the random motion due to turbulence. Mathematically,

$$\bar{\mathbf{u}}(t,\ X_t) = \bar{\mathbf{U}} + \nabla\mathbf{D} = \begin{cases} \bar{U}(t,x,y,z) + \partial D_x / \partial x \\ \bar{V}(t,x,y,z) + \partial D_y / \partial y \\ \bar{W}(t,x,y,z) - w_s + \partial D_z / \partial z \end{cases} \tag{2}$$

$$\boldsymbol{\sigma}(t,\ \mathbf{X}_t) = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} \tag{3}$$

Equation (1) can be developed as a stochastic partial differential equation (SPDE) by combining with the advection-diffusion equation.

2.2 Stochastic Diffusion Jump Particle Tracking Model

The governing equation that describes the jumps due to extreme events can be made by adding the jump term to a Langevin equation or an Ito process. It can be written as

$$\mathrm{d}\mathbf{X}_t = \underbrace{\bar{\mathbf{u}}(t,\mathbf{X}_t)\mathrm{d}t}_{\text{drift term}} + \underbrace{\boldsymbol{\sigma}(t,\mathbf{X}_t)\mathrm{d}\mathbf{B}_t}_{\substack{\text{random term} \\ \text{due to turbulence}}} + \underbrace{\mathbf{h}(t,\mathbf{X}_t)d\mathbf{P}_t}_{\text{jump term}} \quad \text{or} \tag{4a}$$

$$\mathbf{V}_p = \frac{d\mathbf{X}_t}{dt} = \underbrace{\bar{\mathbf{u}}(t,\mathbf{X}_t)}_{\text{drift term}} + \underbrace{\boldsymbol{\sigma}(t,\mathbf{X}_t)\frac{d\mathbf{B}_t}{dt}}_{\substack{\text{random term} \\ \text{due to turbulence}}} + \underbrace{\mathbf{h}(t,\mathbf{X}_t)\frac{d\mathbf{P}_t}{dt}}_{\text{jump term}} \tag{4b}$$

where $\mathbf{h}(t,X_t)$ is the jump amplification factor and $\mathbf{P}_t$ denotes a Poisson process so that the inter-arrival time between extreme events is exponentially distributed. To solve equation (4), Newton's second law of motion is introduced as

$$m_p \frac{d\mathbf{V}_p}{dt} = \mathbf{F} = \mathbf{F}_r + \mathbf{F}_e \tag{5}$$

where $\mathbf{F}_r$ denotes forces exerted on the particle in the regular flow field and $\mathbf{F}_e$ denotes additional forces due to extreme events. As $\mathbf{F}_e$ moves the particle from one state to another state, mathematically, it can be expressed as

$$\mathbf{F}_e = m_p \frac{\mathbf{u}_f(t) - \mathbf{V}_p(t_0)}{\tau_p} \tag{6}$$

where $\mathbf{u}_f(t)$ is the increased mean drift flow velocity due to an extreme flow event, $\mathbf{V}_p(t_0)$ is particle velocity before the extreme event and τ_p is the particle relaxation time.
From equations (5) and (6), the particle velocity can be represented by

$$\mathbf{V}_p = \mathbf{V}_{p0} + \int_0^T \frac{F_r}{m_p} d\tau + \int_0^T \frac{\mathbf{u}_f(t) - \mathbf{V}_p(t_0)}{\tau_p} d\tau \tag{7}$$

By comparing equation (4b) with equation (7), we find that the jump term corresponds to the third term of the right-hand side in equation (7). Thus, assuming that only one extreme event occurs within a time interval [t, t+dt], the jump term can be rewritten as

$$\mathbf{h}(t,\mathbf{X}_t) = \int_t^{t+T} \left(\int_0^T \frac{\mathbf{u}_f(t) - \mathbf{V}_p(t_0)}{\tau_p} d\tau \right) dt \tag{8}$$

3. RELAXATION TIME

In aerodynamic engineering, according to Owen (1969), the particle relaxation time is a characteristic time for a particle to transit from one state to another state in air stream. From Newton's law,

$$m_p \frac{dV_p}{dt} = m_p g - F_D = m_p g - 3\pi\mu d_p V_p \quad \text{with} \quad V_p(t=0)=0$$

the particle velocity and relaxation time can be expressed as $V_p = \left\{\rho_p d_p{}^2 g(1-e^{-t/\tau_p})\right\}/(18\mu)$ and $\tau_p = \left(\rho_p d_p{}^2\right)/(18\mu)$. For sediment particle movement in response to an extreme event, the relaxation time can be regarded as the time needed for a particle to move from the regular flow state to the extreme flow state. However, the sediment transport in the flow field has different dominant forces exerted on the sediment particle compared to the pneumatic transport. Therefore it is necessary to define the relaxation time appropriate to the situation of change from the regular flow state to the extreme flow state.

We can derive the relaxation time from the force balance equation. The force balance in the *x*-direction can take the effect of the external force due to extreme events into account as follows:

$$\rho_s \forall_p \frac{dV_p}{dt} = \underbrace{-\frac{1}{2}\rho C_D A_p V_r^2}_{\text{drag force}} \tag{9}$$

where ρ_s is the particle density, $\forall_p$ is the particle volume, A_p is the projected area of the particle, $\mathbf{V}_p$ is the particle velocity, $\mathbf{u}_f$ is the fluid velocity, C_D is the drag coefficient, $\forall_p = (4/3)\pi\left(d_p/2\right)^3$, $A_p = \pi\left(d_p/2\right)^2$, $\mathrm{Re}_p = \left(\rho d_p \left|V_p - u_f\right|\right)/\mu$ and $C_D = \left(24/\mathrm{Re}_p\right)\left(1+0.152\,\mathrm{Re}_p^{0.5}+0.0151\,\mathrm{Re}_p\right)$ Representing $\psi(\mathrm{Re}_p)$ by $1+0.152\,\mathrm{Re}_p^{0.5}+0.0151\,\mathrm{Re}_p$, we can rearrange the equation as a partial differential equation of V_r.

$$\frac{dV_r}{dt} + \frac{18\mu\psi(\mathrm{Re}_p)}{\rho_s d_p^2} V_r + \frac{du_f}{dt} = 0 \tag{10}$$

The general solution of the partial differential equation is

$$V_r(t) = \frac{\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{du_f}{dt}}{1+\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{d}{dt}} + c e^{-\frac{18\mu\psi(\mathrm{Re}_p)}{\rho_s d_p^2}t} \tag{11}$$

Given the initial condition, $V_r = u_{fo}(t) - u_f(0)$ at $t=0$, we can obtain

$$V_r(t) = \frac{\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{du_f}{dt}}{1+\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{d}{dt}} + \left[u_{fo}(t) - u_f(0) - \frac{\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{du_f}{dt}}{1+\frac{\rho_s d_p^2}{18\mu\psi(\mathrm{Re}_p)}\frac{d}{dt}}\right] e^{-\frac{18\mu\psi(\mathrm{Re}_p)}{\rho_s d_p^2}t} \tag{12}$$

The relaxation time is defined in a traditional way from the equation (12) as

$$\tau_p = \frac{\rho_s d_p{}^2}{18\mu\psi(\mathrm{Re}_p)}$$

We find that the particle relaxation time is affected by the particle properties such as particle size, density and particle Reynolds number.

4. DISCUSSION

As the derived relaxation time formula in the previous section, the relaxation time is determined by the particle properties and forces exerted on particles. Firstly, as particle size or density increases, the relaxation time increases proportionally as in Figure 1 and Figure 2. The x-axis and y-axis of the figures respectively represent the Φ scale and the dimensionless relaxation time. The Φ scale is used for the characterization of the grain size distribution of sediment as suggested by Krumbein (1936). As the definition of Φ scale, i.e., $\Phi = -\log_2(d_p)$ where d_p is the diameter of the sediment (mm), it is noted that larger particles have the smaller Φ values. Furthermore, the extent of the effect of each force on time lag is comparable from the Figure 1 and Figure 2. The main force that causes the relaxation of sediment particles is the drag force. Added mass also plays a subordinate role in the relaxation of sediment particles.

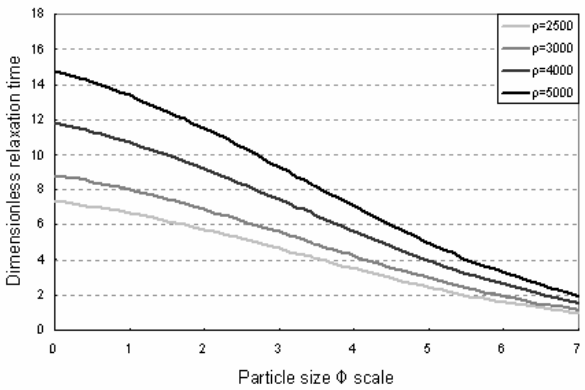

Figure 1 Dimensionless Relaxation Time (only Drag Force)

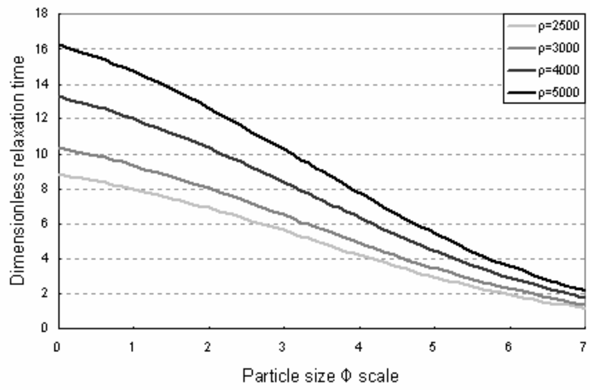

Figure 2 Dimensionless Relaxation Time (Drag Force and Added Mass)

4. APPLICATIONS

The sediment particle trajectories in a flow are presented in the following examples using the proposed stochastic diffusion jump particle tracking model. As the proposed stochastic diffusion jump model newly considers the effect of the relaxation time, we can compare the results of the proposed model in the following scenarios: 1) in the absence of the relaxation time, and 2) in the absence of extreme flow events. Besides, the resultant trajectories of the stochastic diffusion model are also added for comparison. The governing equation of the stochastic diffusion model is a Langevin equation or an Ito process, which is equivalent to the equation used in the random walk particle tracking model.

The flow conditions of an example are given as follows: it is a regular flow which has the mean drift velocity $\bar{u} = 0.1$ m/s. The diffusivity is $\sigma = 0.1$ m^2/s. The occurrence of the extreme events can be represented by a Poisson process with a rate of 0.1/s, i.e., the mean frequency of occurrence $\lambda = 10$. The magnitude of the extreme events is assumed to have the mean drift velocity increased to $\bar{u} = 2$ m/s. Particle properties are as follows: particle diameter is 1 mm and particle density is 4000 kg/m^3. The total simulation time is 100 s , and the time step is 0.006 s. The examples were performed by the MATLAB program that provides the random number generation (randn) function. In order to show the stochastic results effectively, we performed 800 iterative simulations and computed the respective ensemble means and variances of those scenarios. Figure 3 shows the particle trajectories in the one-dimensional flow field during the total simulation time. The relative time in the figure is defined as the time normalized by the total simulation time. The dashed-dotted, dotted and solid lines respectively display the results from the stochastic diffusion model without extreme events, the stochastic diffusion jump model with extreme events and the stochastic diffusion jump model considering relaxation time. The ensemble means represented by bold lines have probabilistically reflected the stochastic scenarios represented by fine lines.

Figure 3 demonstrates the differences in the mean particle trajectories among the stochastic diffusion model (i.e., particle tracking model), stochastic diffusion jump model and stochastic diffusion jump model with the relaxation time. In particular, the differences are well explained by the ensemble mean lines. The dashed-dotted line shows how well the stochastic notion describes the diffusion term due to turbulence as random fluctuation in a regular flow. The dotted line demonstrates that the additional term for jumps due to extreme events can be modeled by introducing the concept of a stochastic process. As the ensemble means of dotted lines manifest, the overall particle velocity is increasing when extreme flow events occur. It can mainly affect the erosion/deposition of sediment such as downstream bridge scouring. The solid line explains the effect of time lag due to the inertia effect of the particle and shows the modified result of the dotted line. As the solid line displayed under the dotted line, the relaxation time can be thought of as a factor that diminishes the impact of the extreme flow events. It can be reasonably explained that the overall velocity of sediment particles is reduced by the relaxation during a given period. The model of solid lines enables us to simulate the varying effect of particle size and density on particle relaxation time. As demonstrated in the simulations, the proposed stochastic diffusion jump model can reckon probabilistic properties in the sediment transport, which could not be included in the traditional sediment transport model.

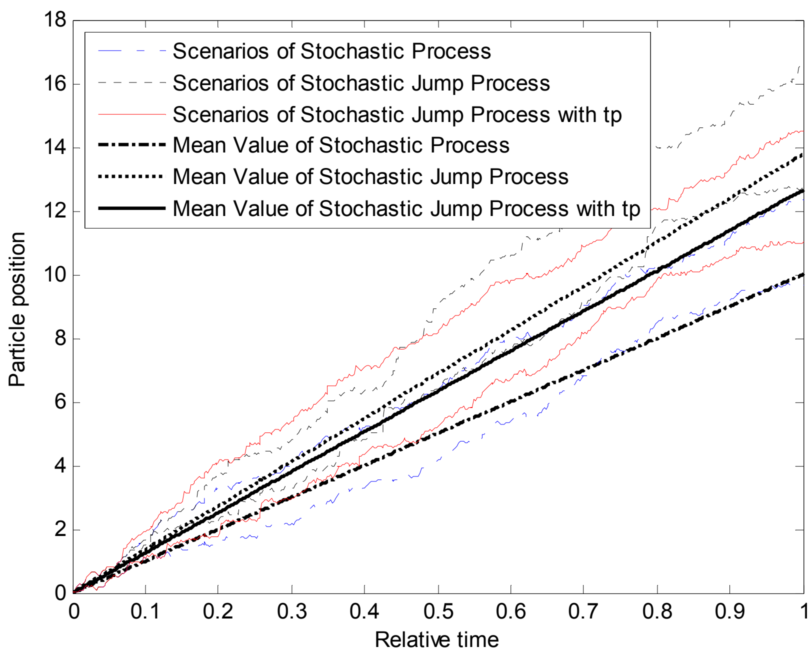

Figure 3. Sediment Particle Trajectories and Ensemble Mean in a 1-D flow

The variances of iterative simulation results of particle trajectories are delineated in Figure 4. The figure conveys the fact that the uncertainties of the trajectories are increasing when the effect of jumps term is larger. The uncertainties due to jump terms decrease when considering the relaxation time because the relaxation of sediment particles reduces the impact of extreme flow events.

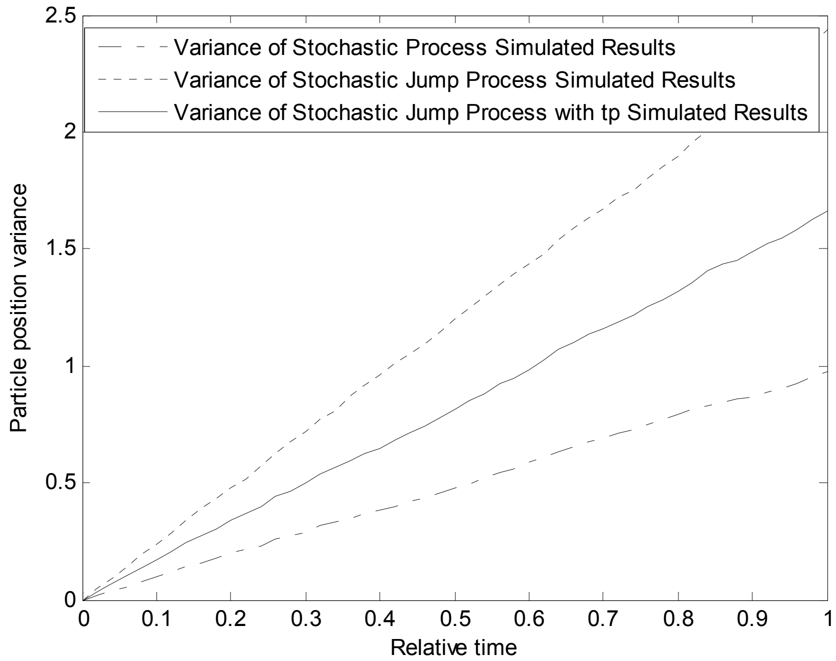

Figure 4. Variances of Sediment Particle Trajectories in a 1-D flow

5. CONCLUSIONS

In order to describe the sediment transport, many researchers have traditionally used the numerical analysis of differential transport equations. However, the traditional governing equations do not explicitly include the statistical characteristics of sediment particles. In this paper, we propose a stochastic diffusion jump model with a particle relaxation time as an alternative approach for sediment transport. This study shows the accuracy of the proposed stochastic diffusion jump particle tracking model by comparing with results of the stochastic diffusion model that has the same governing equations as the random walk particle tracking model. Furthermore, the study accounts for the time lag of sediment particles in response to the extreme flow by introducing a physical term, the particle relaxation time.

The proposed stochastic diffusion jump model is at its developmental stage. In this paper, we have noted the following: Firstly, the stochastic model can elucidate the stochastic characteristics of particle movement. Since the particle movement is intrinsically stochastic, the proposed model may produce the results closer to the reality. Secondly, the proposed model shows a great potential for modeling the sediment transport in extreme event flows as well as in regular flows. Lastly, the jump term due to extreme events can take the particle relaxation time into account. Although there has recently been a study of sediment transport model considering the lag effects (Wu et al., 2006), it has used a conceptual correction factor. When extreme events occur, particles tend to move along the flow after a certain short time period because of the inertia effect of particles. Using the relaxation time, the acceleration can be computed so that retardation of particle trajectory can be modeled. Hence, the relaxation time also enables the proposed model to generate a more realistic result.

It is concluded that the proposed stochastic diffusion jump model can show the probabilistic properties of sediment transport and the most probable pathline in addition to the overall tendency of sediment transport. However, more work is needed. The probability and magnitude of the extreme events from the frequency analysis can be used to quantify the parameter associated with the Poisson process in the proposed model. The applicability of the relaxation time to real case examples should be further investigated.

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