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Dimitriou, D.; Demetriou, J.

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ENERGY LOSS IN SUBMERGED HYDRAULIC JUMP WITHIN INCLINED CHANNEL

Dimitriou D.[1] and Demetriou J.[2]

Abstract: *In this paper the mechanical energy and loss of energy in the submerged hydraulic jump within rectangular inclined (angle φ, $3^o \le \varphi \le 15^o$) open channels are presented, analyzed, discussed and compared to corresponding free jump quantities. The energy and loss of energy calculations are based on previous measurements of both jumps' lengths and conjugate depths' ratios. The loss of energy in the submerged jump is much smaller than in corresponding free jump, showing that the submerged jump is not an effective means to dissipate any excessive flow energy when compared to much larger free jump energy loss.*

Keywords: *Submerged Jumps. Energy Loss.*

INTRODUCTION

The water flow under a sluice gate and subsequent hydraulic jump (when formed under suitable conditions) belong to some of the most important hydraulic phenomena which are appearing in practice within open water channels. Sometimes the hydraulic jump is free and some other times is submerged, depending on the downstream flow conditions, i.e. on the free or obstructed downstream channel flow. In the last case an obstacle such as a solid body (for example another sluice gate or a weir) is raising the water level just after the sluice gate and the entire flow becomes submerged. The higher part of any hydraulic jump – under appropriate conditions – is permanently remaining at its place although the flow under it is steadily developing along the open channel.

Fig. 1 schematically shows the general flow case where the rectangular channel is inclined (angle φ) to the horizon – with a slope J_o=sinφ – and includes a sharp edged and very thin sluice gate (perpendicular to the channel floor) which has the same width as the channel, and a lower aperture α. The water discharge per unit width is q, the most important depths are, p_1, p_2, for the submerged jump, and d_1, d_2, for the free jump. Both jumps are schematically presented in Fig. 1 and show the difference of the two states of flow. The submerged jump has an inclined length L and the free jump a corresponding length L_d.For the submerged jump

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1 Dr Mechanical Engineer, Davy Process Technology, London W2 6LE, UK,
Email: dimitrios.dimitriou@davyprotech.com
2 Associate Professor (RD), School of Civil Engineering, National Technical University of Athens, Greece, 12 Polykarpou St., N. Smyrni, Athens, 17123, Greece. Email : idimit@central.ntua.gr (corresponding author).

the control volume is included between cross sections 1 and 2, where the local pressures are assumed to be hydrostatic. Cross section 1 is considered as coinciding with the contracted cross section of the free jump – at a distance x_1.The submerged jump has a mean free surface profile starting from the outer face of the sluice gate, followed by a local fall along x_1,and then it turns (along L) towards the downstream horizontal. Beyond x_1, the lower limit of the submerged jump has a complicated form also ending at p_2 depth, while between the upper free surface and the lower limit a roller is created with its recirculating flow. The discharge q goes across p_1 and p_2, while at cross section 1 the roller's water is increasing the pressure on depth p_1 and the entire pressure distribution (column $t+p_1$)) is considered as hydrostatic, with a resultant force along the flow direction (x) $0.5 \cdot \gamma \cdot (t + p_1)^2 \cdot \cos\varphi$, where γ=specific water weight.

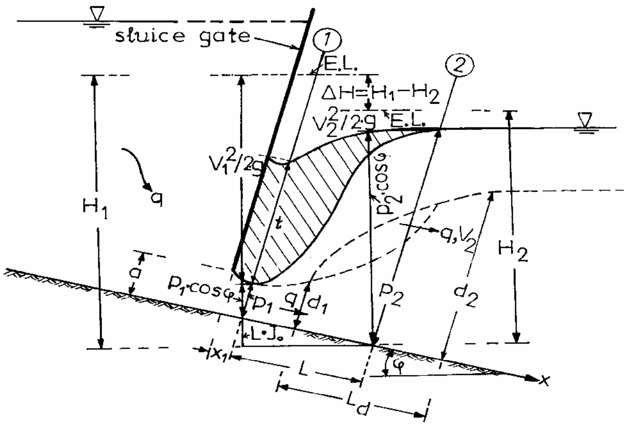

Fig. 1. Submerged and free jump geometry.

The one-dimensional continuity equation is $q = p_1 \cdot V_1 = p_2 \cdot V_2$, while the momentum equation along x-with negligible tractive force on the channel boundaries – is

$$0.5 \cdot \gamma [(t + p_1)^2 - p_2^2] \cdot \cos\varphi + W \cdot J_o = \rho \cdot q^2 \cdot [(1/p_2) - (1/p_1)] \quad (1)$$

The above equation has been solved in the past (Demetriou, 2006), for the theoretical ratio $\lambda_t=p_2/p_1$, after the experimental determination of the weight W among sluice gate, channel floor, cross section 2 and free surface. This theoretical ratio has also been successfully compared to corresponding (experimentally determined) ratio λ_e.

Fig. 1 also illustrates the mechanical energy (per unit water weight) H_1 at section 1 (depth p_1), and H_2 at section 2 (depth p_2), while $\Delta H = H_1-H_2$ is the local loss of energy between conjugate depths p_1 and p_2, which is due both to tractive stresses (for $\varphi>0^0$) and internal friction. The present paper is dealing with H_1, H_2 and ΔH and the comparison of them with corresponding quantities in the free jump, both in inclined rectangular channels. The most important parameters are $\lambda=p_2/p_1$, L/p_2 (experimentally determined) and the Froude numbers Fp_1(section 1)and Fp_2(section 2), generally with

$$Fp=q/g^{1/2}\cdot(corresponding\ depth)^{3/2} \quad (2)$$

where $Fp_1>1$ and $Fp_2<1$. $Fp_2=q/g^{1/2}\cdot p_2^{3/2}$ was mainly used here, while all Reynolds numbers had large enough values (turbulent flows).

For the free hydraulic jump the conjugate depths d_1 and d_2 (and their ratio $\delta=d_2/d_1$), the length L_d/d_2 and Froude number $Fd_2=q/g^{1/2}\cdot d_2^{3/2}=Fd_1\cdot(d_1/d_2)^{3/2}$ are used, while any comparison with the submerged jump is meant with $Fp_2=Fd_2=Fr_2$. The points which appear on the following figures came out as results of energy computations, based on the pertinent experimental data – and in combination with onedimensional energy expressions.

PREVIOUS EXPERIMENTAL RESULTS

2.1 For the submerged hydraulic jump within inclined rectangular channels, Retsinis et.al, (2005) and Demetriou et al (2006), have presented the following experimental equations:

a) For the jump length (with φ in degrees and $Fp_2=Fr_2$),

$$L'=\frac{L}{p_2}\cdot\frac{\cos\varphi}{(2.7\cdot\varphi+30.8)\cdot Fr_2}=[(-2.9053+0.0069\cdot\varphi-0.0018\cdot\varphi^2)\cdot Fr_2+ +(-0.03\cdot\varphi+1.25)] \quad (3)$$

The above length L is the distance between the sluice gate and cross section 2, while the distance x_1 (between sluice gate and cross section 1) was measured as $x_1\cong1.7\cdot\alpha$. Since x_1 is rather small – in comparison with L, it is reasonable to consider the distance 1-2 as approximately equal to L.

b) For the conjugate depth's ratio (with φ in degrees)

$$\lambda=p_2/p_1=1-\{[(L')^2-1]/B_1\}$$

where,

$$B_1=[1.776+0.022\cdot\varphi+0.224\cdot e^{-\varphi}]\cdot Fr_2^{(1.840-0.105\cdot\varphi^{0.5}+0.16\cdot e^{-\varphi})} \quad (4)$$

The above equations are used here for $\varphi=3^o$-6^o-9^o-12^o-15^o , $0.08\leq Fr_2\leq 0.15$ for $\varphi=9^o$, and smaller Fr_2 ranges for other angles φ.

2.2. For the free hydraulic jump, Demetriou (2005), has experimentally given the following equations:

a) For the jump length (with φ in degrees and $Fd_1=Fr_1$),

$$L_d/d_2 = [7.69-0.094\cdot Fr_1-(6.27/Fr_1)]\cdot \cos\varphi^{(3.35\cdot J_o^{-1.3}-2)} \quad (5)$$

b) For the conjugate depths' ratio, the following equation was verified,

$$\delta=d_2/d_1=0.5\cdot[(1+8\cdot Fr_1^2)^{1/2}-1]\cdot e^{3.5\cdot Jo} \quad (6)$$

The above equations hold for $0^o \leq \varphi \leq 16^o$, $2 \leq Fr_1 \leq 19$ for $\varphi=0^o$, and smaller $Fr_1(\geq 2)$ ranges for other angles φ. Eq (6) gives exactly the same δ vs Fr_1 lines as Chow's (1959), graphical straight lines.

RESULTS. ANALYSIS AND DISCUSSION

For the submerged jump the mechanical energy H_1 is (Fig. 1), $H_1 = L\cdot J_o+p_1\cdot\cos\varphi+(q^2/p_1^2\cdot 2\cdot g)$, or, with the use of $Fr_2(= Fp_2)$, $\lambda=p_2/p_1$, and after division by p_1,

$$H_1/p_1= \lambda \cdot (L/p_2)\cdot J_o+\cos\varphi+0.5\cdot\lambda^3\cdot Fr_2^2 \quad (7)$$

Also, $H_2=p_2\cdot\cos\varphi+(q^2/p^2{}_2\cdot 2\cdot g)$, or

$$H_2/p_1 = \lambda\cdot[\cos\varphi+0.5\cdot Fr_2^2] \quad (8)$$

The local loss of mechanical energy between p_1 and p_2 is

$$\Delta H/p_1= (H_1/p_1)-(H_2/p_1) = \lambda\cdot(L/p_2)\cdot J_o+(\lambda-1)\cdot\cos\varphi+0.5\cdot Fr_2^2\cdot\lambda\cdot(\lambda^2-1) \quad (9)$$

or, in terms of $\Delta H/H_1$,

$$\Delta H/H_1=(\Delta H/p_1)/(H_1/p_1) \quad (10)$$

Corresponding energies and loss of energies for the non submerged (=free) jump are given by similar equations, although – instead of p_1, p_2, λ, L and $Fp_2(=Fr_2)$ – corresponding quantities d_1, d_2, $\delta=d_2/d_1$, L_d and $Fd_2(=Fr_2)$ are used, while any comparison is meant for the same Froude number.

For the submerged jump Fig. 2 presents H_1/p_1 vs Fr_2 at $\varphi=3^o$, where the corresponding (solid) curve is freely extrapolated (dashed-short lines): H_1/p_1 is increasing with Fr_2, for example from 6.9 (at $Fr_2=0.17$) to 7.3 (at $Fr_2=0.20$), i.e. the percentage increase is $(7.3-6.9)\cdot 100/6.9 \cong 8\%$.

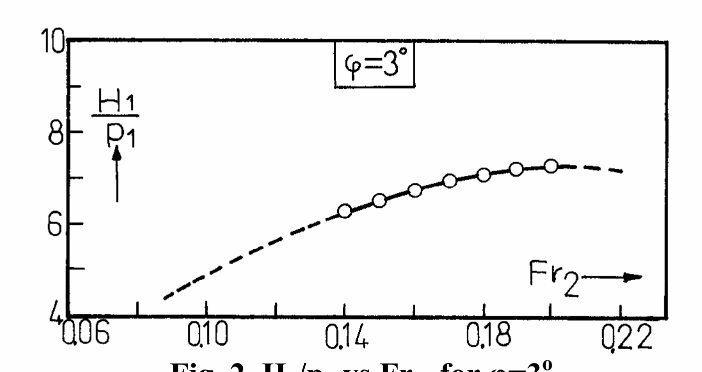

Fig. 2. H_1/p_1 vs Fr_2 for φ=3°.

The same trends present H_1/p_1 vs Fr_2 for φ = 6°-9°-12°-15° in Figs 3,4,5, with the only difference (not visible in Fig.2) that after a certain Fr_2 value corresponding curves are bending down, while the respective curve for φ=15°presents only a descending branch-in the present field of measurements. However the general H_1/p_1 level is considerably rising from φ= 3° to φ=15°.

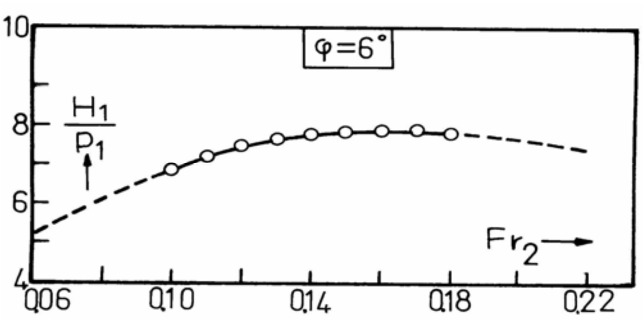

Fig. 3. H_1/p_1 vs Fr_2 for φ=6°.

Fig. 6 presents all previous curves (submerged jumps – solid lines) and all corresponding H_1/d_1 vs Fr_2 (free jumps – dashed lines). Both families of lines are very systematic. For Fr_2=const. H_1/p_1 for submerged jumps are increasing when angle φ is increasing, while to H_1/d_1=const. correspond larger Fr_2 values when φ is decreasing. It is also clear that H_1/d_1 are generally larger than H_1/p_1 for any pair of lines with φ=const., i.e. the dimensionless energy at cross section 1 for submerged jump, is lower than the dimensionless energy at corresponding cross section for free (= non submerged) jump (at d_1) – in the present field of measurements.As a simple example for φ=9° and Fr_2=0.13 $H_1/p_1 \cong 10$ for submerged jump, while $H_1/d_1 \cong 17$ for free jump, i.e. there is a percentage energy increase in free jump of (17-10)·100/10=70%.However, although the present measurements had a rather narrow Fr_2 range, it may be predicted that any pair of corresponding lines – for the same angle φ – do not meet between them but they are incompatible curves.

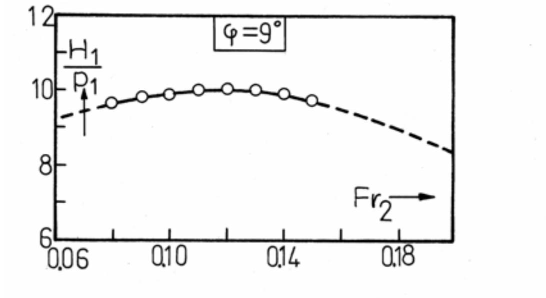

Fig. 4. H_1/p_1 vs Fr_2 for φ=9°.

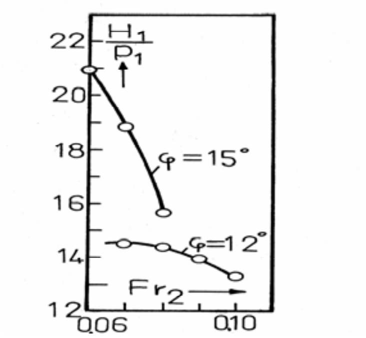

Fig. 5. H_1/p_1 vs Fr_2 for φ=12° and φ=15°.

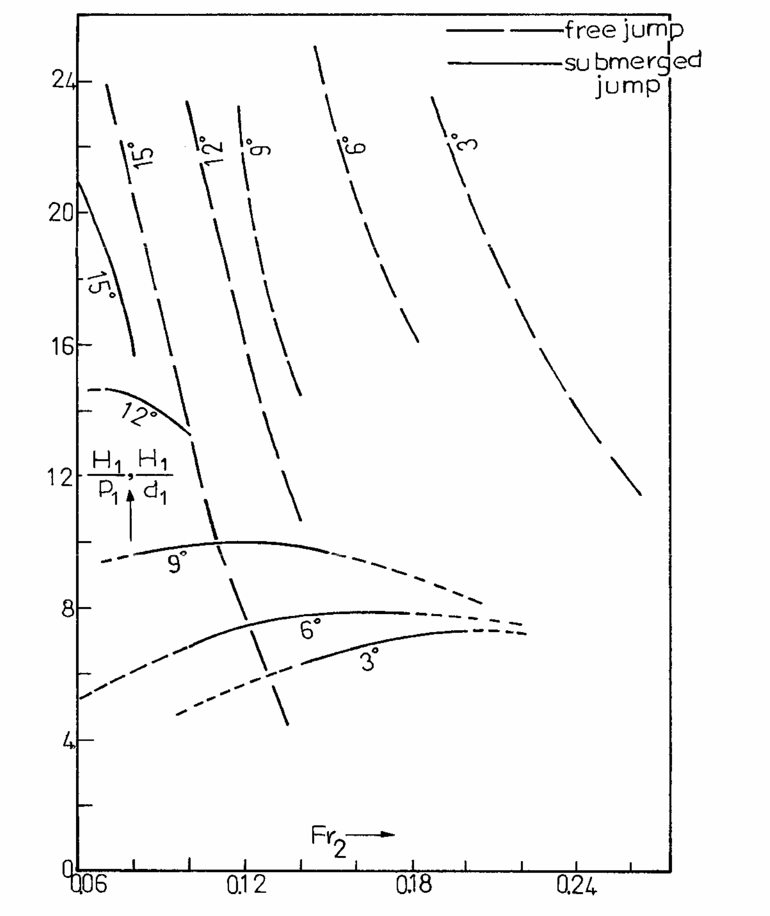

Fig. 6. H_1/p_1 and H_1/d_1 vs Fr_2 for submerged and free jump, $3^o \leq \varphi \leq 15^o$.

Fig. 7 shows the dimensionless energy loss $\Delta H/p_1$ vs Fr_2 at angle φ=3° for submerged jumps. When Fr_2 is increasing $\Delta H/p_1$ are also increasing : As an example, for Fr_2=0.17 $\Delta H/p_1$ is 0.15, while for Fr_2 = 0.20 $\Delta H/p_1$ becomes 1, i.e. the percentage increase is now much larger, (1-0.15)·100/0.15≅565%.

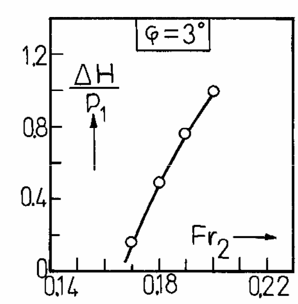

Fig. 7. ΔH/p_1 vs Fr_2 for φ=3°.

Next Figs.8,9,10 and 11 present ΔH/$p_1$vsFr_2 for φ=6°-9°-12°-15°,which also show the same trends as in Fig.7,but the general level of the dimensionless losses is strongly increasing with angle φ.

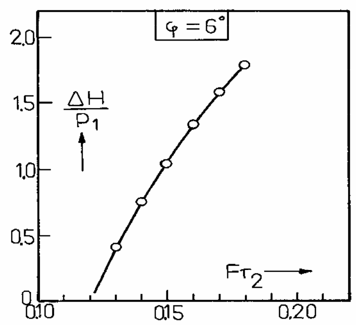

Fig. 8. ΔH/p_1 vs Fr_2 for φ=6°.

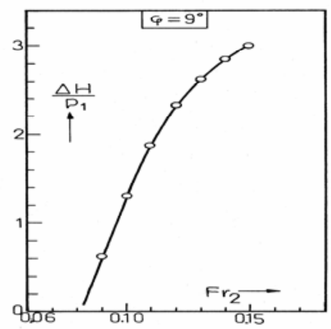

Fig. 9. ΔH/p_1 vs Fr_2 for φ=9°.

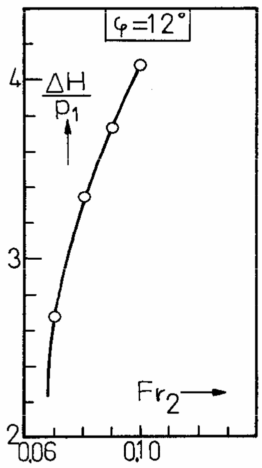

Fig. 10. ΔH/p_1 vs Fr_2 for φ=12°.

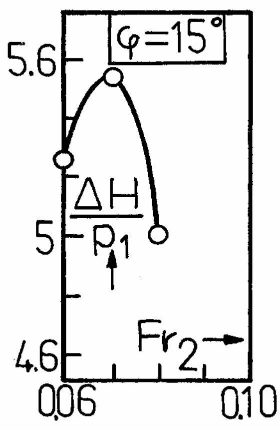

Fig. 11. ΔH/p_1 vs Fr_2 for φ=15°.

Furthermore, Fig. 12 presents all previous $\Delta H/p_1$ vs Fr_2 curves (solid lines) for submerged jumps and all $\Delta H/d_1$ vs Fr_2 curves for free jumps (dashed lines). Both families of curves are very systematic. For Fr_2=const. $\Delta H/p_1$ for submerged jumps are largely increasing when angle φ is increasing. For example for $Fr_2=0.09$ $\Delta H/p_1 \cong 0.6$ ($\varphi=9^o$), while for same Fr_2 $\Delta H/p_1 \cong 3.7$ ($\varphi=12^o$), i.e. there is a percentage energy loss increase of $(3.7\text{-}0.6)\cdot 100/0.6 \cong 515\%$ - when angle φ is increasing with a much smaller rate (~33%).

Finally, the most important result in Fig. 12 comes from a comparison between $\Delta H/p_1$ (submerged jump-solid lines) and $\Delta H/d_1$ (free jump-dashed lines) for the same Fr_2 and angle φ: $\Delta H/d_1$ for any pair of φ = const. appears to be much larger than $\Delta H/p_1$, i.e. the dimensionless energy loss between conjugate jumps' depths is considerably smaller for submerged jumps than for corresponding free jumps – in the present field of measurements. As a simple example for $\varphi=9^o$ and $Fr_2=0.13$ $\Delta H/p_1 \cong 2.6$ for submerged jump, while $\Delta H/d_1 \cong 5.2$ for free jump, i.e. there is a percentage energy loss increase in free jump of $(5.2\text{-}2.6)\cdot 100/2.6=100\%$. This fact looks rather reasonable if considering the flow nature of submerged jump-which is a rather calm flow phenomenon, and of the free jump which is far more violent: The submerged jump is not an effective means to dissipate excessive mechanical flow energy, although it presents some external similarities to the free jump.

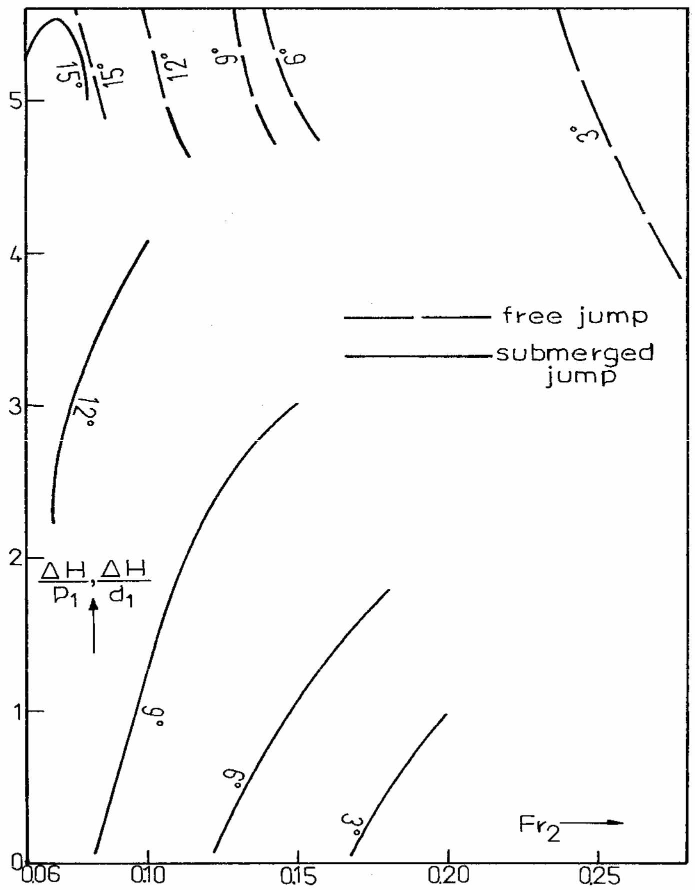

Fig. 12. $\Delta H/p_1$ and $\Delta H/d_1$ vs Fr_2 for submerged and free jump, $3^o \leq \varphi \leq 15^o$.

CONCLUSIONS

In this paper the mechanical energy and loss of energy in the submerged hydraulic jump within rectangular inclined (angle φ, with $3^o \leq \varphi \leq 15^o$) open channels are presented, analyzed, discussed and compared to corresponding free jump quantities. The energy and loss of energy calculations are based on previous measurements by the second author with other authors, concerning jump length and conjugate depths' ratio, both for submerged and free jumps. Fig. 1 shows corresponding flow geometries, eq. (1) presents the one-dimensional momentum equation, while eq. 2 shows the main flow parameter, Froude number. Eqs. (3) to (6) give the results of the previous measurements, while eqs.(7) to (10) describe the corresponding energy and loss of energy. Figs. 2 to 6 present the dimensionless energy at the beginning of submerged and free jumps, while Figs. 7 to 12 show the dimensionless energy loss of both hydraulic jumps: The loss of energy in submerged jumps is much smaller than in free jumps and in this sense the submerged jump is not an effective means for the excessive flow energy dissipation – when compared to the free jump.

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