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# Assimilation of Initial Condition of Tsunami Using Adjoint Equation Method

# ASSIMILATION OF INITIAL CONDITION OF TSUNAMI USING ADJOINT EQUATION METHOD

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## Introduction

To reduce damage of tsunami, we need to understand the behavior of tsunami. Especially, initial wave of tsunami is very important to express the exact behavior of tsunami. However, it is difficult to observe the initial wave of tsunami in a direct manner. The purpose of this study is to present an assimilation method of initial wave of tsunami. In this research, initial condition of tsunami is assimilated with the observed water elevation on the coast line.

## Methodology

The shallow water equation is used for the state equation. The behavior of tsunami can be expressed by the shallow water equation.
The momentum and continuity equations are denoted by eqs (1) and (2)

$$\dot{u}_i + g\eta_{,i} = 0 \tag{1}$$

$$\dot{\eta} + hu_{i,i} = 0 \tag{2}$$

As the discretization technique, the finite element method based on the liner interpolation is used. Two-step explicit scheme is applied to the temporal discretization technique. Assimilation of the initial wave of tsunami is the purpose of this study. To do this the performance function showed be introduced, which consists of the square sum of the discrepancy between computed and observed water elevations at observation points. The problem is to find the initial water elevation so as to minimize the performance function *J*, which is expressed as eq (3).

$$J = \frac{1}{2}\int_{t_0}^{t_f} (\eta_\lambda - \eta_\lambda^{obj}) Q_{\lambda\mu} (\eta_\mu - \eta_\mu^{obj}) dt \tag{3}$$

The performance function is extended by adding the inner products between the Lagrange multipliers and the state equation, which is expressed as eq (4)

$$J^* = J + \int_{t_0}^{t_f} u_{\alpha i}^* (M_{\alpha\beta}\dot{u}_{\beta i} + gA_{\alpha\mu i}\eta_\mu) dt + \int_{t_0}^{t_f} \eta_\lambda^* (N_{\lambda\mu}\dot{\eta}_\mu + hA_{\lambda\beta i}u_\beta) dt \tag{4}$$

In order to minimize the performance function, the stationary condition eq (5) showed be valid

$$\delta J^* = 0 \tag{5}$$

Then, the first-order adjoint equation, terminal condition and the gradient of extended performance function are obtained. As the minimization technique, the weighted gradient method is applied. The modified performance function can be express as follows.

$$K = J^* + \frac{1}{2}\int_{t_0}^{t_f} (\eta_\lambda^{(l+1)} - \eta_\lambda^{(l)}) W_{\lambda\mu} (\eta_\mu^{(l+1)} - \eta_\mu^{(l)}) dt \tag{6}$$

## Numerical study results

We carried out the assimilation of initial wave of the tsunami of The Tohoku earthquake.

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The tsunami was higher than 15 meters. Computational domain set as in figure (1). Figure (2) is assimilated result.

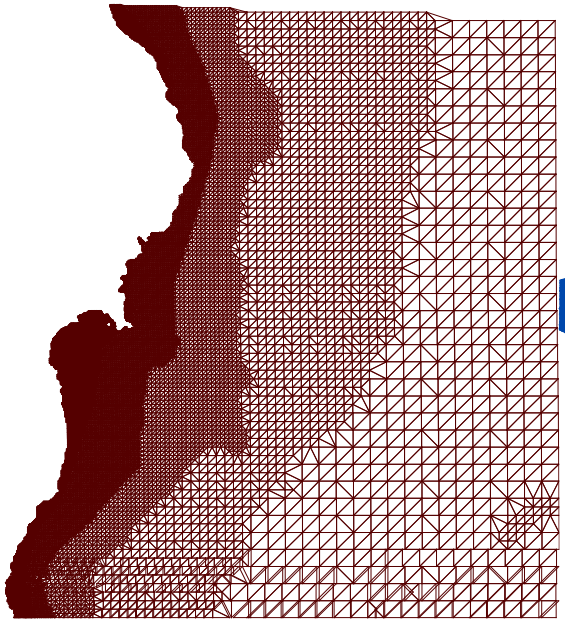

Figure (1) Computation domain

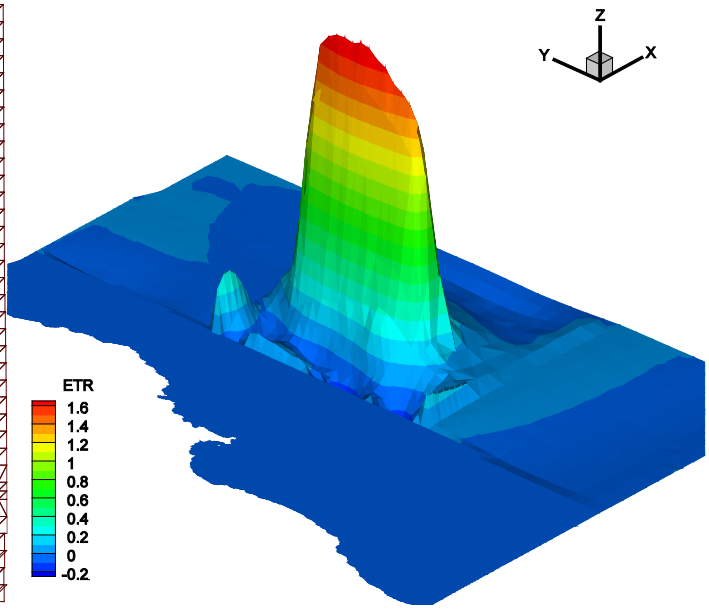


Figure (2) Assimilation results

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