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## DISTRIBUTED WATER BALANCE MODEL IN WATERSHED COUPLING WITH DYNAMIC AND DIFFUSIVE RIVER FLOW ROUTING

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## ABSTRACT

The accurate methods of predicting rivers' flood capacity and water level during flooding are essential to protect property and human lives in the floodplains and the surrounding areas of the natural rivers.

This paper focuses on the application of coupling the river dynamic and the river diffusive modelling techniques in the flat and the steep areas; respectively, to predict the flood discharge and the water depths in the natural rivers. Using the current model, the dynamic-diffusive river flow model, the discharge and the water level at any location of the streams network can be estimated. Observed data in a part of Arakawa River basin in Kanto area, Japan, are used to validate the current model. The simulated results of the discharge and the water levels show good and acceptable agreements with the observed data. The diffusive model is suitable in the hill and steep areas where the dynamic model is suitable for the compound and the natural rivers with high water depths, mild slopes and located on flat areas. The new dynamic-diffusive river flow model is suitable for the natural rivers and gives high efficient results.

Keywords: natural channels, dynamic model, diffusive model, river's floodplains.

## 1. INTRODUCTION

Most of the natural rivers and channels consist of a deep main channel and one or two shallow floodplains. The floodplains' areas created by river floods are often used for many purposes as agricultural, commercial, sports activity, housing, and rice cropping in the alluvial plains (Myers, 1997; River Bureau-Japan; Seckin, 2004). The channels network has been defined as composed of channel segments arranged in bracing configuration, with individual segments connected at junctions to form loops at treelike dendritic structures. For analyzing the discharge and water level quantitatively, numerous distributed and lump hydrological models have been developed such as System Hydrologique European (SHE) model, Institute of Hydrology Distributed Hydrological Model (IHDM) and Institute of Industrial Science Distributed Hydrological Model (IISDHM) (Jha, 2000a; 2000b). Many of the developed models routed the overland flow with kinematics' wave model equations and some of them are not physical distributed models. Luo (2000) developed a distributed water-balance model in a large–scale complex watershed, 2-D physically-based computer model, LUO-TAMAI model and applied it to Tone River basin, Kanto area, Japan.

Many investigators have simulated the channel networks by considering these systems

as combinations of several independent channels or as a main channel having tributaries with distributed lateral inflows (Amein, 1970; Fread, 1973; Franz, 1997; NOAA, 2002; Fan, 2006). **Jha** et al. (2000a; 2000b) developed a simultaneous solution for **1-D** unsteady flow routing a network of rivers which can be used with the complete distributed hydrological model (**IISDHM** model) and with the simple rainfall-runoff model or can stand alone as a river routing model. Moreover, many studies have been conducted to study the discharge capacity in the natural and compound channels and to study the effects of the roughness and the vegetations in the river floodplains on the estimated discharge and water depths values using the steady or the unsteady flow equations (Wormleaton, 1982, 1990; Chow, 1988; Myers, 1997; Bousmar, 1999; Helmino, 2002; Abidin, 2004); where, most of them studied only the single channel's flow.

The Japanese rivers can be characterized as; the rivers are prone to flooding because they flow rapidly, due to the steepness of the slopes along their basins and the relative shortness. The ration of the peak flow discharge to the basin area is relatively large, ranging from 10 times to as mush as 100 times of others countries (River Bureau-Japan). During flooding, rivers often overflow their banks and floodplains and may damage their environments. Therefore, accurate methods and new engineering techniques, to predict the rivers flood capacity and the rivers water depths during flooding, to protect the rivers' banks and floodplains and to keep them as close as possible to their natural state, are essential and required (Seckin, 2004).

In **LUO-TAMAI** model, the channel networks have been simulated by using the diffusive wave model equations considering the river cross-sections as rectangular sections and at the conjunction points, the upstream frequently consists of the main channel upstream part plus two tributaries and the downstream consists of the main channel downstream part and one downstream tributary (Luo, 2000, 2003, 2007; Yagisawa, 2005). Overland grid cells and channel grid cells are different with respect to their flow status and hydraulic characteristics. The channel flow always has a certain flow course and maintains a base flow during all or much of the water year and the hydraulic roughness is relatively small; where, the overland area doesn't have a definite flow course or a steady base flow and the hydraulic roughness is relatively large; however, these differences may be diminished during flooding and inundation (Luo, 2007; Dept. of the Army U.S, 1993).

The new current model (Distributed water balance model with River Diffusive and Dynamic wave Models: hereafter, **DRDDM**) can be used together with the overland flow model (**LUO-TAMAI** model) as a complete hydrological model; where, the overland flow and others hydrological models are important in the flood forecasting, to study the flow in the natural and compound channels with/out rough floodplains located in hill or flat watershed, or can be used to study the flow in the rivers networks only in case that, the upstream discharge hydrographs and the main river's downstream stage hydrograph are known. When it is used with the hydrological models, the upstream and the downstream boundary conditions can be generated using the continuity and Manning's equations.

## 2. METHODOLOGY AND MATEMATICAL BASIS

## 2.1 Overland flow and LUO-TAMAI model

**LUO-TAMAI** model includes evapotransipration, infiltration, groundwater and rivergroundwater exchange models. The main features of this model are 1)- both the overland grids and the channel grids are placed in the same physical frame and governed by the same set of equations; 2)- the channel grids are not the boundary conditions of the overland grids but grids with the same properties as the overland grids. The diffusive model was validated and applied successfully to Tone River basin, Kanto Area, Japan.

For the surface flow, the **2-D** diffusive wave approximation of the free surface flow equations is utilized as the governing differential equations. The diffusive wave equations can be written as;

$$\frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} + \frac{\partial (h)}{\partial t} = q$$
(1a)

$$\frac{\partial z}{\partial x} + S_{fx} = 0 \tag{1b}$$

$$\frac{\partial z}{\partial y} + S_{fy} = 0 \tag{1c}$$

Where *u* and *v* are *x* and *y* flow velocity components, respectively; *h* is the water depth; *z* is the water surface elevation; *q* is the lateral flow in the vertical direction;  $S_{fx}$  and  $S_{fy}$  are the friction slopes in *x* and *y* directions which can be written as;

$$S_{fx} = n_x^2 u \left| u \right| \cdot h^{-4/3} \tag{2a}$$

$$S_{fy} = n_y^2 v |v| \cdot h^{-4/3}$$
(2b)

Where  $n_x$  and  $n_y$  are Manning coefficients in x and y directions, respectively.

The finial finite difference equations for the diffusive model for the surface flow can be written as;

$$\frac{1}{\Delta x \Delta y} \sum_{m=1}^{m} u_m \overline{A}_m + \frac{\Delta h}{\Delta t} \frac{A_g}{\Delta x \Delta y} = q$$
(3a)

$$u_{m}\overline{A}_{m} = \frac{\overline{A}_{m}\overline{R}_{m}^{2/3}}{\overline{n}_{m}\sqrt{L|\Delta z_{m}|}}\Delta z_{m} = \overline{A}_{m}B_{m}\Delta z_{m}$$
(3b)

$$\Delta z_m = z_{ij} - z_{ij}^m, \quad B_m = \overline{R}_m^{2/3} / \left( \overline{n}_m \sqrt{L |\Delta z_m|} \right)$$
(3c)

Where m=4 for overland grids and m=9 for river grids.

For more effective convergence of the numerical solutions, the *SIMPLE* algorithm is adopted to solve these equations.

## 2.2 1-D Full Dynamic River Flow Model

**LUO-TAMAI** model computes the discharge and the run-off from the watershed considering the streams cross sections as rectangular sections while the most natural rivers and natural streams contain floodplains with/out vegetations. The flow physics of the natural and irregular channels with floodplains and vegetations differ remarkably from the ones of the

regular channels (without floodplains). These factors affect on the water surface level, where water surface level is decreased or increased according to the dimension and the geometry of the channels, on the flood peak time and on the cost of the channels construction and the safety against flood.

The river unsteady flow can be simulated by two well known partial differential equations, which express the conservation of the flow mass and flow momentum, the unsteady flow equations, St. Venant equations with the lateral flow. The two equations, the continuity and the momentum equations, can be expressed respectively as;

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L \tag{4}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\left(\frac{\partial h}{\partial x} - S_o + S_f\right) = 0$$
(5)

Where, Q is the discharge; A is the cross-sectional area; x is the distance along the river axe, t is the time;  $q_L$  is the lateral in/outflow per river unity length; g is the gravity; h is the water depth;  $S_o$  is the river bed slope and  $S_f$  is the friction slope which can be expressed as;

$$S_f = Q|Q|/K^2 \tag{6}$$

$$K = \sum \left( \frac{1}{n} A \cdot R^{2/3} \right) \tag{7}$$

Where K is the conveyance factor, n is Manning roughness Coefficient and R is the hydraulic radius.

Substitution of the finite difference approximations of the derivative and non derivative terms into **Eqs.** 4 and 5 yields a system of two nonlinear first order first degree partial differential equations of the hyperbolic type having two independent variables, the river distance x and the time t, and two dependent variables, the water depth h and the discharge Q. This system of the equations can be solved numerically by writing them in the finite difference form; which, can be solved by the implicit finite difference techniques. The implicit finite difference method advances the solution of Saint-Venant equations from one time line to the next one simultaneously for all the points along the river axis x.

In the Weight Four Point Scheme, the continuous distance-time solution domain, x-t, in which the solutions of the water depth h and the discharge Q are considered, is represented by a rectangular net of discretized points, equal or unequal intervals of  $\Delta x$  and the same  $\Delta t$ , along the distance x and the time t axes, respectively.

For the river networks suppose that there are N cross sections in a single river; therefore, there are (N-1) reaches, when the nonlinear equations are considered for (N-1) reaches it yields  $2^*(N-1)$  equations with 2N unknowns at the new time step  $(h_{j+1}, Q_{j+1})$ . Using two additional equations, U.S and D.S boundary conditions, the result is a system of 2N non-linear equations in 2N unknowns which can be solved by Newton Raphson iteration method for every time step.

## 2.3 The Initial and the Boundary Conditions

The initial conditions can be obtained from; 1)- The computed water depth and discharge which have been saved from the previous flood event unsteady flow simulation;

and 2)- From the observed water level and the discharge at each cross section; where, the river gauge stations are located.

All downstream discharges are determined by the summation of the flows from the upstream to downstream boundaries, including tributaries inflow to the main rivers, and the lateral in/outflow along the river course. The water depth at each cross section is calculated from the steady flow state and from the observed water surface level at rivers' gauge stations.

The upstream boundary conditions are required at the upstream ends of all streams. For all streams, which are not connected to others reaches or to storage areas, the upstream boundary in most cases is either the flow or the stage hydrograph. For the dynamic river flow model, the classic upstream boundary conditions is the flow hydrograph, where the stage hydrograph can be used when the flow gauge stations are not available at upstream ends or the quality of flow data is in question. For the downstream rivers' parts, which are simulated by the dynamic river flow model, the upstream boundary conditions are the discharge of the upstream river grid which calculated by the diffusive river flow model. For the upstream river parts, which simulated by the diffusive river flow model, the upstream boundary condition is taken as the stage hydrograph in case of the availability of the flow gauge station or the river grid is not the last grid of the stream; otherwise, the continuity and Manning equations can be used to calculate the water depth at the time step (j+1).

The downstream boundary conditions are four types; stage hydrograph, flow hydrograph, rating curve, and Manning's equation. In the current dynamic-diffusive river flow model, the stage hydrograph is used at the main river downstream end. For the tributaries, the downstream boundary conditions are taken as the water depths calculated from the water levels at the junctions points of these tributaries with their main rivers at the new time step (j+1).

## 2.4 Lateral In/Outflow

Lateral in/outflow can come from various sources as tributaries, diversions, reservoirs, storage areas, infiltrations, evaporations, water supply pump stations, precipitations, watershed's runoffs and river-groundwater exchanges. The current river dynamic-diffusive flow model considers the lateral in/outflow as the sum of all previous sources and considers the tributaries' lateral flow as a uniform flow along the grid of connection with its main river.

#### 2.5 Friction Slope

Fread (1978, 1988) in DWOPER and DAMBRK assumed that the friction slope is equal to the water surface slope and UNET (U.S. Army Corps of Engineers 1991b, 1993) used the friction slope at the last cross section. Those two assumptions, which produce slightly different results, are reasonable. In the current dynamic part of the river flow model the friction slope term can be expressed as in Eq. 6 where the term Q|Q| has the magnitude of  $Q^2$  and the sign +ve or -ve depending on whether the flow is downstream or upstream the river; respectively.

## 2.6 Arakawa River Network and Watershed

Arakawa River is one of the most important catchments in Japan, with about 2940 km<sup>2</sup> watershed area. An area of about 380 km<sup>2</sup> of Arakawa river watershed extended from km 42.0 to km 86.0 of Arakawa River contains two rivers, the main, Arakawa River, and a branch, Iruma River with a length of 9.60 km is considered in the cancellations. This part of the whole watershed is divided to 369 grids of about 1.0x1.0 km size; according to this size, Arakawa

River has 43 cross-sections and 5 gauge stations (G1 $\sim$ G5) and Iruma river has 10 crosssections and 2 gauge stations (G6 and G7). Every River is divided to two parts, the upstream part which is simulated by the diffusive model and the downstream part which is simulated by the dynamic model as shown in **Table 1**.

Rivers Gauge Stations					Dynamic model part		Diffusive model part		Avg. So
No	River	Cross- section	Floodplain Elevation	km	From (km)	To (km)	From (km)	To (km)	(m/km)
G1	Arakawa	2	7.70	41.89					0 562
G2	Arakawa	14	12.95	53.63					0.793
G3	Arakawa	28	22.30	68.3	41.89	66.40	66.40	85.6	1.71
G4	Arakawa	35		76.53					2.74
G5	Arakawa	43		85.6					
G6	Iruma	7	11.05	5.83	0.00	5.40	5.40	9.63	0.231
G7	Iruma	10		9.63					1.642

**Table 1** The Gauge Stations, the Rivers Parts and the Rivers Bed Slopes.

Every cross section, which contains the main channel and two floodplains, is represented by 11 pairs of x and y coordinates points as shown in **Figure 2**. The cubic spline interpolation technique is used to calculate the properties of the river cross-sections and their derivative values.



Figure 1 a)- Natural and 11 points river cross section, b)- Area and perimeter values and their variation with the water depth using the cubic spline technique.

## 3. RESULTS AND DISCUSSION

## 3.1 Effectiveness of the Diffusive River Flow Model with the Compound Channels

In the diffusive model, the compound channels cross sections are considered as rectangular sections so that the most important variables are the water depth, the roughness coefficients and the channel width. Figure 2a shows the simulated discharge values using the old diffusive river flow model without any modification for the rivers widths, for the whole basin as mentioned in section 2.6. There are two cases, small and big flood events Flood1 and

**Flood2** respectively. For the small flood **Flood1**, the calculated discharge values approach the observed values with little differences. For the big flood, **Flood2**, in case of considering only the main channel width, as the water runs into the floodplains, the water dissipated on the floodplain area and the main channel cross section can't load the water amount, then the calculated discharge values are smaller than the observed ones. The diffusive model is not suitable for the simulation of the river flow in the compound and the natural rivers with floodplains.



Figure 2 a)- The observed and the simulated discharge values for the diffusive river flow model and b)- Arakawa River longitudinal section with 4 Floods' initial and maximum observed water surface levels.

#### 3.2 Dynamic-Diffusive River Flow Model (DRDDM) with the Compound Channels

**Figure 2b** shows Arakawa River's longitudinal section which includes the initial water surface levels at all gauge stations and the maximum water surface levels during four floods events measured at the same gauge stations. In most floods events, the water depth increases rapidly, the water overcomes the main channel and runs into and covers the river floodplains with big depths. From the same figure we can notice that, the slopes of Arakawa river's upstream reaches are steep with small water depths; where, the slopes of the downstream reaches are mild slopes with relatively high water depths. The big floodplains' areas and wide widths, as shown in **Figure 1**, affect the simulation of the flood discharge and water depths as they can't be neglected in the river flow calculations and simulations. Some modifications to the diffusive river flow model, to be adaptive with the natural and compound channels with/out floodplains, have been done. The modified diffusive river flow model was applied to simulate some floods events, the simulated values are still far away from the observed ones for both the discharge and the water depth especially at the river downstream reaches and gauge stations as shown in **Figures 3** and **4**.

In DRDDM model, the modified diffusive model is applied to the rivers upstream reaches where the dynamic river flow model is applied to the rivers downstream reaches and the upstream reaches discharge values are the upstream boundary conditions of the dynamic model. The simulated river's discharge and water depth values using the new DRDDM model approach the observed ones with little differences as shown in **Figures 3** and **4**.

The new DRDDM model coupled with other hydrological models mentioned on section 2.1 solves many difficulties of the flow routing in the natural and compound channels out/with vegetated floodplains located in hill or flat areas and gives good and acceptable results.



(c) October 2004

(d) September 2007

Figure 3 Observed "OB" and simulated discharge values for diffusive "Diff" and DRDDM models at gauge stations No. 1, 3 and 6 for four flood events.



**Figure 4** Observed "OB" and simulated water surface level for the DRDDM and the diffusive "Diff" river flow models at gauge stations No. 2, 3 and 6 for two flood events.

#### CONCLUSION

The diffusive model is not suitable for the simulation of the river flow in the compound and natural rivers with floodplains especially at big flood events when the water covers and runs into the channel floodplains. The distributed water balance with dynamic-diffusive river flow (DRDDM) model can be useful in solving various difficulties of the flow routing in the natural and the compound channels with/out vegetated floodplains locate in the hill or flat areas and gives good and acceptable results.

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