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Experimental and Numerical Study of Tsunami Wave Propagation and Run-Up on Sloping Beaches

EXPERIMENTAL AND NUMERICAL STUDY OF TSUNAMI WAVE PROPAGATION AND RUN-UP ON SLOPING BEACHES

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ABSTRACT

In this paper, the propagation and run-up kinematics of tsunami-like solitary waves on sloping beaches are investigated experimentally and numerically. Laboratory experiments concerning breaking solitary waves travelling toward two comparatively sloping beaches (1:20 & 1:60) in a large scale wavetank (300 m × 5 m × 5.2 m) are presented to observe the wave evolution behaviors and the subsequent run-up phenomena. The detailed processes of a breaking solitary wave evolving on a 1:60 mild slope are particularly addressed. An empirical formula for reasonably estimating the maximum run-up heights of breaking solitary waves on a wide range of beach slopes (1:15-1:60) is also suggested. Additionally, numerical modelings based on a highly-nonlinear Boussinesq equations are first calibrated against the previous laboratory works and the present experimental data well and then be utilized to extend and confirm our experimental findings. The maximum run-up/run-down properties of breaking and non-breaking solitary waves on different plane beaches are examined through numerical experiments. Overall, present analyses indicate that the maximum run-up height of non-breaking wave increases with the decrease of slope angle, while that of breaking wave decreases with the growth of bed slope. Our empirical formula is also validated to be applicable.

Keywords: Tsunami, Experiment, Solitary wave, Evolution, Run-up, Run-down, Boussinesq

1. INTRODUCTION

The terrific tsunami catastrophes in Indian Ocean area in 2004 dramatically awake academic notice among a wide spectrum of hydromechanics experts during the past years (e.g. Liu et al. 2005; Arcas and Titov, 2006; Yomamoto et al, 2006). These field survey reports clearly point out that the inland inundation of tsunami disaster has a direct relevance to the run-up/run-down phenomena on the nearshore region. Especially, the island countries of East-Asia (e.g. Japan; Korea; Taiwan; Philippines) located near the intersection of Eurasian and Philippine plates are possibly suffered conspicuous extreme waves (e.g. tsunami and storm surge) propagating from the Pacific Ocean triggered by a undersea earthquake with considerable energy (e.g. Wang and Liu, 2006; Liu et al., 2007). It is therefore believed that the understanding of tsunami-wave mechanisms is of crucial importance for the tsunami hazard mitigation system establishments among these regions.

Figure 1 Laboratory image of a solitary-wave-type tsunami propagating toward a 1:60 sloping beach in the present supertank (offshore view).

In the past decades, it can be seen that the solitary-type waves were typically referred to model various physical properties of tsunami waves on a sloping beach owing to their water-wave hydrodynamic similarities, such as wave evolution mechanics (e.g. Synolakis, 1991; Skjelbreia and Synolakis, 1993; Grilli et al., 1997; Hsiao et al., 2008), run-up/run-down kinematics (e.g. Synolakis, 1987; Zelt, 1991; Tadepalli and Synolakis, 1994; Titov and Synolakis, 1995; Lin et al., 1999; Li and Raichlen, 2001, 2002, Hsiao et al., 2008; Fuhrman and Madsen, 2008) and inherent fluid structure dynamics (e.g. Lin et al., 1999; Jensen et al., 2003; Ting, 2006, 2008). Plentiful and ponderable information which is of great importance in nearshore solitary wave hydrodynamics has been remarkably reported and enables us to well explore the tsunami-wave mechanics. However, because of the limited length of wave flume and therefore the performance of solitary waves propagating over a mild sloping beach is either infeasible for a travelling wave to fully develop or to be limited to a very shallow water depth (Skjelerbia and Synolakis, 1993). Available experiments on solitary wave propagation upon a mild slope with a deep water depth are relatively reported. Further, numerical modelings based on the shallow water-wave equations might inappropriately account for wave run-up/run-down problems owing to the physical drawbacks of hydrostatic assumption (Zelt, 1991; Titov and Synolakis, 1995).

More recently, Hwang et al. (2007) and Hsiao et al. (2008) systematically reported experimental analyses on breaking solitary waves propagation upon mild sloping beaches (1:20 & 1:40 & 1:60) in a large scale wave tank (Fig. 1) at Tainan Hydraulics Laboratory (THL). This tank has a spectacular dimension scale (i.e. 300 m in length, 5.0 m in width and 5.2 m in depth) and, particularly, is capable of quantitatively and qualitatively mimicking the so-called such "tsunami" extreme waves propagating towards onshore structure which responds to the physical phenomena more close to a natural circumstance. It also indicated that we can possibly observe the detailed solitary wave evolution mechanisms from deep to shallow water regions, especially for a mild slope and deep water depth which are of near impossibility in a general laboratory tank.

The main goal of this paper is to investigate the propagation and run-up kinematics of solitary-wave-type tsunamis climbing up sloping beaches. Both experimental and numerical analyses are described. Laboratory data measured on two comparatively mild sloping beaches (1:20 & 1:60) are presented. The wave evolution characteristics of breaking solitary waves on a 1:60 slope are particularly addressed. A well-validated highly-nonlinear Boussinesq model, namely, COULWAVE (COrnell University Long and intermediate WAVE model), is also applied to extend and confirm our experimental findings [see Lynett et al. (2002) for model details]. By recreating the available laboratory experiments, the model simulation competence is reasonably calibrated both for breaking and non-breaking solitary waves. The effects of wave nonlinearity and bed slope correlated with the maximum run-up/run-down heights are numerically examined and discussed. This paper is organized as follows. The laboratory setup and the measured data of the present tsunami experiments are briefly described in Section 2. Section 3 analyzes the run-up/run-down properties of breaking and non-breaking solitary waves through a set of numerical experiments. Eventually section 4 outlines our findings obtained in this article.

Table 1 Laboratory setup and wave conditions.

Beach slope	Offshore water depth (m)	Wave nonlinearity	Start of beach slope (m)	Location of reference gauge (m)
$\cot\beta$	h_o	$\varepsilon = H_o / h_o$		
20	1.75	0.06~0.245	$x = 197$	$x = 178.8$
60	1.2, 2.2, 2.9	0.011~0.338	$x = 50$	$x = 24$
* Note that the present coordinate system of $(x,z)=(0,0)$ (m) in the present supertank is defined as the absolute locations at wave generator and still water level with downstream and upward directions, respectively.				

2. LABORATORY EXPERIMENTS

The present experiments were carried out in a large scale wave tank (300 m × 5 m × 5.2 m) at Tainan Hydraulics Laboratory of National Cheng Kung University. The near-perfect solitary waves were generated at one end of the flume by a programmable wavemaker using the generation method of Goring (1978) for all tests. There are two impermeable sloping beaches (1:20 & 1:60) employed in the present study. Detailed experimental conditions are summarized in Table 1. In all the experiments the free surface elevation and the shoreline motion were recorded by the gauges deployed in the tank (Fig 1) and our visual estimations. We note that all waves produced of the present laboratory trials were eventually broken during the course of run-up on both sloping bottoms. The wave breaking commencement and the plunging breaker were reasonably determined based on our laboratory observations by also comparing the well-known breaking indices of Grilli et al. (1997) (not shown here). Interesting readers can obtain more experimental validations and discussions in Hsiao et al. (2008).

Figure 2 depicts the detailed amplitude evolution processes of breaking solitary waves climbing up a 1:60 sloping beach based on a 1:60 slope experiments. Note that the measured wave amplitude is represented as a function of breaking water depth as suggested by Synolakis and Skjelbreia (1993). They speculated that the amplitude distributions on a plane beach with a bed slope less than 1:50 can be divided into at least four zones and also pointed out that the wave amplitude distribution in each zone can be well-described by a simple power-law-type formula as follows,

$$\frac{\eta_{\max}}{h_b} \sim \left(\frac{h}{h_b}\right)^n \tag{1}$$

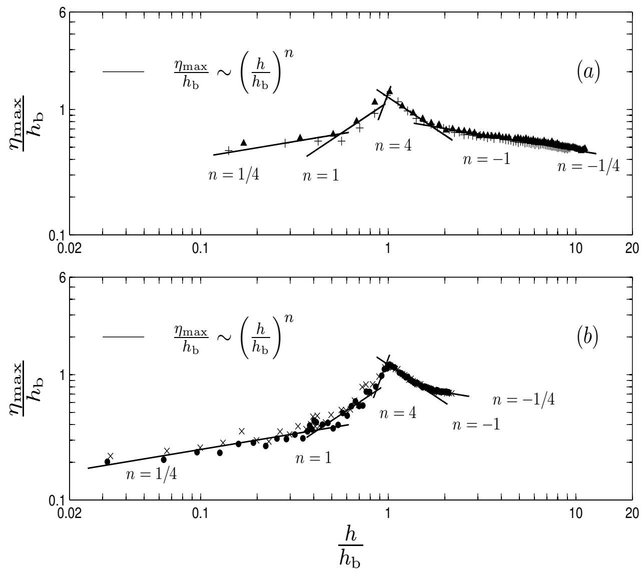

Figure 2 Evolution of wave amplitude of a breaking solitary wave climbs up a 1:60 sloping beach. [(a): $\varepsilon = 0.041$(▲); $\varepsilon = 0.052$(+), (b): $\varepsilon = 0.322$(×); $\varepsilon = 0.338$(●)].

where $\eta_{\max}$ represents the maximum local surface elevation, h denotes the local water depth, h_b stands for the breaking water depth and n is a coefficient to be discussed later. Apparently, in Fig. 2 there are abundant laboratory data composed of five zones during the entire evolution course. Our laboratory results reveal that the conclusions of Synolakis and Skjelbreia (1993) are quite applicable for a gentle slope (i.e. n = -1/4, -1, 4, 1) and noticeably a further bore front propagation region after the outer surf zone does exists for both the small or strong wave nonlinearities [Figs 2(a) and 2(b)]. We also found that in this regime the data suggest the wave amplitudes decrease approximately in proportion to a quarter power of the local water depth (i.e. n = 1/4). The same experimental finding is also given by Ting's (2006).

Figure 3 further shows the maximum run-up heights obtained in the present experiments. The maximum run-up height R_u in this paper is defined as the maximum vertical distance from the run-up tongue of water shoreline to the still water level and it is applied throughout this article. For the sake of comparison, the available laboratory data (see the caption of Fig. 3) are also shown to analyze and it is also worthy to emphasize that all the laboratory data in Fig. 3 are relevant to breaking solitary waves on different slopes. Evidently, the laboratory results in Fig. 3 exhibit that for a given slope the run-up height increases as the growth of wave nonlinearity while the run-up height decreases as the reduction of slope angle for a specific wave nonlinearity. An asymptotic formula for predicting the maximum run-up height based on the nonlinear least-square method and the laboratory data listed in Fig. 3 is empirically obtained as Eq. (2)

$$\frac{R_u}{h_o} = 7.712(\cot\beta)^{-0.632}(\sin\varepsilon)^{0.618}, \quad 0 < \varepsilon \le 0.5 \tag{2}$$

Obviously, fairly good agreements between the predictions of Eq. (2) and the laboratory data are obtained, indicating that the present formula is capable of estimating the reasonable run-up heights of braking solitary waves on a sloping bed with a wide range of beach slopes (i.e. $15 \le \cot\beta \le 60$). The small discrepancies between the model predictions and the laboratory results are also found on account of the equal weighting used in the least-square algorithm.

Nevertheless, the predictive capability of Eq. (2) is reasonably good based on the comparisons shown in Fig. 3 even though the physical clarification in the formula derivation is absent [also see Hsiao et al. (2008) for comparisons against other available models].

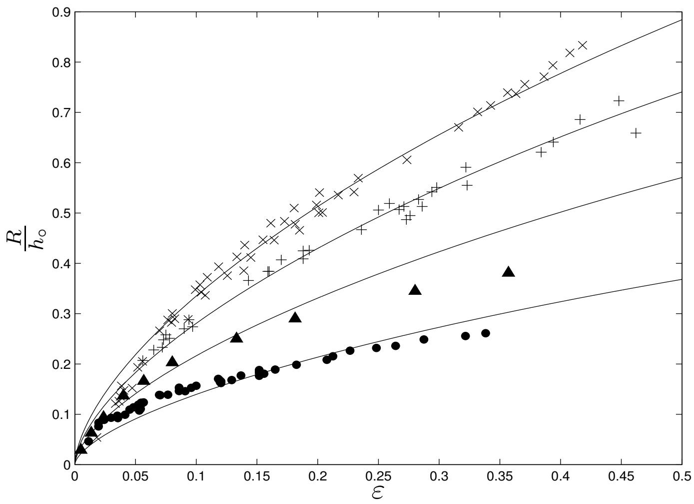

Fig. 3 Normalized run-up height versus wave nonlinearity of breaking solitary waves on a plane slope. [Experimental data of 1:15 slope by Li and Raichlen (2002)(×); experimental data of 1:19.85 slope by Synolakis (1987)(+); experimental data of 1:30 slope by Briggs et al. (1995)(▲); experimental data of 1:60 slope of present experiments (●); Eq. (2) (solid line)].

3. NUMERICAL MODELLINGS

This section numerically investigates the run-up/run-down phenomena of breaking and non-breaking solitary waves on sloping beaches by using COULWAVE. Model simulation capability is firstly calibrated against the available laboratory works and the present experimental data, including the examination of moving boundary technique (see Lynett et al., 2002) for the non-breaking wave (i.e. Zelt, 1991; Li, 2000) and the validation of energy dissipation algorithm for the breaking wave (i.e. Li, 2000 & present experiments). A set of numerical experiments are then presented. The bottom friction effects are also discussed.

3.1 NON-BREAKING SOLITARY WAVES

Figure 4 represents the comparisons of our numerical results and the laboratory data of Zelt (1991) and Li (2000) for the water surface elevation [Fig. 4(a)], the shoreline movement [Fig. 4(b)] and the run-up height [Fig. 4(c)], respectively. The simulation setup in the present cases are identical to Zelt (1991) and Li (2000), and $\Delta x = 0.016$ (m) and $\Delta t = 0.002$ (sec) are appropriately chosen in the model calculation. Note that the breaking model is not implemented in this section since the laboratory reports of Zelt (1991) and Li (2000) both pointed out that the waves do not break during the shoreline deformation processes. Overall, the COULWAVE can excellently capture the kinematics of non-breaking solitary waves climbing up a sloping bed for the different experimental cases compared with the laboratory results and the analytic solutions (i.e. run-up law; Synolakis, 1987). The moving boundary algorithm utilized in the present model is verified to be satisfactory and the convergence capability of it is also confirmed by carrying out the different grid sizes with the same numerical setup [Figs. 4(a) and (b)]. Noticeably, the consideration of bottom friction effects in the model calculation does no significantly affects the run-up heights for non-breaking waves [Fig. 4(c)]. This numerical consequence also consists with the conclusions of Liu and Cho (1994) and Lynett et al. (2002), indicating that the COULWAVE can be convincingly applied in studying the wave evolution and run-up problems of non-breaking solitary waves propagation on the sloping bottom with no need of any *ad-hoc* energy dissipation terms.

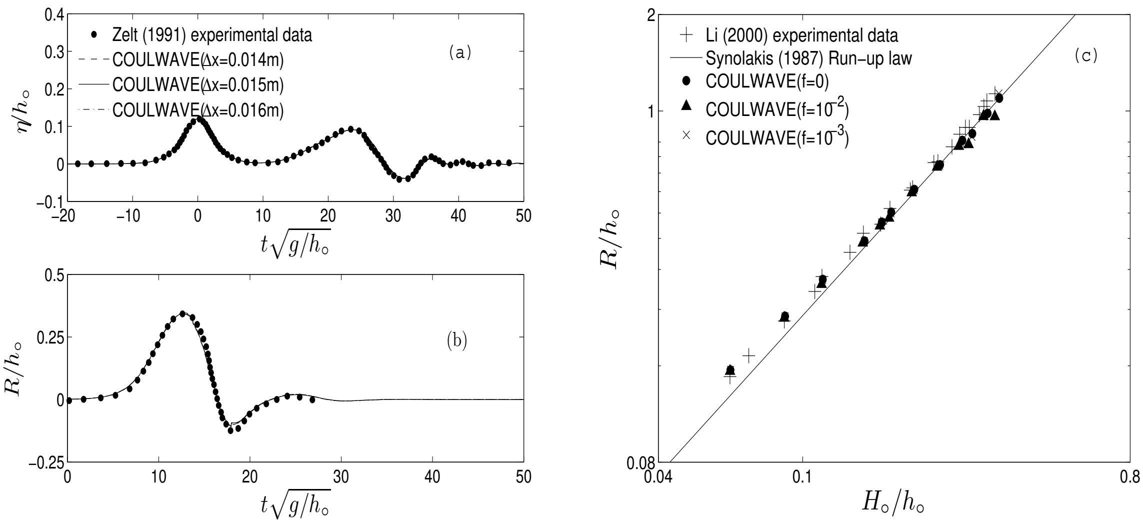

Figure 4 Simulations of non-breaking solitary waves climb up a sloping beach. [(a), time history of surface elevation; (b), shoreline movement; (c), maximum run-up height correlated with different bottom friction coefficients, f. Note that the subplots (a) and (b) are of the case of a 1:2.75 sloping beach of Zelt (1991), subplot (c) is the case of a 1:2.08 sloping beach of Li (2000)].

3.2 BREAKING SOLITARY WAVES

It is well-known that the wave breaking mechanisms in the Boussinesq-type numerical models should be empirically calibrated with experimental data for a range of setup to provide a reasonable calculation (e.g. Kennedy et al., 2000; Lynett et al., 2002). In which the bottom friction are of great importance in a model prediction of wave run-up upon a gentle slope because the thin layer of leading bore fronts dominates the run-up tongue and the long course of swash movement.

Figure 5 shows that the present numerical results are in favorably good agreements with the present laboratory data for the evolution of a breaking solitary wave propagating up a 1:60 plane slope. Note that the bottom friction coefficient $f = 0.0025$, $\Delta x = 0.1$ (m) and $\Delta t = 0.01$ (sec) are selected in the following numerical experiments. Strikingly, the COULWAVE can reasonably capture the wave nonlinearity and the phase dispersion effect through the comparisons of spatial wave undulations between the present numerical and experimental results, suggesting that the breaking model in the COULWAVE can satisfactorily mimic the energy dissipations induced by wave breaking and bottom friction.

Figure 6 accounts for that the model predictions of run-up heights are in reasonable agreements with the experimental data (i.e. Li, 2000 & Synolakis, 1987& Present experiments) by applying the same bottom friction coefficients and the simulation conditions

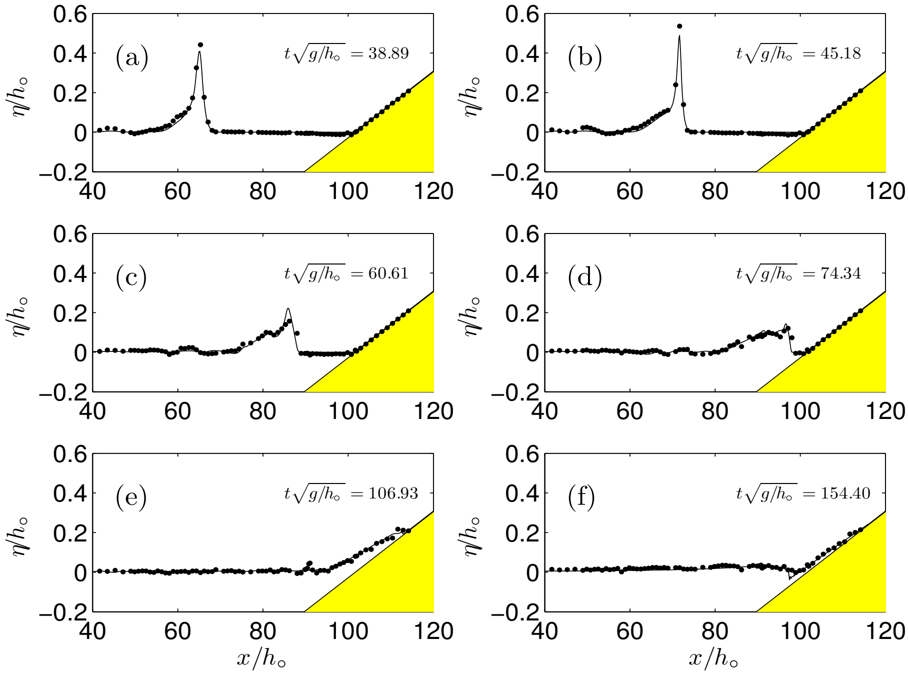

Figure 5 Simulation of a breaking solitary wave climbs up a 1:60 sloping beach ($\varepsilon = 0.338$). [Laboratory data of Hsiao et al. (2008a) (●); COULWAVE (solid line)].

[i.e. $f = 0.0025$; $\Delta x = 0.1$ (m); $\Delta t = 0.01$ (sec)]. Note that the maximum deviation of the calculated run-up height is about 10% compared with the corresponding measurement data. This discrepancy is partly due to the same considerations of the bottom friction effects in two different laboratory environments. The scale effects between present 1:20 slope experiments and 1:19.85 slope of Synolakis (1987) are found with insignificant correlations [see Hsiao et al. (2008a,b) for more discussions].

4. DISCUSSIONS AND CONCLUSION

Overall, it is convincing that the COULWAVE can reasonably predict the run-up heights both for breaking and non-breaking solitary waves based on the above discussions (Figs. 4, 5 and 6). Consequently, Fig. 7 numerically concerns the effects of bed slope and wave nonlinearity on the maximum run-up heights both for breaking and non-breaking solitary waves using COULWAVE. The simulation conditions for the given numerical experiments are specified in the Table 2. For the calculation efficiency and validity, the implementation of wave breaking model is dependent on the breaking criteria of Synolakis (1987) for our numerical wave conditions. The reference wave amplitude is obtained by the numerical gauge dynamically located in the position as suggested by Li (2000) to minimize the reflection effects. Also the effect of bottom friction is not considered for the non-breaking cases according to the former analyses [see Fig. 4(c)] while the same bottom friction coefficient of breaking cases are included as the descriptions in Section 3.2.

Figure 7 summarizes the calculated results of run-up and run-down of the present numerical experiments. The breaking limitation of Li and Raichlen (2001), the run-up law of Synolakis (1987), the modified run-up law of Li and Raichlen (2001) , the corresponding laboratory data (see caption of Fig. 8) and the empirical formula of Eq. (2) are also

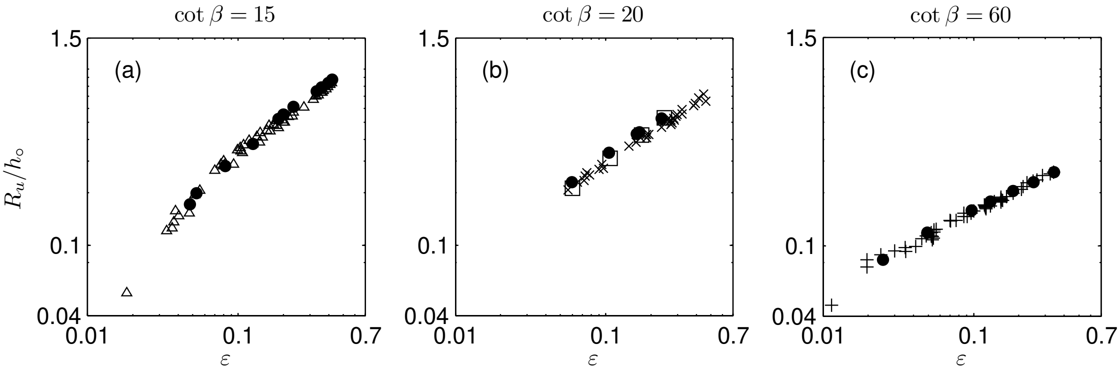

Fig. 6 Comparison of maximum run-up height of breaking solitary waves for different wave nonlinearities versus bed slopes [1:15 slope data of Li & Raichlen (2002) (△); 1:19.85 slope data of Synolakis (1987) (x); 1:20 slope data of present experiments (□); 1:60 slope data of present experiments (+); COULWAVE (●)].

incorporated into the figures for a comparison. Overall, results in the Fig. 7 indicate that the maximum run-up/run-down height increases as the slope angle decreases for non-breaking waves, while run-up/run-down height decreases with the decrease of beach slope for breaking waves. The same conclusions are also given in Li and Raichlen (2002) (only for run-up). We also found that in the Fig. 7(a) favorably good agreements are found between our numerical results, the available experimental data, the analytic approaches and the results calculated by Eq. (2). Although the overall results show quite promising, there are no available data to validate our run-down results for such a wide range of wave conditions and beach slopes. Nevertheless, it is instructive to see how the run-down changes with the wave nonlinearities and beach slopes. Clearly, the run-down heights are insensitive to initial wave nonlinearity as the beach slope is less than $\cot\beta = 25$. It is not unexpected because the long course of wave propagation on gentle slope will generally result in the energy dissipation and therefore the near saturation on the shoreline is achieved. On the other hand, the run-down is significantly influenced by wave nonlinearity as $\cot\beta < 20$. In particular, while the run-down heights increase as beach slope decreases until the maximum run-up is reached for nonbreaking events, the run-down heights decreases with the decrease of beach slopes for breaking events. This tendency is similar to that of run-up phenomena.

As a conclusive note, we experimentally and numerically describe the evolution and run-up kinematics of breaking and non-breaking solitary waves on a uniform slope. Our laboratory data not only confirm the conclusions of Synolakis and Skjelbreia (1993) but also demonstrate another decay regime after the outer surf zone. The amplitude distribution in this new region can be appropriately modeled by a power-law-type formula in proportion to a quarter power of the local water depth. An easily implemented formula for estimating the maximum run-up heights of breaking solitary waves are suggested. The COULWAVE simulations are well-calibrated the available experiments and the present laboratory data for both breaking and non-breaking waves. Numerical experiments are then performed to extend and confirm our findings. Based on the present analyses, it is convincingly concluded that the maximum run-up height of non-breaking solitary wave increases with the decrease of slope angle, while that of breaking wave decreases with the growth of bed slope. Nevertheless, more experimental data are needed to confirm our findings.

Table 2 Numerical experiment conditions of Fig. 7

h_o	Δx	Δt	f
1.0 (m)	0.1 (m)	0.01 (sec)	0.0025 (only for breaking cases)
Note that in our numerical tank the numerical slope starts at $x = 20$ (m) downstream of the wavemaker.			

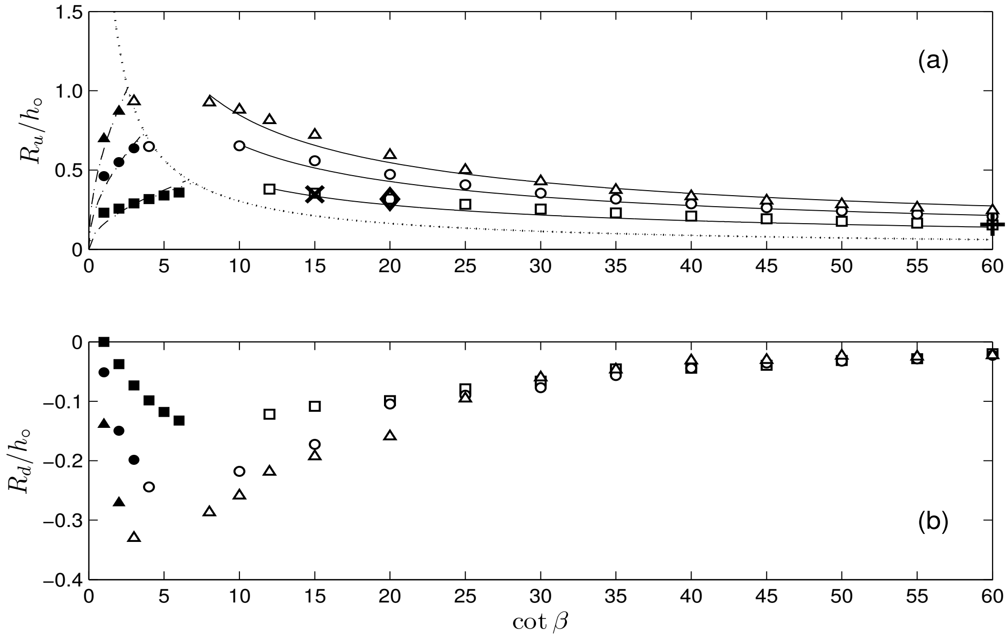

Figure 7 Numerical experiments of maximum (a) run-up and (b)run-down heights of breaking and non-breaking solitary waves for different wave nonlinearities versus bed slopes.

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