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A Methodological Framework for Evaluating Uncertainty in Hydrodynamic Models

A METHODOLOGICAL FRAMEWORK FOR EVALUATING UNCERTAINTY IN HYDRODYNAMIC MODELS

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ABSTRACT

Model uncertainty analysis is an important component of a numerical modeling study used to quantitate the confidence that may be associated with model predictions. Uncertainties in model parameters, input data, and different model structures may all propagate through a numerical model, resulting in uncertainties in model output. While quantification of model uncertainty is undeniably important, general issues remain with regard to the reliability of alternative methods and strategies to evaluate different sources of uncertainty during the modeling process. This investigation provides a discussion about the relevance of uncertainty analysis in hydrodynamic modeling, identifying the fundamental sources of uncertainty in practical studies, and describing a set of effective strategies for their evaluation. The methods presented have been successfully implemented in previous hydrodynamic studies and in other fields of study such as hydrological modeling, ecological modeling, and weather forecast. Although not evaluated in this paper, it is proposed that these methods for quantification of the individual sources of uncertainty can be used in a strategic way to provide estimations of total model uncertainty, under the assumption that this uncertainty can be expressed as a linear combination of the uncertainty caused by the individual sources of error. Ultimately, it is expected the present paper to be the base for the development of a methodological framework for the evaluation of uncertainty in hydrodynamic modeling studies.

1. INTRODUCTION

Hydrodynamic models (HMs) are useful tools for representing the governing processes of fluid motion and scalar transport (e.g. tracers, salinity, temperature, etc.) in natural and man-made water systems. HMs also provide a fundamental base for the development of more complex models of water quality, ecology, and sediment transport. Given their relevance, HMs are increasingly used around the globe to support research, policy analysis, and decision making by designers, stakeholders, and governmental agencies. Some practical examples of the use of these models include the design of restoration and ecological conservation programs (e.g. for the TMDL program in the USA), the design of hydraulic structures (e.g. levees and channels) for prevention of flood disasters (e.g. Savant et al. 2010), and the evaluation of the ecological impacts that may result from the withdrawal of water from natural systems for human consumption (e.g. Sucsy et al. 2010)

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Although in principle HMs provide an accurate mathematical representation of the physics of fluid motion, in the practice the results of these models are subject to uncertainty due to the existence of several sources of errors during a modeling process. The most relevant sources of errors can be grouped in: 1) Input data errors, 2) Model structure errors, and 3) Errors in the estimation of model parameter values (e.g., Horritt and Bates 2001; Horritt and Bates 2002; Johnson 1996; Pappenberger et al. 2005; Sehnert et al. 2009; Somlyódy 1983; Thompson et al. 2008). Input data errors result from imprecise measurement techniques and procedures to collect field data, inadequate equipment precision, and also as a result of the interpolation and extrapolation of data given the infeasibility of collecting field records at the spatial and temporal resolution required by the model. Model structure errors on the other hand refer to the inaccuracies that may result from the inappropriate selection of a model dimensionality (i.e. 1D, 2D or 3D), spatial resolution, turbulent closure scheme, etc. Finally, errors in the estimation of model parameter values result from the difficulty of determining an optimum set of parameter values such as roughness coefficients and dispersion coefficients at the spatial resolution of the model.

If not quantified and comprehensively reported through an uncertainty analysis, the uncertainty resulting from the above sources of errors (also known in conjunction as model uncertainty) can negatively impact the effectiveness of the management programs and decisions based on the use of mathematical models (Beven 2009; McIntyre et al. 2002; Refsgaard and Henriksen 2004; Refsgaard et al. 2007; Walker et al. 2003).

Although uncertainty analysis has received considerable attention during the last two decades by the water resources community, in the field of hydrodynamic modeling several issues remain unresolved with regard to the reliability of the available methods and strategies to evaluate different sources of uncertainty during the modeling process. These issues evidence the need for studies evaluating alternative methods for the independent quantification of the different sources of errors, and also the need for strategic protocols for applying uncertainty analysis as a fundamental component of any hydrodynamic modeling study.

The purpose of this investigation is to discuss the relevance of uncertainty analysis in hydrodynamic modeling. The document describes the main sources of uncertainty in practical studies, and presents a set of specific methods for their quantification. Such methods have been selected based upon the discussions and experiences reported in previous investigations in the field of hydrodynamic modeling and other fields of study such as hydrological modeling, water quality modeling, and weather forecast. The discussions presented in this paper (motivated by studies indicating the need for uncertainty analysis in hydrodynamic modeling such as the uncertainty analysis of the St. Johns River, FL (Sucsy et al. 2010)), are part of an ongoing investigation intended to develop a methodological framework for the quantification of uncertainty in hydrodynamic modeling studies.

It is important to indicate that although only a description of the selected methods for uncertainty analysis is presented in this document, useful references are provided to more detailed explanations of the methods and also to practical investigations where the methods have been implemented and evaluated.

The paper is organized as follows: Section 2 discusses if model uncertainty is or not a linear problem. Such discusion is fundamental for the explanation of the methods and strategies presented in this document. Section 3 discusses the importance of uncertainty analysis in hydrodynamic modeling, individually describing the impacts of the most relevant sources of errors encountered in practical applications. Section 4 presents a set of potential methods for the evaluation of the principal sources of uncertainty indicating the main advantages and limitations of the methods. Section 5 indicates how these methods can be used strategically to obtain model uncertainty estimates, and finally, section 6 presents the conclusions of the investigation.

2. IS MODEL UNCERTAINTY A (NON)LINEAR PROBLEM?

One of the most relevant questions during an uncertainty analysis is whether model uncertainty is a linear or a non-linear problem. Although in the practice it is possible to identify (at least qualitatively) the principal sources of errors in a hydrodynamic modeling study (i.e. errors in input data, model structure, and model parameters), it is extremely difficult to know whether the combined effects of these errors on the model outputs (i.e. model uncertainty) are or not a linear superposition of the uncertainty caused by the individual contribution of errors.

If model uncertainty is a linear problem, then it is reasonable to think in a framework for uncertainty analysis which first, evaluates individually the uncertainty caused by the different sources of error, and then, compute by linear superposition the resulting total uncertainty. Figure 1 shows a schematic representation of the model uncertainty assuming that it can be considered as a linear problem.

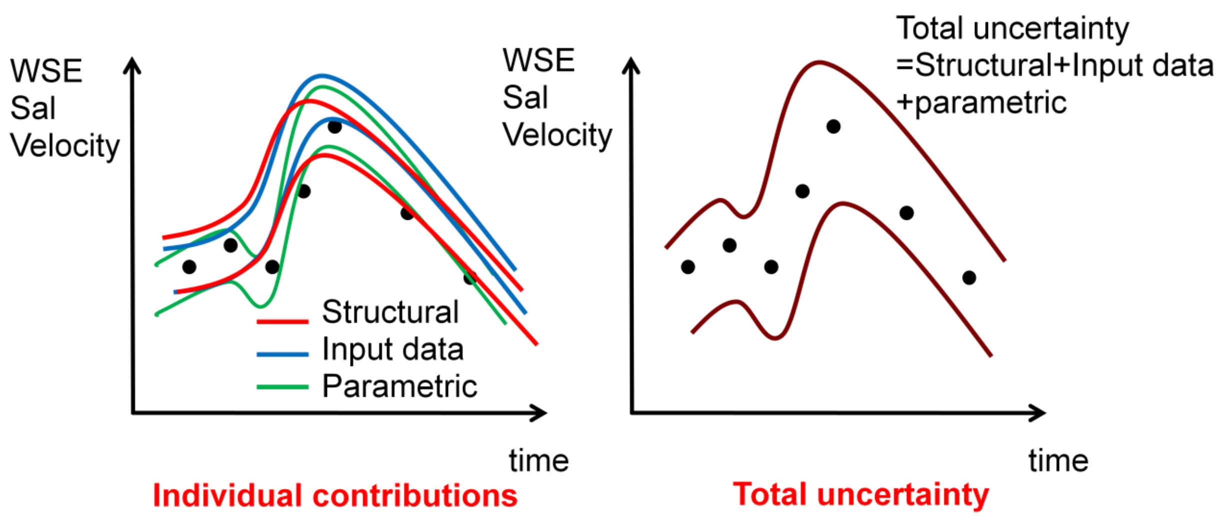

Figure 1. Schematic representation of model uncertainty. Total uncertainty is considered as a linear superposition of the individual effects of the different sources of errors. The vertical axis denotes typical hydrodynamic variables evaluated during a modeling exercise such as velocity, water surface elevations (WSE) and salinity (Sal).

However, if model uncertainty is a non-linear problem, then more complex strategies (or stronger assumptions) are required to quantify total model uncertainty. Presently, most studies involving uncertainty analysis implicitly assume that total uncertainty has nonlinear properties. This way, it is commonly assumed that evaluation of one source of uncertainty is representative of other sources of uncertainty more complex to evaluate. For example, studies based on the use of the Generalized Likelihood Uncertainty Estimation method (GLUE) developed by Beven and Binley (1992) assume that the uncertainty resulting from the errors in the model structure and boundary conditions can be evaluated implicitly by quantification of the parametric uncertainty. Another example is the use of the Sources of UNcertainty Global Assessment using Split SamplES (SUNGLASSES) method introduced by van Grievsgen and Meixner (2004) which increases the parameter uncertainty to a level that compensates the errors in the model structure and other non identifiable sources of uncertainty.

The idea of treating total uncertainty as a nonlinear combination of several sources of error (potentially captured or reflected by only one source of uncertainty such as parametric uncertainty) is attractive and reduces the efforts in evaluating other existing sources of uncertainty. However, in reality such assumptions can lead to unrepresentative magnitudes of the total uncertainty and over confidence in the model results. Note for example that uncertainty analysis over insensitive model parameters can lead to a high confidence in the model outputs (represented by 5-95% confidence bounds) even if the input data (e.g. boundary conditions) contains appreciable uncertainty. As results of these limitations, a linear combination of the principal sources of uncertainty although more complex, may be more appropriate resulting in a more conservative estimate of total uncertainty. We believe therefore that an appropriate representation of total uncertainty is to consider it as a linear problem and also that different methods can be used (where appropriate) to evaluate the individual sources of uncertainty in a hydrodynamic model application.

3. RELEVANCE OF UNCERTAINTY ANALYSIS IN HYDRODYNAMIC MODELING

As indicated in the introduction section, the existence of different sources of errors in hydrodynamic modeling can have a negative impact in the usefulness of HMs as supporting tools for decision making. This section is intended to provide a brief outline about the importance of uncertainty analysis in hydrodynamic modeling.

3.1 Input data uncertainty

Evaluation of the uncertainty resulting from errors in input data is not only useful to communicate to stakeholders and decision makers the limitations of the model's predictions due to the use of imprecise input information (e.g. due to spatial or temporal resolution, instrument precision, etc), but also important to identify and/or design strategies to improve the quality or availability of this information (by better collection techniques or instruments), and therefore the reliability of the model and the confidence on its predictions. However, evaluation of input data uncertainty is rarely conducted in hydrodynamic modeling studies despite it is accepted that errors in the input data can have important impacts on the model's predictive skills. Somlyódy (1983) for example concluded that uncertainty in the specification of the wind direction of a 1D hydrodynamic model of Lake Balaton in Central Europe could lead to errors of up to $\pm$ 15cm and $\pm$ 1500m^3s^{-1} in the predictions of water surface levels and flow discharges respectively. Likewise, Rueda et al., (2009) showed that uncertainty in the construction of spatially varied wind fields based upon single point measurements could lead to errors of up to one order of magnitude in the energy input to a 3D hydrodynamic model of the Salton Sea (a wind driven lake), California, resulting in a limited capacity of the model to correctly reproduce and predict the lake's mixing characteristics (in terms of stratification strength and duration).

Regarding evaluation methods, it is observed that most available studies have implemented Monte Carlo Simulations (MCS) as the strategy for evaluating the impacts of input data. MCS has the advantages of not being limited by the degree of non linearity of the model, and also of being flexible to be computationally parallelized. However, an important limitation of MCS is that is computationally intensive, and typically a large number of simulations are required to obtain unbiased statistical results. The above limitations preclude the use of MCS for practical purposes.

An alternative strategy for evaluation of input data uncertainty recently introduced in the field of hydrodynamic modeling is the First Order Variance Analysis (FOVA) (Blumberg and Georgas 2008; Sucsy et al. 2010; Thompson et al. 2008). FOVA is computationally inexpensive and has the notable feature of providing sensitivity estimates of the model's output to the input variables under analysis. Although further research is necessary to evaluate the benefits and limitations of the method in hydrodynamic modeling, Blumberg and Georgas (2008) and Sucsy et al. (2010) have shown that the method is effective identifying the input variables that have the highest contribution to model uncertainty, and also effective providing uncertainty estimates (in terms of uncertainty bounds) around model predictions. A more detailed description of this method is provided in the next section.

3.2 Model structure uncertainty

Structural uncertainty can be arguably the most complex and difficult to assess source of uncertainty in hydrodynamic modeling. However, studies investigating the real impacts that it may have on the predictions of HMs or about strategies to quantify it, are very limited. In the practice, model results can be substantially more sensitive to uncertainty in model structure than to uncertainty over parameter values or input data errors. As example, a recent study conducted by Sehnert et al., (2009) on the evaluation of structural uncertainty due to discretization resolution and dimensionality of a hydrodynamic polder model for flood control, shows that variations in the spatial resolution of the model can lead to differences of up to 150% on the predictions of flow velocity in the system. Obviously these differences can have negative impacts (for management purposes) if these predictions are used to drive other models for sediment transport or water quality analysis.

Presently, the methods for evaluation of model structure uncertainty are less developed than the methods for evaluation of other sources of uncertainty such as errors in the input data or model parameters. Existing strategies from other fields such as climate change modeling, hydrologic modeling, and ecological modeling, are mostly based on the use of multiple model structures to construct ensemble predictions (IPCC 2007), and also (in a lesser extent) based on formal statistical methods such as Bayes factors to guide the selection of alternative model structures (e.g. Link and Barker 2006; Marshall et al. 2007; Meyer et al. 2003; Min et al. 2007). However, to the author's knowledge none of these methods have been applied in hydrodynamic studies, mostly due to the computational burden, and effort required to construct and evaluate different model structures for a given hydrodynamic system.

3.3. Parametric uncertainty

Model parameters such as roughness coefficients and turbulent dispersion coefficients have considerable uncertainty given that they are lumped over spatial and temporal resolutions where important natural variations exist. As a result, parametric uncertainty is considered one of the most important sources of uncertainty in the practice of hydrodynamic modeling (Aronica et al. 1998; Johnson 1996; Pappenberger et al. 2005; Straatsma and Huthoff 2011; Werner et al. 2005).

Parametric uncertainty can have important impacts on model predictions. Warmink et al. (2010) for example showed that uncertainties in the estimation of the bed roughness of a 2D hydrodynamic model of the River Rhine could lead to differences of up to 70 cm in the prediction of flood levels, negatively impacting the effectiveness of the flood control structures designed using these models.

Many different approaches have been implemented in the past to investigate the importance of parametric uncertainty in water resources modeling. Some of the most relevant applied techniques include Monte Carlo Simulations (MCS), the Generalized Likelihood Uncertainty Estimation Method (GLUE), and Bayesian Monte Carlo analysis (BMC). However, in hydrodynamic modeling, evaluation of this source of uncertainty remains more the exception than the rule (e.g. Pappenberger et al. 2005; Romanowicz et al. 1996; Warmink et al. 2010), and further research is necessary to identify and evaluate these and other potential strategies for effective quantification of parametric uncertainty.

4. A SET OF STRATEGIES FOR EVALUATION OF UNCERTAINTY

In this section we present a set of methods for uncertainty analysis which can be effectively incorporated in any hydrodynamic study. These methods were selected based upon existing studies evaluating uncertainty in hydrodynamic modeling and other fields of study, and also based on ongoing studies implementing some of these methods in selected hydrodynamic models of the Gulf Coast area in the United States (e.g. Camacho et al. 2012).

4.3 First order variance analysis (FOVA)

The First Order Variance Analysis (FOVA) is a method for evaluation of input data uncertainty and parametric uncertainty. The method is relevant in the practice given the reduced computational requirements to obtain uncertainty estimates, and also because it provides a measure of model sensitivity to the input variable or parameter under analysis.

The fundamental idea behind the FOVA method is to construct a Taylor series expansion truncated at the first order term of the function $\mathrm{F(X)}$ that predicts the evolution of a given output variable $\mathrm{O = F(X)}$. The expansion is performed around a given point (typically the mean) of the input variables or parameters $\mathrm{X = (x_1, x_2, \dots, x_n)}$ under analysis i.e.,

$$F(X) = f(x_{1e}, x_{2e} \dots x_{pe}) + \sum_{i=1}^{p} \left. \frac{\partial F}{\partial x_i} \right|_{x_i = x_{ie}} (x_i - x_{ie}) \tag{1}$$

where p is the number of input variables or parameters under study; $\mathrm{x_{ie}}$ the value of the *i-th* input variable at the expansion or unperturbed point e; and $\partial \mathrm{F} / \partial \mathrm{x_i}$ the local change of the output variable O due to changes in the input variable $\mathrm{x_i}$. The expected value and variance of the output variable O are estimated based on eq. 1 as (Blumberg and Georgas 2008),

$$E[O] = E[F(X)] = f(x_{1e}, x_{2e} \dots x_{pe}) \tag{2}$$

$$Var(O) = \sigma_O^2 = \sum_{i=1}^{p} \left[\left. \frac{\partial F}{\partial x_i} \right|_{x_i = xe} \right]^2 E(x_i - x_{ie})^2 + 2 \sum_{i=1}^{p} \sum_{j=1, j \neq i}^{p} \left[\left. \frac{\partial F}{\partial x_i} \right|_{x_i = x_{ie}} \left. \frac{\partial F}{\partial x_j} \right|_{x_j = x_{je}} \right] E\left[(x_i - x_{ie})(x_j - x_{je}) \right] \tag{3}$$

If the input variables are statistically independent, then eq. 3 becomes:

$$Var(O) = \sigma_O^2 = \sum_{i=1}^{p} \left[\left. \frac{\partial F}{\partial x_i} \right|_{x_i = x_{ie}} \sigma_i \right]^2 \tag{4}$$

where σ_0^2 is the variance of the predictions of the output variable; and $\mathrm{\sigma_i}$ the standard deviation of the *i-th* input variable. The terms $\left[\partial \mathrm{F} / \partial \mathrm{x_i}|_{\mathrm{x_i = x_{ie}}}\right]^2$ are sensitivity coefficients which describe how the output variable varies with perturbations in the input variables. The derivative term

∂F can be evaluated numerically using a finite difference scheme as illustrated by Blumberg and Georgas (2008).

A limitation of the FOVA method is that it is based on linear assumptions about the response of the model to any perturbation in the model parameters or input variables. Therefore, in the practice such perturbations must be small and consistent with the assumption of linearity.

Few hydrodynamic modeling studies have implemented FOVA for the evaluation of input data or parametric uncertainty. However, the existing studies suggest that the method is effective in: a) identifying the most relevant input variables or parameters contributing to the output uncertainty, b) evaluating the degree of sensitivity of the model's output to any variable under analysis, and c) quantifying error bounds on model predictions (e.g. Blumberg and Georgas 2008).

An illustrative example of the use of the method for practical purposes is presented in a recent study by Sucsy et al. (2010) who implemented FOVA to evaluate the uncertainty in selected outputs of the 3D hydrodynamic model of the St. Johns River, FL, (SJR) due to errors in the specification of input variables. The uncertainty analysis was part of a water supply impact study conducted by the St. Johns River Water Management District (SJRWMD) to identify and evaluate the ecological effects on the SJR caused by withdrawals of water in the middle and upper regions of the river for public water supply. Sucsy et al. used FOVA motivated by the advantages of the method described above, and also given the practical limitations of alternative strategies such as Monte Carlo Simulations (e.g. computational burden and subjectivity in the specification of probability distributions for the input variables) for its use in a complex 3D hydrodynamic model such as the SJR model (which incorporates riverine, lacustrine and estuarine systems).

As discussed by Sucsy et al., FOVA is inexpensive computationally. Given N model output variables and P input variables, FOVA only requires a total number of NP+1 simulations to provide meaningful information about uncertainty estimates (the additional one corresponds to the unperturbed parameters or input variables) compared to the hundreds or thousands model simulations required by MCS for the same purpose. The computational burden was important in Sucsy et al.'s study given that the uncertainty analysis quantified the impacts of errors in eleven input variables on the model predictions of water levels, current speed, and salinity. The input variables considered in their study included depth, bottom roughness, wind speed, freshwater discharges, ocean tide, rain, evaporation, tributary salinity, ocean salinity, and groundwater salinity.

The impacts and importance of errors in the input variables were evaluated by means of the sensitivity coefficients described in eq. 4, after increasing each input variable in 10% of the unperturbed value. These coefficients were evaluated spatially as well temporally using the hydrodynamic model results, and a forward finite difference procedure to determine the term ∂F in eq. 4. In addition, confidence intervals on model predictions were computed based on $\pm$ 1 standard deviation (68% limits of confidence) derived based on eq. 4.

Sucsy et al.'s investigation constitutes an important example of the use of FOVA for practical purposes in hydrodynamic studies. One of the most relevant results of their study regarding the method is the advantage of FOVA to provide sensitivity estimates and uncertainty bounds at the same time (note that sensitivity analysis and uncertainty analysis are typically evaluated using individual methods in most modeling studies in water resources). Sucsy et al.'s investigation and the discussions presented by Zhang and Yu (2004), Blumberg and Georgas (2008), and Thomson et al. (2008) motivates the selection of this method for its use as an effective strategy for evaluation of input data uncertainty in hydrodynamic modeling.

4.1 Bayesian Monte Carlo (BMC)

Bayesian Monte Carlo is a strategy designed to evaluate parametric uncertainty. In essence, BMC aims to quantify the degree of support that observed data has in favor of a specific set of parameter values. However, the method differs from traditional calibration strategies in that instead of determining the "best" set of parameter values, it determines the complete posterior probability distribution of the model parameters.

To formulate the method, consider a model $M(\boldsymbol{\theta})$ with a parameter vector $\boldsymbol{\theta}$ expressed by $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_p]$ for which there exist an a-priory set of probability distributions $p(\boldsymbol{\theta}) = [p(\theta_1), p(\theta_2), \dots, p(\theta_p)]$ and observed information (D) relative to the system's behavior (e.g., water levels, velocities, salinity profiles, etc). Based upon the above information, Bayes theorem can be used to infer the posterior distribution of the model's parameters constrained to the available observations by,

$$p(\theta|D) = \frac{L(D|\theta)p(\theta)}{p(D)} \tag{5}$$

The term $L(D|\theta)$ is known as the likelihood function, while $p(D)$ is a normalizing constant (c) that ensure that the cumulative function of the posterior probability density $p(\theta|D)$ is equal to unity. Of fundamental importance in the formulation of a Bayesian framework for the assessment of parametric uncertainty is the definition of the likelihood function $L(D|\theta)$ and the prior distributions of the parameters. Both elements may influence the inference process and therefore the resulting shape of the posterior distributions if the observed data is scarce or limited.

The likelihood function $L(D|\theta)$ is determined based upon the nature of the residuals between the observations and the model results i.e. $L(\varepsilon|\mathrm{M}(\theta))$. The simplest model of the errors assumes they are independent, normally distributed and with zero mean. This additive error model can be expressed as (Beven 2009),

$$\mathrm{D} = \mathrm{M}(\theta) + \varepsilon; \quad \varepsilon = \mathrm{N}(0, \sigma_\mathrm{e}) \tag{6}$$

where $\mathrm{M}(\theta)$ is the model output given the parameter vector θ and a set of specific forcing conditions, and $\varepsilon = \mathrm{N}(0, \sigma_\mathrm{e})$ is the error term normally distributed with variance σ_e. The final form of the likelihood function for this additive error model is given by,

$$L(\varepsilon|\mathrm{M}(\theta)) = (2\pi\sigma_\mathrm{e}^2)^{-\mathrm{n}/2} \exp\left[-\frac{1}{2\sigma_\mathrm{e}^2}\{\textstyle\sum_{\mathrm{t}=1}^{\mathrm{n}}(\varepsilon_\mathrm{t})^2\}\right] \tag{7}$$

eq. 5 is numerically computed by Monte Carlo simulations using parameter samples drawn from the a-priori parameter distributions. Then, the resulting posterior joint probability distribution $p(\boldsymbol{\theta}|\boldsymbol{D})$ can be used to obtain the marginal distributions of the individual model parameters, single point parameter estimates, and also confidence limits on model predictions.

To date, the effectiveness, advantages, and disadvantages of the BMC method for practical purposes in hydrodynamic studies are topics that require further research. However, a preliminary application of the BMC method in the hydrodynamic models of the St. Louis Bay estuary, MS, and Weeks Bay Estuary, AL, (Gulf Coast Area of the United States), suggest that although the method can be computationally intensive, it is effective evaluating the model uncertainty resulting from errors associated to the estimation of the model parameters (Camacho et al. 2012). For practical purposes though, the use of alternative strategies such as Latin Hypercube Sampling (LHS) may potentially reduce the computational burden of the BMC method.

4.2 Markov Chain Monte Carlo (MCMC)

Markov Chain Monte Carlo (MCMC) is an alternative method for evaluation of parametric uncertainty. In principle, MCMC uses the same principles of Bayesian inference implemented in

BMC, although it differs from BMC in that it draws the parameter samples directly from the posterior joint distribution instead of the prior parameter distribution. Fundamentally, MCMC aims to construct a Markov Chain of model parameters whose stationary distribution is exactly the posterior distribution. To achieve such objective, MCMC requires the use of a "smart" sampler. One of the most commonly used samplers is the Metropolis algorithm (Metropolis et al. 1953), which construct the Markov Chain based on the following procedure: First, a set of parameter values are drawn from a feasible parametric space $\boldsymbol{\theta}^{(1)}$. Then an alternative set of parameter values $\boldsymbol{\theta}^{*}$ are drawn from a proposal distribution $q(\boldsymbol{\theta}^{*}|\boldsymbol{\theta}^{t-1})$ based on the previous value of the model parameters $\boldsymbol{\theta}^{t-1}$. The proposal distribution is assumed to be symmetric with mean $\boldsymbol{\theta}^{t-1}$ and variance σ_q^2. The set of parameters $\boldsymbol{\theta}^{*}$ are accepted if the likelihood ratio given by

$$R = \min\left(1, \frac{L(\boldsymbol{y}|\boldsymbol{\theta}^{*})}{L(\boldsymbol{y}|\boldsymbol{\theta}^{t-1})}\right) \tag{8}$$

is greater than a random number a generated from a normal distribution with zero mean and variance of one i.e. $\mathrm{R} > [\mathrm{a} = \mathrm{N}(0,1)]$.

In eq. 8, $\mathrm{L}(\mathrm{y}|\theta^{\mathrm{set}})$ is the likelihood function given for example by eq. 7, evaluated based upon a set of observations $\boldsymbol{y}$, and the model's response for the set of parameters $\boldsymbol{\theta}^{*}$ or $\boldsymbol{\theta}^{t-1}$. If the set of alternative parameters are accepted, they becomes the new state of the sampler $\boldsymbol{\theta}^{t}$. Otherwise the previous state becomes the new state $\boldsymbol{\theta}^{t} = \boldsymbol{\theta}^{t-1}$. In either case, the process is repeated until it is obtained a desired number of samples for the inference process.

5. STRATEGY FOR EVALUATION OF MODEL UNCERTAINTY

The methods presented in section 4 can be used independently to obtain uncertainty estimates due to input data uncertainty and model parameter uncertainty, and then the results of each individual analysis summed up to obtain the total model uncertainty. This strategy is based on the assumption that model uncertainty can be represented as a linear superposition of the individual sources of error (section 2). Note that each of the methods presented in section 4 provide uncertainty estimates in terms of confidence limits. Therefore, it is also possible to represent model uncertainty in terms of confidence limits.

A missing component in the previous analysis though, is the evaluation of model structure uncertainty for which it is difficult to identify an effective, yet computationally feasible evaluation strategy. Preliminarily, analysis of model structure uncertainty can be conducted implicitly by the modeler by selecting an appropriate model structure that is consistent with the availability of observed data to evaluate the performance of the model, and also consistent with the requirements of the project. Otherwise, the modeler could compute the standard deviation of the differences (residuals) between the model outputs of several model structures and report such differences as model structure uncertainty (e.g. Radwan et al. 2004).

6. CONCLUSION

We have discussed the importance of uncertainty analysis in hydrodynamic modeling, describing the principal sources of error encountered in practical applications. We also presented a set of effective methods for evaluation of the individual sources of uncertainty which were selected based on a review of strategies previously applied in hydrodynamic modeling studies, as well as in other fields such as hydrologic modeling, water quality modeling, and weather forecast. The methods considered in this study are used under the assumption that model uncertainty can be represented as a linear superposition of the individual sources of error. Therefore, input data uncertainty, model structure uncertainty, and parametric uncertainty can be evaluated individually before obtaining measures of total model uncertainty.

An evaluation of the proposed strategy for evaluation of uncertainty in a practical case study is currently under development. However, the existing discussions about the methods presented in this document suggest that the proposed strategy can be effective for practical purposes.

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