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NUMERICAL SIMULATION OF BUBBLE PLUME IN THE DENSITY STRATIFIED WATER

Bao-Shi Shiau [1,2], Jun-De Hsui[3]

[1] Research Fellow, Institute of Physics, Academia Sinica,
Taipei, 115, Taiwan, e-mail: bsshiau@gate.sinica.edu.tw
[2] Professor, Department of Harbor and River Engineering, National Taiwan Ocean University
Keelung, 202, Taiwan, e-mail: b0085@mail.ntou.edu.tw
[3] Master, Department of Harbor and River Engineering, National Taiwan Ocean University
Keelung, 202, Taiwan, e-mail: b0085@mail.ntou.edu.tw

ABSTRACT

A numerical model is developed in this study to investigate the bubble plume characteristics in the density stratified water. Based on the mass, momentum, and buoyancy conservation laws together with the assumption of Gaussian distribution profiles for the velocity and density deficiency, we adopt integral approach to derive three ordinary differential equations as the governing equation. These ordinary differential equations together with proper initial conditions are solved numerically by utilizing the fifth order Runge-Kutta-Fehlberg method. The calculated bubble plume radius and upward velocity are good in comparison with field measurements of Fannelop & Sjoen (1980). Results show that: (1) The bubble plume radius increases but terminal rise height decreases, when the gas discharge flow rate decreases. (2) Ambient water with strong density stratification will inhibit the bubble plume rise and reduce its terminal rise height. (3) Water density stratification affects the plume terminal rise height more significant in several orders of magnitude than that of gas flow rate does.

Keywords: bubble plume, density stratified water

1. INTRODUCTION

In civil and environmental engineering, bubble plume has many applications over the years. For example: bubble breakwaters, anti-freeze in harbors, bubble curtains for the oil slick on water surface, and destratification system that is to introduce aeration and to inhibit eutrophication as improving water quality. It is known that the temperature for water storage reservoirs or coastal water in summer is higher than in winter. So a thermal stratification or density stratification then occurred. In general, warm surface lighter water is separated from cool denser bottom water by an intermediate region of sharp gradient that forms a barrier and inhibits the vertical mass and heat transfer. Under such circumstances, upper layer water dissolved oxygen is hard to transfer to the lower layer. This leads to anaerobic condition for the bottom water and deteriorates the water quality.

In order to improve the bottom layer water quality, it is necessary to destroy stratification which prevents the dissolved oxygen transfer vertically. The most generally used engineering method is directly to inject gas into the water columns from the bottom upward. As placing at bottom a point source of gas which is released into a water column, the gas forms the bubble and rises upward owing to buoyancy. We call it bubble plume. Generally speaking, a bubble plume is different from a single phase plume driven by a normal source of buoyancy in some respects. Such as (1) the volume flow rate of gas will increase with height,

(2) the bubbles will rise faster than the liquid. Chen and Rodi (1980) had made a thorough review of the single phase jet and plume. And for bubble plume, previous works like Kobus (1968), Fannelop & Sjoen (1980), and Sjoen (1982) were done in homogeneous waters. Therefore, in this study we intend to develop a simple model to simulate bubble plume in the ambient water with density stratification. The effects of ambient water density stratification and discharged gas flow rate on the global characteristics of bubble plume are investigated numerically.

2. FORMULATION OF THE MATHEMATICAL MODEL FOR BUBBLE PLUME

Consider a continuous point source of gas at the seabed which is discharged into the ambient sea water, the gas forms a bubble and rises upward owing to buoyancy (see Figure 1). The mass, momentum and buoyancy conservation laws are applied to formulate the model.

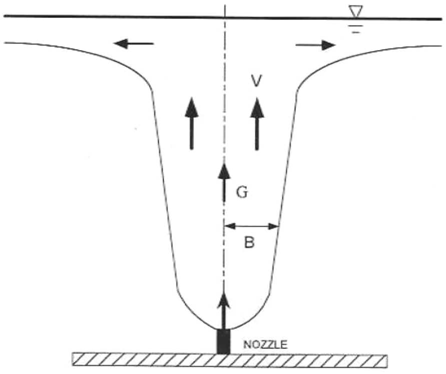

Figure 1 Schematic diagram for the bubble plume

Some basic assumptions are made for deriving the governing equations. (1) The ambient flow is steady and incompressible; (2) The bubble plume is with a round cross section, (3) The bubble plume is axial symmetric; (4) The Boussinesq approximation is valid; (5) The bubble plume has been fully developed, a dynamic similarity exist. The Gaussian distributions for axial velocity and density deficiency in each cross-section can be obtained as follows:

$$v(r,z) = v(z)\exp(-\frac{r^2}{b^2}) \tag{1}$$

$$g'(r,z) = g'(z)\exp(-\frac{r^2}{\lambda^2 b^2}) \tag{2}$$

where b is the effective radius of bubble plume; $v(r,z)$ is the axial velocity at a height of z and a radius of r; $g`(r,z)$ is density deficiency and can be expressed as:

$$g'(z) = \frac{g[\rho_0(z) - \rho(z)]}{\rho_r} \tag{3}$$

where g is gravity acceleration; ρ is bubble plume density; ρ_0 is ambient fluid density; ρ_r is the reference density.

From the above assumptions, we first derive the equation for mass conservation around a horizontal element of the plume

$$\frac{d}{dz}[\int_0^\infty v\exp(-\frac{r^2}{b^2})2\pi r dr] = 2\pi b u_e \qquad (4)$$

Here u_e is the entrainment velocity and $u_e = \alpha v$. α is the entrainment coefficient. Substituting the entrainment velocity and integrating equation (4) yields,

$$\frac{d}{dz}(b^2 v) = 2\alpha b v \qquad (5)$$

The momentum equation is obtained from momentum conservation for the same horizontal element of the plume.

$$\frac{d}{dz}[\int_0^\infty \rho v^2 \exp(-\frac{2r^2}{b^2})2\pi r dr] = \int_0^\infty \rho_r g \ \exp(-\frac{r^2}{\lambda^2 b^2})2\pi r dr \qquad (6)$$

In combination with the Boussinesq approximation yields,

$$\frac{d}{dz}(\frac{1}{2}b^2 v^2) = \lambda^2 b^2 g \qquad (7)$$

We concern two effects for conservation of buoyancy. One is the stratification effect in the ambient fluid and the other is the increase of the volume flow rate of gas with the height. Consider a plume rising in a stratified fluid, buoyancy conservation around a horizontal element of the plume gets

$$\frac{d}{dz}\{\int_0^\infty v(r,z)[\rho_0(z) - \rho(r,z)]2\pi r dr\} = \pi b^2 v \frac{d\rho_0}{dz} \qquad (8)$$

or

$$[\frac{d}{dz}(\frac{\pi\lambda^2 b^2 v g}{\lambda^2+1})]_{start} = -\pi b^2 v N^2(z) \qquad (9)$$

where $N(z) = (-\frac{g}{\rho_r}\frac{d\rho_0}{dz})^{1/2}$ is the local buoyancy frequency. The relation between the actual volume flow rate of bubbles at any height and total average bubble rise velocity is

$$Q(z) = \frac{Q_0 P_a}{(H_T - z)\rho_r g} = \int_0^\infty [v(r,z) + u_s]\frac{g\ (r,z)}{g}2\pi r dr \qquad (10)$$

Where Q_0 is the flow rate when air rises to the water surface; P_a is the atmospheric pressure; ρ_r is the reference density; H_T is total pressure head at the nozzle; u_s is the slip velocity;

Therefore effect of gas expansion is expressed as

$$[\frac{d}{dz}(\frac{\pi\lambda^2 b^2 vg}{\lambda^2+1})]_{\exp ansion}=\frac{d}{dz}[\frac{Q_0 P_a v}{(H_T-z)\rho_r(v+u_B)}] \tag{11}$$

Where λ is Schmidt number.

Equations (9) and (11) denoting the two effects and they are combined together as:

$$\frac{d}{dz}(\frac{\pi\lambda^2 b^2 vg}{\lambda^2+1})=-\pi b^2 vN^2+\frac{d}{dz}[\frac{Q_0 P_a v}{(H_T-z)\rho_r(v+u_B)}] \tag{12}$$

The bubble plume model is then completed described by equations (5), (7) and (12), together with appropriate boundary conditions. These governing equations are now non-dimensionalized as dimensionless variables Z, B,V,G by proper scaling parameters.

$$z=H_T Z \tag{13.a}$$

$$b=2\alpha H_T B \tag{13.b}$$

$$v=u_B M^{1/3} V \tag{13.c}$$

$$g=(\frac{u_B^2 M^{2/3}}{\lambda^2 H_T})G \tag{13.d}$$

$$M=\frac{Q_0 P_a(\lambda^2+1)}{4\pi\alpha^2\rho_r H_T^2 u_B^3} \tag{13.e}$$

$$u_B=u_s(\lambda^2+1) \tag{13.f}$$

In substituting these non-dimensionalized variables shown in equations (13.a) to (13.f), equations (5), (7) and (12) then become

$$\frac{d}{dZ}(B^2 V)=BV \tag{14}$$

$$\frac{d}{dZ}(BV)=\frac{BG}{V} \tag{15}$$

$$\frac{d}{dZ}(B^2 VG)=-CB^2 V+\frac{d}{dZ}[\frac{V}{(1-Z)(V+M^{-1/3})}] \tag{16}$$

where

$$C=\frac{N^2(\lambda^2+1)H_T^2}{u_B^2 M^{2/3}} \tag{17}$$

In combination the conservation laws with the integral method to treat the bubble plume, we obtain equations (14), (15) and (16) which form a complete ordinary differential equation set. These equations together with proper initial conditions constitute a mathematical model which describes the behavior of a point source bubble plume in the density stratified water.

3. NUMERICAL SLOUTION OF THE BUBBLE PLUME MODEL

The governing equations (equations 14 to 16) are ordinary differential equations (ODE). Together with appropriate initial conditions they can be solved by using the Runge-Kutta-Fehlberg method which is efficient and precise especially for solving ODE.

To find the solution, it is necessary to search for an appropriate way of initializing the integration of equations (14) to (16). Morton et al. (1956) indicated that the plume was not influenced significantly by the stratification at the location very near the orifice. The initial values of B^2V and BV at very near the orifice of the plume can be obtained by putting $C = 0$. Here B is the non-dimensional bubble plume radius; and V is the non-dimensional upward velocity of bubble plume. We get the power series solution for B and V. Since the values of B, V and G near the orifice (Z is small) will be zero, therefore, we start the initial values of B, V and G from $Z = 0.025$. Here Z is the non-dimensional height; And G is the non-dimensional buoyancy of the bubble plume. B, V and G for consecutive height, Z, can be obtained numerically by Rung-Kutta-Fehlberg method with choosing turbulent Schmidt number $\lambda = 0.3$ (suggestion of Schladow, (1992)), and turbulent entrainment coefficient $\alpha = 0.083$ (suggestion of List, (1982)).

The gas flow rate Q_0 is related with source strength, M as seen from equation (13.e). As suggested by Schladow (1992), we adopt $\lambda = 0.3$, $u_s = 0.3$. In accordance with the experimental results of turbulent plume, List (1982) found $\alpha = 0.083$. So parameters M and C reduce to

$$M = 4570.349 Q_B / H_T \tag{18}$$

$$C = 10.19 N^2 H_T^2 / M^{2/3} \tag{19}$$

4. RESULTS AND DISCUSSION

4.1 Verification of the bubble plume model

For verifying present model, we compare filed data of Fannelop and Sjoen (1980). They measured the data in 10 m depth of homogeneous water with 0.005 normal m^3/s gas flow rate. These conditions correspond to $C = 0$, $M = 1.14$. Figure 2 shows the bubble plume radius variation with heights. And figure 3 is the bubble plume velocity versus height. Results exhibit that the plume velocity is good match with the filed data except at lower height $Z < 0.1$. The trend for plume radius growth with height is in consistency with filed data but is somewhat a little larger. In general, the present model is simple and it is also efficiently in calculation of bubble plume. Numerical model prediction results are good as in comparison with field data.

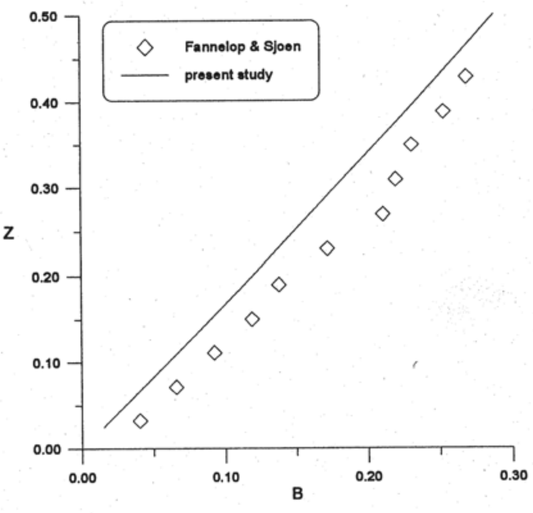

Figure 2 Comparison of computed bubble plume radius with field measured data

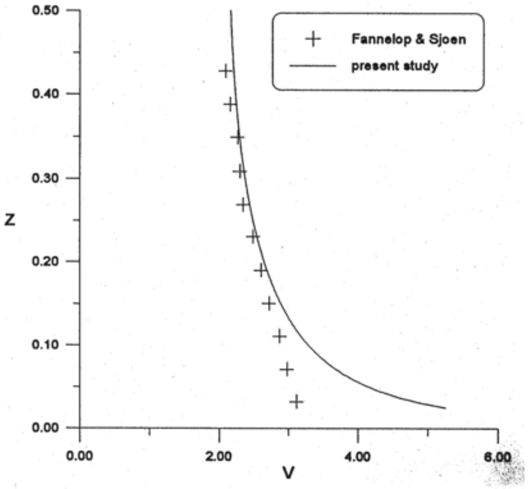

Figure 3 Comparison of computed bubble plume upward velocity with field measured data

4.2 Effect of density stratification, C

Figure 4 shows the bubble plume radius variations with height for different C (C = 0~30000) at gas flow rate M=0.1 and 10. It can be seen that the bubble plume rises upward with enlarging its radius. As it rises to some height and stops, then it spreads out horizontally. At this height the radius suddenly becomes very large. And this height is called terminal rise height. When the terminal rise height is reached, bubble plume is divergent and does not go upward again. For C = 0, i.e., non-stratified ambient fluid, the plume radius, B is about 0.45 as the bubble plume reaches its terminal rise height. As C increases, i.e., the more the ambient water density stratification gradient is, the less rise height of bubble plume is. The terminal rise height decreases. So the divergent phenomenon occurs at a lower height as C increases.

Bubble plume upward velocity for various C at M = 0.1 and 10 are shown in figure 5. The plume upward velocity decreases as plume rises. The suppression effect on the rise of plume by density stratification reducing its upward velocity is clearly displayed in Figure, especially for larger density stratification case.

Figure 6 exhibits bubble plume buoyancy for various C at M = 0.1 and 10. The shape of the curves is somewhat like that of shown in figure 5. Plume buoyancy decreases as C increases. From figure 4, it is known that as the terminal rise height is reached, bubble plume spreads out horizontally and diverges. So the buoyancy becomes zero and moreover is negative.

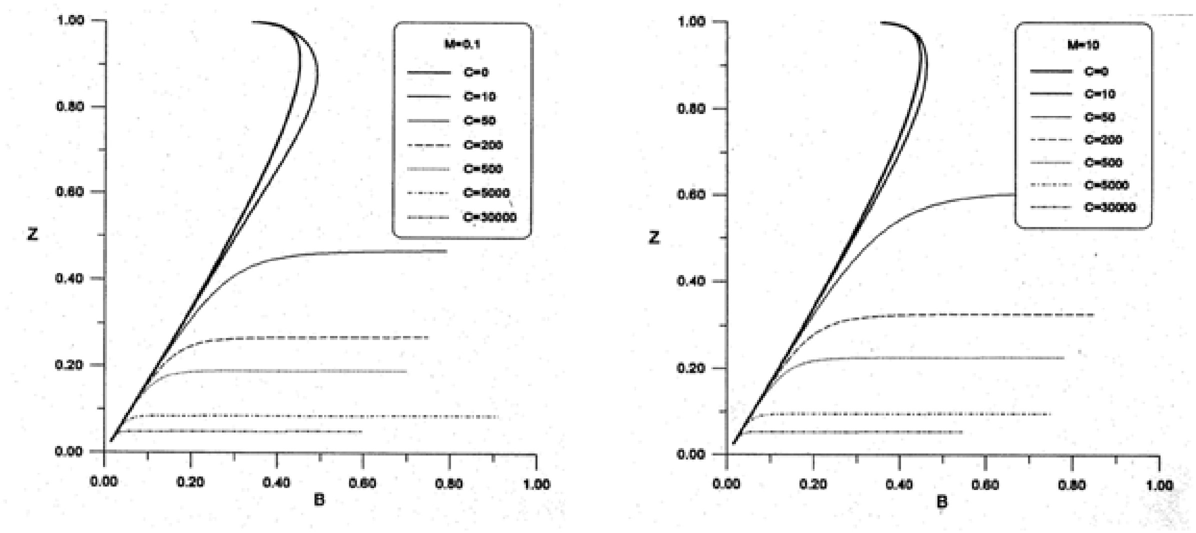

Figure 4 Bubble plume radius variations with height for different C; M=0.1 and 10

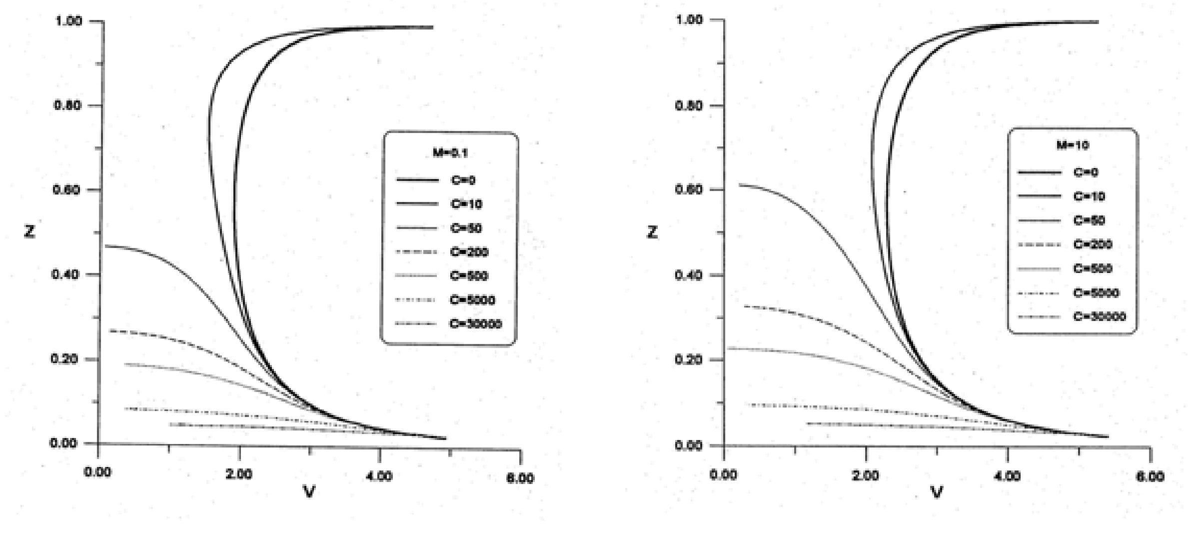

Figure 5 Bubble plume upward velocity variations with height for different C; M=0.1 and 10

4.3 Effect of gas discharged strength, M

The bubble plume radius versus height for different M at C = 10 and 500 are presented in figure 7. Results exhibit that bubble plumes rise at a higher positions and diverge for the case of a larger M value. This means that bubble plume with a larger gas flow rate will rise higher. But at the same height position, the plume radius becomes small as M increases.

Figure 8 is the bubble plume upward velocity for various M at C = 10 and 500. Higher value of M (i.e. larger gas flow rate) has larger plume upward velocity. But we note that the plume upward velocity finally becomes very small for each M value. This is meaningful when they are compared with plume radius variations shown in figure 7. For different gas flow rate cases shown in figure 7, bubble plumes rise and reach to the terminal rise height. So they spread out horizontally and diverge. The upward velocity then becomes small.

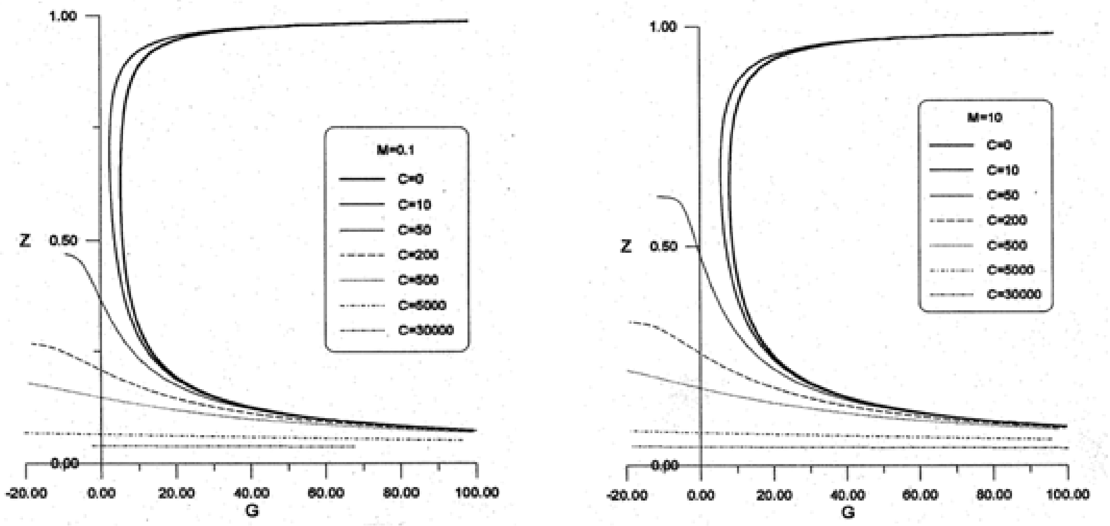

Figure 6 Bubble plume buoyancy variations with height for different C; M=0.1 and 10

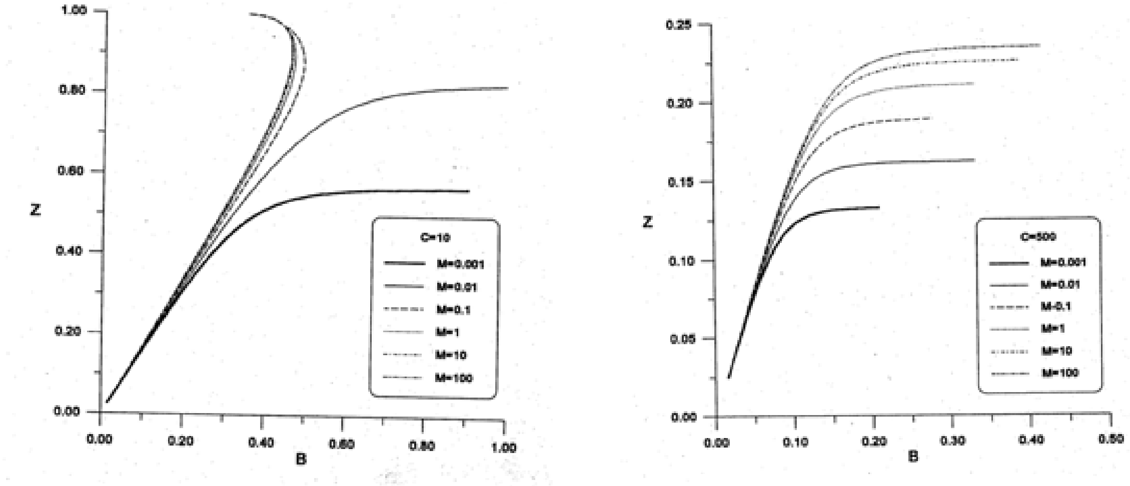

Figure 7 Bubble plume radius versus height for different M at C = 10 and 500

The bubble plume buoyancy for various M at C = 10 and 500 is shown in figure 9. Bubble plume with lower value of M (i.e. smaller gas flow rate) effectively has less buoyancy. The plume buoyancy diminishes at terminal rise height where bubble plume is divergent.

4.4 Terminal rise height of bubble plume

As mentioned in the above, bubble plume rises to some height and plume buoyancy becomes very small. It then collapses and spreads out horizontally. The height in which occurring divergent phenomenon is called terminal rise height. As shown in the above sections, gas flow rate and ambient fluid density stratification have a great influence on this height. Figure 10 exhibits the terminal rise height of bubble plume as functions of density stratification of water under different gas flow rates. Smaller C and larger M value case have a larger terminal rise height.

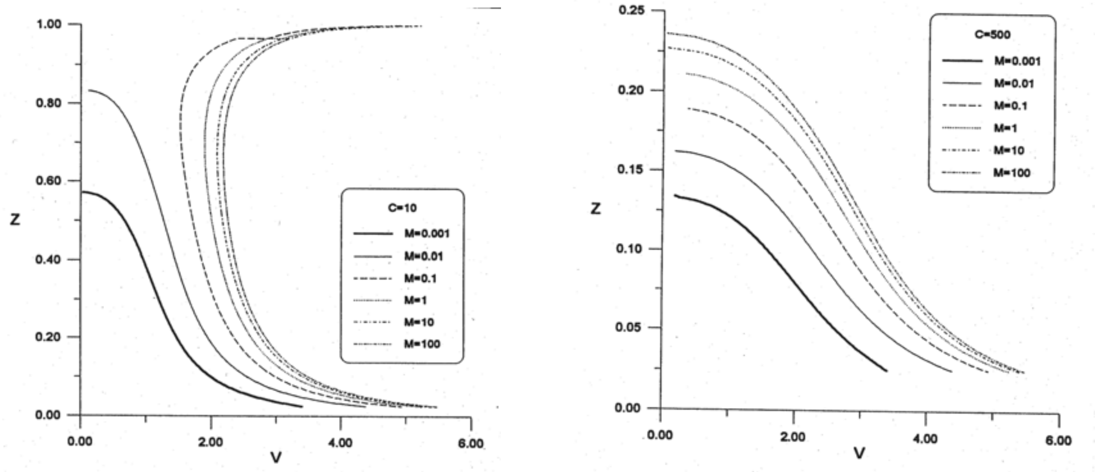

Figure 8 Bubble plume upward velocity versus height for different M at C = 10 and 500

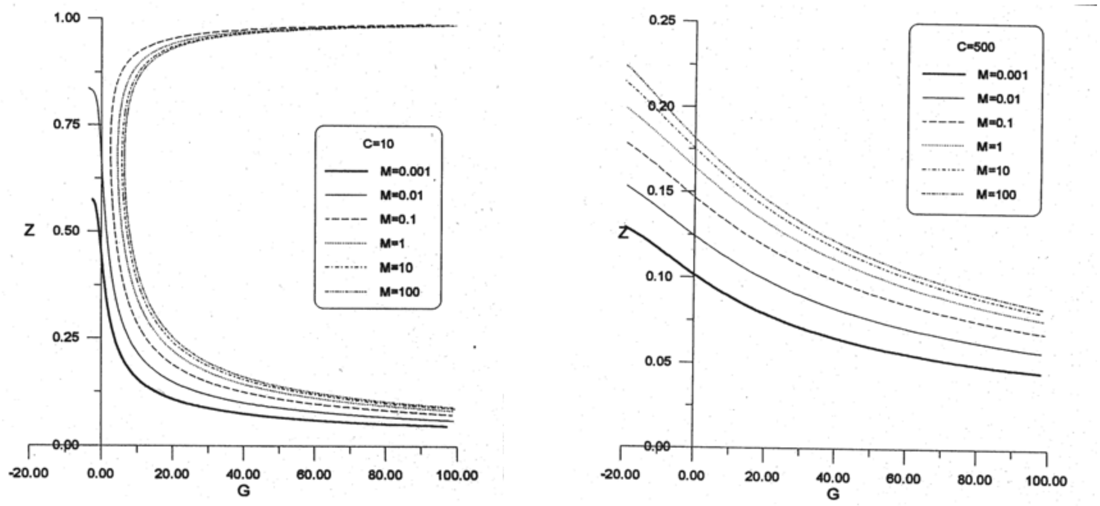

Figure 9 Bubble plume buoyancy versus height for different M at C = 10 and 500

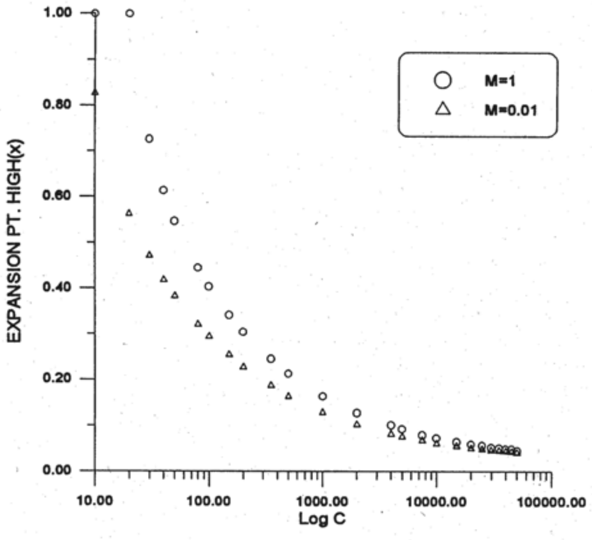

Figure 10 Terminal rise height of bubble plume as functions of density stratification of water

Results shown in figure 10 indicate that for very large C value, the terminal rise height does not change even increasing M value. Also results shown in figure 10 depict effects of stratification and gas flow rate on the bubble plume characteristics. As may be expected, the stratification inhibits the rise of bubble plume and its growth. The terminal rise of height of bubble plume becomes small with increasing ambient water stratification. When the stratification becomes very large, the terminal rise height varies insignificantly even increases gas flow rate in several orders of magnitude. In summary, ambient water density stratification has a stronger effect on the rise of bubble plume than that of the gas flow rate from the source.

5. CONCLUSION

In this study, a numerical model is developed and used to investigate the bubble plume characteristics in the density stratified water. Based on the mass, momentum, and buoyancy conservation laws together with the assumption of Gaussian distribution profiles for the velocity and density deficiency, we adopt integral approach to derive three ordinary differential equations as the governing equation. These ordinary differential equations together with proper initial conditions are solved numerically by utilizing the fifth order Runge-Kutta-Fehlberg method.

The calculated bubble plume radius and upward velocity are found in good comparison with the field measurements of Fannelop & Sjoen (1980). Numerical results show that: (1) The bubble plume radius increases but terminal rise height decreases, when the gas discharge flow rate decreases. (2) Ambient water with strong density stratification will inhibit the bubble plume rise and reduce its terminal rise height. (3) Water density stratification affects the plume terminal rise height more significant in several orders of magnitude than that of gas flow rate does.

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