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DEVELOPMENT OF A MATHEMATICAL MODEL FOR DETERMINING TEMPERATURE OF A LAKE BASED ON CLIMATIC VARIATIONS AND ITS VALIDATION USING MEASURED DATA

S N Jha[1] R Manivanan[2] P G Saptarishi[3]

***Abstract**: The temperature is an important factor for many uses of water and its suitability as a habitat for aquatic life. Thermal stratification can have pronounced effect on water quality. Consequently understanding temperature and heat modelling is an important facet of water quality modelling. The climatic factors play an important role in determining the temperature of water body. In this paper a mathematical model for determining the temperature of a lake was developed with climatic factors. The model was applied for Panshet lake and the results were compared with the field data. The results indicated that there was close agreement in the model predicted temperature values and the field observed data. The model may be applied to the other lakes of India.*

***Keywords**: temperature; heat budget; lake; model*

INTRODUCTION

A lake is a body of water or other liquid of considerable size surrounded entirely by land. A vast majority of lakes on Earth are fresh water, and most lie in the Northern Hemisphere at higher latitudes. Many lakes are artificial and are constructed for hydro-electric power supply, recreational purposes, industrial use, agricultural use, and domestic water supply. Temperature is a physical property of a system that underlies the common notions of hot and cold; something that is hotter generally has the greater temperature. Temperature is one of the principal parameters of thermodynamics. Temperature plays an important role in almost all fields of science, including physics, chemistry, and biology. Many physical properties of materials including the phase (solid, liquid, gaseous or plasma), density, solubility, vapor pressure, and electrical conductivity depend on the temperature. Temperature also plays an important role in determining the rate and extent to which chemical reactions occur. This is one reason why the human body has several elaborate mechanisms for maintaining the temperature at 37 °C, since temperatures only a few degrees higher can result in harmful reactions with serious consequences. Temperature also controls the type and quantity of thermal radiation emitted from a surface. Thermal changes can have pronounced effect on water quality. Consequently understanding temperature and heat modelling is an important facet of water quality modelling. The climatic factors play an important role in determining the temperature of water body. In this

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paper a Heat Budget model for determining the temperature of Panshet lake was developed. Runge-Kutta 4th order numerical method was applied for solving the equations.

STUDY AREA

The Panshet lake (Fig1) located at the upstream of the Bhima Basin in Pune, Maharashtra, India lies between the latitude 18^0 22' 12" N and 18^0 19' 54.7" N and longitude between 73^0 36" 40" E and 73^0 29' 56.3" E. Panshet reservoir constructed for the purpose of drinking water supply and irrigation. The total length of the reservoir is approximately 16 Km with meandering in nature with the width of 1.5Km (Fig.2). The reservoir was constructed in the year 1971. The area of Catchments is 122 km^2. It receives average annual rainfall of483 cm., Reservoir Area at FRL is 15 Km^2. The Gross Storage Capacity and Effective Storage Capacity are 304,000,000 m^3 and 295,000,000 m^3 respectively (CWPRS Technical ReportNo.3919)

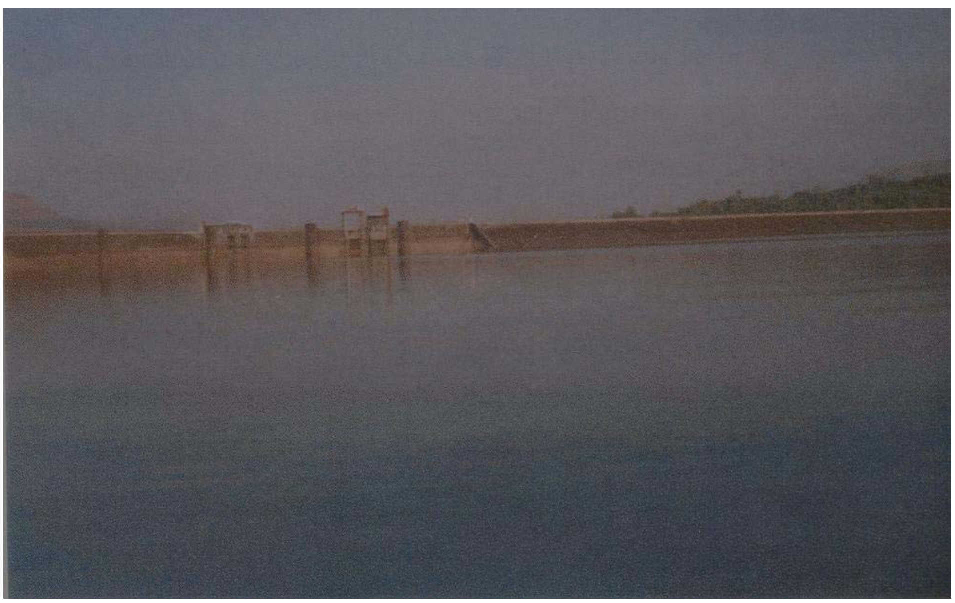

Fig.1 A View of Panshet lake

The catchments area of the Panshet reservoir is surrounded by hills, which are extension of the main ridges of western ghat and about heights of 12000m to 13000m. The slope of the ridges on both sides of the reservoir is very steep. The water of the reservoir is used for the drinking and irrigational purposes. In this paper, a mathematical model for determining the temperature of Panshet lake was developed using various climatic factors. The computed results were compared with the field data. The results indicated that there was close agreement in the model predicted temperature values and the field observed data

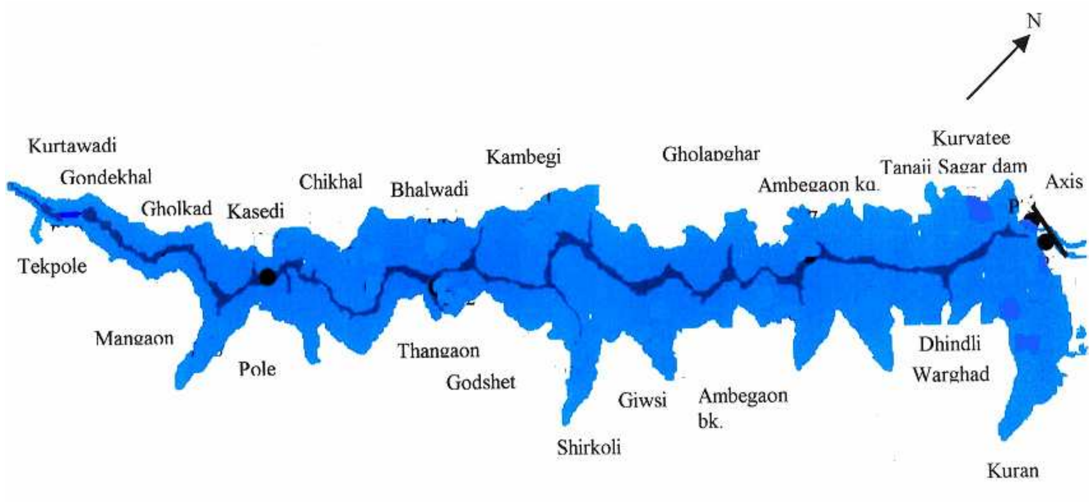

Fig.2 Map of Panshet lake

METHODOLOGY

The mathematical modeling of the transport and fate of heat in natural waters has been subject of extensive study. The total heat budget for a water body includes the effects of inflows (rivers, discharges), outflows, heat generated by chemical-biological reactions, heat exchange with the stream bed, and atmospheric heat exchange at the water surface. In all practicality, however, the dominant process controlling the heat budget is the atmospheric heat exchange. In addition, however, it is also important to include the proper boundary conditions for advective exchange (e.g., rivers, thermal discharges, or tidal flows) when the relative source temperature and rate of advective exchange is great enough to affect the temperature distribution of the water body.
The temperature models have been applied in connection with many different kinds of impact analysis. More often than not, there is some sort of reservoir involved. Thus, either discharge timing, release temperature, release volume, or some combination of all three, may be analyzed for their effect on downstream water temperatures. Thermal loading, as from power plants or waste discharge facilities is an important area of research

Presently there are many models which can help in estimation of temperature. Some of these are.
1. Heat transport - predicts average mean daily and diurnal water temperatures as a function of stream distance.

2. Solar model - predicts solar radiation penetrating the water as a function of latitude, time of year, and meteorological conditions.

3. Shade model - predicts interception of solar radiation due to topography and riparian vegetation.

4. Meteorological model - predicts changes in air temperature, relative humidity, and atmospheric pressure as a function of watershed elevation.

5. Regression model - aids in filling missing water temperature data or smoothing that data.

6. Heat flux model – based on arithmetic sum of various meteorological components.

However, of the various ways to model temperature the most important and accurate one is heat flux model. By convention, heat flux is defined as the arithmetic sum of the following components,

Net Heat Flux = Solar Radiation +Atmospheric Radiation +Vegetative (And Topographic) Radiation +Evaporation +Convection +Conduction +Friction - Water's Back Radiation

Atmospheric radiation results from short wave radiation from the sun passing through the atmosphere, some of which is reflected back into space, but some is absorbed and re-radiated, both by the atmosphere and clouds, as long wave radiation. The same thing happens with the local topography and streamside vegetation. Air temperature and wind speed affect these processes, as do the emissivity of the media themselves, which could be considered the efficiency of energy conversion from short to long wave.

Evaporation is controlled by the air temperature with respect to that of the water, and the amount of moisture in the air, relative humidity. Water that evaporates cools the water that is left behind; water that condenses warms the receiving body. In the heat flux equation, only the sign changes.

Convection is that process whereby heat is moved directly from the air into the water (or vice versa), not by radiation, but by direct molecular stimulation. Again, this process is a function of air temperature relative to that of the water, and atmospheric pressure that is itself a function of elevation. Convection is enhanced by wind speed.

Conduction is that process of moving heat to or from the streambed. It is a function of the so-called thermal gradient, which is a measure of the insulating capacity of the bed material and the relative temperatures of the water and bed.

Friction is governed by the stream gradient, obviously a function of the elevations, but also the discharge and the stream width. Finally, the water itself gives off radiation much like that from the atmosphere, clouds, vegetation, and topography. This is referred to as back radiation and is a function of the water temperature and the water's emissivity.

The transfer of energy which occurs at the air-water interface is generally handled in one of two ways in river, lake, and estuary models. A simplified approach is to input temperature values directly and avoid a more complete formulation of the energy transfer phenomena. This approach is most often applied to those aquatic systems where the temperature can be readily measured. Alternatively, and quite conveniently, the various energy transfer phenomena which occur at the air-water interface can be considered in a heat budget formulation.

Estimation of the various heat flux components has been the subject of many theoretical and experimental studies in the late 1960's and early 1970s. Most of the derived equations rely heavily on empirical coefficients. These formulations have been reviewed extensively by the Tennessee Valley Authority (1972), Ryan and Harlernan (1973), Edinger et al. .(1974), and Paily et al. (1974). Edinger et al (1974) provide an excellent and comprehensive report. Thomann and Mueller (1987) have summarized it in terms of water quality modeling.

In a complete atmospheric heat budget formulation, the net external heat flux is most often formulated as an algebraic sum of several component energy fluxes (e.g., Baca and Arnett, 1976; U.S. Army Corps of Engineers, 1974; Thomann et al., 1975; Edinger and Buchak, 1978; Ryan and Harleman, 1973; TVA, 1972). These flux components can be calculated within the models from semi— theoretical relations, empirical equations, and basic meteorological data. Depending on the algebraic formulation used for the net heat flux term and the particular empirical expressions chosen for each component, all or some of the following meteorological data may be required: atmospheric pressure, cloud cover, wind speed and direction, wet and dry bulb air temperatures, dew point temperature, short wave solar radiation, relative humidity, water temperature, latitude, and longitude. Many a times heat budget models are also useful in estimation of evaporation from the water body (Saur and Anderson, 1956).

In this paper Surface heat flux has been modeled as combination of five process

1. Solar short wave radiation J_{sn}
2. Atmospheric longwave radiation J_{an}
3. Water longwave radiation J_{br}
4. Conduction and convection J_c
5. Evaporation and Condensation J_e

The following assumptions were made:-

1. Water body is well mixed in the lake
2. Climatic condition are constant for a month
3. Negligible exchange of heat with sediment

Thus total Surface Heat flux can be represented as

$$J = J_{sn} + J_{an} - (J_{br} + J_c + J_e) \quad (1)$$

Where

J_{sn} = Net Solar Radiation(cal/cm^2/day)

It is measured directly and its value depends on

1. Solar Altitude
2. Scattering and Absorption
3. Reflection
4. Shading

J_{an} = Net Atmospheric longwave Radiation

It is represented by Modified Stefan Boltzmann Law

$$J_{an} = \sigma(T_{air} + 273)^4 \quad (A + 0.031\sqrt{e_{air}}) \quad (1 - R_L) \quad (2)$$

Stefan Boltzmann Law — Atmospheric attenutation — Reflection

Where σ = Stefan Boltzmann Constant

A = coefficient (0.5-0.7)

e_{air} = air vapor pressure (mmHg)

R_L = Reflection coefficient

J_{br} = Back radiation from water

This is also represented by Stefan Boltzmann Law

$$J_{br} = \varepsilon\sigma(T+273)^4 \qquad (3)$$

Where ε= emissivity of water

J_c = Conduction

In case of water, conduction is insignificant while advection/convection is important and needs to be considered in the model

$$J_c = c_1 f(U_W)(T_s - T_{air}) \qquad (4)$$

where c_1 is bowen coefficient

$f(U_W)$ is wind factor

J_e = Evaporation

$$J_e = f(U_W)(e_s - e_{air}) \qquad (5)$$

Where e_s is saturation vapor pressure at water surface and

e_{air} is Vapour pressure in overlying air (mmHg)

Combining equation 1 to 5 we get

$$J = J_{sn} + \sigma(T_{air}+273)^4(A+0.031\sqrt{e_{air}})(1-R_L) - \varepsilon\sigma(T+273)^4 - c_1 f(U_W)(T_s - T_{air}) - f(U_W)(e_s - e_{air}) \qquad (6)$$

As accumulation is defined as change of Heat in the system over time t

Accumulation = ΔH/Δt

Accumulation = ΔℓCVT/ Δt

(As T=H/ℓ.C.V)

Where T- Temperature

ℓ- Density

C- Specific Heat

V- volume

H -heat

Since density of water, specific heat and volume of the water body remains relatively constant therefore

Accumulation = ℓCVdT/dt

Assuming that the system is completely mixed, as in case of lakes in actual field condition, the heat balance for the system can be expressed as

Accumulation= inflow- outflow± surface heat exchange

$$\ell CV dT/dt = Q\ell CT_{in}(t) - Q\ell CT_{out} \pm A_s J \quad (7)$$

where Q – flow(m^3/day)

Using equation 1 and 7 steady state heat balance equation becomes

$$0 = Q\ell CT_{in}/A_s + J_{sn} + \sigma(T_{air} + 273)^4(A + 0.031\sqrt{e_{air}})(1-R_L) - \varepsilon\sigma(T+273)^4 - c_1 f(U_W)(T_s - T_{air}) - f(U_W)(e_s - e_{air}) - Q\ell CT/A_s \quad (8)$$

Similarly the time variable heat balance can be written as

$$dT_s/dt = QT_{in}/V + J_{sn}/\ell CH + \sigma(T_{air} + 273)^4(A + 0.031\sqrt{e_{air}})(1-R_L)/\ \ell CH - \varepsilon\sigma(T+273)^4/\ \ell CH - c_1 f(U_W)(T_s - T_{air})/\ \ell CH - f(U_W)(e_s - e_{air})/\ \ell CH - QT_s/V \quad (9)$$

Equation 9 was used to simulate time variable solution in the model. Runge Kutta 4th order numerical method was used for simulation. The data ranges given in table 1 was used for prediction of temperature variation in a year.

Table -1 Data ranges used in Heat Budget Model of Panshet lake

Net Solar radiations	380-607cal/cm^2/day
Air temperature	24-35 ^{0}C
Wind speed	2.5 – 10.2 km/hr
Relative Humidity	35-82%

RESULTS AND DISCUSSION

The results obtained from the model indicate that the temperature rises steeply from January to March from 18 C to 24 C. It further rises up to 28 C by the end of May thereafter the temperature decreases back to 25 C by the end of September, this is in line of expectation also as there is an increase in air temperature (28- 35^{0}C) and solar radiation (443 -603 cal/cm^2/day) from January to April. In May though air temperature decreases marginally the water temperature rises up to 27.7 ^{0}C mainly due to increase in solar radiation. From June to August due to the South-West monsoon there is decrease in air temperature (From 28^{0}C to 24^{0}C) and also in net solar radiation resulting in decrease in temperature of lake upto 25^{0}C. There is a small increase in temperature in the Month of September and October by 1.3^{0}C due to marginal increase in air temperature but the downtrend of lake temperature continues in the Month of November and December as lake water temperature decrease from 26.3 to 18. 9^{0}C due to prevailing cold conditions resulting in lower atmospheric radiations. The predicted data matches with the field observed data for temperature obtained in the month of March, April and September.

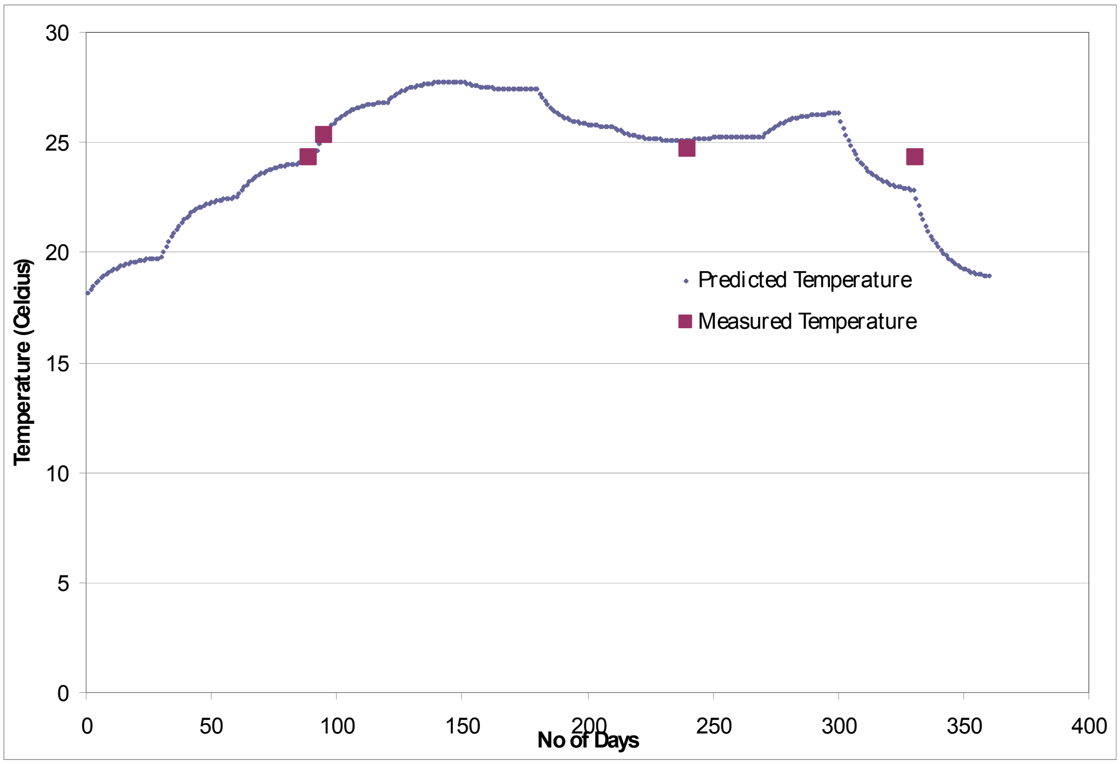

Fig 3 The predicted and observed values of the Panshet lake temperaure

However the model predicts slightly lower temperature for the month of December which can be attributed to the well mixed condition of lake assumed in the model. Fig.3 shows the predicted and observed values of the Panshet lake temperature for the complete year starting from 1st January. Many ecologically important lakes have been modeled by the heat budget method and the computed lake water temperatures were in good agreement with observed temperatures. **(**Kazuro Momii and Yuji Ito 2008). Similarly, the present study results are in concurrence with the above authors.

CONCLUSION

It can be concluded from this study that the heat budget model can be suitably applied in case of Panshet lake to predict the annual variation of the water temperature. The results indicated that there was close agreement in the model predicted temperature values and the field observed data. As the model relies heavily on the data of climatic factors, it is necessary to take due care in the field collection of data. Further, continuous monitoring of the lake water temperature and climatic data is helpful in tuning of the developed model. The model may be applied in case of the other lakes with their climatic data.

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