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DRAG FORCE MINIMIZING SHAPE IDENTIFICATION OF BODY LOCATED IN COMPRESSIBLE FLUID FLOW

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Abstract: *This paper presents a numerical method of shape identification of a body located in compressible viscous flow. The purpose of this research is to identify the optimal shape that minimizes the fluid forces subjected to the body. The formulation of the shape identification is based on the optimal control theory. The finite element method is used for calculation of fluid flow. In this paper, optimal control is treated as fluid force minimization. The first thing that should be carried out in the optimal control theory is to define a performance function which expresses the optimal shape. The performance function must be minimized satisfying the state equation. In the research, the Lagrange multipliers are introduced for the constraint conditions. To avoid the break down of calculation caused by destruction of elements, the finite element mesh is reconstructed in the identification process. The grid generation scheme based on Delaunay triangulation is applied to the reconstruction of finite element mesh. In this paper, an optimized shape of the body is obtained by computation.*

Keywords: *finite element method, shape identification, optimal control theory*

INTRODUCTION

It is well known that the shape of the body in incompressible viscous flow field which has the minimum drag force is the streamline configuration. These are suggested by only experiments and the numerical computations are not sufficiently carried out about this problem. The purpose of this study is to determine an drag minimum shape of the body located in an compressible viscous flow applying a formulation of the shape identification to a numerical simulation.

The shape optimization in the incompressible Stokes flow as originated by O. Pironneau (1973, 1974) and Glowinski(1975). Finite element methods have been successfully used to solve a shape determination of a body in incompressible flows (He et al , 1997, Okumura and Kawahara, 2000, Oawa and kawahara, 2003, Mohamadi and Pironneau, 2004, Katamine et al., 2005, Yagi and Kawahara, 2005, 2007, Nojima and Kawahara, 2006, Azegami and Takechi, 2006).

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In this study, the shape determination problem in a compressible flow is solved in three dimensional domain by the finite element method and the gradient method. For reduction of the huge computational time of the three dimensional computation, the parallel computing is used. The computational domain is decomposed to 512 sub-domains. Three-dimensional finite element mesh generation system, which is based on Delaunay triangulation method is also introduced (Bowyer 1981, Nojima 2006) for three-dimensional computation.

SHAPE IDENTIFICATION

Compressible Viscous Flow

Using the indicial notation and the summation convention with repeated indices, the basic equation of a compressible viscous fluid flow described by the Cartesian coordinate x_i can be expressed by the following compressible Navier-Stokes equations:

$$\dot{\mathbf{U}} + \mathbf{A}_j \mathbf{U}_{,j} - (\mathbf{K}_{ji}\mathbf{U}_{,i})_{,j} = 0 \qquad \text{in } \Omega, \tag{1}$$

where Ω is the computational domain assuming that the compressible viscous fluid flow occupies Ω and $\mathbf{A}_j$ and $\mathbf{K}_{ji}$ are Jacobian matrices as follows:

$$\mathbf{A}_j = \frac{\partial \mathbf{F}_j^a}{\partial \mathbf{U}}, \tag{2}$$

$$\mathbf{K}_{ji}\mathbf{U}_{,i} = \mathbf{F}_j^d, \tag{3}$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho e \end{bmatrix} \equiv \begin{bmatrix} \rho \\ m_1 \\ m_2 \\ m_3 \\ \rho e \end{bmatrix}, \quad \mathbf{F}_j^a = \begin{bmatrix} \rho u_j \\ u_j \rho u_1 + \delta_{1j} p \\ u_j \rho u_2 + \delta_{2j} p \\ u_j \rho u_3 + \delta_{3j} p \\ u_j(\rho e + p) \end{bmatrix}, \quad \mathbf{F}_j^d = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{1j} \\ \tau_{2j} \\ \tau_{3j} \\ u_j \tau_{ij} - q_j \end{bmatrix}, \tag{4}$$

where ρ is the density; u_i, the velocity; e, the total energy density; p, the pressure; q_j, the heat flux; and τ_{ij}, the viscous stress tensor. The Kronecker delta function is denoted by δ_{ij}, where

$$\delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j). \end{cases} \tag{5}$$

The superposed dot denotes the partial derivative with respect to time t. The subscripted comma denotes partial differentiation. The total energy density e is expressed as follows:

$$e = \varepsilon + \frac{u_i u_i}{2}, \tag{6}$$

which is the sum of the internal energy density ε and the kinetic energy density. Since an ideal gas is assumed, the state equation becomes

$$p = (\gamma - 1)\rho\varepsilon, \tag{7}$$

where γ is the ratio of specific heat. The following relationships are introduced:

$$\tau_{ij} = \mu(u_{i,j} + u_{j,i}) + \lambda^* u_{k,k}\delta_{ij}, \tag{8}$$

$$\lambda^* = -\frac{2}{3}\mu, \tag{9}$$

$$\varepsilon = c_v\theta, \tag{10}$$

$$q_j = -\frac{\gamma\mu}{Pr}\varepsilon_{,j}, \tag{11}$$

$$c_v = [\gamma(\gamma-1)M_\infty^2]^{-1}, \tag{12}$$

where c_v is the specific heat at constant volume; μ, the viscosity coefficient; Pr, the Prandtl number; M_∞, the free-stream Mach number; and Re, the Reynolds number. The Sutherland's viscosity is described as follows:

$$\mu = \theta^{\frac{3}{2}}\frac{\theta_\infty + C}{\theta_\infty\theta + C}, \tag{13}$$

$$C = 110, \tag{14}$$

where θ is the temperature, and $\theta_\infty[K]$ is the temperature scale. The Jacobian matrices $\mathbf{A}_j$ and $\mathbf{K}_{ji}$ are expressed as follows:

$$\mathbf{A}_j = \begin{bmatrix} 0 & \delta_{j1} & \delta_{j2} & \delta_{j3} & 0 \\ \frac{1}{2}\delta_{j1}\bar{\gamma}u^2 - u_ju_1 & \delta_{j1}u_1 - \delta_{j1}\bar{\gamma}u_1 + u_j & \delta_{j2}u_1 - \delta_{j1}\bar{\gamma}u_2 & \delta_{j3}u_1 - \delta_{j1}\bar{\gamma}u_3 & \delta_{j1}\bar{\gamma} \\ \frac{1}{2}\delta_{j2}\bar{\gamma}u^2 - u_ju_2 & \delta_{j1}u_2 - \delta_{j2}\bar{\gamma}u_1 & \delta_{j2}u_2 - \delta_{j2}\bar{\gamma}u_2 + u_j & \delta_{j3}u_2 - \delta_{j2}\bar{\gamma}u_3 & \delta_{j2}\bar{\gamma} \\ \frac{1}{2}\delta_{j3}\bar{\gamma}u^2 - u_ju_3 & \delta_{j1}u_3 - \delta_{j3}\bar{\gamma}u_1 & \delta_{j2}u_3 - \delta_{j3}\bar{\gamma}u_2 & \delta_{j3}u_3 - \delta_{j3}\bar{\gamma}u_3 + u_j & \delta_{j3}\bar{\gamma} \\ (\bar{\gamma}u^2 - \gamma e)u_j & \delta_{j1}\bar{\varepsilon} - \bar{\gamma}u_ju_1 & \delta_{j2}\bar{\varepsilon} - \bar{\gamma}u_ju_2 & \delta_{j3}\bar{\varepsilon} - \bar{\gamma}u_ju_3 & \bar{\gamma}u_j \end{bmatrix}, \tag{15}$$

where

$$\bar{\gamma} = \gamma - 1, \quad u^2 = u_iu_i, \quad \bar{\varepsilon} = \gamma e - \bar{\gamma}\frac{u_iu_i}{2}. \tag{16}$$

$$
\mathbf{K}_{ji} = \left[\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
-\{\mu(\frac{m_1}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{1j}) + \lambda^*\frac{m_j}{\rho^2}\delta_{1i}\} & \frac{\mu}{\rho}(\delta_{ji} + \delta_{1j}\delta_{1i}) + \frac{\lambda^*}{\rho}\delta_{1j}\delta_{1i} & \frac{\mu}{\rho}\delta_{1j}\delta_{2i} + \frac{\lambda^*}{\rho}\delta_{2j}\delta_{1i} & \frac{\mu}{\rho}\delta_{1j}\delta_{3i} + \frac{\lambda^*}{\rho}\delta_{3j}\delta_{1i} & 0 \\
-\{\mu(\frac{m_2}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{2j}) + \lambda^*\frac{m_j}{\rho^2}\delta_{2i}\} & \frac{\mu}{\rho}\delta_{2j}\delta_{1i} + \frac{\lambda^*}{\rho}\delta_{1j}\delta_{2i} & \frac{\mu}{\rho}(\delta_{ji} + \delta_{2j}\delta_{2i}) + \frac{\lambda^*}{\rho}\delta_{2j}\delta_{2i} & \frac{\mu}{\rho}\delta_{2j}\delta_{3i} + \frac{\lambda^*}{\rho}\delta_{3j}\delta_{2i} & 0 \\
-\{\mu(\frac{m_3}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{3j}) + \lambda^*\frac{m_j}{\rho^2}\delta_{3i}\} & \frac{\mu}{\rho}\delta_{3j}\delta_{1i} + \frac{\lambda^*}{\rho}\delta_{1j}\delta_{3i} & \frac{\mu}{\rho}\delta_{3j}\delta_{2i} + \frac{\lambda^*}{\rho}\delta_{2j}\delta_{3i} & \frac{\mu}{\rho}(\delta_{ji} + \delta_{3j}\delta_{3i}) + \frac{\lambda^*}{\rho}\delta_{3j}\delta_{3i} & 0 \\
-\{\mu\frac{m_k m_k}{\rho^3}\delta_{ji} + \frac{m_j m_i}{\rho^3}(\mu + \lambda^*) + \lambda^*\frac{m_i m_j}{\rho^3} + \frac{\gamma\mu}{Pr}(\frac{e}{\rho} - \frac{m_k m_k}{\rho^3}\delta_{ji})\} & \mu\frac{m_1}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{1i} + \lambda^*\frac{m_k}{\rho^2}\delta_{1j}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_1}{\rho^2}\delta_{ji} & \mu\frac{m_2}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{2i} + \lambda^*\frac{m_k}{\rho^2}\delta_{2j}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_2}{\rho^2}\delta_{ji} & \mu\frac{m_3}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{3i} + \lambda^*\frac{m_k}{\rho^2}\delta_{3j}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_3}{\rho^2}\delta_{ji} & \frac{\gamma\mu}{Pr}\frac{1}{\rho}\delta_{ji}
\end{array} \right]. \tag{17}
$$

The typical optimization problem is shown in Fig. 1 Computational domain., where a solid body with boundary Γ_B is placed in an external flow. The initial condition is expressed as

$$\mathbf{U} = \hat{\mathbf{U}}(x_i, 0), \quad \text{in} \quad \Omega, \tag{18}$$

where the superscripted ˆ denotes a constant value. The boundary conditions for Γ_O and Γ_B are defined as follows:

$$
\begin{aligned}
(u_2, \tau_{2j}, q_n) &= (0, 0, 0) && \text{on} \quad \Gamma_{\mathrm{S1}} \times \mathbf{I}, \\
(u_3, \tau_{3j}, q_n) &= (0, 0, 0) && \text{on} \quad \Gamma_{\mathrm{S2}} \times \mathbf{I}, \\
(p_0, \tau_{1j}, q_n) &= (\hat{p}_0, 0, 0) && \text{on} \quad \Gamma_{\mathrm{O}} \times \mathbf{I}, \\
(u_i, q_n) &= (0, 0) && \text{on} \quad \Gamma_{\mathrm{B}} \times \mathbf{I},
\end{aligned} \tag{19}
$$

where $\mathbf{I}$ is the total time interval, and p_0 is the static pressure. In this study, the boundary condition for inflow is defined by the Riemann invariant and the isentropic change. The boundary condition for Γ_I is expressed as follows:

$$
\begin{aligned}
u_t &= \hat{u}_t, && \text{on} \quad \Gamma_{\mathrm{I}} \times \mathbf{I}, \\
R_\infty &= u_n - \frac{2c}{\gamma - 1}, && \text{on} \quad \Gamma_{\mathrm{I}} \times \mathbf{I}, \\
c^2 &= \gamma(\gamma - 1)(e - \frac{1}{2}u_i u_i), && \text{on} \quad \Gamma_{\mathrm{I}} \times \mathbf{I}, \\
S &= ln(p\rho^{-\gamma}), && \text{on} \quad \Gamma_{\mathrm{I}} \times \mathbf{I},
\end{aligned} \tag{20}
$$

where u_n, u_t, c, R_∞, and S are the normal velocity, tangent velocity, sound speed, Riemann invariant, and entropy, respectively.

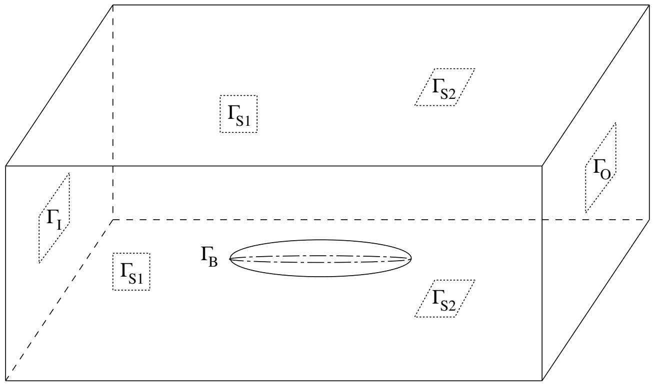

Fig. 1 Computational domain.

Performance Function

A fluid force reduction problem is formulated and solved, where the fluid force acting on the surface of a body is used in the performance function. Geometric coordinates of the surface of the body are selected such that they minimize the performance function J, which is defined by the square sum of the fluid forces as follows:

$$J = \frac{1}{2}\int_I (F_i - \bar{F}_i)Q_{ij}(F_j - \bar{F}_j)dt, \tag{21}$$

$$F_i = -\int_{\Gamma_B} t_i d\Gamma, \tag{22}$$

$$t_i = (\tau_{ij} - p\delta_{ij})n_j, \tag{23}$$

where t_i is the traction acting on the body; F_i, the fluid force; $\bar{F}_i$, the target fluid force, which is normally zero; Q_{ij}, the weighting constant; and I, the duration for which optimum computation is carried out. The extended performance function J^* can be obtained as follows:

$$\begin{aligned} J^* =& \frac{1}{2}\int_I (F_i - \bar{F}_i)Q_{ij}(F_j - \bar{F}_j)dt \\ &+ \int_I \int_\Omega \mathbf{\Lambda}^T \cdot \{\dot{\mathbf{U}} + \mathbf{A}_j \mathbf{U}_{,j} - (\mathbf{K}_{ji}\mathbf{U}_{,i})_{,j}\} d\Omega dt, \end{aligned} \tag{24}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} \xi \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \zeta \end{bmatrix}. \tag{25}$$

Here, ξ, η_i, and ζ are the Lagrange multipliers, and they are also known as adjoint variables.

Adjoint Equations

The solution of the optimal control problem under the constraint conditions of the basic equation (1) is used to determine the stationary condition of J^* instead of that of J with respect to the surface coordinates. The necessary condition required to satisfy the optimal condition can be derived from the first variation of J^* as follows:

$$\delta J^* = 0. \tag{26}$$

The first variation of J^* yields the following result:

$$\begin{aligned}
\delta J^* = &\int_I \int_\Omega \delta \mathbf{U} \cdot \{-\dot{\mathbf{\Lambda}} - (\bar{\mathbf{A}}_j \mathbf{\Lambda})_{,j} + \bar{\mathbf{A}}_{j,j}\mathbf{\Lambda} - (\bar{\mathbf{K}}^{\mathbf{1}}_{ji}\mathbf{\Lambda}_{,i})_{,j} + \bar{\mathbf{K}}^{\mathbf{1}}_{ji,j}\mathbf{\Lambda}_{,i} - \bar{\mathbf{K}}^{\mathbf{2}}_{ji,j}\mathbf{\Lambda}_{,i} \\
&+ \int_I \int_\Gamma \bar{\xi}\delta\rho d\Gamma dt + \int_I \int_{\Gamma_{S1}} \bar{\eta}_1 \delta m_1 d\Gamma dt + \int_I \int_{\Gamma_{S1}} \bar{\eta}_3 \delta m_3 d\Gamma dt \\
&+ \int_I \int_{\Gamma_{S2}} \bar{\eta}_1 \delta m_1 d\Gamma dt + \int_I \int_{\Gamma_{S2}} \bar{\eta}_2 \delta m_2 d\Gamma dt + \int_I \int_{\Gamma_O + \Gamma_B} \bar{\eta}_i \delta m_i d\Gamma dt \\
&+ \int_I \int_{\Gamma_{S1}+\Gamma_{S2}+\Gamma_O} \bar{\zeta}\delta(\rho e) d\Gamma dt + \int_I \int_{\Gamma_B} \zeta_{,j}\delta q n_j d\Gamma dt \\
&- \int_I \int_{\Gamma_I} \eta_i \delta\tau_{ij} n_j d\Gamma dt - \int_I \int_{\Gamma_{S1}} \eta_2 \delta\tau_{2j} n_j d\Gamma dt - \int_I \int_{\Gamma_{S2}} \eta_3 \delta\tau_{3j} n_j d\Gamma dt \\
&- \int_I \int_{\Gamma_I} \zeta \delta q_j n_j d\Gamma dt - \int_I \int_{\Gamma_I} \zeta u_i \delta\tau_{ij} n_j d\Gamma dt \\
&+ \int_I \int_{\Gamma_B} \{-\eta_i - (F_j - \bar{F}_j) Q_{ij}\} \delta t_i d\Gamma dt \\
&+ \int_\Omega \mathbf{\Lambda}(x_i, t_f) \cdot \delta \mathbf{U}(x_i, t_f)^T d\Omega = 0,
\end{aligned} \tag{27}$$

where

$$\bar{\mathbf{A}}_j = \begin{bmatrix}
0 & \frac{1}{2}\delta_{j1}\bar{\gamma}u^2 - u_j u_1 & \frac{1}{2}\delta_{j2}\bar{\gamma}u^2 - u_j u_2 & \frac{1}{2}\delta_{j3}\bar{\gamma}u^2 - u_j u_3 & (\bar{\gamma}u^2 - \gamma e)u_j \\
\delta_{j1} & \delta_{j1}u_1 - \delta_{j1}\bar{\gamma}u_1 + u_j & \delta_{j1}u_2 - \delta_{j2}\bar{\gamma}u_1 & \delta_{j1}u_3 - \delta_{j3}\bar{\gamma}u_1 & \delta_{j1}\bar{\varepsilon} - \bar{\gamma}u_j u_1 \\
\delta_{j2} & \delta_{j2}u_1 - \delta_{j1}\bar{\gamma}u_2 & \delta_{j2}u_2 - \delta_{j2}\bar{\gamma}u_2 + u_j & \delta_{j2}u_3 - \delta_{j3}\bar{\gamma}u_2 & \delta_{j2}\bar{\varepsilon} - \bar{\gamma}u_j u_2 \\
\delta_{j3} & \delta_{j3}u_1 - \delta_{j1}\bar{\gamma}u_3 & \delta_{j3}u_2 - \delta_{j2}\bar{\gamma}u_3 & \delta_{j3}u_3 - \delta_{j3}\bar{\gamma}u_3 + u_j & \delta_{j3}\bar{\varepsilon} - \bar{\gamma}u_j u_3 \\
0 & \delta_{j1}\bar{\gamma} & \delta_{j2}\bar{\gamma} & \delta_{j3}\bar{\gamma} & \bar{\gamma}u_j
\end{bmatrix}, \tag{28}$$

$$\bar{\mathbf{K}}^{\mathbf{1}}_{ji} = \begin{bmatrix} 0 & -\{\mu(\frac{m_1}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{j1}) + \lambda^*\frac{m_j}{\rho^2}\delta_{1i}\} & -\{\mu(\frac{m_2}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{j2}) + \lambda^*\frac{m_j}{\rho^2}\delta_{2i}\} & -\{\mu(\frac{m_3}{\rho^2}\delta_{ji} + \frac{m_i}{\rho^2}\delta_{j3}) + \lambda^*\frac{m_j}{\rho^2}\delta_{3i}\} & -\{\mu\frac{m_k m_k}{\rho^3}\delta_{ji} + \frac{m_j m_i}{\rho^3}(\mu+\lambda^*) + \lambda^*\frac{m_i m_j}{\rho^3} + \frac{\gamma\mu}{Pr}(\frac{e}{\rho} - \frac{m_k m_k}{\rho^3})\delta_{ji}\} \\ 0 & \frac{\mu}{\rho}(\delta_{ji} + \delta_{j1}\delta_{1i}) + \frac{\lambda^*}{\rho}\delta_{j1}\delta_{1i} & \frac{\mu}{\rho}\delta_{j2}\delta_{1i} + \frac{\lambda^*}{\rho}\delta_{j1}\delta_{2i} & \frac{\mu}{\rho}\delta_{j3}\delta_{1i} + \frac{\lambda^*}{\rho}\delta_{j1}\delta_{3i} & \mu\frac{m_1}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{1i} + \lambda^*\frac{m_k}{\rho^2}\delta_{j1}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_1}{\rho^2}\delta_{ji} \\ 0 & \frac{\mu}{\rho}\delta_{j1}\delta_{2i} + \frac{\lambda^*}{\rho}\delta_{j2}\delta_{1i} & \frac{\mu}{\rho}(\delta_{ji} + \delta_{j2}\delta_{2i}) + \frac{\lambda^*}{\rho}\delta_{j2}\delta_{2i} & \frac{\mu}{\rho}\delta_{j3}\delta_{2i} + \frac{\lambda^*}{\rho}\delta_{j2}\delta_{3i} & \mu\frac{m_2}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{2i} + \lambda^*\frac{m_k}{\rho^2}\delta_{j2}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_2}{\rho^2}\delta_{ji} \\ 0 & \frac{\mu}{\rho}\delta_{j1}\delta_{3i} + \frac{\lambda^*}{\rho}\delta_{j3}\delta_{1i} & \frac{\mu}{\rho}\delta_{j2}\delta_{3i} + \frac{\lambda^*}{\rho}\delta_{j3}\delta_{2i} & \frac{\mu}{\rho}(\delta_{ji} + \delta_{j3}\delta_{3i}) + \frac{\lambda^*}{\rho}\delta_{j3}\delta_{3i} & \mu\frac{m_3}{\rho^2}\delta_{ji} + \mu\frac{m_j}{\rho^2}\delta_{3i} + \lambda^*\frac{m_k}{\rho^2}\delta_{j3}\delta_{ki} - \frac{\gamma\mu}{Pr}\frac{m_3}{\rho^2}\delta_{ji} \\ 0 & 0 & 0 & 0 & \frac{\gamma\mu}{Pr}\frac{1}{\rho}\delta_{ji} \end{bmatrix}, \tag{29}$$

$$\bar{\mathbf{K}}^{\mathbf{2}}_{11,1} = \begin{bmatrix} 0 & 0 & 0 & 0 & (\mu-\lambda^*)\frac{m_l}{\rho^2}\{(\frac{m_1}{\rho})_{,l} - (\frac{m_k}{\rho})_{,k}\delta_{l1}\} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}\{(\frac{m_1}{\rho})_{,1} - (\frac{m_k}{\rho})_{,k}\} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_1}{\rho})_{,2} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_1}{\rho})_{,3} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{30}$$

$$\bar{\mathbf{K}}^{\mathbf{2}}_{22,2} = \begin{bmatrix} 0 & 0 & 0 & 0 & (\mu-\lambda^*)\frac{m_l}{\rho^2}\{(\frac{m_2}{\rho})_{,l} - (\frac{m_k}{\rho})_{,k}\delta_{l2}\} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_2}{\rho})_{,1} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}\{(\frac{m_2}{\rho})_{,2} - (\frac{m_k}{\rho})_{,k}\} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_2}{\rho})_{,3} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{31}$$

$$\bar{\mathbf{K}}^{\mathbf{2}}_{33,3} = \begin{bmatrix} 0 & 0 & 0 & 0 & (\mu-\lambda^*)\frac{m_l}{\rho^2}\{(\frac{m_3}{\rho})_{,l} - (\frac{m_k}{\rho})_{,k}\delta_{l3}\} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_3}{\rho})_{,1} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}(\frac{m_3}{\rho})_{,2} \\ 0 & 0 & 0 & 0 & -(\mu-\lambda^*)\frac{1}{\rho}\{(\frac{m_3}{\rho})_{,3} - (\frac{m_k}{\rho})_{,k}\} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{32}$$

$$\bar{\mathbf{K}}^{\mathbf{2}}_{12,1} = \bar{\mathbf{K}}^{\mathbf{2}}_{13,1} = \bar{\mathbf{K}}^{\mathbf{2}}_{21,2} = \bar{\mathbf{K}}^{\mathbf{2}}_{23,2} = \bar{\mathbf{K}}^{\mathbf{2}}_{31,3} = \bar{\mathbf{K}}^{\mathbf{2}}_{32,3} = \mathbf{0}, \tag{33}$$

$$\begin{aligned}
\bar{\xi} &= \{-u_i u_j + \frac{1}{2}\bar{\gamma} u_k u_k \delta_{ij})\eta_i + (\bar{\gamma} u_k u_k - \gamma e) u_j \zeta\} n_j \\
&\quad - \{\mu(\eta_{i,j} + \eta_{j,i}) + \lambda^* \eta_{k,k}\delta_{ij} + \mu(\frac{m_i}{\rho}\zeta_{,j} + \frac{m_j}{\rho}\zeta_{,i}) + \lambda^* \frac{m_k}{\rho}\zeta_{,k}\delta_{ij}\} \frac{m_i}{\rho^2} n_j, \\
\bar{\eta}_i &= \{\xi\delta_{ij} + (u_k\delta_{ij} - \bar{\gamma} u_i \delta_{kj})\eta_k + \eta_i u_j + (\bar{\varepsilon}\delta_{ij} - \bar{\gamma} u_i u_j)\zeta\} n_j \\
&\quad + \{\mu(\eta_{i,j} + \eta_{j,i}) + \lambda^* \eta_{k,k}\delta_{ij} + \mu(\frac{m_i}{\rho}\zeta_{,j} + \frac{m_j}{\rho}\zeta_{,i}) + \lambda^* \frac{m_k}{\rho}\zeta_{,k}\delta_{ij}\} \frac{1}{\rho} n_j, \\
\bar{\zeta} &= (\bar{\gamma}\delta_{ij}\eta_i + \gamma\zeta u_j) n_j.
\end{aligned} \tag{34}$$

By setting each term to zero so as to satisfy the optimal condition, the adjoint equations, boundary conditions, and terminal conditions for adjoint variables can be derived as follows:

$$-\dot{\mathbf{\Lambda}} - (\bar{\mathbf{A}}_j \mathbf{\Lambda})_{,j} + \bar{\mathbf{A}}_{j,j}\mathbf{\Lambda} - (\bar{\mathbf{K}}^{\mathbf{1}}_{ji}\mathbf{\Lambda}_{,i})_{,j} + \bar{\mathbf{K}}^{\mathbf{1}}_{ji,j}\mathbf{\Lambda}_{,i} - \bar{\mathbf{K}}^{\mathbf{2}}_{ji,j}\mathbf{\Lambda}_{,i} = 0, \text{ in } \Omega \times \mathbf{I}, \tag{35}$$

$$\begin{aligned}
(\eta_i, \zeta) &= (0,0) && \text{on } \Gamma_{\mathrm{I}} \times \mathbf{I}, \\
(\bar{\xi}, \bar{\eta}_i, \bar{\zeta}) &= (0,0,0), && \text{on } \Gamma_{\mathrm{O}} \times \mathbf{I}, \\
(\bar{\xi}, \bar{\eta}_1, \eta_2, \bar{\eta}_3, \bar{\zeta}) &= (0,0,0,0,0), && \text{on } \Gamma_{\mathrm{S}_1} \times \mathbf{I}, \\
(\bar{\xi}, \bar{\eta}_1, \bar{\eta}_2, \eta_3, \bar{\zeta}) &= (0,0,0,0,0), && \text{on } \Gamma_{\mathrm{S}_2} \times \mathbf{I}, \\
(\eta_i, \zeta_n) &= (-(F_j - \bar{F}_j)Q_{ij}, 0), && \text{on } \Gamma_{\mathrm{B}} \times \mathbf{I}, \\
\mathbf{\Lambda}(x_i, t_f) &= 0, && \text{in } \Omega,
\end{aligned} \tag{36}$$

where ζ_n is the flux of $\zeta_{,j}$ along the normal direction n_j. The following relationship is used for deriving the variation of J^* with respect to δx_i:

$$\delta m_i = \delta(\rho u_i) = u_i \delta\rho + \rho\delta u_i = u_i\delta\rho + \rho u_{i,l}\delta x_l. \tag{37}$$

The gradient of J^* with respect to the surface coordinates is obtained by solving the compressible Navier-Stokes equations and adjoint equations, and it is expressed as follows:

$$grad(J^*)_l = \{\rho\xi\delta_{ij} + (\rho e + p)\zeta\delta_{ij} + \mu(\eta_{i,j} + \eta_{j,i}) + \lambda^*\eta_{k,k}\delta_{ij}\} u_{i,l} n_j. \tag{38}$$

Minimization Method

In this study, a weighted gradient method is used for the minimization algorithm. The modified performance function K can be expressed as follows:

$$K = J^* - \frac{1}{2}\int_I \int_{\Gamma_B} (x_i^{(l+1)} - x_i^{(l)}) W_{ij}^{(l)} (x_j^{(l+1)} - x_j^{(l)}) d\Gamma dt + a^* \{ \sum_{e=1}^{m} A_e^{(l+1)}(x_k) - A_0 \}, \quad (39)$$

$$W_{ij}^{(l)} = W^{(l)} \delta_{ij}, \quad (40)$$

where l represents the iteration cycle, and $W^{(l)}$ represents the weighting constant, which is updated during iteration. The third term represents a constant volume condition and a^* represents the Lagrange multiplier. Under the stationary condition $\delta K = 0$, we obtain the following equation:

$$W_{ij}^{(l)} x_j^{(l+1)} = W_{ij}^{(l)} x_j^{(l)} + G_i + a^* \frac{\partial}{\partial x_i} \sum_{e=1}^{m} A_e^{(l+1)}(x_k). \quad (41)$$

The surface coordinates $x_i^{(l)}$ are updated using to equation (41).

NUMERICAL EXAMPLE and CONCLUSION

The 3D shape determination problem is solved. The shape of the body is modified to an appropriate shape in order to minimize the fluid force F_i. The pre-assigned fluid force $\overline{F}_i$ in equation (24) is set to zero. To minimize the drag force, Q_{11}, Q_{22}, and Q_{33} are set to 1.0, 0.0, and 0.0, respectively. The value of the parameters are $\Theta = 1.0$, $\Delta t = 0.005$, $\varepsilon = 1.0$, $W = 0.001$, and $\overline{E} = 10^{-6}$. The finite element mesh, shown in Fig. 2, consists of 1,56,937 nodes and 9,088,753 linear elements. Parallel computing based on the domain decomposition method is applied to the computation. The computational domain divided into 512 sub-domains. HA8000 cluster system, which is developed at the University of Tokyo, is used to solve the 3D shape determination problem.

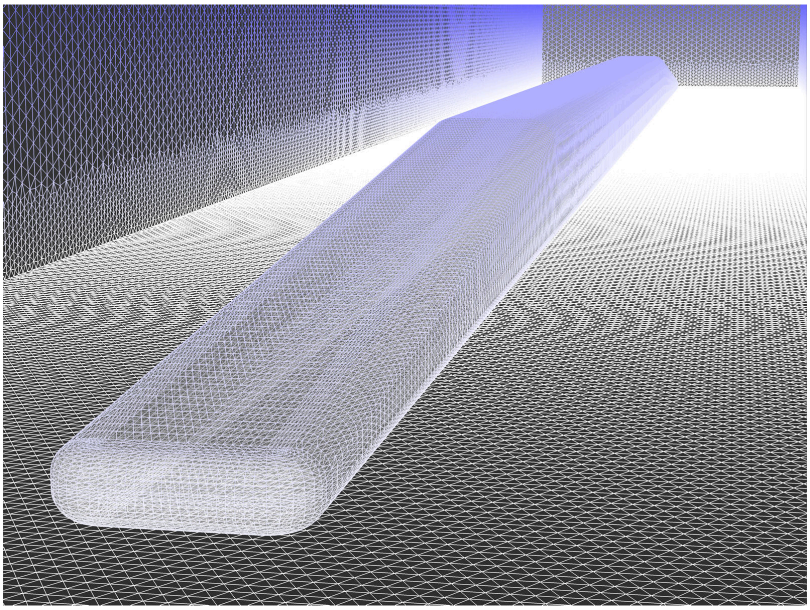

Fig. 2 Finite element mesh of a car.

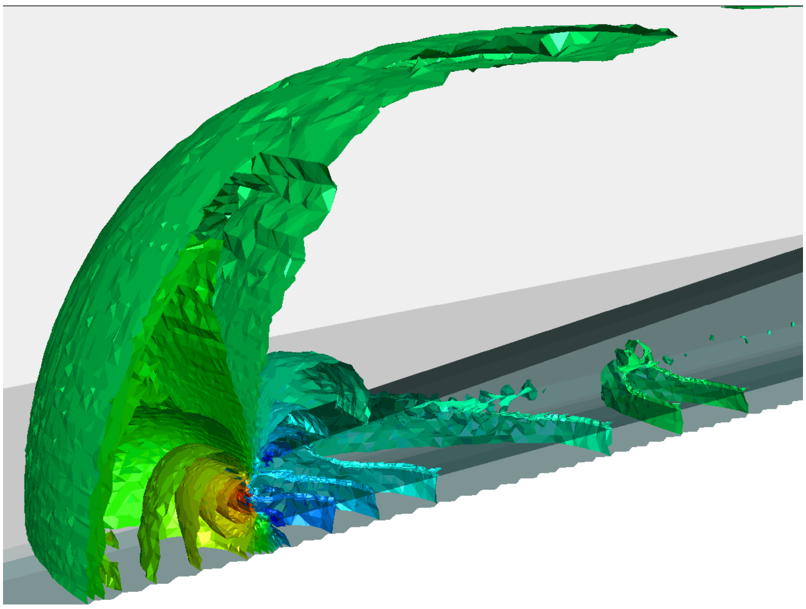

Fig. 3 Pressure distribution at front-nose.

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