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# A METHOD OF DISCHARGE ESTIMATION BASED ON LATERAL VELOCITY DISTRIBUTION FUNCTION APPLIED TO ULTRASONIC VELOCITY METER SYSTEM

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***Abstract***: *Lateral velocity distribution (LDM) based on the beta density function proposed by Seo and Baek (2004) is applied to compute discharge using the velocity data obtained from the ultrasonic velocity meter. Shape parameters $\alpha$ and $\beta$ are estimated by using the 5 lateral velocity distribution datasets measured by an ADCP. Vertical velocity distribution is assumed to follow common one-sixth power law. Edge discharges are estimated by using the nearest velocities. Resultant discharge by LDM is roughly in accordance with the dam discharge. The method shows better accuracy for the high flow over 200m$^3$/s with relative difference of 7.4% from the dam discharge. In a practical and economic view, LDM is considered as an accurate method to get discharge with a little effort.*

***Keywords***: *lateral velocity distribution; LDM; discharge; UVM;*

## INTRODUCTION

Automatic and continuous measurement methods have been widely introduced to streamflow monitoring field in place of traditional stage-discharge relationship. Among them, ultrasonic velocity meter (UVM) which as a one of the oldest methods, only gives information on line velocity has been still used due to its simplicity and robustness for natural condition.

Discharge measurement by using the single path UVM generally needs relationship between line velocity which is directly measured by the UVM and mean velocity of the cross-section which is obtained by making direct discharge measurements. Then conversion coefficient $k$ value is commonly used to represent the relationship. And cross-section area calculated from stage-area relationship is used to compute final discharge. For establishment of the relationship between the line and the mean velocities to derive the value $k$, generally at least a dozen direct discharge measurements need to be made for a range of discharge as wide as possible. However, the work needs much time and manpower. So, there have been various effort to make this work efficient.

In this study, we propose a new method to compute discharge based on a probabilistic density function to reproduce lateral velocity distribution. By using this method discharge in the whole

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cross-section can be computed. It only needs several direct lateral velocity measurements for parameter estimation. The procedures of and the results by this new method are described and presented in the following paragraphs.

## SITE

In this study we use a UVM installed at the midstream of the Dalcheon River which is located near the center of the South Korea. It is a cobble-bed river which is stable even during high flow. Its flow is regulated by the upstream dam for hydroelectric power generation and flood control. Dam releases discharges ranging from 6 to over 1,000$m^3$/s. Its discharge is to be used as a reference value to measured discharge by the UVM.

In the study site, a double layer UVM is installed across the river. But only a lower sensor is used for this study. It is installed at the lower part in the measurement cross-section for unceasing monitoring throughout a year. Figure 1 shows cross-section and plan view of the installation site.

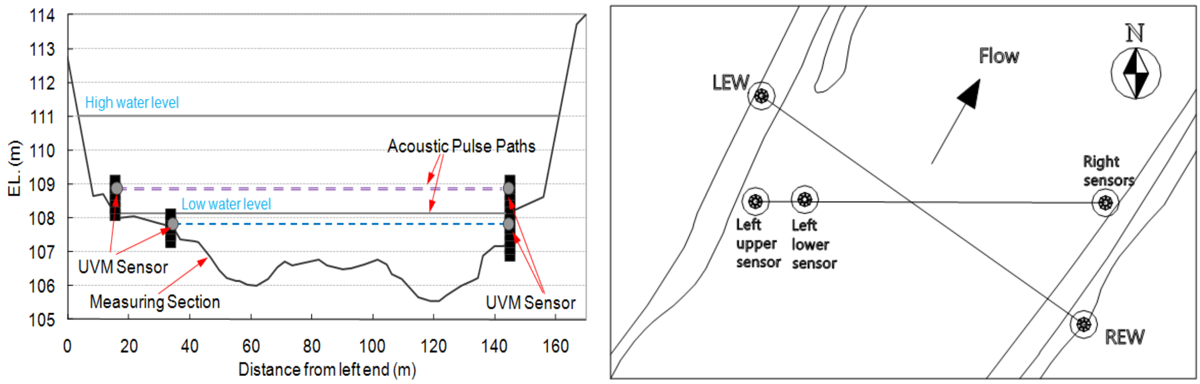


**Figure 1. Cross-section and plan view of the UVM installed site**

## LATERAL VELOCITY DISTIRIBUTION

The UVM only gives the line velocity across the river for discharge measurement. It is regarded as integration of lateral velocity distribution along in the narrow water layer through which acoustic beam penetrates. To obtain cross-sectional velocity distribution for discharge, there should be two elements: lateral and vertical. For the lateral velocity distribution, beta density function (BDF) is used for this study. In this study, we call this method as lateral distribution method (LDM). Parameters in the BDF are estimated by using the measured velocity distribution acquired with an ADCP for the acoustic beam layer. For the vertical velocity distribution, simple power law is adopted.

### Beta Density Function

BDF for the lateral velocity distribution is previously used by Seo and Baek (2004). It is known that the BDF well reproduce the lateral velocity distribution pattern in the natural channels. The BDF is established by combination of gamma probability density function. It can reproduce both symmetric and asymmetric distribution. General equation for the BDF is just as Eq. (1).

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1 \quad (1)$$

where, $\alpha$ and $\beta$ are parameters which determine the shape of the distribution. $\Gamma(\alpha)$ and $\Gamma(\beta)$ are defined by Eq. (2) and Eq. (3).

$$\Gamma(\alpha, x) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \quad \alpha > 0 \quad (2)$$

$$\Gamma(\beta, x) = \int_0^\infty x^{\beta-1} e^{-x} dx, \quad \beta > 0 \quad (3)$$

**Dimensionless Lateral Velocity Distribution**

Dimensionless lateral velocity distribution along in the acoustic beam layer can be expressed in the form of the BDF like Eq. (4).

$$\frac{u}{U} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} (y')^{\alpha-1}(1-y')^{\beta-1}, \quad 0 < y' < 1 \quad (4)$$

in which $u$ is a point velocity at the UVM height, whose position is $y'$ in the lateral direction, $U$ is laterally integrated velocity and $y'$ is dimensionless lateral distance from the left end.

**Parameter Estimation**

Parameters $\alpha$ and $\beta$ may be determined by two ways. First, if there is laterally measured velocity dataset at the UVM height, they can be estimated by least square method. But there no velocity distribution data, they may be estimated from only two figures: maximum velocity and its horizontal position along the width. In this study, ADCP transect data were used to determine the parameters. Cell velocity data at the UVM height are extracted considering depth and cell size of the ADCP data. Figure 2 shows lateral velocities for the 5 cases measured by ADCP in the cross-section. Used ADCP dataset and best fitted values for the parameters $\alpha$ and $\beta$ are shown in Table 1.

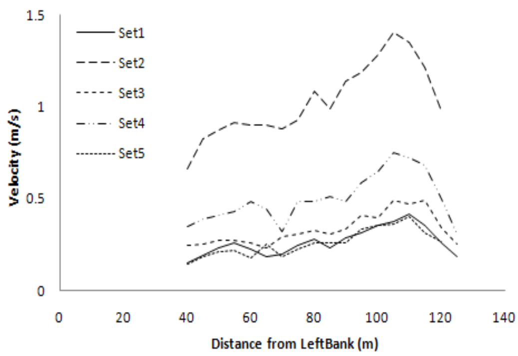


**Figure 2 Measured lateral velocity distribution by the ADCP**

**Table 1 Parameter estimation for the measured dataset**

| Data set | Discharge ($m^3/s$) | Width | Water level (m) | Best fitted | | | Mean value applied | | | Difference |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $\alpha$ | $\beta$ | RMSE | $\alpha$ | $\beta$ | RMSE | |
| #1 | 46.2 | 117.5 | 108.38 | 1.58 | 1.16 | 0.788 | 1.52 | 1.15 | 0.795 | 0.007 |
| #2 | 257.2 | 128 | 109.16 | 1.47 | 1.17 | 0.564 | 1.52 | 1.15 | 0.6 | 0.036 |
| #3 | 58.9 | 119 | 108.44 | 1.47 | 1.11 | 0.752 | 1.52 | 1.15 | 0.768 | 0.016 |
| #4 | 97.7 | 124 | 108.61 | 1.48 | 1.14 | 0.776 | 1.52 | 1.15 | 0.777 | 0.001 |
| #5 | 43 | 118 | 108.44 | 1.6 | 1.17 | 0.753 | 1.52 | 1.15 | 0.768 | 0.015 |
| Average 1.52 | | | | | 1.15 | 0.727 | | | 0.741 | 0.015 |

After judgment from the fitted parameters shown in Table 1, which have only slight difference from each other, the parameters $\alpha$ and $\beta$ are assumed to be approximately the same for whole discharge range. Thus, mean values of 1.52 and 1.15 for the parameters $\alpha$ and $\beta$, respectively are used to compute the lateral velocity distribution in the acoustic beam layer.

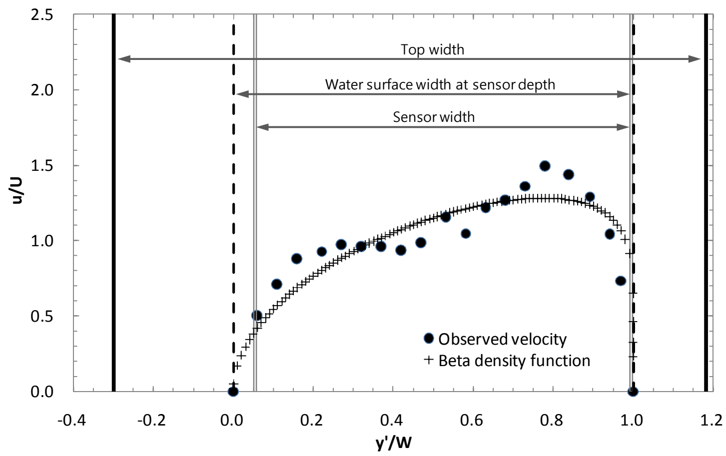


**Figure 3 The BDF fitted to measured lateral velocity distribution (case #2)**

## VERTICAL VELOCITY DISTRIBUTION

To get depth-averaged velocities for all the discretized lateral positions, it is necessary to establish vertical velocity distribution for them. In this study, simple power law in which one-sixth exponent is adopted is assumed. According to Chen (1991) and Simpson and Oltmann (1990) power law equivalent of Manning's formulae for open channels is expressed as Eq. (5). From the computed lateral velocity for a single position in the acoustic beam layer, depth-averaged velocity for it can be expressed as Eq. (5).

$$v / v_* = 9.5(z / z_0)^{1/6}, \quad (5)$$

here, $z$ is distance to the channel bed, $z_0$ is bottom roughness height, $v$ is velocity at distance $z$ from the bed, $v_*$ is shear velocity.
For a condition in which there are known velocities in the vertical, Eq. (5) can be rewritten as Eq. (6).

$$v = az^{1/6}, \quad (6)$$

in which $a$ can be determined from at least one velocity value in the vertical. From the computed lateral velocity for a single discretized position in the acoustic beam layer, depth-averaged velocity for it can be expressed as Eq. (7).

$$V_i = \frac{6}{7} u_i \left( \frac{d_i}{h_i} \right)^{1/6}, \quad (7)$$

where $V_i$ is depth-averaged velocity for the laterally discretized vertical, $u_i$ is the calculated velocity by Eq. (4) for the laterally discretized real coordinate (distance) from the left sensor, $d_i$ and $h_i$ are depth and the height of the acoustic beam layer in the vertical.

## DISCHARGE COMPUTATION

The lateral velocity distribution can only be calculated for the width between the two acoustic transducers. For this width, discharge is calculated by multiplying the depth-averaged velocity by sub-areas defined by the product of the discretized width and depth. Finally, remaining area of the both sides beyond the sensor position should be estimated. For these areas, traditional triangular computation method expressed by Eq. (8) is used.

$$Q_{L,R} = 0.707 V_{1,n} A_{L,R}, \quad (8)$$

In which, $Q_{L,R}$ is edge discharge near the left or the right bank, $V_{1,n}$ is the first or the last velocity near the sensor, and $A_{L,R}$ is edge area.

## RESULTS AND DISCUSSION

Figure 4 shows time-series discharge data of the two during the flood season of July 2009. The one is dam released discharge which serves as reference and the other is computed discharge by LDM in this study. LDM discharge is in a good agreement with dam discharge, but over approximately 450m$^3$/s it shows no data because acoustic signals could not detected over this discharge.
Figure 5 shows one to one comparison between the dam and LDM discharges for various flow condition which maintain relatively constant dam release. It can be easily noted that the LDM gives slightly larger discharge than the dam does, especially for the low flow. Due to this, RMS error for the low flow comes to about 20%. However, for discharge larger than 100m$^3$/s, RMS error decreases to 11.7% and for over 200m$^3$/s, it diminishes to only 7.4%. Judging from this

result, it is inferred that the LDM model under the condition mentioned above gives more accurate discharge for the relatively high flow.

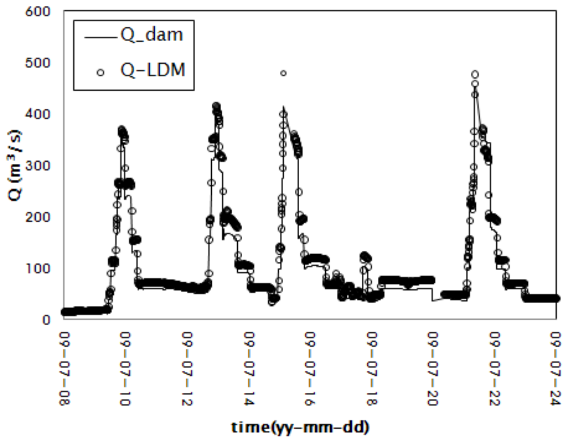


**Figure 4 Time-series discharge comparison**

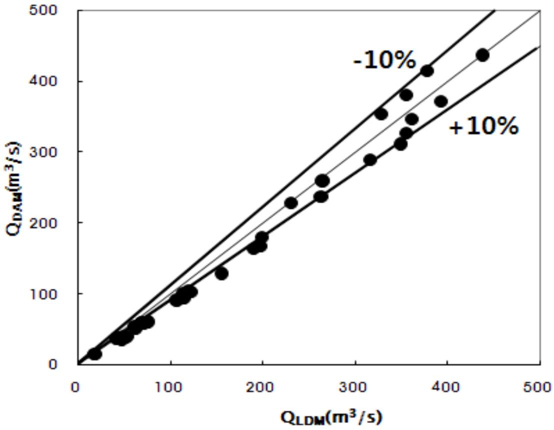


**Figure 5 Comparison of the LDM discharge with the dam discharge**

Under the consideration that the LDM in this study has two fixed assumptions on the lateral and vertical velocity distribution and only 5 times of direct measurements are used to estimate the parameters, this result is very good. Conversely speaking, its assumptions are roughly suitable for the real condition. Especially, lateral velocity distribution pattern seems to be relatively consistent for the entire discharge range. Nevertheless, the LDM are not so accurate for the low flow. There should be a detailed analysis.
Though not tested, effect of parameter estimation based on the fewer cases of data on accuracy of computed discharge is necessary to be evaluated. In this study, we used 5 data for parameter fitting. Resultant $\alpha$ and $\beta$ values for the 5 cases are nearly the same. So, with a fewer direct measurement, largely the same result is expected to be obtained. In this point, this method is economic and applicable for discharge computation.

## SUMMARY AND CONCLUSIONS

In this study, lateral velocity distribution (LDM) based on the beta density function proposed by Seo and Baek (2004) is applied to compute discharge with the ultrasonic velocity meter data. Shape parameters $\alpha$ and $\beta$ are estimated by using the 5 lateral velocity distribution datasets measured by ADCP. Vertical velocity distribution is assumed to follow power law with a common one-sixth exponent. Edge discharges are estimated by using the nearest velocities. Discharge computed by using the LDM are roughly in accordance with the dam released discharge. The method shows large errors for the low flow, while it shows better accuracy over 200m$^3$/s by showing 7.4% of relative difference from the dam discharge. Under the condition that the LDM has some rather fixed assumption concerning lateral and vertical velocity distribution together with relatively few direct measurements, it is considered as a practical and economic way to get accurate discharge, especially for the high flow using the ultrasonic velocity meter.

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