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PROBABILISTIC CHARACTERIZATION OF HYDROLOGIC EXTREMES USING BIVARIATE COPULAS

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Abstract: *The study aims to model the joint distribution of drought duration, severity, and peak using bivariate copulas. A nonparametric method, based on kernel density function is used to determine probability density function (PDF) of drought characteristics. Drought occurrences are analyzed by the Standardized Precipitation Index (SPI) computed on the mean areal precipitation, aggregated at 12 months, observed in Western Rajasthan meteorological subdivision of India. The study also presents conditional distribution and joint return periods - computed as mean inter-arrival time, taking into account drought characteristics – severity and duration at a time. Application of the proposed methodology shows a good correspondence between empirical and theoretical joint distribution, indicating copulas are adequately modeling drought characteristics and helping in computation of exceedence probabilities of drought events.*

Keywords: *Standardized Precipitation Index; Drought; Bivariate copulas; Return Period.*

INTRODUCTION

In hydrological studies, Drought is a climatic anomaly, characterized by deficient supply of moisture caused due to either from sub-normal rainfall /erratic rainfall distribution / higher water need or a combination of all these factors. Drought is a perennial feature in some regions in India. In India, about 33 per cent of the arable land is considered to be drought-prone (i.e., about 14 per cent of the total land area of the country) and a further 35 per cent can also be affected if rainfall is exceptionally low for extended periods (ESCAP, 1995a). Rajasthan is one of the most drought prone areas of India with eleven districts of the state are in arid region. In the present study the association between the drought characteristics is modeled with bivariate copula. Using Copula function the *n*- dimensional distribution function is linked to its one dimensional margins which is itself a continuous distribution function characterizing the model's dependence structure. Copulas are able to capture the dependence structure of random variables independently from the marginal distributions. Traditionally the drought properties are investigated by univariate frequency analysis (Talaksen et al., 1997; Fernandez and Salas, 1999; Cancelliere and Salas, 2004). Recently, by recognizing the significant stochastic association between the drought variables copulas has been used in multivariate analyses of drought irrespective of the type of marginal distributions they follow (Shiau, 2006; Shiau et al., 2007;

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Serinaldi et al., 2009).

DROUGHT DEFINITION AND CHARACTERISTICS

The Standardized Precipitation Index (SPI) is a common indicator of drought, which requires only rainfall data (McKee etal., 1993). SPI is the simple, spatially invariant, and probabilistic in nature and can be applied to analyze different types of drought phenomenon, such as, meteorological, agricultural, hydrological characteristics etc. The temporal nature of the index is also helpful in understanding drought dynamics, such as onset and ending, which is difficult to track in other indices. Due to its standardized nature, the frequency of extreme events at any location and on any time scale is consistent (Hays etal., 1999). Moreover due to its probabilistic nature, SPI is helpful in carrying out drought risk and decision analysis (Guttman, 1998). The SPI can monitor dry and wet periods over a multiple time scales ranging from 1, 2, 3,..., 72 months. For example, Agricultural users may be interested in shorter time periods, such as SPI for 3-6 months, while hydrologists or water managers might be interested in SPI values for 12 months. Thus, the SPI is a *z*-score and represents an event departure from the mean, expressed in standard deviation units. The threshold for indicating severity of drought based on SPI has been adopted from earlier studies. In practice, the computation of the SPI index in a given year *i* and calendar month *j*, for a *k* time scale requires (McKee et al., 1993).

1. Computation of cumulative precipitation series $X_{ij}^{k}\left(i=1,...,n\right)$ for a particular month of interest *j*, where each term is the sum of the actual monthly precipitation with precipitation of the *k-1* past consecutive months.

2. Fitting of a gamma distribution function to the series.

3. Computing the non-exceedence probabilities corresponding to the cumulative precipitation values. As the two-parameter gamma function is not defined for zero values, so the cumulative probability becomes, $F\left(x\right)=q+\left(1-q\right)G\left(x\right)$; where, $G\left(x\right)$ is the distribution function estimated for nonzero precipitation $\left(x\right)$; and q is the zero precipitation probability from the historical time series.

4. Computate SPI values by transforming those probabilities into standard normal variable values, $SPI_t=\Phi^{-1}\left(F\left(x\right)\right)$.

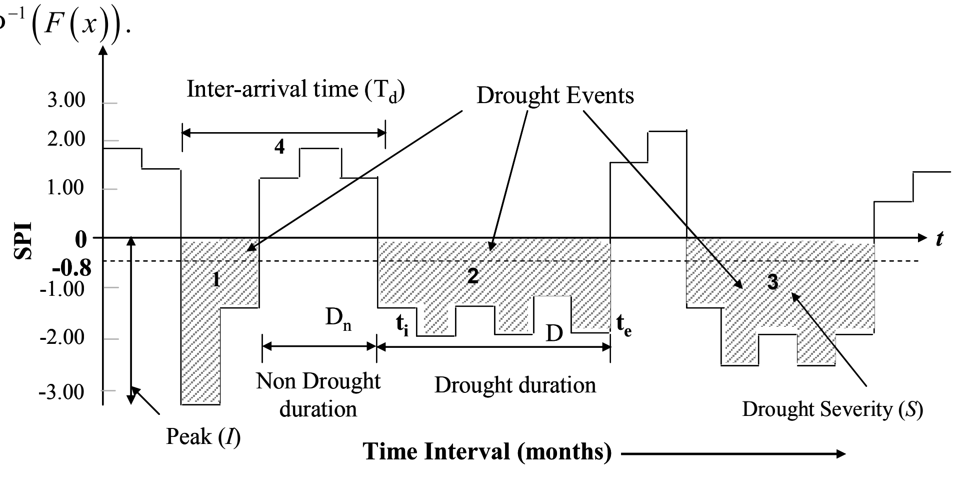


Fig. 1. Drought characteristics using SPI

A drought period is assumed as a consecutive number of intervals where SPI values are less than -0.8. A definition sketch of drought characteristics is shown in Fig. 1. *Drought severity* or *Magnitude* (S) is the cumulative values of SPI within the drought duration. For convenience, drought severity is taken to be positive, which is given by (Shiau, 2006),

$$S = -\sum_{i=1}^{D} SPI_i \tag{1}$$

COPULA

Definition and Properties of Copula

Definition A 2-dimensional copula is a function C, with domain $[0,1]^2$ and range $[0,1]$, and have following properties, (i) $C(u,0)=0 \quad \forall u \in [0,1]^2$ (ii) $C(u,1)=u$; similarly $C(1,v)=v$ $\forall u,v \in [0,1]^2$, and (iii) since $C(u,v)$ is a distribution function, $C(u_2, v_2) - C(u_2, v_1) - C(u_1,v_2) + C(u_1, v_1) > 0$, if $0 \leq u_1 < u_2 \leq 1$ and $0 \leq v_1 < v_2 \leq 1$.

Sklar Theorem In bivariate case, considering the two correlated random variables X and Y with univariate continuous marginal distribution functions $F_x(X) = P(X \leq x)$ and $F_Y(Y) = P(Y \leq y)$, the link between the joint distribution and copula C, is given by Sklar's theorem (1959), $F_{X,Y}(x,y) = C[F_x(x), F_Y(y)]$, such that $\forall x, y$ in $R \in (-\infty, \infty)$, where $F_{X,Y}(x,y)$ is the joint cumulative distribution function (CDF) of the random variables X and Y.

SIMULATION STUDY

The Archimedean class of copula is widely used in hydrology because they are easily generated and are capable of capturing wide ranges of dependence. They can be expressed as (Genest and Mackay, 1986), $C(u,v) = \phi^{[-1]}(\phi(u)+\phi(v))$, for some convex decreasing function on $(0,1]$, where $\phi(\bullet)$ is known as generator of the copula and $\phi^{[-1]}(\bullet)$ is the pseudo inverse of $\phi(\bullet)$. If $\phi(0)=\infty$; the generator is termed as strict, and inverse exists. In this case, from the above expression, the copula is recovered by, $C(u,v) = \phi^{-1}(\phi(u)+\phi(v))$. Non-strict generators are those for which $\phi(0)<\infty$. In this case, the generators are said to have a singular component and analysis should begin by defining a pseudo-inverse, $\phi^{[-1]}$. For example, if $\phi(t)=1-t$, then $\phi^{[-1]}(t) = max(1\text{-}t, 0)$ and $\phi^{[-1]}(\phi(u)+\phi(v)) = max\,(u + v - 1, 0)$.

The two standard nonparametric correlation measures, viz., population versions of Kendall's τ and Spearman's ρ could be expressed in terms of copula function (Schweizer and Wolf, 1981). The population version of Kendall's τ is defined as the probability of concordance minus the probability of discordance and is given as,

$$\tau = \tau_{X,Y} = P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) > 0\right] - P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) < 0\right] \tag{2}$$

If there are c number of concordant pairs and d number of discordant pairs, and out of n paired sample there are ${}^{n}C_2$ different ways of selecting two pairs, the Kendall's τ can be expressed as,

$$\tau = \frac{c-d}{c+d} = \frac{c-d}{{}^{n}C_2} \tag{2a}$$

Spearman's ρ is computed on the ranks of the original data and expressed as,

$$\rho = 1 - \frac{6d_i^2}{n\left(n^2-1\right)} \tag{3}$$

where $d_i = x_i - y_i$ = the difference between the ranks of corresponding values of X_i and Y_i; and n = the number of values in each dataset. The functional forms of different Archimedean copulas, with their generator functions and closed form relationship, Kendall's τ with copula parameter θ are given in the Table 1.

Table 1. Functional forms of various Archimedean copulas along with their $\phi(\bullet)$ functions (source: Nelson, 1999)

Copula	$\boldsymbol{C_\theta(u,v)}$	$\boldsymbol{\varphi_\theta(t)}$	$\boldsymbol{\theta \in}$	$\tau(\ ) \quad \int \frac{(\ )}{'(\ )}$
Clayton	$\left[\max\left(u^{-\theta} + v^{-\theta} - 1, 0\right)\right]^{-1/\theta}$	$\frac{1}{\theta}\left(t^{-\theta} - 1\right)$	$[-1, \infty) \setminus \{0\}$	$\theta/(\theta+2)$
Gumbel-Hougaard	$\exp\left(-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right)$	$(-\ln t)^\theta$	$[1, \infty)$	$(\theta-1)/\theta$

A widely used copula in hydrology is the Plackett's copula which is in a class of its own. The functional form of Plackett copula is given in Eq. (4). There is no closed form relationship exists for Plackett's family of copula and Kendall's τ. The relationship between Spearman's ρ and the copula parameter for Plackett's family of copula is given by Eq. (5), (Nelson, 1999).

$$C_\theta(u,v) = \begin{cases} \dfrac{1+(\theta-1)(u+v) - \sqrt{\left\{1+(\theta-1)(u+v)\right\}^2 - 4uv\theta(\theta-1)}}{2(\theta-1)}, & \theta > 0;\ \theta \neq 1 \\ uv, & \theta = 1 \end{cases} \tag{4}$$

$$\rho(\theta) = (\theta+1)/(\theta-1) - 2\theta\log\theta/(\theta-1)^2 \tag{5}$$

The selection of appropriate copula is done by computing a nonparametric empirical copula and comparing the values to the estimates of parametric copulas. For bivariate case, the empirical copula may be computed as (Deheuvels, 1979).

$$C_n(u,v) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{I}\left(\frac{R_i}{n+1} \le u, \frac{S_i}{n+1} \le v\right) \tag{6}$$

where, n = sample size; $\mathbf{I}(A)$ denotes the indicator variable of the logical expression A and taking the value 0 if A is false and 1 if A is true; R_i and S_i stands for the ranks of the observed drought variables. The denominator $(n+1)$ is used instead of n to avoid numerical problems at the boundaries of $[0,1]^2$. Following goodness of fit tests are performed (Ane and Kharoubi, 2003):

- Anderson – Darling $$AD = \max_{1\le i\le n, 1\le j\le n} \frac{\left|\hat{C}_n\left(\frac{i}{n},\frac{j}{n}\right) - C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\right|}{\sqrt{C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\left[1 - C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\right]}} \qquad (7)$$
- Integrated Anderson-Darling = $$IAD = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\left[\hat{C}_n\left(\frac{i}{n},\frac{j}{n}\right) - C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\right]^2}{C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\left[1 - C_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\right]} \qquad (8)$$
- Cramer-von Mises Distance estimator $$S_n = d_{C_n}(\theta) = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{C}_n\left(\frac{i}{n},\frac{j}{n}\right) - \tilde{C}_{p\theta}\left(\frac{i}{n},\frac{j}{n}\right)\right)^2 \qquad (9)$$

The Cramer-von Mises estimator $\hat{\theta}$ is defined to be θ that minimizes the above formula.

After selecting the appropriate copula model, the joint CDF of drought variable obtained from copula method is compared with their empirical nonexceedance probabilities using Gringorten's plotting position formula. The χ^2 - test is employed to study the theoretical joint distribution obtained using copula functions.

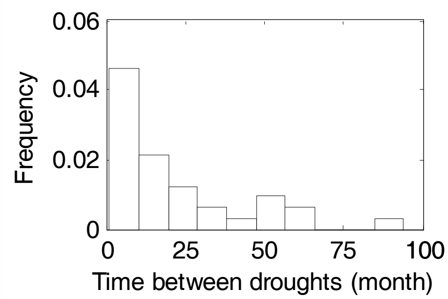
CASE STUDY DEATAILS

Western Rajasthan occupies 196,150 Km^2 or 57.31% of India's total arid zone area. The climate is characterized by low, highly variable and uneven distributed rainfall (annual rainfall varies from 10-40 cm), high wind speed, high evaporation losses, and extremes of seasonal temperatures. The percentage coefficient of variability of rainfall is as high as 60-70%, and probability of drought is more than 46%. The monthly area-weighted precipitation data of 26 rainfall stations in Western Rajasthan meteorological subdivision is obtained for time period of January 1930 to December 2008 from Indian Institute of Tropical Meteorology, Pune (http://www.tropmet.res.in). The monthly SPI-12 series was calculated and 35 drought events were identified. Fig. 2(a) shows location of Western Rajasthan and Fig. 2(b) shows the distribution of the time between the end of one drought and start of another in months. Correlations between the drought characteristics and non-drought time, before/after the drought are small and are not statistically significant at 5% level.

Fitting Marginal Distributions

Drought severity, duration, and peak are fitted with non-parametric univariate kernel density estimator with probability density function, $\hat{f}_{Ker}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$ (10)

where, the function $K(x)$ is called a kernel and h is the bandwidth that controls the variance of the kernel function.



(a) **(b)**

Fig. 2. (a) Scarcity affected district in Rajasthan (Source: http://gis.rajasthan.gov.in); (b) Histogram of months between the end of one drought and start of the next

Drought duration is fitted with Epanechnikov kernel while severity and peak are fitted with Normal kernel, the expressions for which are given as,

Epanechnikov kernel $K(x) = \frac{3}{4}(1 - x^2) \qquad -1 < x < 1$ (11a)

Normal kernel $K(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-x^2}{2}\right\} \qquad -\infty < x < \infty$ (11b)

The bandwidth is chosen as, $h_{opt} = \left(\frac{4}{3n}\right)^{1/5} \sigma$, where σ is the standard deviation and n is the length of the data. Table 1 presents pair-wise association between the drought variable. The Kolmogorov-Smirnov (K-S) goodness-of-fit test is used to detect whether the proposed models can be used to represent the observed data as shown in Table 2.

Table 1. Dependence Characteristics of flood variables

Dependence Measure	Severity-Duration	Duration-Peak	Severity-Peak
Pearson's product moment correlation	0.934 0.	8166	0.8944
Spearman's rank correlation	0.964 0	.7626	0.852
Kendall's τ 0.9	05	0.612	0.714

Table 2. Details of the K-S test for the proposed model

Drought variable and Kernel type	Maximum difference between theoretical model and observed data	Critical values of K-S test at 5% significance level
Duration-Epanechnikov	0.11346	0.23
Severity - Normal	0.1672	0.23
Peak - Normal	0.0962	0.23

Fitting Bivariate Copulas for Drought Variables

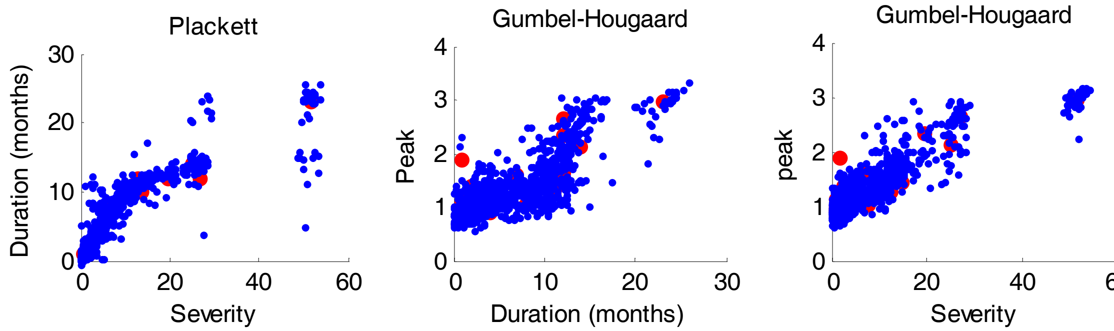
The Clayton, Gumbel-Hougarrd, and Plackett copulas are tested to select the best fitted copula. For each of the first two Archimedean families, Kendall's τ is used for computation of dependent parameter. For the Plackett family, Spearman's ρ has been used. The dependence parameter and corresponding goodness-of-fit test for each copula is presented in Table 3. From Table 3, it can be found that Plackett copula is suitable for modeling Severity-Duration combination and Gumbel-Hougaard is suitable for Duration-peak and Severity-peak combination respectively. To verify the copula based joint distribution, Chi-square goodness-of-fit test at 5% significance level was performed and the results are presented in Table 4. As a further test, a Monte Carlo simulation (Fig. 3) is performed generating margins of 1000 random pairs of (u_i, v_i) chosen from the two copula and transformed back into their original units using the marginal distribution $F_X(x)$ and $F_Y(y)$, and compared with the observed values (i.e., (x_i, y_i) in this case).

Table 3. Dependence parameter of copula and corresponding goodness-of-fit test statistic

	Copula	theta	AD	IAD	CVM
Severity-Duration	Clayton 19.25	2	0.629	0.951	8.518
	Gumbel-Hoagaard	10.626 0	.625	0.996	6.347
	Plackett 258	.90	**0.619**	**0.924**	**3.832**
Severity-Peak	Clayton 5.0		0.174	0.099	6.184
	Gumbel-Hoagaard	3.5	**0.118**	0.081	**2.368**
	Plackett 37.99	5	0.124	**0.053**	6.979
Duration-Peak	Clayton 3	.159	0.447	0.442	5.283
	Gumbel-Hoagaard	2.579	**0.397**	0.406	**3.183**
	Plackett 18.60	1	0.405	**0.349**	6.194

Table 4. Comparison of copula-based joint probability distributions with empirical joint distributions using χ^2 test statistics

χ^2-test statistic	**Severity-Duration**	**Duration-Peak**	**Severity-Peak**
Copula model	Plackett	Gumbel-Hougaard	Gumbel-Hougaard
χ^2-value	**0.3786**	**0.3881**	**1.1718**
cutoff obtained from χ^2-probability table at 5% significance level	11.0705 11	.0705	11.0705



(a) (b) (c)

Fig. 3. Observed versus simulated data for the fitted copula. (Observed data shown in red).

The conditional distribution [Fig. 4(a)] of drought severity given that drought duration exceeding a certain threshold d' is expressed as, (Shiau, 2006)

$$P(S \le s \mid D \ge d') = \frac{P(D \ge d', S \le s)}{P(D \ge d')} = \frac{F_S(s) - F_{D,S}(d', s)}{1 - F_D(d')} = \frac{F_S(s) - C(F_D(d'), F_S(s))}{1 - F_D(d')} \quad (13a)$$

Similarly, conditional distribution [Fig. 4(b)] of drought duration given that drought severity exceeding a certain threshold s' is expressed as,

$$P(D \le d \mid S \ge s') = \frac{P(D \ge d, S \le s')}{P(S \ge s')} = \frac{F_D(d) - F_{D,S}(d, s')}{1 - F_S(s')} = \frac{F_D(d) - C(F_D(d), F_S(s'))}{1 - F_S(s')} \quad (13b)$$

Both Fig. 4(a) and (b) show that the conditional drought severity distribution and the conditional drought duration distribution decreases with drought duration and severity respectively; which are useful information in evaluating the water-supply capability and needed auxiliary water resources during severe droughts for a specific water-supply system (Shiau, 2006).

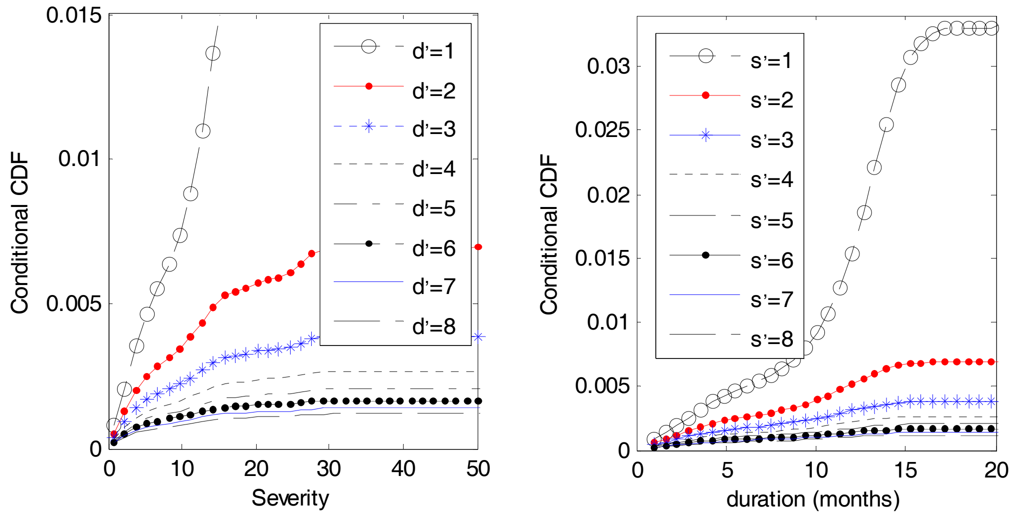


Fig. 4(a) Conditional distribution of drought severity given drought duration exceeding certain threshold; 4(b) Conditional distribution of drought duration given drought severity exceeding certain threshold

Joint Return Periods of Drought Severity and Duration

The return period of a variable is the average elapsed time between occurrences of an event with a certain magnitude or greater and is a standard criterion in water resources system planning and management. When considering drought duration and severity simultaneously, the bivariate return periods (T_{DS} and T_{DS}') can be defined by (i) drou ght du ration ex ceeding a specific value or drought severity exceeding another specific value $(D \geq d \text{ or } S \geq s)$ denoted by T_{DS} ; (ii) drought duration exceeding a specific value and drought severity exceeding anot her specific value $(D \geq d \text{ and } S \geq s)$ denoted by T_{DS}'; Both return periods in te rms of co pula-based bivariate distribution are given by,

$$T_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \tag{14a}$$

$$T_{DS}' = \frac{E(L)}{P(D \geq d \text{ and } S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \tag{14b}$$

where L is the inter-arrival time of droughts as shown in Fig. 1., $E(L)$ is the expected inter-arrival time of droughts estimated from observed data, and $F_D(d)$ and $F_S(s)$ are the cumulative distribution functions of drought duration and severity respectively. The average drought inter-arrival time estimated from the observed data is 26.742 months or 2.23 years. For the 2002-03 drought (July 2002-June 2003) with a duration 12 month and severity 26.96, the return period defined by duration and severity separately are 81.48 and 78 years respectively. For the same drought event, the bivariate return period defined by Eq. 14(a) and 14(b) are 61.07 and 114.67

years respectively. The former is less than both the return period defined by drought duration and severity separately while the later is greater than both the return period. The most severe drought observed during the study period is 1968-70 (September 1968-July 1970), which lasted for 23 months with severity of 51.59 has return period of 16.84 and 40.35 years using Eq. 14 (a) and 14 (b). The contour lines for specific joint return periods, in which either severity or duration exceeded, T_{DS} has no bounds, whereas, the joint return period, in which both severity and duration are exceeded T'_{DS} are described by horizontal and vertical axes.

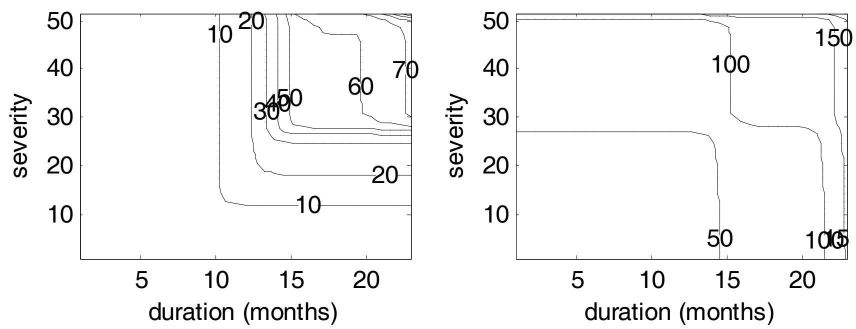


Fig. 5. (a) Joint return period, either severity *or* duration are exceeded, T_{DS} (years); (b) Joint return period, both severity *and* duration are exceeded, T_{DS}' (years)

CONCLUSIONS

Bivariate copula based methodology is developed for modeling drought events and corresponding return periods have been estimated using SPI series computed based on mean areal precipitation observed in 26 rainfall stations in Western Rajasthan meteorological subdivision of India. By using standard statistical tests the best suited copula is selected and it is found that, the Plackett copula performed better for severity –duration; and Gumbel-Hougaard for severity-peak and duration-peak respectively. The bivariate probabilistic properties of droughts, viz., joint and conditional probabilities, as well as joint return periods, have been investigated for comprehensive drought assessment. This study concludes that the copula method is a flexible tool for modeling dependence between hydrometerological variables and provides useful information about droughts; thereby it can help in better water resources system planning and management.

REFERENCES

Ane, T., and Kharoubi, C. 2003. Dependence structure and risk measure. *J. Business*. 76(3), 411-438.

Cancelliere, A., Salas, J.D. 2004. Drought length properties for periodic-stochastic hydrological data. *Water Resour. Research,* 40, W02503.

Deheuvels, P. 1979. La function de dependence empirique et ses proprietes-Un test non parametrique d'independence. *Bull, Cl. Sci. Acad. R.Belg*., 5 (65), 274-292.

ESCAP. 1995a. *State of the Environment in the Asia-Pacific, 1995*. Bangkok.

Fernandez, B., Salas, J.D., 1999. Return period and risk of hydrologic events I: mathematical formulation. *J. Hydrol. Engg.* 4(4), 297-307.

Genest, C., Mackay, J. 1986. The joy of copulas: bivariate distributions with uniform marginals. *Am. Stat.*, 40, 280-283.

Guttman, N.B. 1999. Accepting thestandardized precipitation index: A calculation algorithm. *J. American Water Resour. Association*, 35(2), 311-322.

Hayes, M.J., Svoboda, M.D., Wilhite, D.A. and Vanyarkho, O.V. 1999. Monitoring the 1996 drought using thestandardized precipitation index. *Bull. American Meteorol. Soc*. 80, 429-438.

McKee, T.B., Doesken, N.J. and Kliest, J. 1993. The relationship of drought frequency and duration to time scales. *Proc. Eighth Conference on Applied Climatology*, Anaheim, California, 179-184.

Nelson, R.B. 1999. *An introduction to copulas*. Springer, New York.

Schweizer, B., Wolff, E.F. 1981. On nonparametric measures of dependence for random variables, *Ann. Stat.*, 9, 879-885.

Serinaldi, F., Bonaccorso, B. Cancelliere, A. and Grimaldi, S. 2009. Probabilistic characterization of drought properties through copulas, *Physics and Chemistry of the Earth*, 34, 596-605.

Shiau, J.T. 2006. Fitting drought duration and severity with two-dimensional copulas. *Water Resour. Manage.*, 20, 795-815.

Shiau, J.T., Feng, S. andNadarjah, S., 2007. Assessment of hydrological droughts for the Yellow River, China, using copulas. *Hydrol. Processes*, 21(16), 2157-2163.

Sklar, A. (1959). Functions de repartition a n dimensions et leurs marges. *Publ Inst. Stat. Univ. Paris*, 8, 229-231.

Tallaksen, L.M., Madsen, H. and Clausen, B. 1997. On the definition and modeling of stream drought duration and deficit volume. *Hydrol Sciences J*, 42(1), 15-33.