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CRITICAL APPRAISAL OF VARIOUS TECHNIQUES USED FOR VELOCITY DISTRIBUTION IN OPEN CHANNEL FLOW

Mrutyunjaya Sahu.¹, Ramakar Jha², Raysing Meena³

Abstract: *Distribution of velocity in the longitudinal as well as lateral velocity component is one of the basic aspects in open channel flow. It directly relates to various flow properties like shear stress distribution, secondary flows, surface profile estimation, channel capacity measurement and host of other flow entities. The knowledge of velocity distribution helps to know the velocity magnitude at each point across the flow cross-section. It has been found that velocity distribution in various types of channels varies with the shape, type and patterns of channels. In straight channel velocity distribution varies with different width-depth ratio, whereas in meandering channel velocity distribution varies with aspect ratio, sinuosity, meandering. Compound channel are all the way different and velocity distribution is a combination of flood plain and main channel (straight or meandering). It is found essential to study various methods used for accurate estimation of velocity distribution in various natural and artificial open channels. In the present work, critical appraisal of different approaches used for velocity distributions in channels are discussed. It has been found from the review that most commonly methods used by different researches globally are Prandtl, Theodore Von Kármán, Nikurádse, Coles because it is simple, robust and easy to use for computing velocity distribution in open channels.*

Keywords: *Velocity Distribution; Velocity; Open channel flow; Critical Analysis.*

INTRODUCTION

The fluvial process in natural and artificial channels has long been concerned for river engineers. But the central problem of open channel flow like design of channels and canals, the transportation of sediment due to various flow processes, flood mitigation efforts with a particular cross-section required the flow variables like flow depth, average velocity, flow. To solve the typical problems of open channel flow knowledge of velocity distribution in a flow cross-section is required to find out related variables like boundary shear stress, flow resistance. The variability increases in natural system from straight to meanders due to presence of unseen phenomena like flow variability, secondary current, which make the analysis of the current problem more complex. From earlier ages to present conditions various researchers provided investigations to ease the problem. In straight the bed roughness, channel section, wall roughness, width depth ratio effects the distribution of velocity. The knowledge of variables are required to solve the problem of the open channel flows.

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STATE-OF-ART

In the 17th century, several eminent researchers like Descartes, Pascal, Newton and Boyle laid the foundations of modern mathematics and physics. They perceived very logical pattern in various aspects of mechanics. During 18th century, further development progressed on both experiments and in the analysis. For example Poleni (1717) investigated the concept of discharge coefficient, which was in French language. Thereafter, German thinker Henri de pitot (1732) invented Pitot tube to measure flow velocity. Antoine chez, followed by Eytelwein and Woltmannm(1790), developed a rational equation to describe flow in streams. Woltmann venture used Bernoulli's work as a basis for developing the principles of flow measurement. Hagen constructed experiments to investigate the effects of temperature on pipe flow. His understanding of the nature of viscosity was limited to Newton's idea, yet so careful to his work that the result were within 1% of modern measurements. He probably also the first person to observe turbulence in the fluid flow.

The 19th century was the period of further advancement. French doctor Poiseuille (1891) was also making observation in pipe flows, which led to development of laminar flow phenomenon. At this stage the study of fluid flows are subdivided into classical hydrodynamics and experimental hydraulics. Navier, Stokes, Schwarz, Christoffel and other hydrodynamicists all contributed to the development of a formidable array of mathematical equations and methods. The rapid growth of industry in the 19th and 20th centuries was done by producing a demand for a better understanding of fluid flow phenomenon. The real break through came with the work of Prandtl (1901) that flow was divided into two independent parts. On one hand the free fluid was considered, which can be treated as inviscid (which obey the law of hydrodynamics) and on the other hand the transition layer on the fixed boundary was used.

Further, during 20th century, significant development in this field of study took place. Prandtl and Karman published a series of papers in 1920s and 1930s, covering various aspects of boundary layer theory and turbulence. In 1930s, the efforts of Nikuradse (1933), Moody (1944), Colebrook (1939) and others resulted in clear understanding of pipe flow in particular. This led directly to modern methods for estimating flows in pipes and channels. Theoretical investigations of Prandtl and Karman on flow through pipes and the experimental studies of Nikuradse (1933) have led to rationalize formulas for velocity distribution for turbulent flows over flat plates and circular pipes. The formula paved the way for further development of formulas to open channel flows. Although similarities exist between the flow through pipes and flow through open channel but certain basic factors like dimension, cross-section of channel, and non uniform distribution of shear along wetted perimeter, distinguishes the open channel from pipe flow. The spatial distribution of the longitudinal velocity component in a cross section is one of the basic properties of an open channel flow. It is directly related or interacts with other flow properties such as the shear stress distribution, secondary flow. The velocity distribution in pipe flow is initiated by the well-known universal law of velocity distribution in the turbulent boundary layer was deduced by Prandtl (1925) using the mixing-length hypothesis and by von Karman (1930). Prandtl (1932) developed the general form of velocity distribution, which is generally considered as **P-vK law**, this law was derived by assuming the "shear stress is constant", and can be applied near bed, but has been applied in outer flow region with modification of von karman constant like Milikan(1939) ,

Vanoni (1941).

Vanoni (1946) and Einstein and Chen (1955) modified von Karman constant in universal P-vK law for sediment laden flow and noticed that von Karman constant becomes smaller with increase of sediment concentration and greater for clear water (>0.4).

The P-vK law was derived by taking shear stress as constant whereas the shear stress is not constant in turbulent layer (outer zone) in open channel flow. Milikan (1939) suggested that actual velocity distribution consists of logarithmic part and correction part, where the correction part considers the outer layer into account.

Coles (1956) proposed a semi-empirical equation of velocity distribution, which can be applied to outer region and wall region of plate and open channel. He generalized the logarithmic formula of the wall with tried wake function, $w(y/\delta)$. This formula is asymptotic to the logarithmic equation of the wall as the distance y approaches the wall. This is basic formulation towards outer layer region.

Zagustin & Zagustin (1969) proposed an analytical solution for turbulent flow in smooth pipes based on a new concept of balance of pulsation energy.

Daily *et.al.* (1966) assumed the P-vK law for smooth boundaries to get the surface velocity. They used experimental data to arrive at two-form of velocity defect law, one of which applied to $y/Y_w < 0.15$ and other $y/Y_w > 0.15$, where y/Y_w is scaled length. But later Monin and Yaglom (1971) discovered from experimental results that velocity distribution have some important aspects in both rough and smooth walls. The experimentation in whole turbulent boundary layer shows that results deviate from logarithmic equation in outer region.

Coleman (1981) proposed that the velocity equation for sediment-laden flow consists of two parts, as originally discussed by Coles for clear-water flow. Also he has revealed that the von Karman coefficient is independent of sediment concentration and the wake strength II is a function of the global Richardson number, which is the ratio between potential energy and kinetic energy.

Naot (1982) demonstrated the use of an algebraic stressed model for the calculation of secondary current. From early sand experiment by Nikuradse (1933), it is evident that rough wall are associated with high turbulent shear stress and hence P-vK law is completely different from that of suitable smooth wall. Wang and Nickerson (1972) showed that transition from rough wall characteristics to smooth wall is completely with in a distance smaller than ten times the roughness height step. Since the turbulent normal stress and turbulent energy near the wall also follow the transition, strong gradients in the normal stresses is formed which is responsible for secondary current.

Coleman and Alonso (1983) developed an equation, which predicts the velocity profile in the viscous sub-layer, the buffer zone, the logarithmic or inertial zone, and the outer or wake region in conduit. The channel flows provided that secondary or cross flow is weak or absent. Sarma *et al.*

(1983) studied velocity distribution in a smooth rectangular channel by dividing the channel into four regions. Region 1 comprises of the inner region of the bed and the outer region of the side wall. Region 2 belongs to the inner regions of both the bed and the side wall. Region 3 consists of the inner region of the side wall and the outer region of the bed. Region 4 forms the outer regions of both the bed and the side wall. The aspect ratio is varied from 2.0 to 8.0 and the Froude number from 0.2 to 0.7. Experiments included the aspect ratio of 1.0 also for Froude numbers of 0.2 and 0.3. Further, Samaga et al.(1986) developed a model in alluvial channel using two layer model, where in region $d_a < y/\delta < 0.2$, where d_a = arithmetic mean size sediment water mixture, velocity distribution is assumed to be parabolic. But for region $y/\delta > 0.2$ it was logarithmic. In their approach velocity and eddy viscosity is constant at $y/\delta = 0.2$ but shear stress is discontinuous at $y/\delta = 0.2$.

Nezu & Nakagawa (1984) investigated experimentally the turbulent structure and their currents in air conduit by considering an essential interaction between secondary currents and bed form. They measured accurately by hot-wire anemometers all three components of the velocity. The structure of secondary currents was examined through the equations of mean flow vorticity and mean flow energy.

Naot (1985) designed eight cases to study the hydrodynamic response of open channel flow to wall roughness lateral homogeneity. The response of the secondary current to the wall roughness heterogeneity. Joe C. Willis (1985) based on the equations of motion for uniform flow and a parabolic distribution of eddy viscosity over the turbulent portion of the boundary layer. Effects of increased roughness are accounted for by a shift of the distribution of eddy viscosity toward the flow boundary. The resulting velocity distribution agrees with published data and has an advantage over the classical logarithmic distribution for flow over rough surfaces in that the lower velocity limit is zero at distance measurements near the boundary.

Chen (1991) represented turbulent velocity profile by Power law relations. Recommended using $m_{PL} = 1/7$ for hydraulically smooth flows and $m_{PL} = 1/6$ for hydraulically rough flows. Where m_{PL} is power law index.

Tominaga & Nezu (1992) measured velocity with a fiber-optic laser-Doppler anemometer in steep open-channel flows over smooth and incompletely rough beds. As velocity profile in steep open channel is necessary for solving the problems of soil erosion and sediment transport, and he observed the integral constant A in the log law coincided with the usual value of 5.29 in subcritical flows. Swamee (1993) presented a generalized equation for velocity distribution in the inner law region of a turbulent boundary layer. The equation includes linear and logarithmic velocity distributions and it is valid for hydraulically smooth and rough boundaries and the transition range in between.

Kirkgoz *et al.* (1998) conducted experiments in 12 different test conditions with Reynolds number ranging from 28,026 to 136,842 in a rectangular laboratory channel. From the experiment they observed that the fully developed turbulent boundary layer along the centerline or axis of the channel, which develops up to the free surface for a flow aspect ratio $b/h >= 3$, where b/h is width ratio, In turbulent inner regions of developing and fully developed boundary flows. The measured

velocity profiles agree well with the logarithmic ‘law of the wall’ distribution when the coefficients in expression are 2.44 and 5.5, respectively. The “wake” effect becomes important in the velocity profile of the fully developed boundary layer. A reasonable agreement between the modified velocity defect law and the experimental profile in the inner and outer region is obtained with profile parameter of 0.1 in the cole’s **law of the wake**.

Sarma *et al.* (2000) tried to formulate the velocity distribution law in open channel flows by taking generalized version of binary version of velocity distribution, which combines the logarithmic law of the inner region and parabolic law of the outer region. The law developed by taking velocity-dip in to account.

Wilkerson *et al.* (2005), using data from three previous studies, developed two models for predicting depth-averaged velocity distributions in straight trapezoidal channels that are not wide, where the banks exert form drag on the fluid and thereby control the depth-averaged velocity distribution. The data they used for developing the model are free from the effect of secondary current. Yang *et al.* (2005) derived dip modified log law taking in to the negative Reynolds shear stress near the free surface. The new law consists of a combination of two logarithmic distances; one from bed and other from free surface and a dip correction factor α . This law is able to produce velocity nears a corner. Also it incorporates dip- phenomenon both in central and corner portion.

Cheng (2007) derived power law as first order approximation of power law, and its index is computed as a function of Reynolds number as well as relative roughness height. As log law is generally applied in near bed region, also it is assumed that shear velocity is global velocity scale and can be applicable to both inner and outer region, when Reynolds number increases there is a region of overlap between power law and log law and both the condition of both region holds. The range of overlap is quite narrow about 20% of the flow depth. Power law not only applicable to the overlap region but also be applied in outer region as explained by Hinze (1975) and Bergstrom DJ *et al.* (2001). Knight *et al.* (2007) used Shiono and Knight method (SKM) , which is a new approach to calculating the lateral distributions of depth-averaged velocity and boundary shear stress for flows in straight prismatic channels, also accounted secondary flow effect. This method used to analyzed in straight trapezoidal open channel. The number of secondary current varies with aspect ratio. It is three for aspect ratio less than equal to 2.2 and four for aspect ratio greater than equal 4. Afzal *et al.* (2007) analyzed power law velocity profile in fully developed turbulent pipe and channel flows in terms of the envelope of the friction factor. This model gives good approximation for low Reynolds number in designed process of actual system compared to log law.

Modified wake law was developed by Guo *et al.* (2008) incorporates three components i.e. (i) the law of the wall due to constant bed stress.(ii) the law of the wake that reflects the effect of gravity (iii) the cubic correction near the maximum velocity.

CATAGORIZATION OF DIFFERENT METHODOLOGIES:

Velocity distribution in open channel flows with reference to previous studies indicates some

remarkable influencing factor such as roughness of the bed, wall of the channel. Channel geometry, and secondary current. Here According to their influence on velocity distribution and methodologies adopted by various researchers these factors can be categorized under described sub headings.

P-vK Law

This law was derived by assuming the “shear stress is constant”, applied near bed, but has been applied in outer flow region with modification of von karman constant. Prandtl (1932) developed the general form of velocity distribution, which is generally considered as P-vK law, here shear stress was considered to be constant through-out as well as shear velocity globally. This law is applied near bed region where viscous flow is predominant, skeptical about outer layer where turbulence of flow and Reynolds’s stress associated with it makes the shear stress variable, and which violet the fundamental assumption of the **P-vK Law**. But near bed in small roughness or smooth surface this can be applicable. Also as described above this method can be applied for full depth in center line of wide channel where corner shear flow and secondary circulation effect is very less as described by Nezu *et al.* (1984).

Power law

This law has advantage over P-vK law although derived on the basis of empirical relations. This law overlap with log law in the range of about 20% of flow depth can used to full depth of flow as noticed by Hinze (1975) and Bergstrom *et al.* (2001) . Recently Afzal *et al.* (2007) developed a model using this method where power law index a and pre-factor C are shown as the function of the friction Reynolds number, as well as the function of the alternate variable the non-dimensional friction velocity and Cheng N.S. (2007) derived power law as first order approximation of power law, and its index is computed as a function of Reynolds number as well as relative roughness height. This law yet to incorporate the variations due to free surface and upper edge boundary roughness. Also these methods are silent about the effects of secondary current.

Multi shear flow zones

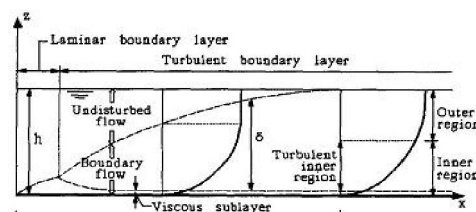


Fig.1. Showing multi-shear flow zones with flow depth

Flows in natural streams like flows in open channel in rectangular or trapezoidal channel and conduits, belong to general class of bounded shear flows. The boundary layer of the bounded shear flows consists of two regions; a near-wall region and an outer region near free surface, where inner region is controlled by inner variables, namely kinematic viscosity, friction velocity. The inner layer is further divided by the viscous sub layer, in which viscous stresses are dominant; and the log-law region, in which velocity distribution obeys logarithmic laws. Again the viscous sub layer consists of two parts; the linear sub layer, in which velocity distribution is linear; and the buffer layer, which provides a smooth transition between the linear and logarithmic velocity distribution. The outer region or wake region, which is controlled by the scale and intensity or

intermittency of its ambient turbulence and nature of its pressure gradient that determine the relative thickness of wake region. Here some methods adopted by researchers according to full depth of flows are described.

Coles (1956) founded and added wake effect in outer layer for free surface flow. Joe C. Willis (1985) through parabolic eddy viscosity distribution tried to incorporate near bed velocity distribution due to roughness. Sarma *et al.* (1983) have done a four region division approach to comprehend all flow zones. Cardoso *et al.* (1989) noticed that wake strength is not universal but is dependent in , the secondary currents, flow history , and inactive turbulence components , hence outer region variations are significant. Swamee (1993) generalized equation for velocity distribution in the inner law region of a turbulent boundary layer. The equation includes linear and logarithmic velocity distributions and it is valid for hydraulically smooth and rough boundaries and the transition range in between. M. salih kirkgoz *et al.* (1998) used modified velocity defect law in outer region and obtained with profile parameter of 0.1 in the cole’s “law of the wake”. Sarma *et al.* (2000) tried to formulate the velocity distribution law in open channel flows by taking generalized binary version of velocity distribution, which combines the logarithmic law of the inner region and parabolic law of the outer region. Shu-qing yang (2005) derived dip modified log law taking in to the negative Reynolds shear stress near the free surface. The new law consists of a combination of two logarithmic distances; one from bed and other from free surface and a dip correction factor α . Also it incorporates dip- phenomenon both in central and corner portion. Modified wake law was developed by Guo *et al.* (2008) incorporates three components i.e. (i) the law of the wall due to constant bed stress.(ii) the law of the wake that reflects the effect of gravity (iii) the cubic correction near the maximum velocity.

ANALYSIS

As described above different methodologies adopted by various researchers are categorized according to three groups. Those are Power law, P-vK Law and models developed by taking multishear flow zone in to consideration. Here a comparison between recently developed models like Shu-qing-yang (2005), N.S.Cheng (2005) using power law model, Guo and Jullien (2008) using MLWL model where impact of shear flow in different zones is taken in to consideration is presented with published data of Coleman(1986) and Sarma *et al.*(2000).

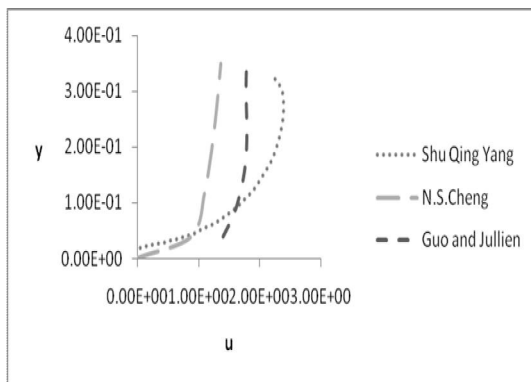


Fig.1. Comparison of models with Sarma(2000) data

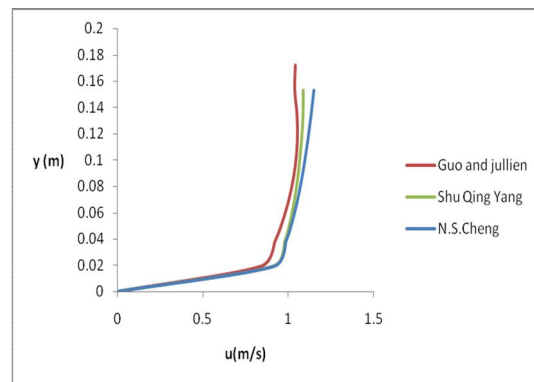


Fig.2. Comparison of models with Coleman(1986) data

The above comparison shows that power law model developed by N.S.Cheng (2007) cannot able

to incorporate any variations in outer layer. But model developed by Shu Qing Yang (2005) and guo and Jullien (2008) are taking outer layer variations due to secondary circulation or wake.

CRITICAL APPRAISAL

In this section, advantages, shortcomings and limitations of different methods used for estimating velocity distribution in straight open channels are discussed. The governing equations used are also shown in Table 1.

Table 1. Critical Appraisal of different Techniques used for Velocity Distribution

AUTHOR	GOVERNING EQUATION	CRITICAL ANALYSIS
Prandtl (1925- 1932)	$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A$ $\frac{u}{u_*} = \frac{1}{k} \ln \frac{y}{k_s} + A'$	Fundamental equation established using mixing length hypothesis suitable for near bed region and laminar flow zone where shear stress is taken as constant for the full depth, neglected roughness and dip-phenomena in to account. Later Milikan, Vanoni, Coles, Daily et al., Kirkgoz, Jullien and Guo modified to incorporate all variables effecting open channel flows.
Von Karrman (1930)	$\frac{u}{u_*} = \frac{u_{\max}}{u_*} + \frac{1}{k} \left[\ln \left\{ 1 - \sqrt{\frac{1-y}{\delta}} \right\} + \left\{ \sqrt{1 - \frac{y}{\delta}} \right\} \right]$	Mixing length models forms the basis for determining the velocity profile for shear flows. However modified approach was developed by the author with Prandtl.
Milikan (1939)	$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A$	Developed taking outer region into consideration. But method is not able to formulate the outer region. It is the essential to incorporate both outer and inner region of flow.
Vanoni (1941)	$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A$	Taken channel dimension and multiphase flow into consideration and developed the model by varying Karman constant to incorporate the model for full depth in both inner and outer region. But neglected variation in velocity profile due to wake strength, dip-phenomena, and aspect ratio.
Einstein and Chine (1955)	$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A$ $\frac{u}{u_*} = \frac{1}{k} \ln \frac{y}{k_s} + A'$	Examined the velocity distributions of sediment-laden flow. The method is found suitable for sediment laden flow in comparison to clear water flow, because they found the value of the von Karman coefficient became smaller with the increase of sediment concentration greater than 0.4 for clear water.

Coles (1956)	$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A + \frac{\Pi}{k\omega} \left(\frac{y}{\delta} \right)$ $\omega \left(\frac{y}{\delta} \right) = 2 \sin^2 \left(\frac{\pi y}{2\delta} \right)$	It is a semi-empirical equation, developed for the use of both outer and inner region. However, the authors failed to take roughness of near bed and velocity-dip due to secondary current in to account.
Daily <i>et al.</i> (1966)	$\frac{u(Y_w) - u(y)}{u} = -3.74 \cdot \ln \left(\frac{y}{Y_w} \right), y/Y_w$	Surface velocity was derived using velocity-defect model. Model could not accommodated roughness effect of bed and secondary circulation.
Zagustin & Zagusin (1969)	$\frac{u_m - u_z}{u} = \frac{2}{k} \tanh^{-1} \left(\frac{d_0 - 2z}{d_0} \right)^{3/2}$	Gave an analytical solution for turbulent flow in smooth pipes based on a new concept of balance of pulsation energy for outer region of flow.
Sarma <i>et al.</i> (1983)	Wall region: $\frac{u}{u_*} = C \left(\frac{zu}{\nu} \right)^n$ Inner region of the bed: $\frac{u}{u_*} = \left(\frac{zu_*}{\nu} \right)^n$ Outer region of bed: $\frac{u_m - u}{u_*} = K_b \left(1 - \alpha - \frac{z}{D} \right)^2$	A new approach of division of the channel depth. Outer region and inner region taken in to considering by taking power law function. It provides good approximation for all regions but neglected the channel variation as well as roughness of bed and wall. Although dip factor is considered but model cannot be applied for lateral velocity distribution in outer region of side wall away from bed.
Coleman and Alonso (1983)	$U^+ = \int_0^{y^+} \frac{2}{1 + \left\{ 1 + [2k^+ \Delta t^+] \left[1 - \exp \left(\frac{-t^+ - \Delta t^+}{26} \right) \right] \right\}^{1/2} dt^+}$ $\left(\frac{y^+}{\delta^+} \right)^2 + \left(1 - \frac{y^+}{\delta^+} \right)$ $+ \left(\frac{2\pi}{k} \right) \left(\frac{y^+}{\delta^+} \right)^2 \left[3 - 2 \left(\frac{y^+}{\delta^+} \right) \right]$	Although this model similar to the Coles (1956), but here this model incorporates inner and outer layer as a form of viscous sub layer, buffer zone and outer zone. The model has not considered secondary circulation, which is responsible for dip –phenomenon as well as for aspect ratio.
Nezu & Nakagawa (1984)	Energy equation: $v \frac{\partial K}{\partial y} + w \frac{\partial K}{\partial z} + U \cdot \left(\frac{\partial \bar{u}v}{\partial y} + \frac{\partial \bar{u}w}{\partial z} - \frac{U^2}{a} \right) = \nu \nabla^2 \zeta$ $- \nu \left[\left(\frac{\partial U}{\partial y} \right)^2 + \left(\frac{\partial U}{\partial z} \right)^2 \right]$ Vorticity equation: $\frac{\partial \zeta}{\partial t} + \frac{\partial(\zeta v)}{\partial y} + \frac{\partial(\zeta w)}{\partial z} = g \sin \theta +$ $\nu \left(\frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \zeta}{\partial z^2} \right) + \frac{\partial^2 (\tau_{yx} - \tau_{xy})}{\partial y \partial z} + \frac{\partial^2 \tau_{yz}}{\partial y^2} = \frac{\partial^2 \tau_{yz}}{\partial z^2}$	This is a initiative effort for the analysis and understanding of cellular secondary current. Although this paper highlighted the effect of cellular secondary current but fails to incorporate effects of free surface, bed roughness and aspect ratio.

<p>Joe C. Willis (1985)</p>	$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{W}{\rho} - \frac{1}{\rho} \nabla p + \nu(\nabla \cdot \nabla)V$ <p>Diffusivity distribution:</p> $\varepsilon_t = 6\bar{\varepsilon}_t \frac{y_t}{\delta_t} \left(1 - \frac{y_t}{\delta_t}\right)$ <p>For outer region:</p> $U_m^+ - U_t^+ = -\frac{1}{k} \ln Y_t + 2 \frac{\psi}{k} \cos^2 \frac{\pi Y_t}{2}$	<p>Suitable equation for near bed velocity distribution is developed, which took the effect of roughness by taking parabolic eddy viscosity factor. The model neglected dip-phenomena, variation of aspect ratio and multiphase flow.</p>
<p>Chen (1991)</p>	<p>General form of power law:</p> $u(y) = k_{PL} \cdot u \cdot \left(\frac{y}{y_0}\right)^{m_{PL}}$ <p>and average vertical velocity as</p> $U_w = u \cdot \left(\frac{k_{PL}}{m_{PL} + 1}\right) \cdot \left(\frac{Y_w}{y_0}\right)^{m_{PL}}$	<p>Developed power law model, which used $m_{PL} = 1/7$ for hydraulically smooth flows and $m_{PL} = 1/6$ for hydraulically rough flows. Model neglects aspect ratio, secondary current and channel variation in to account.</p>
<p>Swamee (1993)</p>	$u = u_* \left\{ \left(\frac{v}{u y}\right)^{10/3} + \left[k^{-1} \ln \left(1 + \frac{9u y}{v + 0.3\varepsilon}\right) \right] \right\}$	<p>This is a good model for inner layer region which took roughness effect in to account.</p>
<p>Kirkgoz <i>et.al.</i> (1998)</p>	$\frac{u_m - u}{u_*} = -2.44 \ln \frac{z}{\delta} + 0.488 \cos^2 \left(\frac{\pi z}{2\delta}\right)$ <p>With Cole's parameter $\chi = 0.4$ and $\pi = 0.55$</p> <p>In outer region.</p>	<p>Experimental velocity profiles agree well with the logarithmic 'law of the wall' distribution when the coefficients in expression are 2.44 and 5.5, respectively. Here log law in inner region and velocity defect law in outer region is used to described the fully developed velocity model. In this model variation of shear velocity with aspect ratio in fully developed flow is presented. This model is developed in fully developed flow region.</p>
<p>Sarma <i>et al.</i> (2000)</p>	$U_{y,d} = U_{m,y} - \frac{1.22 D_y^2}{Z_i (1 - D_y - Z_i)}$	<p>Incorporates velocity distribution both in outer and inner region described through binary law. Where inner layer responds to logarithmic and outer layer parabolic law. This model also shown the possibilities of the junction point of the two models adopted. Although it was validated in both subcritical smooth, super critical smooth and subcritical rough, super critical rough region but failed to take aspect ratio in to consideration.</p>

<p>Katul <i>et al.</i> (2002)</p>	$u(y) = 4.5 \cdot \left[1 + \tanh\left(\frac{y - y_r}{y_r}\right) \right]$ $U_w = 4.5 \cdot u \cdot \left[1 + \frac{y_r}{Y_w} \cdot \ln \frac{\cosh\left(1 - \frac{Y_w}{y_r}\right)}{\cosh(1)} \right]$	<p>This model is a inner layer region model where roughness height is equivalent to flow depth or just above it.</p>
<p>Shu-qing yang (2005)</p>	$\frac{u}{u_*} = \frac{1}{k} \ln\left(\frac{y}{y_0}\right) + \frac{\alpha}{k} \ln\left[\frac{1 - \frac{y}{h}}{1 - \frac{y_0}{h}}\right]$	<p>Dip phenomena countered both in corner and central portion. But lack roughness effect due to channel wall and bed neglected.</p>
<p>Wilkerson et al. (2005)</p>	$\frac{U(Z)}{U_0} = \left[\frac{(1 + 0.104Z) - (0.125Z) \exp\left(2.24Z^{-0.582} \cdot \frac{\pm z - z_{oc1}}{YZ}\right)}{\dots} \right]$	<p>Developed model by taking the effect of trapezoidal channel which is not wide. But the data taken are not influenced by secondary circulation. Hence effect of secondary current is absent; according to Nezu et al (1984) secondary current is predominant factor in narrow channel.</p>
<p>Knight et al. (2007)</p>	$\rho g H S_0 - \frac{1}{8} \rho U_d^2 \left(1 + \frac{1}{s^2}\right) + \frac{\partial}{\partial y} \left[\frac{\rho \lambda H^2 \left(\frac{f}{8}\right)^{1/2} \times \frac{\partial U_d}{\partial y}}{\dots} \right]$	<p>It accounts for bed shear, lateral shear, and secondary flow effects via 3 coefficients. This method incorporates the effects of secondary flows by specifying an appropriate value for the Γ parameter depending on the sense of direction of the secondary flows, but ignores channel roughness. No analysis on multi shear flow zones is provided.</p>
<p>Afzal <i>et.al</i> (2007)</p>	$\frac{U_c - U_b}{u_\tau} = -D_b \frac{k}{k + J}$ <p>where $D_b = \frac{1}{k} = \frac{a}{\gamma} \exp(\gamma)$ and</p> $\gamma = \frac{a}{a + b\alpha}$	<p>Model is developed keeping in mind that would provide better representation for lower Reynolds number. The power law velocity profile has been analyzed in terms of the envelope of the friction factor which gives the friction factor log law. However, the variability of model according to multi shear flow region, roughness, secondary current not taken in to account.</p>
<p>Cheng N.S.(2007)</p>	$\frac{u}{u_{ir}} = \left(\frac{y}{y_{ir}}\right)^{1/\ln(y_{ir}/y_0)}$	<p>Here the power law index varies logarithmically. This model validated by taking roughness factor in to account. But neglected effect free surface like wake and dip phenomena.</p>
<p>Jullien and Guo(2008)</p>	$\frac{u}{u_*} = \left(\frac{1}{k} \ln \frac{y u_*}{\nu} + B\right) + \frac{2\Pi}{k} \sin^2 \frac{\pi \xi}{2} - \frac{\xi^3}{3k}$	<p>It is a good fit model for multi shear flow zone. It counters dip phenomena as well as bed roughness. The model validated in Mississippi river.</p>

CONCLUSION

From above discussion it is inferred that prandtl and von karman contribution is widely accepted by various authors across the globe. Although this method have limitations but this basic step has really resolved many critical analytical issues in velocity profiles of open channel flow by modifying the model by various of researchers. Recently Sarma et al(2000), N.S Cheng(2007) Shu-qing Yang(2005), Jullien and Guo (2008) developed models to describe inner layer variation using log law or P-vK law. The variation of the above models for the full depth are verified with the published data of Sarma et al.(2000) and coleman(1986). But Modifie-Log Wake Law developed by Guo and jullen is good fit model , as it fits not only experimental data of coleman(1986),Lyn(1986), Kironto and Graf(1994) and sarma et al.(2000) but also natural conditioned Mississippi river data. on the other hand the power law model of N.S.Cheng although covered almost full depth but variation of outer layer due to secondary current, wake is totally ignored. Model of Shu-qing yang (2005) also a good model which took the account of both inner and outer region variation but MLWL developed by Guo and Jullen (2008) is good fit as well as counter the variation of channel roughness, aspect ratio variation and other flow parameters.

NOTATIONS

A, A' = integral constant

B = additive constant related to wall roughness

* C, n = coefficient and exponent of power

u = horizontal time mean velocity;

u_* = shear velocity;

k = von kármán constant;

y_r = vertical distance from bed at distance r ;

δ = thickness of boundary layer;

u_m = maximum velocity occurring at the centre of the pipe

u_z = velocity at a distance z from bed

z = distance from bed;

k_s = roughness height

ν = kinematic viscosity

Π = wake strength

$\omega(\)$ = wake function

$u(Y_w)$ = surface velocity

$u(y)$ = velocity at a distance y from bed.

Y_w = flow depth at the surface

* K_b = coefficient in the law for outer region of bed

* α = dip factor

** U^+ = scaled local velocity

law

D_y = dimensionless length from free surface to the point where maximum velocity occurs in vertical distance y from side wall.

** k = equivalent sand grain boundary roughness

** y^+ = scaled distance from boundary or dominant viscous layer

** δ^+ = scaled boundary layer thickness

** t^+ = dummy variable for y^+ ,

U, V, W = mean velocity in x, y, z directions

u, v, w = turbulent fluctuations in x, y, z directions

K = mean kinetic energy

k_{pL} = Power law constant

\overline{U}_* = the mean friction velocity averaged in z -directions

ξ = the vorticity term

ε_t = eddy viscosity

y_t = distance from top of dominant viscous layer

δ_t = thickness of fully turbulent portion of boundary layer

U_m^+ = value at U^+ at δ^+

U_t^+ = normalized increase in velocity above upper limit of dominant viscous zone

u_{ir} = velocity in inner region	Y = depth of flow over channel bed
y_{ir} = depth in inner region	y_{ir} = bed-normal location of the upper edge of the overlap region or inner region.
Y_i = relative distance above the top of dominantly viscous layer	ρ = fluid density
$\hat{\xi}$ = normalized distance	H = depth
Ψ = wake strength parameter	S_0 = channel side slope
$U(z)$ = depth averaged velocity of depth z from bed;	f = Darcy-weisbach friction factor
U_{ir} = streamwise velocity at y_{ir} from bed.	U_d = depth- averaged stream velocity
U_0 = the cross-section averaged velocity	λ = dimension less eddy viscosity.
Z = cotangent bank slope	J = non-dimensional friction velocity.

*sarma et al.(1983) notations taken** Coleman and alonso(1983) ^ Guo and jullien (2008)

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