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Okumura, Hiroshi; Maruoka, Akira

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A SEMI-LAGRANGE GALERKIN METHOD FOR SHALLOW WATER EQUATIONS

Hiroshi Okumura[1] and Akira Maruoka[2]

[1] Assistant Professor, Information Technology Centre, University of Toyama
3190 Gofuku, Toyama City, Toyama, 930-8555, Japan, e-mail: okumura@itc.u-toyama.ac.jp
[2] Associate Professor, Dept. of Civil Engineering, Hachinohe National College of Technology
16-1 Uwanotai, Tamonoki, Hachinohe, Aomori, Japan 039-1192, Japan,
e-mail: maru-z@hachinohe-ct.ac.jp

ABSTRACT

A new finite element method, named semi–Lagrange Galerkin (SLG) method, which integrates a semi–Lagrange method into a characteristic/Lagrange Galerkin method is proposed for solving advection–diffusion and shallow water equations. In the present method, the calculation procedure is divided into two phases which are advection and non–advection phases. The advection phase is calculated by the semi–Lagrange method using an Hermitian type element and the non-advection phase is calculated by the Galerkin finite element method using the same Hermite cubic element.

Keywords: characteristic, Lagrange, semi–Lagrange, Galerkin, Hermite element

1. INTRODUCTION

Generally, in a flow problem, advection and diffusion terms are included in a governing equation. That either dominates, or a characteristic of the flow changes. In a numerical simulation of a flow problem, it is necessary to select a suitable numerical method or scheme depending on a characteristic of a flow. Especially, for the advection dominant flow, numerous numerical methods are proposed to overcome a fundamental issue that a numerical solution tends to be instable using the central difference approximation in a sense of the computational fluid dynamics (CFD). These conventional proposed methods can be largely divided into two methods which are the upwind streamline method and the characteristic method. In our research, we focus on the characteristic method that recently many high accurate schemes based on the characteristic method are developed. A representative scheme based on the characteristic method is the CIP (cubic interpolated pseudo-particle/propagation) method that a distribution of upwind physical values can be approximated by a local-Hermite interpolation and a computation of the advection part by the semi-Lagrange method. Additionally, the semi-Lagrange method is one of fractional step procedure which allows us to compute the advection step and the non-advection step separately. In a framework of the finite element method (FEM), the conventional approach for the numerical instability above mentioned is a way of the characteristic/Lagrange Galerkin method that both terms of time derivative and advection terms represent by the Lagrange derivative form can be approximated by the finite difference method (FDM) and the conventional Galerkin method is applied for spatial discretization. In this paper, we focus on the two methods, i.e., the semi-Lagrange method and the characteristic/Lagrange Galerkin method, and we propose new scheme, named

Semi-Lagrange Galerkin (SLG) method, that the SLG method is built the he characteristic/Lagrange Galerkin method into the Semi-Lagrange method. More specifically, the proposed method treats an interpolation of physical values needed to the semi-Lagrange method as a piecewise interpolation in the finite element method, a Hermite-type element, which consists of cubic polynomials given by function values and first order derivatives on each vertex, applies the computation of the advection step by the semi-Lagrange method. Furthermore, a computation of a non-advection step is approximated by the Galerkin method employed the same Hermite element.

Applying the Hermite-type element to the both steps of the advection and the non-advection computations, the SLG method allows us to extend the idea of the CIP method to a non-structure grid, and to compute the non-advection step with high accuracy. Though a degree of freedom of the Hermite-type element includes values of the first order derivatives, it is not necessary to derive a time marching equation for the first order derivatives, appeared in the CIP and the Multi-Moment methods, concerning with a governing equation of a flow problem. Incidentally, the IDO (interpolated differential operator) method, which expanded more high accurate then the CIP method, is the 5th order interpolation for the diffusion term, that method can be considered to be difficult to apply to non-structure grids. Fortunately, the SLG method has an additional feature that an integration of a composite function which is necessary with a characteristic finite element formulation does not appear because a numerical solution by the computation of the advection step is projected in advance by the Semi-Lagrange method. In this paper, for a advection-diffusion equation, using the 10-DOF Hermite type triangular element which consists of cubic polynomials given by function values and first order derivatives on each vertex and a function value on barycenter of a triangle is used in the two dimensional calculation. At last, as an application of the SLG method, we discuss an applicability of the present method for a shallow water equation.

2. ADVECTION DIFFUSION PROBLEM

Let Ω be a bounded domain of $\mathbb{R}^2$ with the boundary $\Gamma = \partial\Omega$. The advection diffusion problem is governed by the following advection diffusion equation to find a scalar function u for $t > 0$

$$\begin{cases} \dfrac{\partial u}{\partial t} + \boldsymbol{a}\cdot\nabla u - \nu\,\Delta u = f & \text{in } \Omega \\ u = g & \text{on } \Gamma \\ u(\boldsymbol{x},\,0) = u^0(\boldsymbol{x}) & \text{in } \Omega \end{cases} \tag{1}$$

where ν is a coefficient of viscosity ($\nu \geq 0$), $f(\boldsymbol{x},\,t)$ is a source term, and $\boldsymbol{a}(\boldsymbol{x},\,t)$ is an advection velocity satisfied the divergence–free condition ($\operatorname{div}\boldsymbol{a} = 0$).

In a characteristic method, the acceleration term (the advection and the time derivative terms) in Eq.1 is described by the following Lagrange derivative form.

$$\dot{u} \equiv \frac{\partial u}{\partial t} + \boldsymbol{a}\cdot\nabla u = \frac{\mathrm{d}}{\mathrm{d}\tau} u(\boldsymbol{X}(\boldsymbol{x},\,t\,;\,\tau),\,\tau)\big|_{\tau=t} \tag{2}$$

where $\boldsymbol{X}(\boldsymbol{x},\,t\,;\,\tau)$, which is a position of fluid particle at time τ with an initial position $\boldsymbol{x}$ at time t, is a characteristic curve of upwind streamlines. It can be solved by the following ordinary derivative equation.

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}\tau} = \boldsymbol{a}\,(\boldsymbol{X}(\boldsymbol{x},\,t\,;\,\tau),\,\tau) \\ \boldsymbol{X}(\boldsymbol{x},\,t\,;\,t) = \boldsymbol{x} \end{cases} \tag{3}$$

3. TEMPORAL DISCRETIZATION OF A CHARACTERISTIC METHOD

3.1 1st Order Backward Difference (B1)

In the discretization of the characteristic method, a finite difference approximation is applicable to Eq.2 directly. Let Δt be a time increment, $\cdot^n$ be a time step, the scheme (Pironneau, 1982) with a convectional 1st order backward difference (named B1 hereafter) can be described as follows:

$$\frac{u^{n+1} - u \circ \boldsymbol{X}^n}{\Delta t} - \nu\,\Delta\,u^{n+1} = f^{n+1} \tag{4}$$

where $\boldsymbol{X}^n\,(\equiv\,\boldsymbol{X}(\boldsymbol{x},\,t^{n+1};\,t^n))$ is a postion of upwind streamlines on the characteristic curve from fluid particle originated at a point $\boldsymbol{x}$, $u \circ \boldsymbol{X}^n$ is a composite function described by a scalar function $\boldsymbol{X}^n$.

3.2 2nd Order Backward Difference (B2)

It is considered that a high accurate order backward difference can be applicable as a high accurate scheme of a temporal discretization. A scheme with a 2nd order backward difference (named B2 hereafter) can be described as follows:

$$\frac{3\,u^{n+1} - 4\,u \circ \boldsymbol{X}^n + u \circ \boldsymbol{X}^{n-1}}{2\,\Delta t} - \nu\,\Delta\,u^{n+1} = f^{n+1} \tag{5}$$

It is noted that a computation of $\boldsymbol{X}^{n-1}$ is required in the case of B2.

3.3 Crank–Nicolson Method (CN)

As another 2nd order scheme in time, Crank–Nicolson (named CN hereafter) can be available (Lui & Tabala, 2002) such that

$$\frac{u^{n+1} - u \circ \boldsymbol{X}^n}{\Delta t} - \frac{\nu}{2}\left(\Delta\,u^{n+1} + \Delta\,u \circ \boldsymbol{X}^n\right) = \frac{1}{2}\left(f^{n+1} + f \circ \boldsymbol{X}^n\right) \tag{6}$$

In Eq.6, the operator Δ in the term $\Delta\,u \circ \boldsymbol{X}^n$ should be evaluated at $\boldsymbol{X}^n$. However an integration-by-part in a similar term $\Delta\,u^{n+1}$ cannot be applied since a weighting function should be evaluated at $\boldsymbol{x}$ on the procedure of a weak formulation. For this problem, Rui & Tabata treat the term $\Delta\,u \circ \boldsymbol{X}^n$ as follows (Rui and Tabata, 2002):

$$\Delta\,u \circ \boldsymbol{X}^n = \frac{\partial}{\partial x_i}\left(\frac{\partial u}{\partial x_i} \circ \boldsymbol{X}^n\right)\bigg|_{\boldsymbol{x}} + \Delta t\,\frac{\partial a_i}{\partial x_j}\,\frac{\partial}{\partial x_i}\left(\frac{\partial u}{\partial x_j} \circ \boldsymbol{X}^n\right)\bigg|_{\boldsymbol{x}} + O(\Delta t^2) \tag{7}$$

3.3 Linear Acceleration Method (LA)

In CN, we ascribe that a treatment of some terms included a function of spatial derivatives is troublesome resulting to appearance of a composite function in the diffusion and source terms, because Eq.1 is evaluated at $\left(\frac{\boldsymbol{x}+\boldsymbol{X}^n}{2},\, t^{n+\frac{1}{2}}\right)$. Applying a linear acceleration method (named LA hereafter) which often used in a strunctural analysis, therefore, we consider that Eq.1 is evaluated at $(\boldsymbol{x},\, t^{n+1})$ in a similar way of B1 and B2.

Taking a linearization and a temporal integration of the acceleration $\dot{u}(\boldsymbol{X}(\boldsymbol{x},\, t^{n+1}\,;\, \tau),\, \tau)$ $(t^n \leq \tau \leq t^{n+1})$ on the characteristic curve, u^{n+1} can be obtained as follows.

$$u^{n+1} = u \circ \boldsymbol{X}^n + \frac{\Delta t}{2}\left(\dot{u} \circ \boldsymbol{X}^n + \dot{u}^{n+1}\right) \tag{8}$$

Furhtermore, evaluationg Eq.1 at $(\boldsymbol{x},\, t^{n+1})$, that can be described such that

$$\dot{u}^{n+1} - \nu\, \Delta\, u^{n+1} = f^{n+1} \tag{9}$$

In LA, Eq.8 and 9 are should be simultaneous to solve. On the other hand, substituting Eq.8 to Eq.9, it is not necessary to solve simultaneously in the following treatment. However it is remarkable to treat a composite function since the diffusion term should to be evaluated at $\boldsymbol{x}$.

$$\dot{u}^{n+1} - \nu\, \Delta \left(u \circ \boldsymbol{X}^n + \frac{\Delta t}{2}\left(\dot{u} \circ \boldsymbol{X}^n + \dot{u}^{n+1}\right)\right) = f^{n+1} \tag{10}$$

In this research, letting the only term of the composite function in Eq.8 be a $\tilde{u}$ at first, we apply the following algorithm to find $\tilde{u}$.

$$\tilde{u} = u \circ \boldsymbol{X}^n + \frac{\Delta t}{2}\, \dot{u} \circ \boldsymbol{X}^n \tag{11}$$

$$\dot{u}^{n+1} - \nu\, \Delta \left(\tilde{u} + \frac{\Delta t}{2}\, \dot{u}^{n+1}\right) = f^{n+1} \tag{12}$$

$$u^{n+1} = \tilde{u} + \frac{\Delta t}{2}\, \dot{u}^{n+1} \tag{13}$$

This algorithm allows us to integrate a composite function in the only Eq.11 in a formulation of a characteristic finite element method. Furthermore, Eq.12 can be formulated by a conventional finite element method.

4. HERMITE TRIANGULAR CUBIC ELEMENT

The domain Ω divedes into a triangular element Ω_e $(1 \leq e \leq N_{el})$. In this research, a 10-DOF Hermitian type triangular element which consists of cubic polynomials given by function values u_α and first order derivatives $\left(\left.\frac{\partial u}{\partial x}\right|_\alpha,\, \left.\frac{\partial u}{\partial y}\right|_\alpha\right)$ on each vertex $\boldsymbol{x}_\alpha$ $(\alpha = 1,\, 2,\, 3)$ and a function value u_e on barycenter $\boldsymbol{x}_e$ of a triangle is used. This element is a complete cubic interpolation, which allows this to describe an interpolation fuction by a area coordinate L_α explicitly.

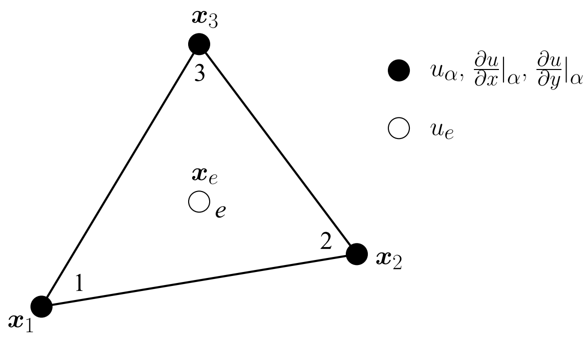

Figure 1: a 10-DOF Hermitian type triangular element

Let u_h be a finite element approximation in Ω, and $u_h|_{\Omega_e}$ be a finite element approximation in Ω_e. Then $u_h|_{\Omega_e}$ can be described as follows.

$$u_h|_{\Omega_e} = \mathbf{H}_e(\boldsymbol{x}_\alpha,\, \boldsymbol{x})^T\, \mathbf{U}_e \tag{14}$$

$$\mathbf{H}_e = \begin{bmatrix} H_{0\alpha}, & H_{x\alpha}, & H_{y\alpha}, & H_{0e} \end{bmatrix}^T \tag{15}$$

$$\mathbf{U}_e = \begin{bmatrix} u_\alpha, & \frac{\partial u}{\partial x}\Big|_\alpha, & \frac{\partial u}{\partial y}\Big|_\alpha, & u_e \end{bmatrix}^T \tag{16}$$

where $\mathbf{H}_e$ is an interpolation function described by the area coordinates L_α as follows.

$$\begin{cases} H_{0\alpha} = L_\alpha^2\,(3 - 2\,L_\alpha) - 7\,L_1\,L_2\,L_3 \\ H_{x\alpha} = L_\alpha^2\,(x_{\beta\alpha}\,L_\beta - x_{\alpha\gamma}\,L_\gamma) - (x_{\beta\alpha} - x_{\alpha\gamma})\,L_1\,L_2\,L_3 \\ H_{y\alpha} = L_\alpha^2\,(y_{\beta\alpha}\,L_\beta - y_{\alpha\gamma}\,L_\gamma) - (y_{\beta\alpha} - y_{\alpha\gamma})\,L_1\,L_2\,L_3 \\ H_{0e} = 27\,L_1\,L_2\,L_3 \end{cases} \tag{17}$$

where $(\alpha,\, \beta,\, \gamma)$ denotes an even permutation of $(1,\, 2,\, 3)$, $x_{\alpha\beta} = x_\alpha - x_\beta$, $y_{\alpha\beta} = y_\alpha - y_\beta$.

For $\boldsymbol{a}$ and f, these are approximate by an interpolation operator Π_h in the same finite element space of u_h.

$$\boldsymbol{a}_h = \Pi_h \boldsymbol{a}, \quad f_h = \Pi_h f \tag{18}$$

5. FURTHER CONSIDERATION FOR SHALLOW WATER EQUATION

5.1 Non–linear Shallow Water Equation

Let Ω be a bounded domain of $\mathbb{R}^1$ with the boundary $\Gamma = \partial\Omega$. Consider a non–linear shallow water equation to find a velocity $u(x,\, t)$ and a water depth $h(x,\, t)$ such that

$$\begin{cases} \dfrac{\partial h}{\partial t} + u\,\dfrac{\partial h}{\partial x} + h\dfrac{\partial u}{\partial x} = 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial t} + u\,\dfrac{\partial u}{\partial x} + g\dfrac{\partial h}{\partial x} = 0 & \text{in } \Omega \end{cases} \tag{19}$$

where the first and the second equations in Eq.19 are a momentum equation and a continuity equation, respectively. These are called primitive equations, and unknown functions u, h are called primitive variables. Eq.19 can be rewritten as the following matrix form.

$$\frac{\partial \boldsymbol{q}}{\partial t} + \mathbf{A}\,\frac{\partial \boldsymbol{q}}{\partial x} = \mathbf{0} \tag{20}$$

where a vector $\boldsymbol{q}$ and a coefficient matrix $\mathbf{A}$ can be described as follows.

$$\boldsymbol{q} = \begin{pmatrix} h \\ u \end{pmatrix} \ , \quad \mathbf{A} = \begin{pmatrix} u & h \\ g & u \end{pmatrix} \tag{21}$$

An eigenvalue $\lambda^{\pm}$ and an eigenvalue matgrix $\boldsymbol{\Lambda}$ can be found in the coefficient matrix $\mathbf{A}$. The eigenvalue matrix $\boldsymbol{\Lambda}$, which diagonal components are an eigenvalue $\lambda^{\pm}$ of the eigenvector matrix $\mathbf{A}$, consists of an eigenvector matrix $\mathbf{R}$ ant it's inverse matrix $\mathbf{R}^{-1}$. For Eq.20, these matrices can be derived such that

$$\begin{cases} \boldsymbol{\Lambda} = \begin{pmatrix} \lambda^{+} & 0 \\ 0 & \lambda^{-} \end{pmatrix}, \quad \lambda^{\pm} = u \pm c \ , \\ \mathbf{R} = \begin{pmatrix} 1 & 1 \\ c & -c \end{pmatrix}, \ \mathbf{R}^{-1} = \dfrac{1}{2} \begin{pmatrix} 1 & \sqrt{h/g} \\ 1 & -\sqrt{h/g} \end{pmatrix} \end{cases}$$

where $c \equiv \sqrt{gh}$ is a wave velocity due to a gravity wave. Assume that the coefficient matrix $\mathbf{A}$ is able to decompose into $\mathbf{A} = \mathbf{R}\,\boldsymbol{\Lambda}\,\mathbf{R}^{-1}$, the matrix $\mathbf{A}$ can be diagonalized as follows.

$$\mathbf{R}^{-1}\,\mathbf{A} = \boldsymbol{\Lambda}\,\mathbf{R}^{-1} \tag{22}$$

Then, Eq.20 is transformed to the following hyperbolic equation.

$$\frac{\partial(\mathbf{R}^{-1}\boldsymbol{q})}{\partial t} + \boldsymbol{\Lambda}\,\frac{\partial(\mathbf{R}^{-1}\boldsymbol{q})}{\partial x} = \frac{\partial \boldsymbol{p}}{\partial t} + \boldsymbol{\Lambda}\,\frac{\partial \boldsymbol{p}}{\partial x} = \mathbf{0} \tag{23}$$

where unknown variables in the hyperbolic equation 23 are the following characteristic variables $\boldsymbol{p}$.

$$\boldsymbol{p} = \begin{pmatrix} p^{+} \\ p^{-} \end{pmatrix} = c \pm \frac{1}{2} u \tag{24}$$

Incidentally, $p^{\pm} = c \pm \frac{1}{2}u$ is called the Riemann invariant.

Furthermore, in the Lagrange description of the hyperbolic equation 23, the following characteristic equation for the non–linear shallow water equation 19 can be obtained.

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = 0 \quad \text{along} \quad \frac{\mathrm{d}X^{\pm}}{\mathrm{d}t} = \lambda^{\pm} \tag{25}$$

In Eq.25, parenthetically, we note that a solution in a certain point is determined a couple of the Riemann invariants which possess a characteristic velocity $\lambda^{\pm}$ along two characteristic curve $X^{\pm}$. Additionally, a relation on the primitive variable $\boldsymbol{q}$ and the characteristic variable $\boldsymbol{p}$ can be expressed as follows.

$$u = p^{+} + p^{-} \ , \quad h = \frac{(p^{+} + p^{-})^2}{4g} \tag{26}$$

5.2 Temporal Discretization

In the discretization of the characteristic method, a finite difference approximation is applicable to Eq.25 directly. Let Δt be a time increment, $\cdot^{n}$ be a time step, a conventional scheme can be described as follows.

$$\frac{\boldsymbol{p}_{n+1} - \boldsymbol{p} \circ X_n^{\pm}}{\Delta t} = \mathbf{0} \tag{27}$$

where $\boldsymbol{X}^n\,(\equiv\;\boldsymbol{X}(\boldsymbol{x},\,t^{n+1};\,t^n))$ is a postion of upwind streamlines on the characteristic curve from fluid particle originated at a point $\boldsymbol{x}$, $\boldsymbol{p}\circ\boldsymbol{X}^n$ is a composite function described by a scalar function $\boldsymbol{X}^n$.

5.3 Hermite-type Element

The domain Ω diviedes into a line element Ω_e $(1 \leq e \leq N_{el})$. In this research, a 4-DOF Hermitian type element which consists of cubic polynomials given by function values $\boldsymbol{p}_\alpha$ and first order derivatives $\frac{\partial \boldsymbol{u}}{\partial x}\big|_\alpha$ on each vertex x_α $(\alpha = 1,\,2)$ is used. This element is a complete cubic interpolation, which allows this to describe an interpolation fuction by a line coordinate L_α explicitly.

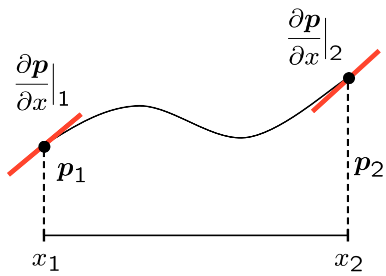

Figure 2: Hermite-type element

Let $\boldsymbol{p}_h$ be a finite element approximation in Ω, $\boldsymbol{p}_h|_{\Omega_e}$ of a finite element approximation in Ω_e can be described as follows.

$$\boldsymbol{p}_h|_{\Omega_e} = \mathbf{H}_e(x_\alpha,\,x)^T\,\mathbf{P}_e \tag{28}$$

$$\mathbf{H}_e = \left[H_{0\alpha},\quad H_{x\alpha}\right]^T,\ \mathbf{P}_e = \left[\boldsymbol{p}_\alpha,\quad \tfrac{\partial \boldsymbol{p}}{\partial x}\big|_\alpha\right]^T \tag{29}$$

where $\mathbf{H}_e$ is an interpolation function which can be described by a line coordinate L_α as follows.

$$H_{0\alpha} = L_\alpha^2\,(3 - 2\,L_\alpha),\quad H_{x\alpha} = x_{\beta\alpha}\,L_\alpha^2\,L_\beta \tag{30}$$

where $(\alpha,\,\beta)$ denotes an even permutation of $(1,\,2)$, $x_{\alpha\beta} = x_\alpha - x_\beta$. In addition, the same approximation as $\boldsymbol{p}_h$ of the eigenvalue (the characteristic velocity) is applied as follows.

$$\lambda_h^\pm = \left(p_h^+ + p_h^-\right) \pm \frac{p_h^+ + p_h^-}{2} \tag{31}$$

5.2 Treatment of a Composite Function

In a formulation of a conventional characteristic finite element method, a problem of an integration included in a composite function remains. Since the composite function $\boldsymbol{p}_h \circ X_n^\pm|_{\Omega_e}$ of the finite element approximation in Eq.28 intersects on a number of elements, some approximation, *e.g.*, a numerical integration, is required for the argued integration. In this research, for the sake of computational simplicity for a treatment of composite functions, a composite function on each element is approximate as one polynomial by reconstructing a Hermite-type cubic line element on vertices $X_n^\pm|_\alpha$ $(\alpha = 1,\,2)$ which are two positions of upwind streamlines along a characteristic curve.

The composite function $\boldsymbol{p}_h \circ X_n^{\pm}|_{\Omega_e}$ is approximate in a similar way of Eq.28

$$\boldsymbol{p}_h \circ X_n^{\pm}\big|_{\Omega_e} \approx \mathbf{H}_e(X_n^{\pm}\big|_{\alpha},\, X_n^{\pm})^T\, \hat{\mathbf{P}}_e \tag{32}$$

$$\hat{\mathbf{P}}_e = \Big[\boldsymbol{p}_h \circ X_n^{\pm}\big|_{\alpha},\quad \tfrac{\partial \boldsymbol{p}_h}{\partial x} \circ X_n^{\pm}\big|_{\alpha}\Big]^T \tag{33}$$

5.4 Upwind Position along a Characteristic Curve

In this research, let $x|_l$ be a position of a node l $(1 \leq l \leq N_{nd})$, a couple of upwind positions $X_n^{\pm}|_l$ is computed as follows.

$$X_n^{\pm}|_l = x|_l - \Delta t\, \lambda^{\pm}|_l \tag{34}$$

5.5 SLG Formulation

In a formulation of the SLG method, a function value and first order derivatives on a node l can be updated as follows.

$$\boldsymbol{p}^{n+1}\big|_l = \boldsymbol{p}_h \circ X_n^{\pm}\big|_l \tag{35}$$

$$\frac{\partial \boldsymbol{p}^{n+1}}{\partial x}\bigg|_l = \left(1 - \Delta t\, \frac{\partial \lambda^{\pm}}{\partial x}\bigg|_l\right) \frac{\partial \boldsymbol{p}_h}{\partial x} \circ X_n^{\pm}\big|_l \tag{36}$$

where $\frac{\partial \lambda^{\pm}}{\partial x}\big|_l$ is first order derivatives composised of the engenvalue $\lambda_h^{\pm}$. Note in passing that the SLG method allows this to compute first order derivatives by using an interpolation of a first order derivative variable and Jacobian, not solving a time marching equation for first order derivatives of a governing equation for example the CIP method.

6. CONCLUSION

This paper presents a new finite element method, named semi–Lagrange Galerkin (SLG) method, which integrates a semi–Lagrange method into a characteristic/Lagrange Galerkin method is proposed for solving advection–diffusion and shallow water equations. The most salient feature of the present method is that the advection phase is calculated by the semi–Lagrange method using an Hermitian type element and the non-advection phase is calculated by the Galerkin finite element method using the same Hermite cubic element.

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