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RAINFALL RUNOFF MODELING OF WALNUT CREEK WATERSHED USING TWO-DIMENSIONAL SHALLOW WATER EQUATIONS

Singh Jaswant[1], Mustafa S. Altinakar[2] and Yan Ding[3]

ABSTRACT

This paper presents the application of a structured finite volume two-dimensional shallow water equations based numerical model to simulate the flow discharge due to high intensity short duration storms in a watershed. The rainfall intensity and infiltration rate are added as source and sink terms in the continuity equation of the shallow water equations. The Green-Ampt method is used for calculation of the infiltration rate. The depth averaged mass and momentum conservation equations are solved using a second order accurate numerical scheme, considering the effects of bottom elevation gradient, bed friction due to land use, and the infiltration process. Hence the model has physically based conceptual and mathematical grounds much stronger than the classical hydrological models. The model considers the spatial variability of rainfall, infiltration parameters and the Manning's friction coefficient. The GIS layers such as DEM, soil type, and land use can be directly used as input for the model. Spatio-temporal distribution of the rainfall over the watershed is determined based on rain gauge records. Three sub-watersheds in the USDA-ARS Walnut Gulch experimental watershed, located in SE Arizona, are selected for this study. The model parameters are first calibrated using a single storm event. Using the calibrated parameters, the model is then applied to two other storms events. The comparison of computed and measured runoff discharge at the outlet of the selected sub-watersheds shows good agreement. The results of this study indicate that this model can be used for modeling rainfall-runoff in large watersheds based on a historical or predicted rainfall distribution.

1. INTRODUCTION

The high intensity short duration storm events produce flash floods which cause severe damage in terms of property and life. The response time is generally very slow in such catastrophic events. Therefore, the prediction of accurate flood discharge hydrographs resulting from such storm events is of crucial importance in designing the mitigation strategy. The prediction of flash flood hydrographs based on advanced mathematical grounds and physically based concepts is the main purpose of this paper.

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A two-dimensional shallow water equation based numerical model is used to predict the runoff hydrograph generated by the storms of high intensity and short duration events in the agricultural watersheds. When rainfall occurs over an area, some portion infiltrates into soil and some amount of this rainfall is lost as the evapo-transpiration. The remaining water flows over the surface as overland flow and gets accumulated in depressions. Over the time, water depth in these depressions builds up and water starts flowing in the form of streams and passes through the watershed outlet. This runoff discharge is usually obtained using simple hydrological models based on empirical formulae, which relate the runoff with precipitation intensity, catchment area, mean slope, soil type and antecedent moisture conditions. Such models use simplified representations of surface runoff, infiltration and channel flow. However, in real life, the overland flow process is far more complex considering the non-linearity and heterogeneity of the governing factors.

Recent developments in the numerical schemes for solving the two-dimensional shallow water equations made it possible to model the water flow over complex topography and extremely small depths including the wet-dry interfaces. In the event of floods generated from the rainfall, a very shallow water flow, called as overland flow, occurs over the surface with wet-dry front propagating in the direction of the bottom gradient. This overland flow is the key process for transportation of sediment and nutrients over the surface of the watershed. The overland flow depth and velocities are highly variable and discontinuous in space and time due to micro-topography and natural spatial variation of soil properties. These characteristics of the overland flow are difficult to be resolved using an empirical model. So some physically based methods are required for better understanding of the overland flow.

In the existing literature, there are numerous studies done on the use of shallow water equations for prediction of the runoff discharge. Recently hydrodynamic models have been used for simulating overland flow and sediment and nutrient transport in agricultural watersheds. Many studies have been reported on using shallow water equations for runoff prediction. For instance; Chow and Bew-Zvi (1973), Hromadha et al. (1987), Zhang and Cuudy (1989), Tayfur et al, (1993) used shallow water equations for developing numerical models for predicting overland flow. A constant rainfall intensity and spatially constant infiltration rate was considered in these studies. Esteves et al. (2000) developed a two-dimensional model based on an explicit finite difference scheme. The model performed well in the field experiments. However, the spatial variability of rainfall and infiltration were not considered. Kivva and Zheleznyak (2005) developed rainfall runoff model for small catchments using shallow water equations. After validating their model against the one-dimensional dam break flows over impermeable bottom, the model was applied to Butenya River Catchment. This model underestimated the observed runoff in some cases. Mignot et al. (2006) used shallow water equations to simulate urban flooding in Nims, France. They used an explicit second order scheme to solve the governing equations and encouraging results were obtained. Singh et al. (2012) developed a two-dimensional numerical model using shallow water equations with spatially and temporally varying rainfall and infiltration. The model performed well in predicting runoff in experiments as well as real life field simulations. Fiedler et al. (2000) developed a numerical model for simulating discontinuous shallow flow over an infiltrating surface. They used MacCormack finite difference solution scheme with spatially varying infiltration rate and temporally constant rainfall. The model produced reasonable agreement with the laboratory experiments. Cea et al. (2008) used shallow water equations based two-dimensional numerical model to predict rainfall-runoff discharge in an experimental watershed and produced a good match between the experiments and simulations.

This paper presents the application of a two-dimensional numerical model, based on shallow water equations with the rainfall and the infiltration as source and sink terms, in the Walnet Creek watershed in Southeastern Arizona. Section 2 describes the governing equations and sources and sink terms used in the model to describe the over land flow process. The details regarding the model development are not thoroughly detailed in this paper. Interested readers can refer Singh et al. 2011 for more details regarding the model development methodology. Section 3 and 4 presents the application of this the selected sub-watersheds in Walnut Gulch watershed in southeastern Arizona. The selected storms for this study are high intensity and short duration. One storm event is used for calibration of the model parameters by comparing with the field observations and then using the calibrated parameters, two more storm events are simulated for the model validation. The simulated results are compared with the field observations. Finally conclusions are presented in the section 5 of this paper.

2. NUMERICAL MODEL

The detailed description of the numerical method, used for solution of the governing equation in this paper, is presented in Singh et al. 2011.This section presents just a brief description of the governing equations, source and sink terms used in this numerical model. This numerical model is based on depth averaged two-dimensional shallow water equations (2D-SWE), also known as St. Venant equations. The 2D-SWE are obtained after vertical integration of 3D Navier-Stokes equations over the water depth.

2.1 Governing Equations

The conservative form of 2D shallow water equations, consisting of a continuity equation with rainfall and infiltration, and two momentum equations for depth-averaged free surface flows, are written as follows:

$$\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{bmatrix}_x + \begin{bmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix}_y = \begin{bmatrix} R - I \\ -gh\frac{\partial B}{\partial x} - S_{fx} \\ -gh\frac{\partial B}{\partial y} - S_{fy} \end{bmatrix}, \tag{1}$$

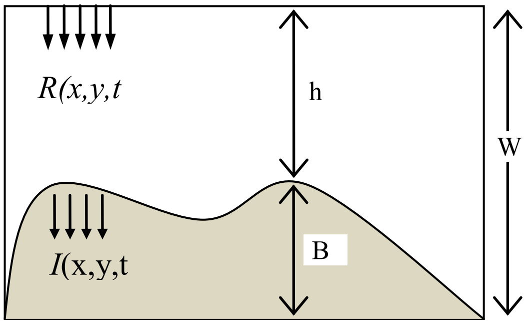

Figure 1 Model variables definition

where x and y are Cartesian coordinates describing the horizontal plane, t is time, $h(x,y,t)$ is water depth, $u(x,y,t)$ and $v(x,y,t)$ are the two components of the depth-averaged velocities in x and y directions, respectively. $R(x,y,t)$ is the rainfall intensity and $I(x,y,t)$ is the infiltration rate. The gravitational acceleration is denoted by g. $B(x,y)$ is the bottom elevation function describing an arbitrary or natural bathymetry, S_{fx} and S_{fy} , the components of the non-linear bottom friction terms due to its roughness in x and y directions.

The water surface elevation is represented by $w(x,y,t)$ such that $w = h+B$ (see Figure 1). The bottom elevation $B(x,y)$ is assumed not to change with time, i.e., fixed bed is considered in this model. By simply replacing water depth (h) by the difference of surface water elevation and bottom elevation ($w-B$), and doing simple algebraic manipulations, Eq.1 is rewritten in terms of $w(x,y,t)$.

$$\begin{bmatrix} w \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ \dfrac{(hu)^2}{w-B}+\dfrac{1}{2}g(w-B)^2 \\ \dfrac{(hu)\,(hv)}{w-B} \end{bmatrix}_x + \begin{bmatrix} hv \\ \dfrac{(hu)\,(hv)}{w-B} \\ \dfrac{(hv)^2}{w-B}+\dfrac{1}{2}g(w-B)^2 \end{bmatrix}_y = \begin{bmatrix} R-I \\ -g(w-B)\dfrac{\partial B}{\partial x}-S_{fx} \\ -g(w-B)\dfrac{\partial B}{\partial y}-S_{fy} \end{bmatrix}, \quad (2)$$

2.2 Definition of Sources and Sinks

Rainfall intensity

The least distance method is used to assign the DEM cells to the nearest rain gauge located in the study area. Spatially and temporally varying rainfall intensity $R(x,y,t)$ data is provided to the model from the rain gauge readings directly.

Infiltration rate

The infiltration rate $I(x,y,t)$ is calculated using the Green-Ampt method . In this method, a wetting front, which is a sharp boundary between the saturated soil above, and the soil at antecedent moisture conditions below, is considered to penetrate into the soil. The equation for calculation of the infiltration rate in mm/hr by the Green-Ampt method is as follows:

$$I = \frac{\partial F}{\partial t} = \eta\, K_s\left(\left(h-\psi_f\right)\frac{\Delta\theta}{\eta\, F}+1\right) \quad (3)$$

where F is the total infiltration (m), K_s is the saturated hydraulic conductivity (m/s), η is the porosity (-), Ψ_f is the dry suction (m), $\Delta\theta$ is the difference of saturated soil moisture and the antecedent soil moisture (-). Brooks and Corey (1964) studied the variation of dry suction with moisture content for many soils in the laboratory, and concluded that Ψ_f can be expressed as a logarithmic function of effective saturation S_e. The effective saturation is the ratio of available moisture to the maximum possible moisture content of the soil and has a range of $0 \le S_e \le 1$.

$$S_e = \frac{\theta-\theta_r}{\eta-\theta_r} \quad (4)$$

Where θ_r is residual moisture content and θ is antecedent moisture content of the soil. The change in moisture content ($\Delta\theta$), as the wetting front passes, can be expressed as following (Chow et al., (1988)).

$$\Delta\theta = (1-S_e)\theta_e \quad (5)$$

Where θ_e is effective porosity.

Bottom friction terms

The bottom friction terms in Eq.1, S_{fx} and S_{fy}, are defined as follows:

$$S_{fx} = gu\sqrt{u^2 + v^2} / C^2, \tag{6}$$

$$S_{fy} = gv\sqrt{u^2 + v^2} / C^2. \tag{7}$$

where C is the Chezy coefficient and is calculated from $C = (h)^{1/6} / n$, in which n is the Manning's coefficient.

2.3 Solution of green-ampt Equation

Total infiltration in Eq. 5 is discretized using the forward Euler method as follows:

$$F_{i,j}^{n+1} = F_{i,j}^{n} + n\Delta t K_s \left(\left(h_{i,j} - \psi_f\right) \frac{\Delta\theta}{nF_{i,j}^{n}} + 1 \right) \tag{8}$$

The infiltration rate and rainfall intensity terms are implemented in the continuity equation of shallow water equations as follows:

$$w_{i,j}^{n+1} = w_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left((H^x)_{i+1/2,j}^{(1)} - (H^x)_{i-1/2,j}^{(1)}\right) - \frac{\Delta t}{\Delta y}\left((H^y)_{i,j+1/2}^{(1)} - (H^y)_{i,j-1/2}^{(1)}\right) + \Delta t R_{i,j} - (F_{i,j}^{n+1} - F_{i,j}^{n}) \tag{9}$$

F_{ij}^{n+1} and F_{ij}^{n} are the cumulative infiltration at the current and next time level. The difference (F_{ij}^{n+1} - F_{ij}^{n}) is the infiltration rate $\Delta t I_{ij}$ during the time step. The method of successive substitution is used for cumulative infiltration F_{ij}^{n} at the first time step with $F_{ij}^{n} = K_s t$ as the first trial. There is published literature for the value of K_s, η, Ψ_f, θ_e for various soil classes. (Chow et al., (1988)). The value of S_e can be found by calibration. If any cell in the DEM is dry, then the infiltration terms are set to zero in Eq. 9. However, the computation of the wetting front movement in the soil layer is continuously updated using Eq. 8. The governing equations are solved using a Godunov-type central upwind scheme (Singh et al. (2011)), which is based on integration over the Riemann fan and is second order accurate in space. This scheme does not require characteristic decomposition and Riemann solvers to calculate inter-cell fluxes. The positivity of the depth is guaranteed at the wet and dry interface by introducing a slope limiter and by keeping the courant number less than 0.25.

3. STUDY AREA DESCRIPTION

This study is conducted using data from the Walnut Gulch watershed operated by the United States Department of Agriculture (USDA), Southwest Water Research Center (SWRC). The Walnut Gulch experimental watershed has an approximate area of 150 km^2 and is located in southeastern Arizona. Walnut Gulch is a tributary of San Pedro River, which originates in Sonora, Mexico and flows into the United States as part of the Lower Colorado River Basin (Hernandez et al. 2000). The climate of the area is semi-arid with annual precipitation of approximately 345 mm and highly spatially and temporally varying precipitation patters dominated by the North American Monsoon.

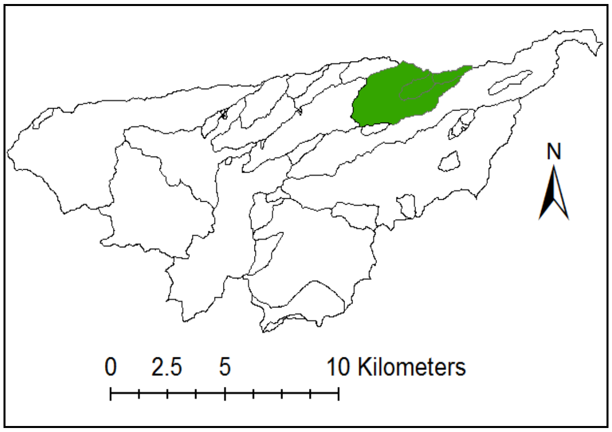

Figure 2 Walnut Gulch watershed with study area indicated in green color

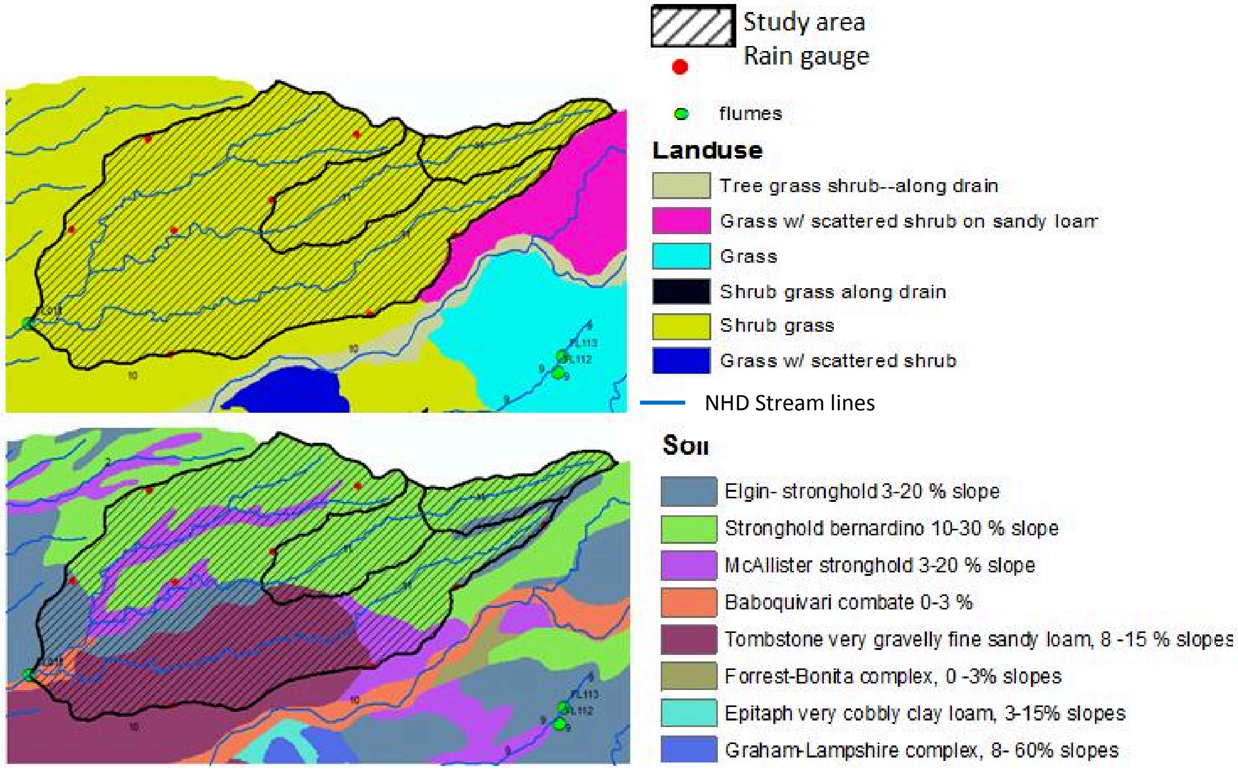

Figure 3 Landuse and soil types in the selected study area of the Walnut Gulch watershed.

Walnut Gulch watershed has 119 rain gauges which are almost uniformly distributed over the entire watershed. These rain gauge stations provide long term climatic information for hydrologic research. In the northeast side of the Walnut Gulch watershed, three adjoining sub-watersheds (see Figure 2) having a total area of 7.74 Km^2 are selected for this study. There are nine rain gauges located in the selected area. One measuring flume is located at the outlet of this sub-watershed and one flume is also located at the outlet to measure runoff. Rainfall data, runoff discharge data, were extracted from Southwest Watershed Research Center (SWRC) long term rainfall and runoff database. GIS layers of DEM dataset, soil type and landuse data are provided at the online data access service of SWRC. These data layers were taken directly from SWRC database for this study.

The soils are calcareous, generally well drained, gravelly loams with large proportion of rocks and gravel at the surface. The soils in this watershed are characterized by STATSGO as AZ061, a complex composed of very gravelly loam (60%), a gravelly fine sandy loam (25%(and a very find sandy loam (15%) (Hernandez et al. 2000). The spatial distribution of soil types and landuse in the study area with their nomenclature is shown in Figure 3. The main landuse in the watershed are grazing with minimum and limited urbanization. The landuse in the selected study area are characterized as grassland composed of tree shrub grass along the drain, grass with scattered shrubs, shrub grass etc. The predominant landuse is shrub grass in the selected study area. The NHD stream lines are also shown in the Figure 3.

GIS layers such as topography DEM (digital elevation model), soil type and landuse downloaded from SWRC database are given as direct input to the model. The model uses the method of least distance from the location of rain gauge to assign the corresponding rain gauges to the DEM cells. The temporally varying rainfall intensity from different rain gauges located in the study area can be given as input to the model. The infiltration parameters for different soil type needs to be calibrated using field or laboratory observed data. The landuse information is used for spatial distribution of varying Manning's *n*.

4. RESULTS AND DISCUSSION

4.1 Computational Conditions

A uniform computational grid of 30 m x 30 m is used. All other shape files for landuse and soil type were also of the same resolution. The initial conditions were assumed to be dry in the whole study area. The time step is governed by the CFL criteria and Courant number is 0.25 for all the cases (Refer Singh et al. 2011). The boundary conditions are open type for all the sides. The discharge is summed up along a cross-section at the location of the flume. The precipitation data (rainfall intensity) for all the rain gauges in the study area were interpolated linearly for the time step and were assigned to the corresponding cells based on the method of least distance. This study was carried in two steps. In the first step, the model parameters were calibrated using a field observed data for one storm. The model parameters were selected based on the best agreement with the observations. In the second step, using the selected parameters, two storm events were simulated and the results were compared with the observations.

4.2 Model Calibration

The model parameters, namely porosity, effective porosity, dry suction, hydraulic conductivity, effective saturation, to calculate the infiltration rate and the Manning's friction coefficients need to be calibrated based on the observed runoff hydrograph. The comparison of the simulated and the observed data is based on the visual observation. An event dated 8/1/2002 is chosen for the calibration purpose. The selection of a storm event for calibration is very important. It should be selected in such a way that all the rain gauges in the selected area are having rainfall so that the runoff contribution from every cell can be considered. The influence of landuse is considered only in terms of Manning's friction coefficient. The shrub grass is the only landuse contributing in the runoff at the watershed outlet. A value of 0.056 for Manning's friction coefficient was found to produce the closest match between the simulated and observed runoff. The soil parameters finally selected based on the closest match between the simulated and field observed runoff at the outlet of the study area on 8/1/2002, are shown in Table 1.The simulated runoff discharge was found to be most sensitive for the hydraulic conductivity and Manning's friction coefficients for this numerical model.

The comparison of the simulated and the field observed runoff discharge for this storm event is shown in Figure 4. The rainfall intensity from the gauge covering maximum area is shown in the graph. It is observed that the model produced slightly higher values of peak runoff discharge than the observed values. However, time to peak discharge was in good agreement with the observations. The recession limb is also well captured by the model. At this stage, the closest possible reason that can be attributed to this over-estimation is that some water gets lost in evapo-transpiration over the field. Presently, the evapo-transpiration is not considered in the model computations. As a result of that the model slightly over-estimated the runoff discharge.

Table 1. Calibrated model Parameters for Green-Ampt method

Sr. No.	Infiltration parameters Soil type	Porosity	Effective Porosity	Dry Suction (cm)	Hydraulic Conductivity (cm/h)	Effective saturation
1	Elgin	0.35	0.410	21.5	1.35	0.30
2	Stronghold	0.20	0.330	19.35	1.55	0.20
3	McAllister	0.31	0.341	13.50	1.63	0.35
4	Baboquivari	0.27	0.310	17.81	1.49	0.31
5	Tombstone	0.18	0.260	12.78	0.85	0.15
6	Forest-Bonita	0.32	0.325	13.68	0.53	0.25
7	Epitah	0.41	0.486	12.68	1.65	0.23
8	Gram-Lampshire	0.31	0.326	10.58	1.36	0.32

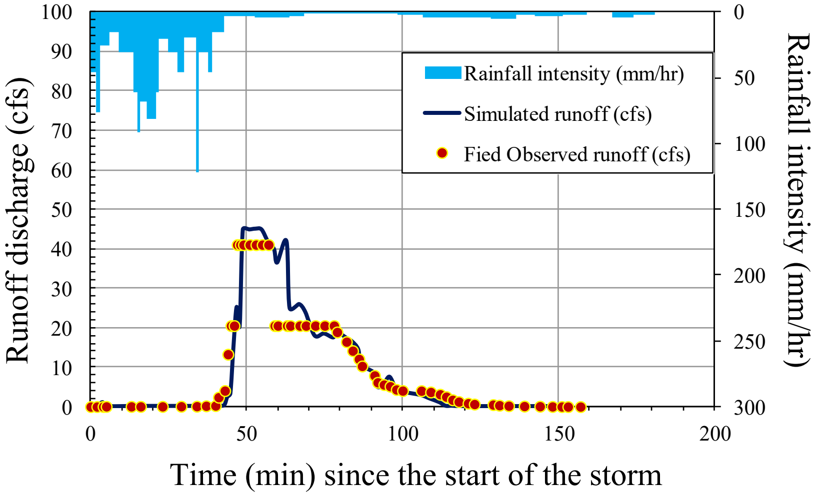

Figure 4 Comparison of simulated and field observed runoff discharge on 8/1/2002. (Calibration)

4.3 Model Validation

After calibration, the model is applied to other storm events. Two storm events dated 8/4/2002 and 8/6/2002 are selected for the model validation. Using the calibrated parameters and the same computational conditions as in the calibration case, two simulations cases were carried out. The simulated runoff discharge at the outlet of the study area is compared with the field observations for both the events. These results are shown in Figure 5 in which the rainfall intensity from the largest area covering raingauge is presented along with the simulated and field measured runoff discharge. The upper graph shows the comparison of the simulated and the field observed runoff hydrographs at the location of the measuring flume on 8/4/2002. It can be observed that simulated rising limb of the runoff hydrograph agreed well with the field observations. The peak discharge and time to peak are also simulated to a good accuracy. However, the recession limb is slightly over estimated. The overall shape, time to peak and duration of the simulation runoff hydrograph matched well with the field observations.

For another event dated 8/6/2002, the simulated and the observed runoff discharge is shown in Figure 5. It can be observed that the model simulates the runoff discharge up to a good accuracy. There is slight over-estimation towards the recession side of the hydrograph. The over-estimation of the recession limb of the hydrograph can be attributed to fact that, in the field, some water is lost in the evapo-transpiration, which is not considered in this model.

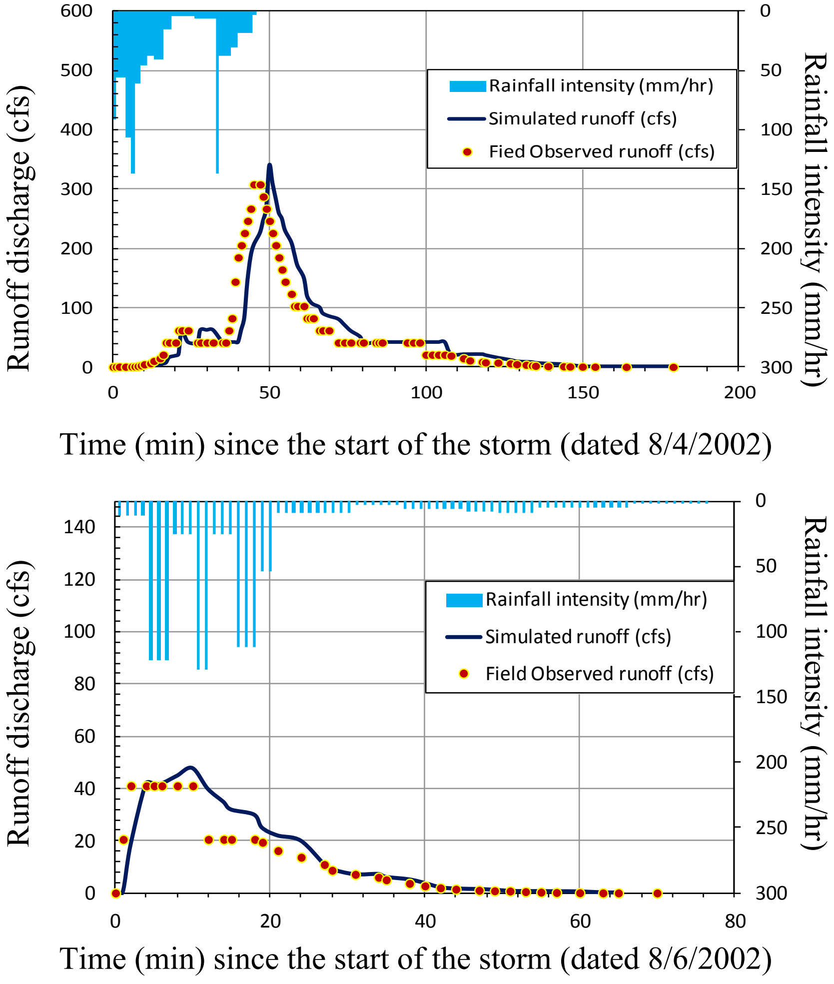

Figure 5. Comparison of simulated and field observed runoff discharge on 8/4/2002 and 8/6/2002.

The calibration of the parameters can also affect the results. Since visual comparison of the observed and the simulated hydrograph were carried out, there is some scope for improvement in the calibration of the parameters as well. Overall, the comparison of the simulated and field observed runoff discharge hydrographs shows that this model can be applied to predict runoff discharge in real life case up to a reasonable accuracy.

5. CONCLUSIONS

A two-dimensional numerical model is used to simulate the overland flow in a selected area in Walnut Gulch watershed, SE, Arizona. The model is based on the shallow water equations with rainfall and infiltration as the source and the sink terms. The governing equations are solved using a well balanced central explicit scheme. The Green-Ampt method is used for infiltration. The spatial and temporal variability of rainfall and spatial variability of infiltration is considered in this model. The model is GIS compatible and shape files of the DEM, soil type and landuse were directly given as input to the model. The model parameters were calibrated using the observed values of runoff at the outlet for one event. The model parameters producing the best agreement with the field observation were selected. Using these parameters, the model is further validated to simulate two more storm events. The comparison of the simulated runoff hydrograph and the field measured runoff hydrograph showed a reasonably good match. The future development in this model can include sediment transport and erosion by the overland flow.

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