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FRACTAL CONCEPTS IN MODELLING OF GEOMORPHOLOGIC INSTANTANAEOUS UNIT HYDROGRAPH

Jairaj.P.G.[1] and Sumi.R.[2]

Abstract : *Generally most of the catchments are ungauged and for such catchments Geomorphologic Instantaneous Unit Hydrograph is used for the computation of hydrologic response as it mainly depends on the geomorphologic properties of the catchment. The present study takes into account the fractal property of the stream network in developing an Analytical Geomorphologic Instantaneous Unit Hydrograph (AGIUH) model to estimate the runoff from a catchment. In AGIUH model a geomorphologic transfer function is used in determining the runoff from the catchment. As the geomorphologic transfer function represents the distribution of hydraulic pathways through which water travels inside the river networks, it can be easily estimated by the convolution of the probability distribution of individual drains through which water travels. This AGIUH model was applied to two field catchments, and the simulated runoff was compared with the observed values and the model performance was evaluated.*

Keywords : *Runoff; GIUH; transfer function; fractals*

INTRODUCTION

Rainfall runoff modelling is the conversion of rainfall into runoff at the basin outlet. Even though various techniques are developed to model this complex process, there is no universally accepted method. Most of the methods were developed for the gauged catchments where critical data for the rainfall runoff modelling are easily available. But for the planning, development and operation of various water resources schemes, estimation of runoff from ungauged catchments is also very much necessary. For such catchments, a method which incorporates the geomorphologic characteristics of the catchment to obtain the catchment response, through modelling using a Geomorphologic Instantaneous unit Hydrograph (GIUH) is introduced.

The modelling of rainfall runoff process using geomorphologic instantaneous unit hydrograph were reported by Rakesh Kumar et al (2007) Fleurant et al (2006) and the geomorphological theory of hydrologic response by Rinaldo and Iturbe (1996). In the present study an Analytical Geomorphologic Instantaneous Unit Hydrograph (AGIUH) is discussed, which uses the fractal

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property in the computation of the direct runoff hydrograph. The performance of the model was studied by applying the model to two field catchments of Walnut Gulch of Arizona, USA. A comparison of the simulated runoff values with that of with the observed values was also done. The methodology and the details pertaining to the model application are discussed in the subsequent sections.

METHODOLOGY

The purpose of rainfall-runoff modelling is the conversion of rainfall into runoff at the river basin outlet. This complex process is the result of the combination of multiple factors involved at various spatial scales. The process can be modelled using two functions, namely the production function and the transfer function. The production function makes it possible to determine the net rainfall or rainfall excess (input) actually involved in the runoff production at the river outlet. The transfer function works out the runoff at the basin outlet according to the net rainfall.

Analytical GIUH Model

Analytical Geomorphologic Instantaneous Unit Hydrograph (AGIUH) is a model which uses a geomorphologic transfer function based on general hypothesis of symmetry combined with the fractal properties of branched structures to estimate runoff. This probability density function is simpler in computation than the probability density function of the time of transfer on the drainage basin. The geomorphologic transfer function is the probability density function of hydraulic lengths, and this is shifted to the probability density function of the time of transfer for the estimation of runoff. The computational procedure of the geomorphologic transfer function is discussed subsequently.

The relation between the runoff expressed as production function and transfer function can be written as

$$q(t) = h(t) \times T(t) \tag{1}$$

where $q(t)$ is the rate of flow from unit area (runoff), $h(t)$ is the rainfall in depth units (production function), $T(t)$ is the function corresponding to the distribution of rainfall transfer over the whole basin.

Production function

In order to determine the net rainfall from the gross rainfall, it is necessary to assess the runoff deficit. Among the various losses, the loss due to infiltration is only considered for computing the rainfall excess. The infiltration loss is estimated by making use of the Horton's infiltration curve, defined as

$$f(t) = f_c + (f_0 - f_c)e^{-kt} \tag{2}$$

where $f(t)$ is the infiltration capacity at time t, f_0 is the initial infiltration capacity, f_c is the final infiltration capacity, k is Horton's constant. The value of the mass infiltration $F(t)$ at any time is then worked out in the case of saturating rainfall:

$$F(t) = f_c t + \frac{f_0 - f_c}{k}(1 - e^{-kt}) \tag{3}$$

Net rainfall (RE, rainfall excess) is

$$RE(t) = R(t) - \left[f_0 - k(F(t) - f_c t)\right] \tag{4}$$

Using the above expression the net rainfall *RE(t)* is computed and forms the input for the runoff computation and the function is called the production function.

Transfer function

The transfer function *T(t)* corresponds to the distribution of rainfall transfer time over the whole area of river basin. The transfer function spreads the net rainfall over time and space in the river basin. A transfer function can be modelled using the approach of Geomorphologic Instantaneous Unit hydrograph (GIUH). Generally the transfer function is modelled as a probability density function of the time of transfer on the drainage basin. But in this study a geomorphologic transfer function which is the probability density function of the hydraulic lengths is used. The geomorphologic transfer function is then shifted to the probability density function of the time of transfer.

Geomorphologic Transfer function

The geomorphologic transfer function is the probability density function of hydraulic lengths. The probability density function of individual drains is expressed on the hypothesis of symmetry, combined with fractal geometry as,

$$pdf\ (l_k) = \frac{1}{\sqrt{2\Pi\, \bar{l}_k}} \frac{1}{\sqrt{l_k}} e^{\frac{-l_k}{2\bar{l}_k}}, \ k = 1,2\ldots n \tag{5}$$

where l_k is the hydraulic length of k^{th} order stream

The geomorphologic transfer function for the stream network is estimated by applying the convolution integral of each individual stream to form a hydraulic path.

$$pdf\,(L) = pdf\,(l_1) * pdf\,(l_2) * \ldots * pdf\,(l_n) \tag{6}$$

$$pdf(L) = \frac{1}{\sqrt{2^n\, \Pi_{i=1}^{n}\, \bar{l}_1}} \frac{L^{(\frac{n}{2})-1}}{\Gamma(\frac{n}{2})} e^{\frac{-L}{2\Gamma_n}} \sum_{k=0}^{\infty} \frac{b_n(k)(\frac{n-1}{2})k}{k!(\frac{n}{2})_k} \left[\frac{L}{2(l_n - l_{n-1})}\right]^k \tag{7}$$

where

$$b_n(k) = \begin{cases} 1, & if \quad i = 2 \\ \displaystyle\sum_{j=0}^{k} \frac{b_{i-1}(j)(\frac{i-2}{2})_j}{j!(\frac{i-1}{2})_j} \left[\frac{\bar{l}_{i-1} - \bar{l}_{i-2}\ \bar{l}_{i-1}}{\bar{l}_{i-1} - \bar{l}_i\ \bar{l}_{i-2}}\right]^j, & if \quad i = 3,4,\ldots\ldots. n \end{cases}$$

$$(x)_k = x(x+1)(x+2)\ldots\ldots\ldots\ldots(x+k-1)$$

where L is the hydraulic length, k is the order number, n is the order of the streams.

The order of the streams was assigned following the Strahler's hierarchical classification system concept. Considering an indefinite point on the river basin to represent a raindrop, the path covered between this point and the outlet successively goes over channels of increasing order and the distance covered by the raindrop known as hydraulic length L can be computed as,

$$L = l_0 + \sum_{k=1}^{n} l_k \tag{8}$$

where l_k is the length of the channel of k^{th} order, n is the order of the river network, l_0 is the length of the hill slope.

Hence, the hydraulic length is the added length of n channels. The hydraulic length L in the river network, may be calculated using a vector with n components $(l_1,l_2,...,l_n)$. It is important to note that some length can be nil, a first-order channel might join a third-order one directly, so l_2 would be zero in that particular instance.

Conversion of Geomorphologic transfer function

The geomorphologic transfer function given by Eq. 7 is to be shifted to the transfer function *T(t)* which is the probability density function of time. The shifting is done by dividing the geomorphologic transfer function with the average runoff speed $\ddot{\upsilon}$ in the stream network and this transfer function is used for the computation of runoff from the catchment. Hence the transfer function *T(t)* is given by

$$T(t) = pdf\left(\frac{L}{\upsilon}\right) \tag{9}$$

Combing Eq.7 and Eq.8, the resulting transfer function *T(t)* is,

$$T(t) = \frac{1}{\sqrt{2^n \Pi_{i=1}^{n} \bar{l_1}}} \frac{L^{(\frac{n}{2})-1}}{\Gamma(\frac{n}{2})} e^{\frac{-L}{2\Gamma_n}} \sum_{k=0}^{\infty} \frac{b_n(k)(\frac{n-1}{2})k}{k!(\frac{n}{2})_k} \left[\frac{\upsilon t}{2(\bar{l_n} - \bar{l_{n-1}})}\right]^k \tag{10}$$

where *T(t)* is the Transfer function, n is the order of the stream, L is the hydraulic length.

The details pertaining to the application of the methodology for the computation of runoff for two catchments is discussed in the subsequent sections.

MODEL APPLICATION

The model was applied to the sub watersheds of the Walnut Gulch at Cochise country of Arizona, USA. The Walnut gulch catchment consists of 29 nested watersheds that range in drainage area from 0.002 km^2 and 150km^2 in which sub watershed 7 with a drainage area of 13.25 km^2 and sub watershed 2 with drainage area of 2.36 km^2 were used. The rainfall and runoff data for particular storms occurring in the catchment was available from the South West Water Resource Centre, Arizona. The necessary data of the Walnut sub-catchments used for runoff computation were obtained from the South West Water Resources Centre, Arizona. The catchment characteristics of two sub-watersheds of Walnut gulch are described below.

Catchment Details

Walnut sub-watershed 2: The prevailing type of soil for the Walnut sub-watershed 2 is very gravelly sandy loam and gravelly fine sandy loam. Most of the vegetation in the catchment is shrubs and sparse grass. The infiltration was computed using the Horton's law, with the initial infiltration capacity as 4.5 mm/min and final infiltration capacity as 0.32 mm/min and the value of constant *k* as 0.14 per minute. The average stream flow velocity is given as 1.1 m/s. The input rainfall data used for runoff computations from the catchment is a rainfall having 20 minutes duration.

Walnut sub-watershed 7: The prevailing type of soil in the Walnut sub-watershed 7 is extremely cobbly loam 50% and very gravelly sandy loam and gravelly fine sandy loam for remaining 50%. Most of vegetation in this catchment is shrubs and sparse grass. The infiltration component was worked out using Horton's law making use of initial infiltration capacity as 5mm/min, final infiltration capacity as 0.45mm/min and the constant *k* as 0.13 per minute. The average stream flow velocity is given as 1.3 m/s. The input rainfall data used for runoff computation is a rainfall of 26 minutes duration.

Parameters of GIUH model

The parameters of the model are of a geomorphologic nature, and these are the Strahler's order *n* of the river basin and Horton's ratios. These parameters were assessed from digital elevation model, of the river basin, using the software Arc GIS 9.1. The procedure involves two steps (i) extraction of the parameters from the generated stream network and (ii) Estimation of Horton's ratios for the basins.

Generation of stream network

The stream network was generated from the digital elevation model (DEM) of the respective catchment. The digital elevation model (DEM) of the river catchment forms the basis of the GIS interface. A 10 m DEM of the basin was used for the generation of the stream network. From the available DEM, the catchment area was delineated.

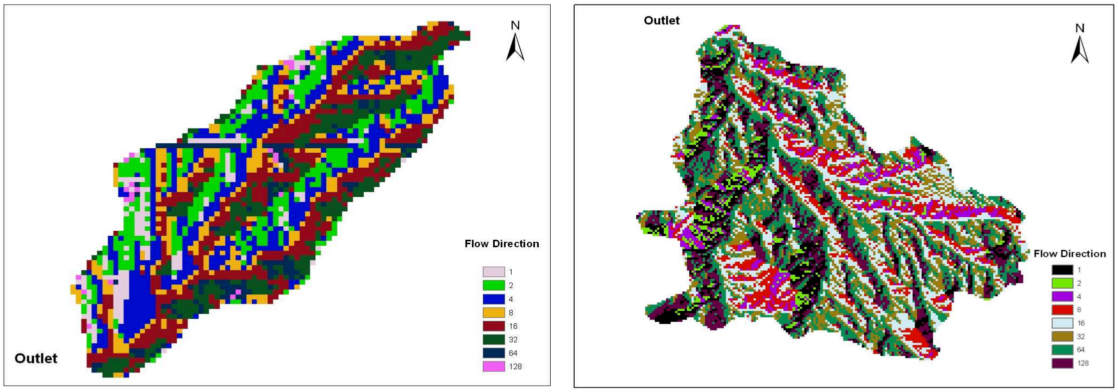

Fig. 1. Flow direction map of Walnut subcatcments 2 and 7

In order to generate the stream network, initially the flow direction map was prepared. This was accomplished by creating a flow direction file. The DEM of the area contains errors called sinks, which are cells in the DEM with a lower value than the surroundings, and these depressions are problematic because any water that flows into them cannot flow out. They were filled up before proceeding to any other operation. The reconditioned DEM was used to create flow direction raster and the flow direction map of the respective sub catchments 2 and 7 as shown in Fig.1. From the generated flow direction maps the flow accumulation maps were created. Flow accumulation function in Arc GIS software was used to create the flow accumulation map of the basins. The flow accumulation function calculates the accumulated weight of all cells flowing into each down slope cell in the output raster. The flow accumulation map of Walnut sub catchments 2 and 7 are shown in Fig. 2.

From this flow accumulation map the river network was identified. The river network was drawn in such a way that two first order streams join to form the second order stream and so on. The stream network of different order in different line weight for the Walnut sub- catchment 2 and 7 are shown in Fig. 3. The generated stream network of sub-watershed 2 was found to be a fourth order network with a total number of 129 streams and that for sub-watershed 7 a fourth order network having 258 streams, the details available in Sumi (2008).

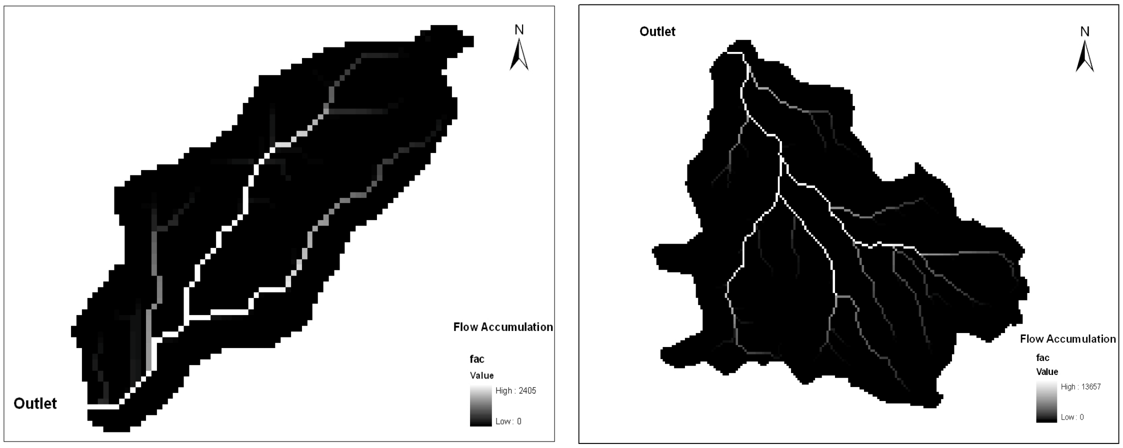

Fig 2 : Flow Accumulation map of Walnut sub-catchments 2 and 7

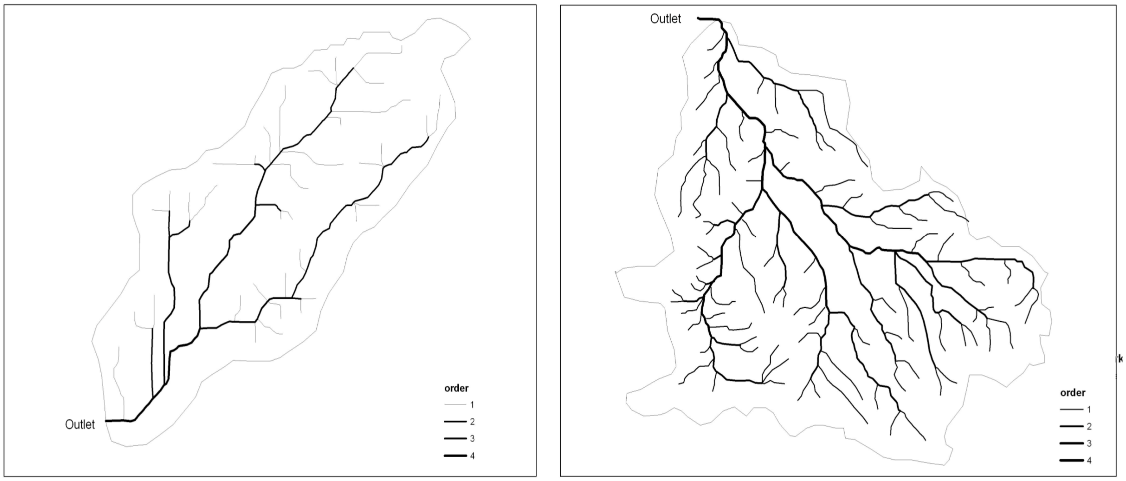

Fig.3. Stream network map of Walnut sub-catchments 2 and 7

Estimation of Horton's ratio

From the generated stream networks of the sub catchments 2 and 7, the values of Horton's ratio's R_A, R_L and R_B for the respective catchments were estimated on the basis of the number of streams, mean length of streams and the contributing area of streams, and are given in Table 1.

Table 1: Horton's Ratio for the Walnut Sub-catchments

River Basin	Estimated Horton Ratios		
	R_A	R_B	R_C
Sub-catchment 2	2.6	2.2	3.3
Sub-catchment 7	3.0	2.4	4.5

Analytical GIUH Model

For the development of analytical GIUH model, the two parameters required are the order of the river basin and its hydraulic length. For the computation of hydraulic length, the river network was subdivided into grids of area 150 m x150 m, and a point was selected in each of these grids representing raindrops falling uniformly over the river basin. The path of the raindrop down the hill slope to the stream network was determined using the elevation values in the Digital Elevation Model of the stream network. Direction of flow of the raindrop is towards the pixels whose elevation value is lower than that of the pixel where it actually falls. Thus the direction of the flow of the raindrop can be identified and the path followed by the raindrop to the first order stream can be expressed as l_0. Finally the distance from this point to the river basin outlet was assessed by identifying the path travelled by a raindrop through different Strahler's orders and their corresponding lengths $l_1, l_2, \ldots. l_n$

The distance travelled by each raindrop to the outlet is measured, the length of the travel of each raindrop in each order stream are identified. The average distance travelled in each order stream was computed which forms the model parameters for the experimental river basins and are shown in Table. 2.

Table 2: Derived values of Model parameters for the Walnut Watershed

Particulars	Subcatchment 2	Subcatchment 7
Area (km^2)	2.26	13.35
Strahler order	4	4
$\overline{l_1}$ (m)	181.74	97.41
$\overline{l_2}$ (m)	561.75	294.67
$\overline{l_3}$ (m)	1118.04	540.14
$\overline{l_4}$ (m)	1770.56	687.28

Computation of Transfer function

The parameters estimated in the above section are used to compute the geomorphologic transfer function for the catchments. The average lengths of each Strahler order stream, hydraulic length and the order of the catchment are the main parameters used for the estimation of the probability density function of hydraulic lengths. The geomorphologic transfer function was estimated using Eq. 7, for the sub-watershed 2 and 7 of Walnut Gulch catchment, where the only variable is hydraulic length (L). The hydraulic length and corresponding probability density function for the sub-watersheds were derived. The hydraulic length is plotted against the obtained probability density function for the sub-catchments as shown in Fig. 4. From the figure it can be seen that the probability density function nearly follows a gamma distribution with parameters $\alpha = 1/2$ and $\beta = 1/2l_k$. This geomorphologic transfer function is then transferred to the probability density function with respect to time using Eq 10.

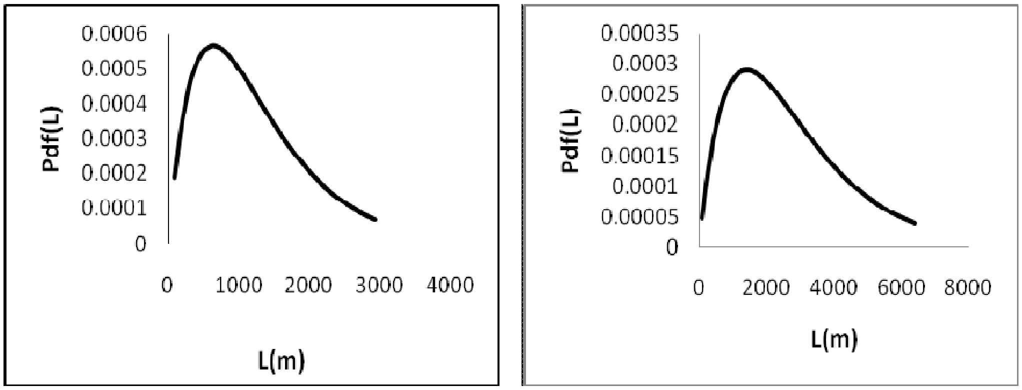

Fig. 4. Probability density function of hydraulic lengths for sub-watershed 2 & 7

Computation of runoff using AGIUH model

The runoff from the two sub catchments i.e. the sub watershed 2 and 7 of the Walnut Gulch was then estimated using the production function which is the rainfall excess for a particular storm and the transfer function, which is the probability density function of the time of travel. The simulated hydrographs using the model is given in Fig. 5.

Model Performance

The hydrologic response for the two sub watersheds 2 and 7 of the Walnut watershed for the particular storms was simulated using AGIUH models and the comparison of the generated runoff was done with the observed runoff values. To check the reliability of the model, the simulated runoff for observed rainfall for the two sub watersheds of the Walnut Gulch were compared with the observed runoff values. The comparison of the simulated and observed runoff hydrographs obtained is shown in Fig. 5.

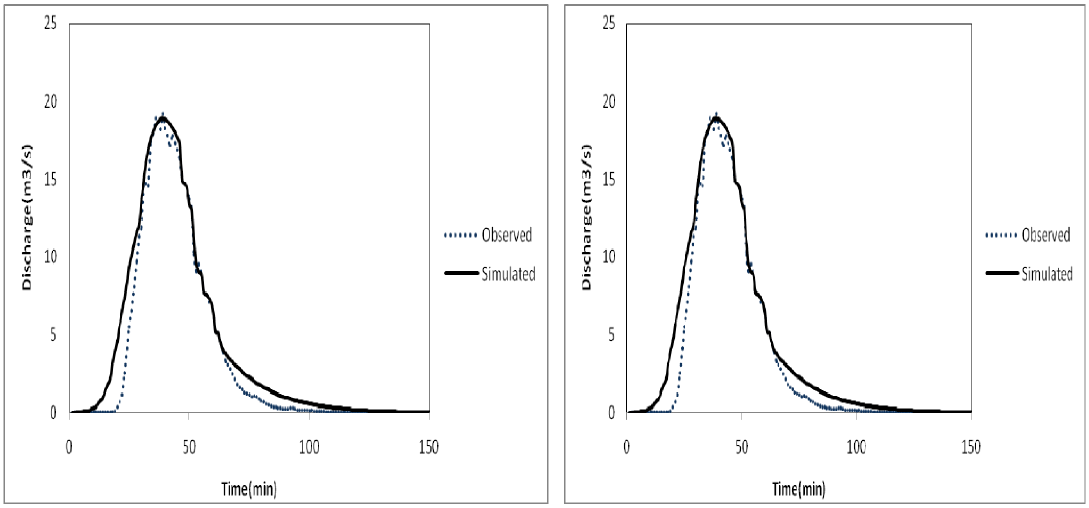

Fig. 5. Simulated vs. observed runoff of Walnut sub-watersheds 2 & 7

The degree of accuracy of runoff simulation is assessed using the Nash and Sutcliffe coefficient *F* defined as,

$$F = 1 - \frac{\sum (Q_{\exp} - Q_{sim})^2}{\sum (Q_{\exp} - \overline{Q_{\exp}})^2} \tag{11}$$

where Q_{exp} is the actual flow rate, Q_{sim} is the simulated flow rate and Q_{exp} is the average value of actual flow rate.

A Nash–Sutcliffe coefficient higher than 0.8 means a good simulation, i.e. close to the observed stream measurements. In this simulation using AGIUH model the Nash-Sutcliffe coefficient obtained for the sub watershed 2 is 0.91, and for sub watershed 7 is 0.925, which shows that the simulation is good.

The output of the AGIUH model showed a very little variation from the observed runoff in both sub-catchments and the model is reliable. It is to be noted that this model is developed taking into account the symmetry hypothesis of fractal geometry. Even though the catchment area of the two watersheds are different, (i.e. Walnut sub watershed 7 is 6 times bigger than the Walnut sub watershed 2), and the scale aspect does not limit the use of the model.

CONCLUSION

The hydrologic implication of fractals was made use of in the development of an Analytical Geomorphologic Instantaneous unit hydrograph (AGIUH) model so as to estimate the runoff from a basin. The AGIUH model was applied to the two field catchments of Walnut Gulch of Arizona, USA, and the simulated runoff compared well with the observed values. The model performance on application to the watersheds gave Nash-Sutcliffe coefficient of greater than 0.9, indicating the efficiency of the model. The AGIUH model developed on the concept of self similarity can be used for catchments of any size and is scale independent.

REFERENCES

Fleurant. C, and P. Boulestreau, 2005, Comment on A geomorphological explanation of unit hydrograph concept. *Hydrological Processes*, 19, 541- 543.

Rakesh Kumar, Chatterjee C, Singh, R.D., and A.K. Lohani, 2007. Runoff estimation for an ungauged catchment using geomorphological instantaneous unit hydrograph. *Hydrological Processes*, 21, 1829 – 1840.

Rinaldo A and I. Rodriguez-Iturbe, 1996. Geomorphological theory of the hydrological response. *Hydrological Processes*, 10, 803 – 829.

Sumi, R. 2008, Fractals and its influence on runoff computations. M.Tech. Thesis, University of Kerala.