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Uncertainty Modeling for Participation of Electric Vehicles in Collaborative Energy Consumption

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***Abstract*—This paper proposes an accurate and efficient probabilistic method for modeling the nonlinear and complex uncertainty effects and mainly focuses on the Electric Vehicle (EV) uncertainty in Peer-to-Peer (P2P) trading. The proposed method captures the uncertainty of the input parameters with a low computational burden and regardless of the probability density function (PDF) shape. To this end, for each uncertain parameter, multitude of random vectors with the specification of corresponding uncertain parameters are generated and a fuzzy membership function is then assigned to each vector. Since the most probable samples occur repeatedly, they are recognized by the superposition of the generated fuzzy membership functions. The simulation results on various case studies indicate the high accuracy of the proposed method in comparison with Monte-Carlo simulation (MCs), Unscented Transformation (UT), and Point Estimate Method (PEM). It also scales down the computational burden compared to MCs. Also, a real-world case study is employed to examine the ability of the method in capturing the uncertainty of EVs' arrival and departure time. The studies on this case reveal that involving EVs in P2P trading augments the amount of energy traded within the prosumers.**

***Index Terms*—EV Uncertainty, P2P trading, Uncertainty Modeling, Vehicle to Home.**

List of symbols and nomenclature

Indexes	
h	house
p	peers
t	time
Variables	
SoC	State of charge
$S_{EV}^{(t,e)}$	State of the charge of the e^{th} EV at time t
$C_{EV}^{(t,e)}$	Charging of the e^{th} EV at time t
$D_{EV}^{(t,e)}$	Discharging of the e^{th} EV at time t
η_{EV}^{c}	EV charging efficiency
η_{EV}^{d}	EV discharging efficiency
α_{EV}	EV charging rate
β_{EV}	EV discharging rate
$G^{(t,h)}$	Grid import of house i at time t
$I_p^{(t,h\leftarrow p)}$	P2P energy import of house h from p at time t
$X_p^{(t,h\rightarrow p)}$	P2P energy export of house h from p at time t
ψ^{p2p}	loss factor
$P_G^{(t)}$	Electricity price
d	Traveled distance of the EV
C_{eff}	Efficiency coefficient of the PEV during driving (km/kWh)
Cap	Capacity of the EV's battery
Parameters	
$t^{departure}$	Departure time of the EV
$t^{arrival}$	Arrival time of the EV
$SoC^{arrival}$	Arrival state of charge
a	Random number between 0 and 1
σ	standard deviation
μ	mean
N	Number o f samples
$X_{i\prime}$	$i^{th} sample$
X'_{ij}	j^{th} element of ascending sorted $X_{i\prime}$
F_j	j^{th} element of superposition vector F
X_1	uncertain parameter
X_2	uncertain parameter
α	Scale parameter
β	Shape parameter
γ_1	skewness
K	Kurtosis

I. Introduction

ELECTRIFICATION of the transportation systems is gaining a crucial role in solutions for environmental problems. Indeed, integrating the electric transportation assets to a grid supplied by renewable energy sources (RESs) will reduce the emission of greenhouse gasses and deal with the scarcity of non-renewable resources. The importance of this issue has been exhibited in many pieces of research, such as [1]–[7].

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Reference [1] reviews six EV charging strategies which are the optimization problem formulations in the vehicle to grid (V2G) or vehicle to home (V2H) programs. It also proposes an algorithm based on the logic of selling electricity back to the grid during peak hours. Paper [2] proposes a mixed-integer optimization that models the EVs as mobile storage. An optimal V2G model is presented by [3] which considers the battery aging of EVs along with requirements of driving patterns. A distributed control algorithm is then employed to implement the proposed strategy. Reference [4] proposes a long short-term memory (LSTM) based EV battery available capacity prediction that is going to help the frequency regulation in a micro-grid. Aggregation of the EVs in a fleet has been presented in [5]–[7] to support grid operation.

A transition from the centralized to decentralized or distributed structures of the grid operation is happening rapidly due to the higher potential for integration of the distributed resources [8]. In this situation, involving EVs, which make up a considerable part of the electric transportation systems in P2P trading programs, paves the way toward providing electrified transportation systems. The related researches to utilizing EVs in P2P trading of electricity are briefly reviewed in the following.

Reference [9] proposes a collaborative energy consumption based on the renewable energy clusters by providing the best match of the EV, demand, and renewable resources. Reference [10] has presented a P2P trading system between local plug-in electric vehicles (PEVs) that trade. An aggregator collects the bids and offers and the demand data from EVs and determines the optimal P2P prices. Paper [11] has proposed P2P trading model to buy and sell electricity among local plug-in hybrid electric vehicles (PHEVs). This study satisfies the prerequisites such as security and privacy by consortium Blockchain. The presented approach has tried to issue security and privacy as well as mobility. It is to mention that, in [11], EVs are the only participants in the P2P market, and the other types of traders have not been considered. Proposing blockchain-based methods can reduce credit costs and enhance renewable energy integration. So, in [12], a trading allocation based on the private chain of blockchain has been provided. Also, Monte Carlo simulation has been used to show the uncertain nature of charging stations' charging demand. Another work [13] employs a private blockchain method to prove transaction records between EVs. This framework relies on a private blockchain-based P2P electricity trading approach to obtain secure electricity trading. Paper [14] has introduced a smart contract and blockchain-based energy trading system that directly provides conditions for direct interaction between providers and EV owners. This framework depends on utility companies for metering and billing to prevent significant infrastructure changes. Although blockchain can facilitate decentralization and security requirements, it does not solve all problems of distributed structures, such as performance efficiency [15]. Hence, Trading strategies for inter vehicles (V2V) was analyzed in [15] which, addresses the problems of conventional blockchain. In [16], a P2P method for energy trading in the local electricity market was utilized. This method can help the PV owners to achieve more accuracy in the forecast. In another study [17], P2P trading through DSO, as the central party, has been presented. DSO keeps the overall data of all users and links prosumers and consumers. In [18], a novel approach for EVs' charging and discharging has been introduced. For the validation process, the presented method has been compared with standard consensus methods. Moreover, a new proof-of-Benefit (PoBen) consensus protocol was proposed that fills the gap of previous consensus methods. The experimental results demonstrated that PoBen method developed the security and sustainability of power fluctuations.

Generally, the uncertainty of a problem can be modeled by different methodologies , such as probabilistic methods [19], possibilistic approaches [20], hybrid methods [21], information gap decision theory [22], robust optimization [23], and interval analysis [24], depending on different factors. For example, probabilistic methods are applicable in cases that the PDF of the uncertain parameters are known. In contrast, possibilistic methods are not based on the PDF and use fuzzy functions to capture the uncertainty. Combination of these two methods are called hybrid models. However, information gap decision theory, robust optimization, and interval analysis, are based on the measurement error or estimation of the parameters, feasibility in the worst case, and uniform PDF, respectively. Since the proposed method can be considered a probabilistic method, more relevant papers are reviewed.

Owing to recent developments in the advanced technologies of electric vehicles, many sources of uncertainty have appeared in the energy systems. This shows the necessity of developing accurate and fast uncertainty modeling methods to enhance the decision-making of energy systems. Monte-Carlo simulation, which is well known for its high accuracy, is very time-consuming [6] and may not be practical in a wide range of applications. This drawback has been partially addressed by some techniques such as PEM [25], UT [26] and various scenario reduction methods. Importance sampling (IS) methods such as Cross-Entropy involve finding a distribution that estimates necessary samples of uncertain elements. In [27], the authors implemented a method using the cross-entropy function to minimize the distance between sampling distribution and the original one. In [28], the stratified sampling Monte Carlo method was employed to calculate the lightning performance of transmission and distribution systems. Non-Sequential Monte Carlo simulation was used in [29] to model statistically dependent time-varying quantities, including renewable energy sources. In order to reach a practical and logical computational burden, scenarios with high similarity or low probability were omitted from the scenario set in [30]. Compression of scenarios by scenario mapping technology was applied on the uncertain behavior of wind power in [31]. Authors in [32] provided a comparison on random sampling, importance sampling inspired method, distance-based method, and stratified scenario sampling as scenario reduction methods. They concluded that the scenario reduction methods could effectively reduce the size of the larger models with a complete set of scenarios. However, for their case study, the distance-based method is the most accurate among the others.

Although these methodologies are accurate and more time efficient, they face some limitations in the complex analysis. For example, PEM is not capable of capturing the uncertainty in the correlated environment [25]. As for UT, it can be employed just when uncertain parameters have a symmetric distribution such as Normal [26].

Furthermore, in these methods, the computational burden is dependent on the number of uncertain parameters and can demand more computational effort than MCs in large-scale problems. This paper proposes a new uncertainty modeling method with high accuracy but a meager computational effort to deal with these problems. The main contributions of the paper are summarized as follows:

- Since the EV owners potentially could participate in the energy communities, this paper aims to analyze the

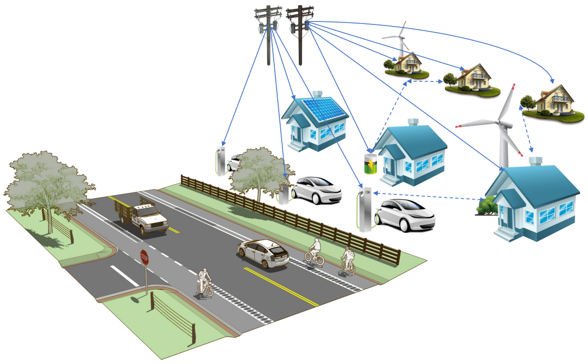

Fig. 1. Schematic illustration of a community made up of buildings - residential or office- as well as EV charging nodes

impact of the uncertain EV owner behaviors on the energy trading in a community.

- The EVs' arrival and departure times usually follow known behaviors. However, these behaviors can be estimated by a combination of the different PDFs. So, the paper's second contribution is to propose a probabilistic uncertainty modeling approach that is not dependent on the features of the PDF, such as correlation, skewness, and so on.
- Analyzing the impact of local energy sharing on the distribution feeders under EV uncertainty is the third contribution of this paper.

The remainder of this paper is organized as follows. The employed P2P trading framework, as well as the uncertain behavior of the EVs, are explained in section II. The proposed uncertainty modeling is then described in section III. Section IV deals with evaluating the performance of the proposed method through four case studies covering a wide area and various situations. Finally, section V wraps the paper up with a conclusion.

II. EV Uncertainty in P2P Trading

In this section, we describe the community-based P2P trading, which has been proposed in [33]. This framework provides the opportunity for energy trading between various prosumers that build a community together. Indeed, the required energy of the prosumers and consumers is provided by the main grid, the renewable sources, as well as P2P trading with other members of the community shown by dashed arrows in Fig. 1. Since the houses in a neighborhood or the buildings in commercial or official centers can form a community, analysis in [3], [34], [35] are employed to model the uncertainties in arrival, sojourn, and departure times of the EVs which are charged near the owners' house or workplace.

Equations (1) to (6) describe the structure of the community-based P2P trading. The objective function (1) minimizes the cost of energy importing from the main grid. Equation (2) models the P2P energy trading between houses h and p, considering the loss factor ψ^{P2P}. Each house can trade with its peers at each time-step. So, equations (3) and (4) show the total Export and export of each house at each time-step, respectively. Since all trades happen within the community, the total amount of imports is proportional to the exports as it is shown in (5).

$$OF = \min \left\{ \sum_h \left(\sum_t \left[p_G^{(t)} \cdot G^{(t,h)} \right] \right) \right\} \tag{1}$$

$$I_p^{(t,h\leftarrow p)} = \psi^{P2P} \cdot X_p^{(t,p\rightarrow h)} \qquad \forall\, p \neq h, \tag{2}$$

$$X^{(t,h)} = \sum_{p \neq h} X_p^{(t,h\rightarrow p)} \tag{3}$$

$$I^{(t,h)} = \sum_{p \neq h} I_p^{(t,h\leftarrow p)} \tag{4}$$

$$\sum_h \psi^{P2P} \cdot X^{(t,h)} = \sum_h I^{(t,h)} \qquad \forall\, t \in T. \tag{5}$$

Finally, (6) balances the input and output energy of each house at each time-step. In these equations, $p_G^{(t)}$, $G^{(t,h)}$, $I_p^{(t,h\leftarrow p)}$, $X_p^{(t,p\rightarrow h)}$ are energy price, grid import of house i, energy import of house h from p and energy export of house p to h, all at time-step t respectively.

$$\overbrace{res^{(t,h)} + G^{(t,h)} + D_{ev}^{(t,h)} + I^{(t,h)}}^{\text{RES + Grid + EV disch. + P2P purchase}} \geq \overbrace{dem^{(t,h)} + C_{ev}^{(t,h)} + X^{(t,h)}}^{\text{Demand + EV charge + P2P sale}} \tag{6}$$

The equations related to battery or EV state of charge (SoC) must be included for the corresponding prosumers, as can be seen in equations (7) to (11).

$$S_{EV}^{(t,e)} = S_{EV}^{(t-1,e)} + \eta_{EV}^c \cdot C_{EV}^{(t,e)} - (1/\eta_{EV}^d) \cdot D_{EV}^{(t,e)} \tag{7}$$

$S_{EV}^{(t,e)}$, $C_{EV}^{(t,e)}$, $D_{EV}^{(t,e)}$, η_{EV}^c, and η_{EV}^d are state of charge, charging, discharging, charging efficiency, and discharging efficiency, respectively. Equations (8) and (9) specify the EVs' state of charge at their arrival and departure times, respectively.

$$S_{EV}^{(t,e)} = S_{EV}{}^{(e)}_{Arrival} + \eta_{EV}^c \cdot C_{EV}^{(t,e)} - (1/\eta_{EV}^d) \cdot D_{EV}^{(t,e)} \quad t = Arrival \tag{8}$$

$$S_{EV}^{(t,e)} = S_{EV}{}^{(e)}_{Departure} \quad t = Departure \tag{9}$$

Finally, Eq. (10) and (11) define the charging and discharging rate of the EVs.

$$0 \leq C_{EV}^{(t,e)} \leq \alpha_{EV} \tag{10}$$

$$0 \leq D_{EV}^{(t,e)} \leq \beta_{EV} \tag{11}$$

As can be seen, the arrival and departure times are required to model the EVs' participation in the community. According to Eq. (12), the departure time can be calculated based on arrival and sojourn times.

$$t^{departure} = t^{arrival} + t^{sojourn} \tag{12}$$

The authors in [34], [35] have analyzed real-world data sets and classified the EVs into three clusters named "Charge near work (CNW)", "Charge near home (CNH)", and "Park to charge" clusters, considering the influence of weekends and seasonal changes. Then, they have fitted distributions to the sojourn time of EVs belong to each cluster. Table I shows the probability density functions of sojourn time for the first and second behavioral clusters[1]. It must be noted that the presented PDFs are based on normalized sojourn time concerning the last column ([min max]) of Table I.

There are different approaches for estimation of a PDF for the EVs' arrival times. In [3], a normal distribution has been assumed for the arrival time of the EVs to work or home. In another approach, according to [35] the arrival times can be uni/multi-modal or skewed in various situations. In ref [36], the arrival times are generated based on a normal distribution. However, the average arrival time of the different EVs is generated by a Pearson distribution.

In this study, we assume a combination of two normal distributions for multi-modal situations and one normal PDF for the uni-modal cases as can be seen in Eq. (13) and (14), respectively.

$$f_{CNH}(x) = \begin{cases} \frac{1}{0.5\sqrt{2\pi}} \exp^{\frac{-(x-33)^2}{2\times 0.5^2}} & a < 0.5 \\ \frac{1}{1\sqrt{2\pi}} \exp^{\frac{-(x-40)^2}{2\times 1^2}} & \text{otherwise} \end{cases} \tag{13}$$

$$f_{CNW}(x) = \frac{1}{\frac{4}{3}\sqrt{2\pi}} \exp^{\frac{-(x-14)^2}{2\times(\frac{4}{3})^2}} \tag{14}$$

Which a is a random number between 0 and 1 with uniform distribution. So, in 50 percent of situations, a normal distribution with $\mu = 33$ and $\sigma = 0.5$ represents the EV arrival for CNH cluster. In the rest, another normal PDF with $\mu = 40$ and $\sigma = 1$ shows the behavior of the arrival time for the mentioned cluster. Also, a normal distribution with $\mu = 14$ and $\sigma = \frac{4}{3}$ is considered for the arrival times of CNW cluster. It is worth noting that the mentioned formulation of the community-based P2P trading is a day ahead schedule with 30 min time-steps (48 time-steps for 24 hours). Therefore, mean and standard deviations in equations (13) and (14) refer to the time step and standard deviation of the EV arrivals in each cluster. It is worth noting that the departure time is a summation of two uncertain parameters with logistic and a combination of normal distributions. As we don't have the exact data for the skewness of the PDFs when they are skewed, we just follow the assumption in [3]. This assumption will not affect the proposed method proficiency, as we will show it's ability in modeling the correlated-uncorrelated and symmetric/non-symmetric uncertain parameters. Moreover, arrival SoC would be the last uncertain parameter related to the EV behavior. Indeed, the arrival state of charge depends on the storage capacity, efficiency coefficient of the EV during driving ([km/kwh]) as well as the traveled distance, and can be calculated according to (15) [37].

$$SoC^{arrival} = 100 - \frac{d}{C_{eff} \times Cap} \tag{15}$$

In this equation, the traveled distance is uncertain and leads to uncertain EVs' state of charge at their arrival. Paper [38] uses a generalized extreme value distribution with $\mu = 17.27$, $\sigma = 0.84$, and $k = -0.06$ to model this uncertain parameter.

[1]We assume the communities contain residential, office or commercial prosumers.

TABLE I
PROBABILITY DENSITY FUNCTION OF SOJOURN TIME FOR "CHARGE NEAR WORK", "CHARGE NEAR HOME" CLUSTERS FOR THE FIRST 24 HOURS [34]

	PDF	Location	Scale	[min max] hours
Charge near work	Logistic	0.27	0.06	[5.00 18.52]
Charge near home	Logistic	0.56	0.08	[0.02 23.99]

III. UNCERTAINTY MODELING

Monte-Carlo simulation is a powerful tool to map the uncertain behavior of the input parameters to output in a process. However, it needs a high number of iterations and can be time-consuming. So, scenario reduction methods that can reduce the required iterations gain importance. This section introduces a heuristic approach to capture the main behavior of the uncertain parameter by a few points. Assume X is an uncertain parameter with an arbitrary PDF, e.g. see the blue curve in Fig. 2-(a-c). A considerable run-time reduction can happen if the most probable samples are selected and other samples that are not as important as previous ones are omitted. To highlight the most probable scenarios of uncertain parameter X, N vectors ${X_i}'_{[1\times n]}$ are generated according to the specifications of the PDF of X by Eq. (16). Then, the mean value (μ_i) and standard deviation (σ_i) of each vector are calculated using (17) and (18).

$$X_i' = random('PDFofX', shape, scale, [1, n]) \tag{16}$$

$$\mu_i = \frac{1}{n}\sum X_i' \qquad \forall i = 1:N \tag{17}$$

$$\sigma_i = \sqrt{\frac{1}{n}\sum |X_i' - u_i|} \qquad \forall i = 1:N \tag{18}$$

Because of probabilistic nature, and may be different in each generated vector in case of low number of samples. Each time, the period of $C_i = [\mu_i - k\sigma_i, \mu_i + k\sigma_i]$ is selected and an arbitrary fuzzy function is set on it. The main reason for producing a reasonably unbiased PDF is that the aggregated fuzzy function would peak around the critical samples. However, the amplitude of the peak may differ depending on the importance of the samples. So, the amplitude of each peak is considered as the selection criteria. For instance, a Gaussian membership function can be indicted by (19).

$$f_{ij}(X_{ij}', \sigma_i, u_i) = e^{-\frac{(X_{ij}')^2}{2\sigma_i{}^2}} \qquad \forall i = 1:length(C_i) \tag{19}$$

X_{ij}' is j^{th} element of ascending sorted X_i'. This period and its corresponded fuzzy membership function for all N vectors have been shown in Fig. 2-(a) and 2-(b) respectively. As can be seen in Fig. 2-(c), the most probable values for uncertain parameter X can be detected by superposition of fuzzy functions. Equation (20) shows the element-wise superposition of produced fuzzy functions with (19).

$$F_j = \sum_{i=1}^{N} f_{ij}(X_{ij}', \sigma_i, u_i) \tag{20}$$

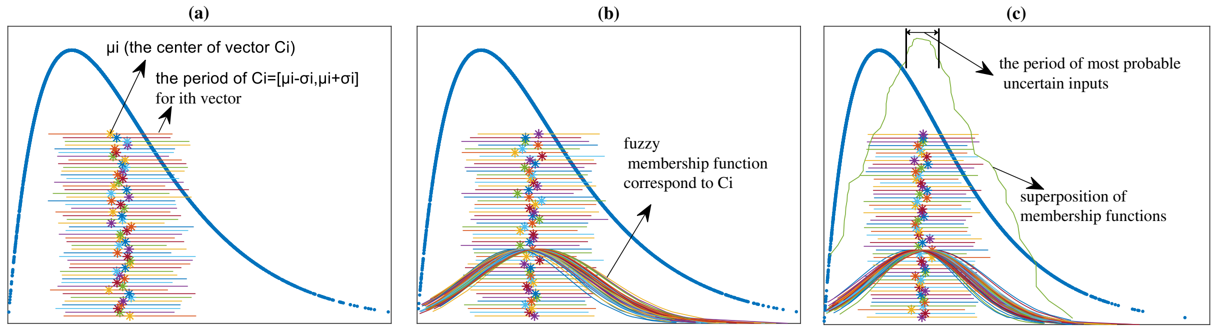

Fig. 2. (a) probability density function and confidence levels of uncertain input in various generations- (b) corresponding fuzzy membership function to each confidence level- (c) superposition of membership functions.

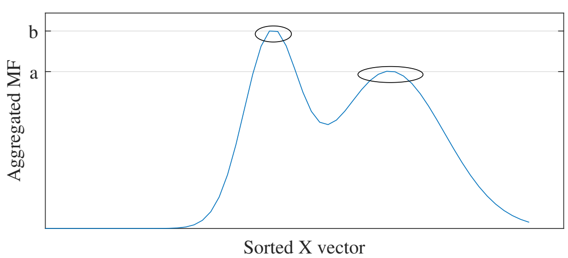

Fig. 3. Multi-modal aggregated fuzzy FM

Where F_j is j^{th} element of superposition vector F. In fact, important samples are further repeated during sampling and more number of fuzzy membership functions will be added together. This leads to the fact that peak of vector F corresponds to most probable sample.

In some situations, the superposition of the fuzzy MFs leads to a multi-modal curve like Fig. 3. In such situations, although the value of one local peak may be lower than the others, the peak states that there are some important scenarios for the uncertain parameter around that area. So, for selecting the most probable samples all points around the local peaks of the F function must be taken into account. Finally, the number of the samples around each peak is proportional to the peak value of F in peak points i.e. a and b in Fig. 3.

Table II provides pseudo code of the presented method.

IV. Assumptions

This section briefly summarizes the assumptions made in this study.

- It is assumed that the arrival time of the CNW EVs, follows a normal distribution.
- It is assumed that the arrival time of the CNH EVs, follows a combination of two normal distribution as can be seen in (13). The histogram of the generated numbers by this equation has two modes and is not symmetric.
- It is assumed that the sojourn times of the CNH and CNW categories follow the distributions shown in table I. The sojourn time is not.
- There is no feed-in tariff in the community model.
- It is assumed that the community manager operates the community independent of the grid operator.
- It is assumed that the residential loads have a constant power factor.

V. Simulation Result

In this paper, two case studies are employed to illustrate the impact of the EV uncertainty on collaborative energy consumption in a community. Since the community is operated by a community manager independent of the grid operator, the first case focuses on the community model and ignores the physical grid. The second case study deals with the propagation of the EV uncertainty on the grid parameters, mainly voltage magnitude. The cases, and results are explained in the following subsections.

A. Case I: EV Uncertainties in collaborative energy consumption

A neighborhood consisted of 25 houses located in the UK has been employed to evaluate the performance of the proposed method. More information can be found in [9]. As can be seen in Table III, some buildings own assets like 4 kWh storage, 2 kW and 4 kW PVs as well as 2.3 kW wind

TABLE II
Proposed algorithm

Step 1	**// Recognition of important samples**
	for $1:N$ times // N is the number of superposition Generate $1 \times n$ vector X_i' with specification of X Fit the fuzzy MF to ascending sorted X_i' vector end add generated fuzzy functions Select the highest period as the most probable samples // Step 1 must be repeated for each uncertain parameter.
Step 2	**//Mapping of input uncertainty to output**
	Calculate the output function for selected inputs
Step 3	**//Calculation of output uncertain behavior**
	Compute the output parameters

TABLE III
DESCRIPTION OF THE COMMUNITY ASSETS IN CASE IV

Method	Houses
2 [kW] PV	2, 7, 8, 9, 16, 20, 24, 25
4 [kW] PV	5, 15, 23
2.3 [kW] Wind	3, 15, 20, 25
4 [kWh] Storage	5, 15, 23

turbines. The real-world data used in [33] is employed for the mentioned assets. According to this data and configuration of the community, 37 % of the annual demand of the whole community is covered by renewable sources, approximately. Besides, EVs that belong to people who are living or working in the community, can participate in P2P trading program. It's worth noting that the simulations of this case study have been done for one month in spring, considering the seasonal impact on the EV availability patterns based on equations (13) and (14). It is assumed that three EVs owned by residents of the neighborhood (CNH cluster) join the community every day. Also, two EV owners who work near that area (CNW cluster) prefer to participate in P2P program of the community. Also, according the model, EVs should be charged to a certain level before they leave. The demand profiles of the first case have been formed based on smart meter energy consumption data in London households, as part of Low Carbon London project https://data.london.gov.uk/dataset/smartmeter-energy-use-data-in-london-households. The information related to the wind and solar profiles, as well as the half-hourly energy prices are accessible through the following GitHub page: https://github.com/LocalEnergyMarkets/PCDGModel-LocalCommunities

In the following, we analyze the EV uncertainties as well as operation in two situations. At first, we assume the EVs have one-way charger and can import energy from the main grid or the other prosumers in the community. In the other situation, the EVs can actively participate in the P2P trading due to their bidirectional chargers.

The calculated confidence levels which are illustrated in Fig. 4 cover the deterministic value of the objective function in Eq. (1) for the simulation period. It shows the ability of the proposed method in mapping the input uncertainty with various distributions or a combination of PDFs like Eq. (13) to the output function. To calculate these confidence intervals, we have only selected 20 samples of each uncertain parameter using the proposed method. Table IV compares the performance of the method in terms of confidence levels for two cases, i.e., 20 scenarios and 5 scenarios. Indeed, the number of scenarios refers to the number of the most probable samples that are recognized by the proposed superposition method. In case of 20 scenarios, relatively higher confidence levels are achieved due to covering more scenarios than the other case shown in Table IV. To better analyze the performance of the proposed method, day 25 -with the biggest standard deviation as can be seen in Fig. 4- has been selected for more investigations. Figure 5 illustrates the calculated histogram of the community cost based on 20 scenarios. Due to the low number of the selected samples, only the most probable scenarios are

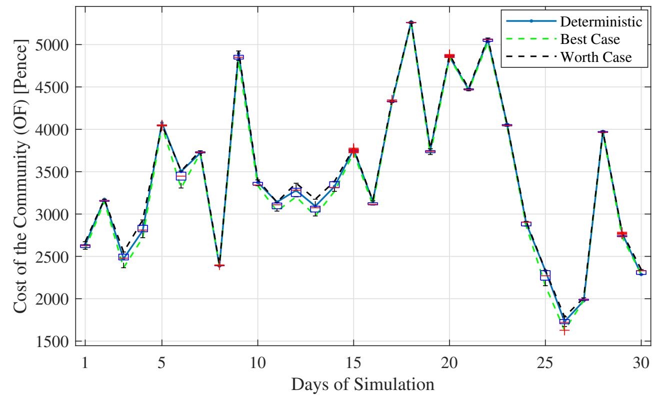

Fig. 4. Comparison of the deterministic and probabilistic community costs for 30 days.

TABLE IV
COMPARISON OF THE RELATIVE CONFIDENCE LEVELS OF 20 SCENARIOS WITH 5 SCENARIOS

Number of Scenarios	Max Confidence [%]	Avg Confidence [%]
20	9.1433	2.7674
5	8.2390	2.0067

illustrated in this histogram. Scenarios with a cost around 2320-2340 [Pence] have repeated 7 times. The deterministic value of the community cost for the same day is 2331 [Pence] which has been covered by the proposed method. Table V reveals the impact of the EVs in the amount of the total P2P trading in the community. As can be seen, involving EVs - even with unidirectional chargers- in P2P trading augments the volume of energy traded. This impact scales up when the EVs can actively participate in the trades, using bidirectional chargers. An interesting finding is the different tendency of the mentioned clusters in P2P trades, as can be seen in Fig. 6. The EVs belong to CNH cluster tend to export energy to their peers, when is possible. It means that in case of using the bidirectional chargers for charging EVs near the houses, the EVs behave like the stationary batteries in their availability. On the other hand, the EVs belong to CNW tend to import energy from the other prosumers. It's due to the simultaneous availability times and PV production periods.

The energy community model has been implemented in Matlab R2019b. Solving the community model for 25 houses and 5 EVs (on a laptop: RAM 32 GB, CPU intel core i 7) on average takes 11 sec. This run-time includes building and solving the model using the linprog solver. It should be noted that the sparse implementation of the model can speed up the building process. However, in this case, the time reported is based on the Matlab optimization toolbox. The proposed method reduces the number of runs (compared to MCs) by eliminating the unimportant sample points.

B. Case II: The impact of EV Uncertainties in collaborative energy consumption on the grid operation

This case study focuses on the impact of energy sharing on the distribution feeder that supplies the end-users. In this

TABLE V
IMPACT OF THE EVS IN P2P TRADING - ONE MONTH RESULT

Total trades [kWh]	No EV	One-way charger	Bidirectional charger
Import-all participants	1088	1154	1356
Export-all participants	1178	1249	1467
Import-EVs	-	139	161.4
Export-EVs	-	0	241.5

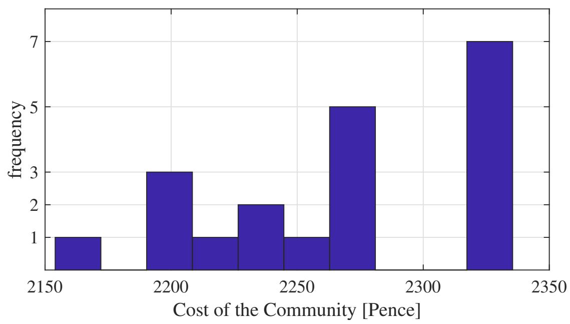

Fig. 5. Histogram of the community cost with 20 scenarios

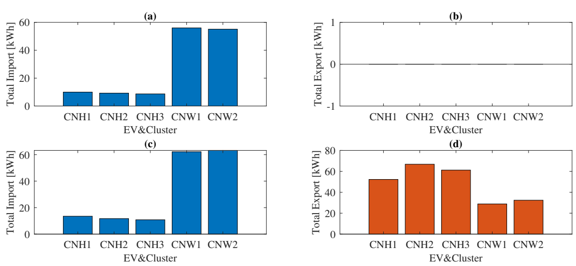

Fig. 6. Total P2P trading of EVs- (a) Import, Unidirectional charger - (b) Export, Unidirectional charger - (c) Import, Bidirectional charge - (d) Export, Bidirectional charger

case, in addition to the demand profiles, prices, and solar profiles, a low voltage distribution feeder is employed to evaluate the energy sharing impact. This feeder (feeder lvgd-2388 connected to the medium voltage grid called mvgd-2) is not a real but a realistic synthetic low voltage grid generated by the Ding0 package[2] in python. The feeder has 84 buses and 83 branches and supplies 27 residential end-users, and the nominal voltage is 0.4 kV. The grid topology and the line characteristics are shown in the appendix. Since this synthetic grid only provides information about one snapshot of the demand and distributed generation, various energy consumption, and generation profiles are assigned to different grid nodes. In addition, there are 5 EVs in the neighborhood. The battery capacity of the EVs in this case is 50 kWh with a round-trip efficiency of 96% as the average of Nissan Leaf, Volkswagen e-Golf and Tesla S [9]. The charging and discharging rate of the EVs are set to 7.3 kWh per hour. It is worth noting that the profiles are based on the same references as the previous case. To analyze the impact of the local energy transactions on the grid, the community is first analyzed under the uncertain behavior of the EVs, regardless of the grid constraints. Then, the ex-post analysis is conducted to understand the impact of

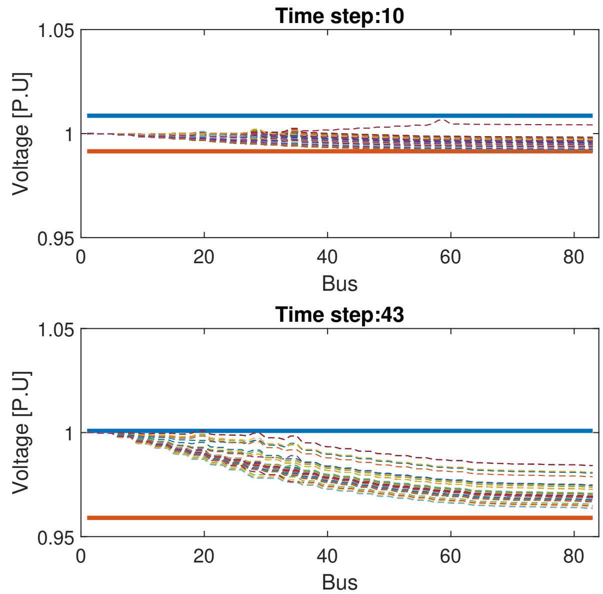

Fig. 7. Comparing the performance of the proposed method with MCs

the EV uncertainty on the grid. In other words, the outcome of the energy community is converted to the active power injections into different nodes of the grid. The reactive power injections also are estimated based on the loads' power factor. The Matpower toolbox[3] then is employed to run the powerflow calculation on the grid. This assumption is based on [39] and [40], that have separated the market and grid layers.

Before dig in the result of this case, it is worth mentioning that the this case is based on 100 scenarios extracted from 1000 scenarios. Figure 7 compares the voltage profiles obtained by each scenario with the minimum and maximum voltage magnitudes given by MCs. Indeed, the solid blue and red lines illustrate the maximum and minimum voltages over the feeder during the day calculated by MCs (850 scenarios). The dashed lines are the voltage profiles for extracted scenarios. Two time steps, representative of low load (10 - 05:00), afternoon (30), and high load (43 - 21:30) are exhibited in figure 7. As can be seen, the voltage profiles lay in the voltage range estimated by MCs. This figure also indicates that energy sharing does not jeopardize the grid in terms of over or under-voltage problems, even under heavy load. Obviously, this is not a general conclusion and is relevant for the case study. However, there might be some situations, especially in the future, that the end-users may experience overvoltage due to either EV participation in energy sharing programs or an increase in the share of renewable generation. Comparing the histograms of the energy imported from the grid for these two cases could be interesting. Figure 8 shows the histograms of the energy imported from the main transformer at 05:00 and 21:30. Although the histograms do not represent a specific PDF, the proposed method provides similar histograms to the ones obtained by MCs.

[2]https://dingo.readthedocs.io/en/dev/welcome.html

[3]https://matpower.org/

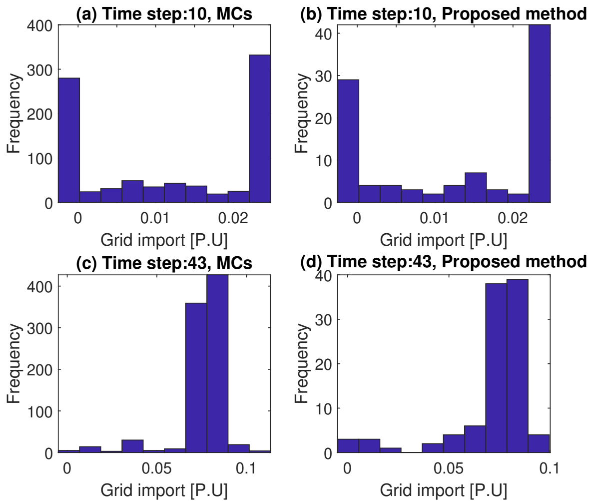

Fig. 8. Comparing the histograms of the grid import, obtained by the proposed method and MCs

VI. Conclusion

This paper proposed a novel and accurate stochastic method for uncertainty modeling based on the superposition of uncertain input parameters. To this end, the most significant and probable samples of the input uncertain parameters are recognized through the superposition of various sampling vectors under fuzzy transformation. After forming some vectors containing samples of the uncertain parameter, an arbitrary fuzzy membership function is assigned to each vector. The simulation results show the appealing performance of the proposed method with high accuracy and a very low computational burden. As another significant feature of the proposed method, it can calculate the output histogram with just a few number of input samples. These findings reveal a promising role for the proposed method in modeling the uncertainty effects in the real practical power system problems. For example, EVs that are being utilized increasingly impose uncertainty on the grid or market operation. Besides, their arrival or sojourn times may have bi-modal and skewed or non-skewed PDFs that can not be modeled by the methods like PEM or UT accurately. But, the proposed method showed satisfactory performance in the described situation. Such a special feature can play an important role in addressing the big issues of computational burden, high complexity and low accuracy in the literature. Taken together, this paper has identified the tendency of the EVs to participate in local P2P tradings. Indeed, the EVs that are charged near the owners' houses tend to export energy to the other prosumers. Because they are connected to the charging stations in the evening and leave the house in the morning. So, case of using bidirectional chargers, their operation is similar to stationary batteries in the sojourn time. It means that active operation of the EVs belong to CNH cluster in P2P trading can increase the flexibility of the local markets. On the other hand, the availability time of the CNW cluster is from the morning when the people go to their offices until they go back to their homes. So, they tend to import energy from the neighborhood buildings that have renewable productions.

To sum up, regarding the input data, the energy price, renewable profiles, and the demand of the houses are available and can be easily found. However, the arrival and departure times of the EVs for a certain period are not available to be considered as the basis of the comparisons. The results have been compared to the artificial EV data generated by the known PDFs.

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References

[1] H. Turker and S. Bacha, "Optimal Minimization of Plug-In Electric Vehicle Charging Cost With Vehicle-to-Home and Vehicle-to-Grid Concepts," in IEEE Transactions on Vehicular Technology, vol. 67, no. 11, pp. 10281-10292, Nov. 2018, doi: 10.1109/TVT.2018.2867428.

[2] Triviño-Cabrera A, Aguado JA, Torre Sd l. Joint routing and scheduling for electric vehicles in smart grids with V2G. Energy 2019;175:113-22.

[3] K. Ginigeme and Z. Wang, "Distributed Optimal Vehicle-To-Grid Approaches With Consideration of Battery Degradation Cost Under Real-Time Pricing," in IEEE Access, vol. 8, pp. 5225-5235, 2020, doi: 10.1109/ACCESS.2019.2963692.

[4] Qingqing Yang, Jianwei Li, Wanke Cao, Shuangqi Li, Jie Lin, Da Huo, Hongwen He. An improved vehicle to the grid method with battery longevity management in a microgrid application. Energy 198 (2020) 1173742.

[5] S. Amamra and J. Marco, "Vehicle-to-Grid Aggregator to Support Power Grid and Reduce Electric Vehicle Charging Cost," in IEEE Access, vol. 7, pp. 178528-178538, 2019, doi: 10.1109/ACCESS.2019.2958664.

[6] A. Kavousi-Fard, T. Niknam and M. Fotuhi-Firuzabad, "Stochastic Reconfiguration and Optimal Coordination of V2G Plug-in Electric Vehicles Considering Correlated Wind Power Generation," in IEEE Transactions on Sustainable Energy, vol. 6, no. 3, pp. 822-830, July 2015, doi: 10.1109/TSTE.2015.2409814.

[7] S. Pirouzi, J. Aghaei, V. Vahidinasab, T. Niknam, A. Khodaei, Robust linear architecture for active/reactive power scheduling of EV integrated smart distribution networks, Elec. Power Syst. Res. 155 (2018) 8e20.

[8] K. E. Antoniadou-Plytaria, I. N. Kouveliotis-Lysikatos, P. S. Georgilakis and N. D. Hatziargyriou, "Distributed and Decentralized Voltage Control of Smart Distribution Networks: Models, Methods, and Future Research," in IEEE Transactions on Smart Grid, vol. 8, no. 6, pp. 2999-3008, Nov. 2017, doi: 10.1109/TSG.2017.2679238.

[9] Hashemipour N., Crespo del Granado P., Aghaei J., "Dynamic allocation of peer-to-peer clusters in virtual local electricity markets: a marketplace for ev flexibility," in Energy, 0360-5442 (2021), p. 121428, https://doi.org/10.1016/j.energy.2021.121428

[10] R. Alvaro-Hermana, J. Fraile-Ardanuy, P. J. Zufiria, L. Knapen and D. Janssens, "Peer to Peer Energy Trading with Electric Vehicles," in IEEE Intelligent Transportation Systems Magazine, vol. 8, no. 3, pp. 33-44, Fall 2016, doi: 10.1109/MITS.2016.2573178.

[11] J. Kang, R. Yu, X. Huang, S. Maharjan, Y. Zhang and E. Hossain, "Enabling Localized Peer-to-Peer Electricity Trading Among Plug-in Hybrid Electric Vehicles Using Consortium Blockchains," in IEEE Transactions on Industrial Informatics, vol. 13, no. 6, pp. 3154-3164, Dec. 2017, doi: 10.1109/TII.2017.2709784.

[12] Ruijiu Jin, Xiangfeng Zhang, Zhijie Wang, Wengang Sun, Xiaoxin Yang, Zhong Shi, 2019. "Blockchain-Enabled Charging Right Trading Among EV Charging Stations," Energies, MDPI, Open Access Journal, vol. 12(20), pages 1-22, October.

[13] H. Liu, Y. Zhang, S. Zheng and Y. Li, "Electric Vehicle Power Trading Mechanism Based on Blockchain and Smart Contract in V2G Network," in IEEE Access, vol. 7, pp. 160546-160558, 2019, doi: 10.1109/ACCESS.2019.2951057.

[14] N. Lasla, M. Al-Ammari, M. Abdallah and M. Younis, "Blockchain Based Trading Platform for Electric Vehicle Charging in Smart Cities," in IEEE Open Journal of Intelligent Transportation Systems, vol. 1, pp. 80-92, 2020, doi: 10.1109/OJITS.2020.3004870.

[15] G. Bansal and A. Bhatia, "A Fast, Secure and Distributed Consensus Mechanism for Energy Trading Among Vehicles using Hashgraph," 2020 International Conference on Information Networking (ICOIN), Barcelona, Spain, 2020, pp. 772-777, doi: 10.1109/ICOIN48656.2020.9016440.

[16] Z. Zhang, R. Li and F. Li, "A Novel Peer-to-Peer Local Electricity Market for Joint Trading of Energy and Uncertainty," in IEEE Transactions on Smart Grid, vol. 11, no. 2, pp. 1205-1215, March 2020, doi: 10.1109/TSG.2019.2933574.

[17] Liu Y, Wu L, Li J. Peer-to-peer (P2P) electricity trading in distribution systems of the future. Electr J 2019;32(4):2–6.

[18] Liu, C., Chai, K.K., Zhang, X. et al. Peer-to-peer electricity trading system: smart contracts based proof-of-benefit consensus protocol. Wireless Netw (2019). https://doi.org/10.1007/s11276-019-01949-0

[19] Dantzig GB.Linear programming under uncertainty. Manag Sci 1955; 1:197–206.

[20] Zadeh L.A. Fuzzy sets. Inf Control, 1965; 8:338–53.

[21] M. Aien, M. Rashidinejad, M. Fotuhi-Firuzabad, "On possibilistic and probabilistic uncertainty assessment of power flow problem: a review and a new approach". Renew Sustain Energy Rev 2014; 37:883–95

[22] Ben-Haim Y. Info-gap decision theory: decision sunder severe uncertainty. 2nd ed. Oxford: Academic Press; 2006.

[23] A. L. Soyster, Technical note—convex programming with set-inclusive constraints and applications to inexact linear programming .OperRes 1973; 21:1154–7.

[24] R. E. Moore, R.B. Kearfott, Cloud MJ. Introduction to Interval Analysis.1st ed. Philadelphia, PA: Society for Industrial and Applied Mathematics; 2009.
%bibitemuncertainty6

[25] L. Liu, W. Li, J. Shen, C. Jin and K. Wen, "Probabilistic Assessment of β for Thermal Unit Using Point Estimate Method Adopted to a Low-Order Primary Frequency Response Model," in IEEE Transactions on Power Systems, vol. 34, no. 3, pp. 1931-1941, May 2019, doi: 10.1109/TPWRS.2018.2889905.

[26] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," in Proceedings of the IEEE, vol. 92, no. 3, pp. 401-422, March 2004, doi: 10.1109/JPROC.2003.823141.

[27] A. M. Leite da Silva and A. M. de Castro, "Risk Assessment in Probabilistic Load Flow via Monte Carlo Simulation and Cross-Entropy Method," in IEEE Transactions on Power Systems, vol. 34, no. 2, pp. 1193-1202, March 2019, doi: 10.1109/TPWRS.2018.2869769.

[28] F. Napolitano, F. Tossani, A. Borghetti and C. A. Nucci, "Lightning Performance Assessment of Power Distribution Lines by Means of Stratified Sampling Monte Carlo Method," in IEEE Transactions on Power Delivery, vol. 33, no. 5, pp. 2571-2577, Oct. 2018, doi: 10.1109/TPWRD.2018.2795743.

[29] C. L. T. Borges and J. A. S. Dias, "A Model to Represent Correlated Time Series in Reliability Evaluation by Non-Sequential Monte Carlo Simulation," in IEEE Transactions on Power Systems, vol. 32, no. 2, pp. 1511-1519, March 2017, doi: 10.1109/TPWRS.2016.2585619.

[30] N. Amjady, J. Aghaei and H. A. Shayanfar, "Stochastic Multiobjective Market Clearing of Joint Energy and Reserves Auctions Ensuring Power System Security," in IEEE Transactions on Power Systems, vol. 24, no. 4, pp. 1841-1854, Nov. 2009, doi: 10.1109/TPWRS.2009.2030364.

[31] E. Du, N. Zhang, C. Kang and Q. Xia, "Scenario Map Based Stochastic Unit Commitment," in IEEE Transactions on Power Systems, vol. 33, no. 5, pp. 4694-4705, Sept. 2018, doi: 10.1109/TPWRS.2018.2799954.

[32] S. Park, Q. Xu and B. F. Hobbs, "Comparing scenario reduction methods for stochastic transmission planning," in IET Generation, Transmission Distribution, vol. 13, no. 7, pp. 1005-1013, 9 4 2019, doi: 10.1049/iet-gtd.2018.6362.

[33] A. Lüth, J.M. Zepter, P. Crespo del Granado, R. Egging, Local electricity market designs for peer-to-peer trading: the role of battery flexibility, Appl. Energy 229 (2018) 1233–1243, doi: 10.1016/j.apenergy.2018.08.004.

[34] C. Develder, N. Sadeghianpourhamami, M. Strobbe and N. Refa, "Quantifying flexibility in EV charging as DR potential: Analysis of two real-world data sets," 2016 IEEE International Conference on Smart Grid Communications (SmartGridComm), Sydney, NSW, 2016, pp. 600-605, doi: 10.1109/SmartGridComm.2016.7778827.

[35] Sadeghianpourhamami, N., Refa, N., Strobbe, M., Develder, C., 2018. Electrical power and energy systems quantitive analysis of electric vehicleflexibility: a data-driven approach. Int. J. Electr. Power Energy Syst. 95, 451–462.http://dx.doi.org/10.1016/j.ijepes.2017.09.007

[36] Versatile electric vehicle charging profile generation tool for home charging scenarios for regional case study, Available online: https://zenodo.org/record/4599635.YX6MHG3MJPY, DOI: 10.5281/zenodo.4599635 (accessed on 2021-10-31)

[37] A. Ahmadian, M. Sedghi, B. Mohammadi-ivatloo, A. Elkamel, M. Aliakbar Golkar and M. Fowler, "Cost-Benefit Analysis of V2G Implementation in Distribution Networks Considering PEVs Battery Degradation," in IEEE Transactions on Sustainable Energy, vol. 9, no. 2, pp. 961-970, April 2018, doi: 10.1109/TSTE.2017.2768437.

[38] Tayarani H, Jahangir H, Nadafianshahamabadi R, Aliakbar Golkar M, Ahmadian A, Elkamel A. Optimal Charging of Plug-In Electric Vehicle: Considering Travel Behavior Uncertainties and Battery Degradation. Applied Sciences. 2019; 9(16):3420. https://doi.org/10.3390/app9163420

[39] W. Tushar, T. K. Saha, C. Yuen, D. Smith and H. V. Poor, "Peer-to-Peer Trading in Electricity Networks: An Overview," in IEEE Transactions on Smart Grid, vol. 11, no. 4, pp. 3185-3200, July 2020, doi:10.1109/TSG.2020.2969657.

[40] Chenghua Zhang and Jianzhong Wu and Yue Zhou and Meng Cheng and Chao Long, "Peer-toPeer energy trading in a Microgrid,"Applied Energy, vol. 220, pp. 1-12, 2018.

[41] Dynge M.F., del Granado P.C., Hashemipour N., Korpås M. Impact of local electricity markets and peer-to-peer trading on low-voltage grid perations Appl. Energy, 301 (2021), Article 117404

Appendix

Low voltage synthetic grid data generated by Ding0 package: The low voltage distribution feeder used in the second case study has been generated by the Ding0 package which is a tool for generating synthetic medium and low voltage grids. The grid topology and the line impedance are presented in the table below. The base values for voltage and power are 0.4 kV and 0.25 MVA, respectively. This grid has 13 solar units connected to different nodes. All of them are models as PQ buses, as they do not have control on the voltage. This feeder supplies 27 residential houses. The ID of this grid is lvgd-2388, connected to the medium voltage grid mvgd-2.

Branch information

From bus	To bus	R [P.U]	X [P.U]
1	2	0.00135625	0.00013296875
15	16	0.0165	0.004227896875
16	17	0.0007015625	0.0001325
18	19	0.009	0.002306125
18	21	0.0066625	0.00326725625
15	18	0.0066625	0.003265625
19	20	0.0007015625	0.0001325
21	22	0.0165	0.004227896875
21	24	0.0066625	0.003265625
22	23	0.0007015625	0.0001325
24	25	0.009	0.002306125
24	27	0.0066625	0.003265625
25	26	0.0007015625	0.0001325
27	28	0.0165	0.004227896875
27	30	0.0066625	0.003265625
28	29	0.0007015625	0.0001325
30	31	0.009	0.002306125
30	33	0.0066625	0.003265625
3	4	0.012628125	0.002390625
1	3	0.02510625	0.00980176875
31	32	0.0007015625	0.0001325
33	34	0.0165	0.004234375
33	36	0.0066625	0.00326725625
34	35	0.0007015625	0.0001325
36	37	0.009	0.0023125
36	39	0.0066625	0.00326725625
37	38	0.0007015625	0.0001325
39	40	0.0165	0.004234375
39	42	0.0066625	0.00326725625
40	41	0.0007015625	0.0001325
42	43	0.009	0.0023125
42	45	0.0066625	0.00326725625
43	44	0.0007015625	0.0001325
45	46	0.0165	0.004234375
45	48	0.0066625	0.00326725625
4	5	0.0007015625	0.0001325
46	47	0.0007015625	0.0001325
48	49	0.009	0.002306125
48	51	0.0066625	0.003265625
49	50	0.0007015625	0.0001325
51	52	0.0165	0.004227896875
51	54	0.0066625	0.003265625
52	53	0.0007015625	0.0001325
54	55	0.009	0.0023125
54	57	0.0066625	0.003265625
55	56	0.0007015625	0.0001325
57	58	0.0165	0.004227896875
57	60	0.0066625	0.003265625
58	59	0.0007015625	0.0001325
60	61	0.009	0.002306125
60	63	0.0066625	0.003265625
6	7	0.012628125	0.002385646875
6	9	0.0066625	0.00326725625
1	6	0.005078125	0.003234375
61	62	0.0007015625	0.0001325
63	64	0.0165	0.004227896875
63	66	0.0066625	0.003265625
64	65	0.0007015625	0.0001325
66	67	0.009	0.002306125
66	69	0.0066625	0.00326725625
67	68	0.0007015625	0.0001325
69	70	0.0165	0.004227896875
69	72	0.0066625	0.003265625
70	71	0.0007015625	0.0001325
72	73	0.009	0.002306125
72	75	0.0066625	0.003265625
73	74	0.0007015625	0.0001325
75	76	0.0165	0.004227896875
75	78	0.0066625	0.003265625
7	8	0.0007015625	0.0001325
76	77	0.0007015625	0.0001325
78	79	0.009	0.0023125
78	81	0.0066625	0.00326725625
79	80	0.0007015625	0.0001325
81	82	0.0165	0.004234375
82	83	0.0007015625	0.0001325
9	10	0.0231515625	0.0043736859375
9	12	0.005078125	0.003234375
10	11	0.0007015625	0.0001325
12	13	0.012628125	0.002390625
12	15	0.0066625	0.00326725625
13	14	0.0007015625	0.0001325