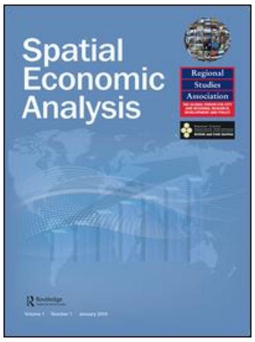

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Machine-learning models for bankruptcy prediction: do industrial variables matter?

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ABSTRACT
We provide a predictive model specifically designed for the Italian economy that classifies solvent and insolvent firms one year in advance using the AIDA Bureau van Dijk data set for the period 2007–15. We apply a full battery of bankruptcy forecasting models, including both traditional and more sophisticated machine-learning techniques, and add to the financial ratios used in the literature a set of industrial/regional variables. We find that XGBoost is the best performer, and that industrial/regional variables are important. Moreover, belonging to a district, having a high mark-up and a greater market share diminish bankruptcy probability.

KEYWORDS
firm distress analysis, machine learning, logistic regression, industrial variables

JEL C45, C52, G33, R11
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INTRODUCTION

Bankruptcy prediction has been intensively studied over the past decades. The theme has been relevant not only for lending institutions, both in deciding whether to grant a loan and in devising policies to monitor existing ones, but also for investors, regulatory authorities, policymakers, managers and so on. More recently with the outburst of Covid-19 pandemic, which has triggered an unprecedented shock to the world economy caused by governments' decisions to lockdown all activities, it becomes important to provide a short-term forecast of the probability of firms going bankrupt, so that policymakers have all the information at their disposal to counteract such a negative shock. Some very recent studies have begun using bankruptcy prediction with this

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task (Bernardi et al., 2021; Carletti et al., 2020). Studying the determinants of firm bankruptcy is thus of vital importance, not only from an economic point of view – the failure of firms represents a cost for employees, entrepreneurs, creditors and for the whole of society – but also from a policy perspective.

The initial contributions to the topic, led by the seminal papers of Altman (1968) and Beaver (1966), focus on critical financial ratios that can help entrepreneurs and fundholders predict insolvency. The literature has departed from these first contributions in two ways. A first departure relates to the variables considered. While the financial nature of default events clearly suggests to primarily look for financial causes, the probability to stay in the market, as well as the financial stability of a firm, is deeply interconnected with the ability to perform well along the economic or industrial aspects of its operation. Thus, it is likely that looking exclusively at financial indicators cannot offer but a partial account of the main determinants of default. Related to this point a first stream of the literature has aimed to determine the causes of firm bankruptcy by looking at variables beyond those that come from accounting books (e.g., Bottazzi et al., 2011; Chava & Jarrow, 2004; Eklund et al., 2020; Gabbianelli, 2018). A second stream of the literature has proposed methodologies and tools to improve firm bankruptcy prediction models. Balcaen and Ooghe (2006) have highlighted the problems related to the classic statistical methodologies for bankruptcy prediction; Kumar and Ravi (2007) have published a comprehensive review of the work done, during the period 1968–2005, in the application of statistical and intelligent techniques to solve the bankruptcy prediction problem faced by banks and firms; and, more recently, Barboza et al. (2017), Gordini (2014) and Zhao et al. (2017), among others, have compared statistical models (logistic regression) with machine learning (ML), whereas Son et al. (2019) have focused on an optimization process to select input variables in intelligent techniques.

The aim and novelty of this paper is to bring together these two streams of the literature to provide a new bankruptcy model for the Italian economy that considers jointly financial ratios and more structural/industrial variables with a special focus on regional aspects. For this purpose, we apply a full battery of bankruptcy forecasting models, which combine more traditional methodologies, such as logistic regression, with more sophisticated techniques based on ML, focusing on AIDA Bureau van Dijk balance sheet information on manufacturing Italian firms for the period 2007–15.

Our results show that indeed incorporating industrial/regional variables into bankruptcy models is beneficial in terms of forecasting performance. The XGBoost technique is the best performer in correctly classifying bankrupt firms. In particular, the percentage of firms correctly classified as bankrupt increases from 87.81% to 89.73% when augmenting the model with industrial/regional variables. Other models, such as weighted logistic regression and neural network (NN), are better in correctly classifying non-bankrupt firms with the augmented specification. Overall, when calculating, through a simulation exercise, the average profit that a bank would receive by granting or denying a loan according to each of the techniques considered, it turns out that for all models the augmented specification increases average profits and that XGBoost with industrial/regional variables ensures the bank has the highest financial gain. Finally, in-sample results on weighted logistic regression show that belonging to an industrial district (ID), having a high mark-up and a high market share have a statistically significant impact in diminishing the probability of default.

The rest of the paper is structured as follows. The next section presents the literature on predictive variables focusing on industrial/regional indicators. The third section describes the data, the fourth section describes the methods and the fifth section describes the evaluation exercise. The sixth section highlights the main results. The final section concludes.

THE LITERATURE ON PREDICTIVE VARIABLES

The list of variables with which to feed the model is crucial in a firm bankruptcy forecasting exercise. The literature has started to focus primarily on financial ratios. The seminal work by Altman (1968) identified a set of financial ratios that were the first under consideration by many researchers and subsequently used in later studies which eventually proposed a very large number of ratios. Courtis (1978), for example, has identified 79 financial ratios that were grouped in three main categories: profitability; managerial performance; and solvency ratios.

The performance and survival of firms, though, might be influenced by several factors external to the firm, that is, the environment, national and international economic conditions. Mensah (1984) noted that different economic environments as well as different sectors lead to different models for the prediction of failures.

Other studies explore the possibility that firms' performance might be influenced not only by financial ratios but also by qualitative variables, that is, quality of management, research and development, market trend, the social importance of the firm, the strength of its bank relationship (Suzuki & Wright, 1985), and its connections with other enterprises (Leoncini et al., 2020; Righi et al., 2019).

Judging from the dates of these contributions, the idea of expanding the initial set of financial ratios is not new to the literature. However, there have been far more recent contributions with the aim of augmenting the financial ratios with other groups of variables and showing the importance of these new variables in increasing the forecasting performance of the model.

For example, Chava and Jarrow (2004) include industries effects in their model; Bottazzi et al. (2011) focus on productivity, profitability and growth as additional variables; Gordini (2014) introduces seven different models dividing the sample by size and geographical area; Mueller and Stegmaier (2015) select size and age; Liang et al. (2016) favour corporate governance indicators; Eklund et al. (2020) introduce the institutional framework; and finally Gabbianelli (2018) adds qualitative variables regarding the territory and the firm–territory relationship.

OUR CHOICE

Our aim is to build a bankruptcy prediction model constructed for the Italian economy. The Italian industrial sector is characterized by a high prevalence of small and medium-sized enterprises (SMEs),[1] usually family-owned firms, whose main source of finance is internal funds and, to a lesser extent, short-term bank loans. Only around 300 manufacturing firms are listed in the stock market in Italy. SMEs are associated with a greater risk of failure compared with larger enterprises and usually the asymmetric information between firms and banks in the case of SMEs is higher, often implying credit shortages (Magri, 2009). The presence of ID, which promote external economies of scale, may help make up for the absence of a firm's growth. The regional disparity between North and South is also very pronounced in terms of economic growth, entrepreneurship opportunities and technological infrastructure. Most of the literature on bankruptcy prediction has focused on medium and large enterprises, whereas just a few papers have tried to build bankruptcy models specific to SMEs. Those that focus on the Italian case, to which our paper is closely related, are even fewer. Some of these focus only on financial ratios (Altman et al., 1994; Altman & Sabato, 2005; Calabrese et al., 2016; Sartori et al., 2016), while others base their analysis only on one or two traditional models (Altman et al., 1994; Altman & Sabato, 2005; Gabbianelli, 2018). The articles that try to incorporate new variables linked to regional and territorial aspects and firm size are those of Gabbianelli (2018), which uses qualitative variables regarding the relationship between the firms and the territory, and Gordini (2014), which considers different models dividing the sample according to firm size and macro-geographical areas. While Gabbianelli (2018) uses only one bankruptcy prediction model, logistic regression, Gordini (2014) compares different methodologies, but does not include size and regions into a comprehensive model. Our contribution to the literature is to compare several state-of-the-art ML techniques and to add industrial/regional indicators to the financial ratios à la Altman, building a comprehensive model with the aim of characterizing the peculiarities of Italian manufacturing firms. We start considering the financial ratios as in Barboza et al. (2017), but differently from that study, we only examine indicators for which we have available and reliable information. In addition, we avoid some variables typical of listed firms, which are few in number in the Italian economy (see Baseline specification in Table A1 in the supplemental data online).

To the financial ratios we then add the following industrial/regional variables: sectors, regional dummies, ID membership, a non-parametric measure of market power (mark-up),[2] and a measure of market share (see augmented specification in Table A1 in the supplemental data online).

District membership

The first contribution on the concept of ID dates back to Marshall (1890), who defines the localization of industry as a 'concentration of many small businesses of a similar character in particular localities'. The disadvantage of the small scale is compensated by the localization externalities that firms belonging to a district enjoy. The key idea is that firms located close to other firms operating in the same industry benefit from reduced transportation costs, the availability of specialized workers and suppliers, and the diffusion of intra-industry knowledge and technological spillovers. According to the literature on ID (Bellandi, 2009; Hart, 2009; Marshall, 1890), these factors enable small firms localized in the same industrial area to benefit from the same economies (external-scale economies) present inside large firms (internal-scale economies).

The Italian revisiting of the Marshallian ID concept introduced by Becattini (1990), Brusco (1982) and Sforzi (1989) highlights more the role of cooperation and the link between social and economic forces that interact within the same geographical area. Trust among district members is central to their ability to cooperate and act collectively.

Alongside this new theoretical definition of ID, a new body of the empirical literature emerged. These works attempt to establish the presence of a 'district effect', that is, they try to identify empirically the agglomeration benefits that firms derive from membership. Signorini (1994) and other research in this field show unanimously that firms in ID do indeed benefit from agglomeration advantages.

Another very vast and more recent stream of the literature focuses on the impact on economic growth (in terms of employment and productivity) of three different types of local externalities: localization economies, Jacob's externalities and urbanization economies. These studies start in the 1990s and cover different countries (Cingano & Schivardi, 2004; Martin et al., 2011), but are not unanimous in their conclusions. More recent contributions have focused on the role of agglomeration in fostering innovation productivity and export (Antonietti & Cainelli, 2011; Boschma & Iammarino, 2009). While these papers all refer to the long-run effects of agglomeration on growth and productivity, short-run effects are less studied. However, an interesting stream of the literature emphasizes the benefits of agglomeration economies over the business cycle with a particular attention to recessions (Brunello & Langella, 2016; Guiso & Schivardi, 2007).

According to Guiso and Schivardi (2007) the intense social interactions within the ID are likely to amplify the responses to negative shocks acting as a social multiplier. A similar result is found by Brunello and Langella (2016) who investigate the impact of agglomeration economies on firm entry during recessions and show that firm entry in ID has declined more during recessions than in comparable areas. On the other hand, social capital,[3] which is found to be highly present in ID (Trigilia, 2001; Soubeyran & Weber, 2002), might increase the trust among firms and between firms and other institutions in the territory. This could, for example, translate into a better access to credit through relationship-lending. The level of trust between district firms and local banks that share the same territory might be crucial in the process of credit supply, given that banks sharing the same territory evaluate firms' solvability, not only implementing a credit scoring approach but also accounting for the entire background of 'soft' and not codified information, which is crucial to fully characterize firms belonging to ID (Alessandrini & Zazzaro, 2009). A higher trust towards district firms could translate into a higher availability of credit that will in turn promote investments and innovation. Though the empirical literature on the role of ID membership on bankruptcy is missing, we have tried to hint at some possible theoretical explanations, related to localization externalities and social capital, that could be potential drivers for reducing the probability of ID firms of exiting the markets.

Mark-up

As a measure of mark-up we use the price cost margin (PCM) defined in Table A1 in the supplemental data online. This indicator is related to the notion of firm profitability, which has been widely considered in the past literature on bankruptcy models.

The reason we introduce this variable in the augmented specification is twofold. The first is related to the fact that the PCM, differently from more traditional indicators of profitability such as return on equity (RoE) and earnings before interest and taxes (EbIT), measures the profit margins related to the core business of the firm, whereas the other two variables comprise both the core business and also the financial and accessory activities.

The second is related to the fact that the PCM, quantifying the mark-up that firms are able to extract from customers, identifies the market power of a firm. An important theoretical feature of this measure is that the higher the market competition, the smaller should be the PCM. In fact, in the absence of barriers to entry, prices should be equal to the marginal costs. A positive and persistent PCM typically suggests that firms have at least a certain degree of market power.

Having a high mark-up implies higher profits and thus more financial resources to increase investments and innovative activities that could reduce production costs (Cassiman & Vanormelingen, 2013).[4] Alternately, a high mark-up might also mean more product diversification (variety) and higher barriers of entry for external firms. These two factors could be potentially important drivers to reduce the firm's probability of going bankrupt.

DATA

Data structure

The analysis is based on balance sheet information on manufacturing Italian firms extracted from AIDA Bureau van Dijk, for the period 2007–15, which allows one to compute the response variable and all the selected covariates with the exception of the ID variable. The latter is obtained merging, through the ZIP code of the firm's operative branch, AIDA with the Industrial District Database provided by the Italian National Statistical Institute (ISTAT).

We construct our response variable based on the AIDA field 'status', that is, we create a dummy variable that takes the value of 1 if the status is 'bankruptcy', and 0 otherwise. For brevity, from here on bankrupt firms will be identified as B (bankrupt), while sound firms will be referred to as NB (not bankrupt).

Table A2 in the supplemental data online reports the different steps of our data set construction. After having cleaned the data to exclude missing observations, inconsistencies and extreme values (step 2), we have to decide on how to construct the sample of B and NB firms. Several papers in the literature have used a balanced sample, that is, they consider the number of B firms one year prior to bankruptcy and then select randomly the same number of NB firms throughout the period considered. This procedure has the advantage of not introducing an imbalance between B and NB firms in the data set, but has the disadvantage of making the results highly dependent on how the researcher selects the control firms.

The choice involves not only which firms to select, but also the balance sheet year to consider. In order to make the exercise more realistic, we propose a different method for downsizing our data set. In accordance with the literature, we consider all B firms one year prior the bankruptcy event, but we make a different choice regarding NB firms. At first, we concentrate on the incumbents, that is, firms that have survived for the entire period considered (2007–15). This choice is driven by the fact that we want the control set of firms to be as healthy as possible.

Once we have selected the incumbents, we need to decide what balance sheet years consider for each of them. Considering nine years would imply a very high imbalance ratio, thus we decide to downsize the NB set by keeping only three different balance sheets for each NB firm, equally spaced in time as explained in step 4 (see Table A2 in the supplemental data online). This reduces the imbalance ratio to 8.59%. The final data set is then composed by 5560 B firms and 64,749 NB year–firm observations. Even if one NB firm is observed in three different points in time, we consider our data set as a cross-section and assume that each observation is as if it were a different firm. This choice is also motivated by the aim of reducing time dependence in our data. In order to further adjust the imbalance ratio, we use a 'class weighted loss function' to perform the classification. Following King and Zeng (2001), we assign different weights to B and NB observations, defined as $w_i = \frac{n}{2n_i}$ with $i = B$, NB, where n_i is the number of observations in the corresponding class; and n is the total number of observations. In our analysis we obtain $w_B = 6.32$ and $w_{NB} = 0.54$.

Summary statistics

The industrial/regional indicators reported in the augmented specification (see Table A1 in the supplemental data online) could be potentially relevant for any country, but are even more important in the Italian context. Italy is characterized by a prevalence of non-listed manufacturing SMEs, ID represent around one-fourth of the Italian productive system, in particular 24.4% of firms belong to ID and 24.5% of employees are employed in ID.[5] Italy is also characterized by regional disparities especially (but not only) between Northern and Southern regions.

Figure 1a reports the default rate by macro-regions, calculated as the geometric mean of defaults rates between 2007 and 2015, and highlights the different propensity of going bankrupt, which is the lowest in the North East region and the highest in the South. Figure 1b emphasizes the disparity in default rates across industrial sectors. Leather and Wood show a default rate which exceeds 2%, whereas the Food industry is characterized by a default rate just above 1%. Default rates are calculated over the total number of active firms downloaded from AIDA Bureau van Dijk.

Figure 2 reports the regional number of district firms (left map) and the regional default rate (right map). Visually the maps seem to point out a negative correlation between belonging to an ID and insolvency.

It seems thus reasonable, given the descriptive statistics presented here, to incorporate these industrial/regional aspects into the bankruptcy forecasting model. One of the aims of the paper is to show that these variables improve the forecasting ability of the model. Table A3 in the supplemental data online reports the summary statistics of the variables considered for B and NB and the correlation matrix between input variables (see Table A4 online).

Models

If one of the purposes of this article is to highlight the importance of industrial/regional variables in forecasting bankruptcy, the second main purpose is to compare methodologies: on one hand, a

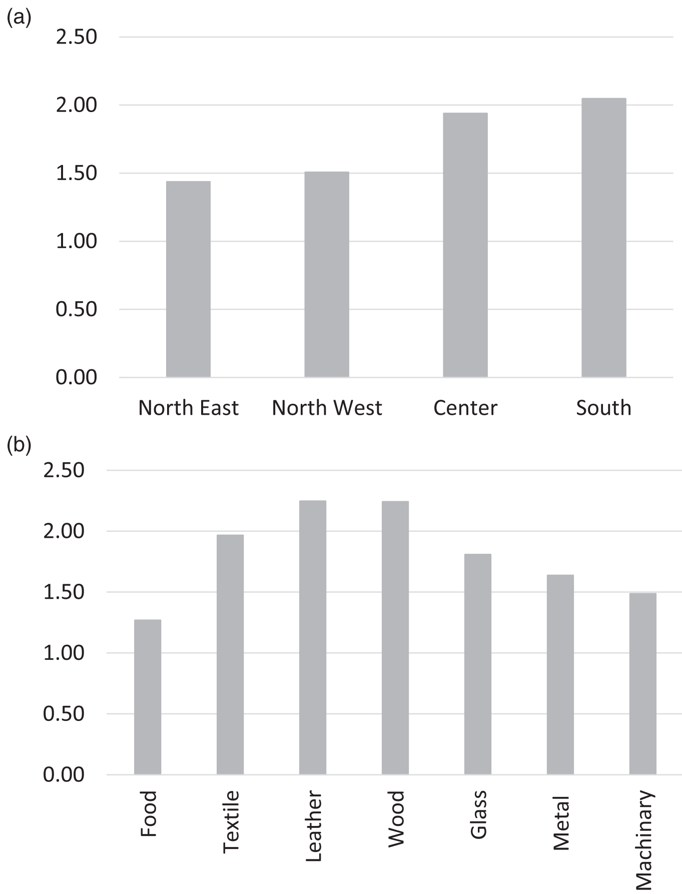

Figure 1. (a) Default rate by macro-region (geometric mean over the period 2007–15); and (b) default rate by sector (geometric mean over the period 2007–15).
Note: The default rate is calculated over the total number of firms (AIDA Bureau van Dijk).

more traditional methodology often used as a benchmark (logistic regression) and, on the other, state-of-the-art ML techniques.

The literature on predictive accuracy comparison across models is very rich. Table 1 reports just a small selection of the most recent papers on the topic. From 1992 to 1998 artificial NN-based techniques started to be central in the field. NN eventually evolved into hybrid models, for example, neuro-fuzzy systems (Chen et al., 2009) and ensembles of NN (Tsai & Wu, 2008). Other important techniques include decision trees and their ensemble variations, particularly random forests (RF) (Kruppa et al., 2013). Support vector machines (SVM), another type of learning machines, are able to perform comparably with NN (see Danenas & Garsva, 2015, for an application of SVM on credit risk).

Ensemble learning has also been widely researched in the context of credit risk, as different authors provide empirically the capability of classifier ensembles to obtain better classification performance. Recent developments include bagging or boosting procedures, in particular feature selection (FS) boosting (Wang et al., 2014) and XGBoost (Son et al., 2019).

We have chosen to focus on NN and techniques based on decision trees. SVMs were also considered, but in the end discarded because due to the high number of observations in our training set, the model training time was prohibitively too long, and therefore not implementable in a real application. In the next section we describe briefly the chosen methodologies.

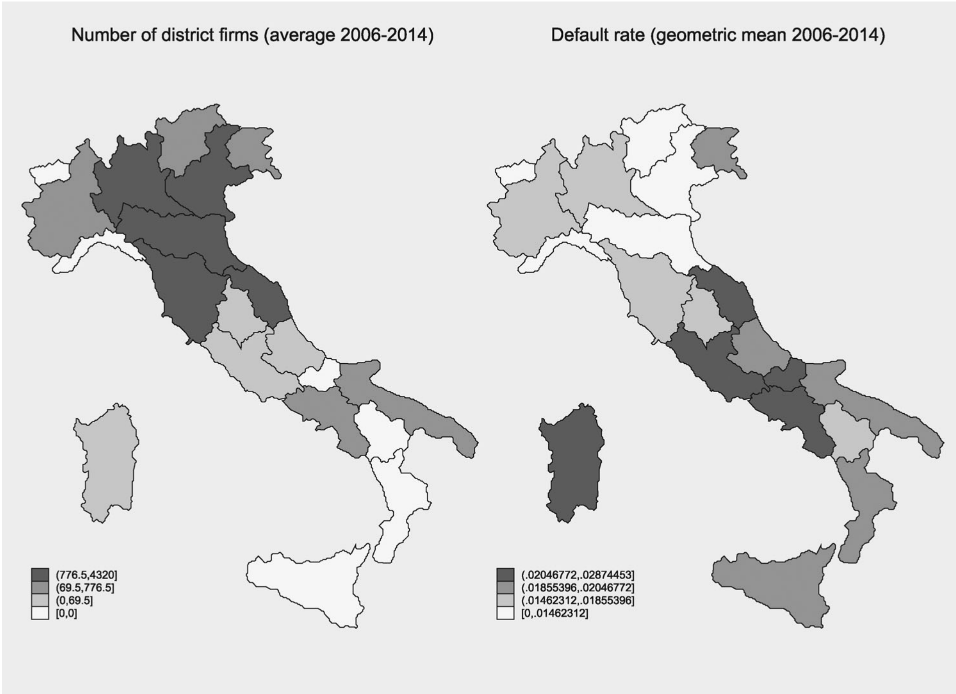

Figure 2. Regional distribution of default rate and number of district firms.

Weighted logistic regression (WLR)

Earlier studies in credit risk modelling employed discriminant analysis to obtain better classification results with the purpose of developing bankruptcy prediction models (Altman, 1968; Beaver, 1966). Starting from the 1980s, logistic regression (LR) has been considered a popular alternative to multivariate analysis for credit risk modelling (Ohlson, 1980).

Here we resume LR to have a benchmark for comparing the more sophisticated techniques that will be presented below. In addition, LR permits one to evaluate the significance of the explanatory variables and the sign of their coefficients, allowing us to give an economic intuition of some important determinants in bankruptcy prediction.

As is well known, through LR we set $Y = 1$ if bankruptcy occurs, 0 otherwise; and we estimate the bankruptcy probability $\pi_i = P(Y_i = 1|X_i = x_i)$ supposing that:

$$\pi_i = \frac{exp(x_i \cdot \beta)}{[1 + exp(x_i \cdot \beta)]}, \tag{1}$$

where $x_i = (x_{i1}, \ldots, x_{ip})$ is the vector of explanatory variables observed for the i-th firm; $x_i \cdot \beta = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_p x_{ip}$; and $\beta_0, \ldots, \beta_p$ are $p + 1$ parameters to be estimated.

It is now worth noting that the log-likelihood function used to estimate the parameters is a sum of n terms, each corresponding to a firm, and consequently it can be split into two parts as follows:

$$\begin{aligned} L &= \sum_{i=1,\ldots,n} [y_i \cdot log(\pi_i) + (1 - y_i) \cdot log(1 - \pi_i)] = \sum_{y_i=1} log(\pi_i) + \sum_{y_i=0} log(1 - \pi_i) \\ &= L_1 + L_0. \end{aligned} \tag{2}$$

If positive events (number of observed $y_i = 1$) are rare, as in our exercise, the estimated probabilities π_i tend to be too small and biased, together with the related standard errors which depend on

Table 1. Predictive performance in the literature: percentage of correct bankruptcy (B); and percentage of correct non-bankruptcy (NB).

		MDA	Logit	Probit	RF	NN	Bagging	Boosting	XGB	FSB	SVM
Present augmented model	Correct default		85.38		84.63	84.67			89.73		
	Correct sound		78.01		83.13	83.32			75.98		
Gabbianelli (2018)	Correct default		72.70								
	Correct sound		92.80								
Barboza et al. (2017)	Correct default	64.66	88.72		83.46	93.23	82.71	81.20			
	Correct sound	52.05	76.16		87.10	72.77	85.70	86.71			
Zhao et al. (2017)	Correct default		69.64		80.36						77.68
	Correct sound		78.91		81.25						75.78
Liang et al. (2016)	Correct default					73.40					82.20
	Correct sound					67.50					80.30
Danenas and Garsva (2015)	Correct default		70.20								
	Correct sound		98.00								
Wang et al. (2014)	Correct default		70.91			71.93		74.64		74.98	
	Correct sound		76.76			74.63		79.84		87.19	
Gordini (2014)	Correct default		78.30								78.70
	Correct sound		59.80								67.90
Laitinen and Suvas (2013)	Correct default		70.24								
	Correct sound		73.44								
Bottazzi et al. (2011)	Correct default			86.67							
	Correct sound			68.39							

		SVM-RBF	LinSVM	PSOLinSVM	RBFN	KNN	CART	NB	KELM	ELM	PSOFKNN	GAs
Present augmented model	Correct default											
	Correct sound											
Gabbianelli (2018)	Correct default											
	Correct sound											
Barboza et al. (2017)	Correct default	78.95	92.48									
	Correct sound	79.78	71.31									
Zhao et al. (2017)	Correct default								84.82	71.43	83.04	
	Correct sound								80.47	73.44	79.69	
Liang et al. (2016)	Correct default					81.50	77.70	76.90				
	Correct sound					67.50	79.60	60.40				
Danenas and Garsva (2015)	Correct default		82.90	77.80	75.00							
	Correct sound		97.30	96.20	96.00							
Wang et al. (2014)	Correct default											
	Correct sound											
Gordini (2014)	Correct default											79.60
	Correct sound											69.50
Laitinen and Suvas (2013)	Correct default											
	Correct sound											
Bottazzi et al. (2011)	Correct default											
	Correct sound											

Note: Reported are some of the contributions on the topic of bankruptcy prediction for which we have a comparison in terms of correct default and correct sound. RF, random forest; NN, neural network; XGB, extreme gradient boosting; FSB, feature selection boosting; SVM, support vector machine; LinSVM, linear support vector machine; SVM-RBF, radial basis function SVM; PSOLinSVM, particle swarm optimization linear SVM; RBFN, radial basis function network; KNN, *k*-nearest neighbour; CART, classification and regression tree; NB, naive Bayes; KELM, kernel extreme learning machine; ELM, extreme learning machine; PSOFKNN, particle swarm optimization enhanced fuzzy *k*-nearest neighbour; and GAs, genetic algorithms.

$\pi_i \cdot (1 - \pi_i)$. To account for this bias, we exploit the aforementioned method proposed in King and Zeng (2001), that is, in order to consider the imbalance ratio, we estimate the parameters maximizing the modified log-likelihood function $L_w = w_1 \cdot L_1 + w_0 \cdot L_0$, where $w_1 = w_B = 6.32$ and $w_0 = w_{NB} = 0.54$. In this light we will refer to this methodology as a WLR.

Machine learning (ML)

Aiming to compare the LR with some state-of-the-art ML techniques, we also perform the classification tasks using NN, RF and XGBoost. All these methods are implemented in Python, with Keras and Scikit-learn packages. With respect to all the models here presented, the hyper-parameter optimization (i.e., number of layers in the NN, shallowness of the trees in the RF, etc.) has been carried out through a grid search. For the sake of conciseness, all tested combinations are not reported; for the implementation details of each technique, including the optimal hyper-parameters resulting from the fine-tuning, see below.

NN are one of the most widespread artificial intelligence methods, widely used for regression, patter recognition and data analysis (LeCun et al., 2015).

For every $i = 1, \ldots, n$, the vector of observed covariates x_i is fed as input into the NN algorithm and elaborated through a sequence of steps ('layers') formed by many 'neurons'. Every neuron j in a layer first computes the weighted sum s_j of the inputs furnished by all the neurons in the preceding layer, and then produces its own output calculating the 'activating function' $f(s_j)$. Such outputs are in turn fed as inputs for the neurons in the following layer, and so on. Weights for the weighted sums are the parameters to be trained. In this exercise we use a fully connected feedforward NN made of three hidden layers, with 16 neurons each, based on the 'relu' activation function $f(s_j) = max(0, s_j)$. All values are obtained through a grid search, as mentioned above.

As it is customary in classification problems, the last layer has a single neuron that generates the response value $\hat{y}_i$ (in our case, the probability for the i-th firm to be bankrupted) using the standard logistic function as activating function.

Generally, weights are estimated minimizing a given loss function, based on the difference between observed and estimated classification for the units in the training set. To consider the imbalance ratio we minimize the weighted binary cross-entropy loss function as follows:

$$-\frac{w_B}{n_B}\sum_{y_i=1} L(y_i, \hat{y}_i) - \frac{w_{NB}}{n_{NB}}\sum_{y_i=0} L(y_i, \hat{y}_i), \tag{3}$$

where w_i and n_i, $i = B, NB$ have been previously defined; and $L(y_i, \hat{y}_i) = y_i \cdot log(\hat{y}_i) + (1 - y_i) \cdot log(1 - \hat{y}_i)$.

RFs, introduced by Breiman (1996), are an 'ensemble method' based on decision tree models and successfully used for firm bankruptcy prediction (Barboza et al., 2017; Bou-Hamad et al., 2011).

Ensemble method means that many ML algorithms are combined together so that the resulting model is more powerful than any single component in the ensemble. In the case of RF, many classification trees are used. The advantage of assembling trees is to obtain a more robust classification and thus to increase forecasting performance (Yeh et al., 2014).

A decision tree is a flow-chart structure able to split the covariates' space in many non-overlapping regions, starting from a unique initial node and following a path made of many partitioning nodes. Every node splits observations according to a given covariate, and every possible path defines a region and leads to a final node ('leaf'), which contains the predicted classification (B or NB). In our study, the RF is implemented with 500 trees built on bootstrapped samples, and each tree is characterized by a maximum depth of 15 internal nodes and by a maximum number of leaf nodes of 20. All values are obtained through a grid search, as mentioned above. The final classification is obtained computing the majority vote among the 500 outputs provided by the trees.

In our RF, at every node we choose as a splitting criterion the heterogeneity Gini index. To treat the imbalanced classes, the splitting criterion is to maximize the following quantity:

$$WID = \frac{n_{node}}{n}\left[G_{node} - \frac{n_{right}}{n_{node}}G_{right} - \frac{n_{left}}{n_{node}}G_{left}\right], \tag{4}$$

where n_{node} is the number of firms in the considered node; and n_{right}/n_{left} are the numbers of firms split in the right/left branch. All these quantities are weighted sums, for example, $n_{node} = w_B \cdot n_{B,node} + w_{NB} \cdot n_{NB,node}$, where $n_{B,node}$ is the number of B training firms observed in the node, and so on.

XGboost (eXtreme Gradient Boosting method), first introduced by Chen and Guestrin (2016), is an extremely performing algorithm to implement gradient-boosted decision trees used for bankruptcy prediction (Zieba et al., 2016). XGB is an ensemble method in which each tree is built sequentially, as opposed to the RFs.

Roughly speaking, XGB acts iteratively as follows: in the first step a (small) tree is built, which provides the (raw) classification $\hat{y}_i^1$ minimizing the cost function $\sum_i L(y_i, \hat{y}_i^1)$. In the second step, XGB tries to improve $\hat{y}_i^1$ by minimizing $\sum_i L(y_i, \hat{y}_i^1 + f_1(x_i))$, in which $f_1(x_i)$ ideally is the best fit among all the possible decision trees based on the x_i's as covariates and the residuals $y_i - \hat{y}_i^1$ as responses. Successive steps are similar. Obviously, it is not possible to check all the possible trees – some approximations are needed.

In our analysis the generated number of trees is equal to 5000 with a maximum depth of 100. We also implement a sampling strategy of the covariates, with a threshold equal to 50%, so that no more than half of the covariates can be considered at each split. All values are obtained through a grid search, as mentioned above.

It is worth noting that XGB does not allow one to specify class weights for the loss function. However, it has a specific parameter, 'the scale positive weights', which can be implemented to account for the imbalance ratio in the data set. Specifically, it can be used to adjust the weights associated with the classification errors of the minority class. In the analysis, we use a scaled weight for the B class equal to 1.0E+10. Given such a high value, we also have to use a low learning rate equal to 9.0E−04.

EVALUATION

In order to measure the predictive performance of our models, we conduct an out-of-sample exercise randomly splitting the whole data set into a training set and a test set (respectively, 75% and 25% of firms). We also implement a stratified split so to reproduce the proportion of B and NB observations in both the training and tests set. In the training set we estimate the models' parameters in the case of WLR and NN and create model instances in the case of RF and XGB. In the test set we verify the predictive performance of each model.

To compare the predictive power of the different models we used, we report the percentage of correctly classified as B and the percentage as correctly classified as NB. Given that we have a classification objective, prediction models are traditionally measured against a confusion matrix depicted in Table 2. We thus calculate the following quantities: T_1 error = FP/(FP + TN) and T_2 error = FN/(FN + TP); correct B =$(1 - T_2)$ and correct NB= $(1 - T_1)$. Given that the training and test sets are randomly selected, to reduce variability we use here a repeated random sub-sampling validation, that is, we randomly split the whole data set into training and test for 200 times, and for every split we estimate the described models. Results are averaged on these 200 repetitions.[6] In addition, in WLR we build a confidence interval around the averaged regression coefficients in order to test significance.

Table 2. Confusion matrix.

		Predicted	
		Bankrupt	**Not bankrupt**
Actual	Bankrupt	TP	FN
	Not bankrupt	FP	TN

Note: TP, true positives; TN, true negatives; FP, false positives; FN, false negatives.

RESULTS

We start by showing the results on the predictive ability of the different models, comparing the two specifications (baseline and augmented).

As already explained in the previous section, the evaluation exercise is out-of-sample, that is, the models are estimated in the training sets and tested in the tests sets. The rationale of this procedure is to mimic the activity of a credit institution which has some information on its client firms, divided into B and NB, and needs to classify a new client as B or NB in order to decide whether or not to grant a new loan. If the credit institution grants a loan to a B firm, which was erroneously classified as NB, it will have a loss on its balance sheet, else if the credit institution does not grant a loan to an NB firm, which was erroneously classified as B, it loses a profit opportunity. The first type of error is what we have previously defined as T_2 and the second is what we have previously defined as T_1. Table 3 shows the complement to unity of T_2 and T_1, namely the percentage of firms correctly classified as B and the percentage of firms correctly classified as NB.

There is usually a trade-off between T_1 and T_2 errors, that is, we cannot expect to minimize both of them at the same time. From a credit institution perspective, though, minimizing the error in classifying as sound a firm that will eventually become insolvent is of crucial relevance, given that the bank has the aim of reducing the number of non-performing loans (NPL) in its balance sheet.

Table 3 compares the different methodologies and the two specifications; it also reports the McNemar test to check whether the classification of B and NB is statistically different in the two specifications.[7] The results show the following. (1) The augmented specification, that is, the introduction of regional/industrial variables, improves both the percentage of

Table 3. Predictive performance: baseline versus augmented.

	Baseline		Augmented	
	Financial ratios		Financial + industrial	
	Correct B	**Correct NB**	**Correct B**	**Correct NB**
Weighted logistic regression	86.16	77.28	85.38	**78.01****
Random forest	83.96	82.98	84.63	83.13
Neural network	85.30	82.52	84.67	**83.32***
XGBoost	87.81	77.51	**89.73****	*75.98****

Note: Correct B = percentage of correctly classified as bankrupt; correct NB = percentage of correctly classified as not bankrupt. Values shown in bold report the statistically significant increases in the percentage of correctly classified of the augmented model compared with the baseline. Values shown in italics report the statistically significant declines in the percentage of correctly classified of the augmented model compared with the baseline. All other percentages are not statistically different from each other. We conduct the McNemar test on the difference between augmented and baseline. Significance levels *10%; **5%; ***1%.

correct B (in the case of XGB) and the percentage of correct NB (in the case of WLR and NN). (2) XGB is the best performer in classifying B correctly and with XGB the percentage of firms correctly classified as B increases from 87.81% to 89.73% when augmenting the model with industrial/regional variables. This is the best result across models. On the other hand, with XGB the percentage of firms correctly classified as NB worsens when augmenting the model with industrial/regional variables. (3) In all other cases the difference in performance between the baseline and the augmented model is not statistically different from zero.

In Table 4, focusing only on the augmented model, which has proven to be a good choice, we show the McNemar statistics comparing each couple of methodologies (WLR, RF, NN and XGB). In every column, the bold value represents the percentage of correctly classified firms of the augmented model (used as a benchmark) and the following values report the percentages related to the other methodologies. We learn that WLR, RF and NN have the same performance in classifying B firms, whereas XGB is better than all other methodologies in correctly classifying B firms. On the other hand, NN beats the other models in classifying correctly NB and XGB (though correctly classifying 76% of NB firms) is worse than the other models. Table 4 provides coherent information to what is reported in Table 3.

The performance of the methodologies reported in this paper are in line with those of the literature, as shown in Table 1. The percentage of correct B and correct NB is similar to other contributions. Table 1 shows that it is very difficult to identify a winning methodology, and all techniques exhibit the trade-off between classifying correctly the two classes. One can notice that our results are improved compared with Bottazzi et al. (2011) and Laitinen and Suvas (2013), and are in line with Barboza et al. (2017). However, while Barboza et al.'s (2017) best model is NN, our best model is XGB; they do very well in classifying B firms, whereas we do better than them in classifying NB.

In addition, our results on the importance of including industrial and regional variables into bankruptcy forecasting models are also in accordance with the literature which has focused on combining financial ratios and other firms' characteristics, such as the relationships between firms and their territory. Gabbianelli (2018), when analysing 141 SMEs located in the Marche region, has shown that developing a default prediction model using jointly quantitative (financial ratios) and qualitative (characteristics of the territory and the relationship firm–territory) variables, increases the predictive accuracy of the model. Similarly, Gordini (2014) finds that when the models are separately calculated according to size and geographical areas, the predictive performance increases compared with a bankruptcy forecasting model based on the entire sample.

Table 4. Predictive performance across methodologies: augmented specification.

	Correct B			Correct NB		
	WLR	**RF**	**NN**	**WLR**	**RF**	**NN**
Benchmark	**85.38**	**84.63**	**84.67**	**78.01**	**83.13**	**83.32**
RF	84.63			83.13***		
NN	84.67	84.67		83.32***	83.32*	
XGB	89.73***	89.73***	89.73***	75.98***	75.98***	75.98***

Note: Correct B = percentage of correctly classified as bankrupt; correct NB = percentage of correctly classified as not bankrupt. Values shown in bold report the percentages of correctly classified referred to the augmented model, which represent our benchmark. We conduct the McNemar test on the difference between method A (benchmark) and method B. Significance levels *10%; **5%; ***1%.

A simulation exercise

From the previous section we have learned that there is no clear-cut solution for which model is best. XGB performs well in classifying insolvent firms, whereas other models, for example, WLR and NN, are more reliable in classifying solvent ones. Since a dominant solution does not exist, to assess the real potentialities provided by each technique, we calculate the expected total gain that a bank would achieve by adopting each method. To this aim we develop a simulation (coded in Python 3.4) based on a net present value (NPV) approach.

Simulation models based on discounted cash flows analysis are typically used either as a way to improve long-term investment decisions (e.g., Kelliher & Mahoney, 2000) or for insolvency risk analysis, especially in the insurance market. On this last topic, and in line with our analysis, Cummins et al. (1999) is one of the first studies to introduce an insolvency testing approach, based on Monte Carlo simulation, applied to insurance companies. Since then, while remaining a niche sector, other similar studies have been proposed (e.g., Casarano et al., 2017; Colombini & Ceccarelli, 2004).

Let m be the method used by the bank; and $\pi(m,1)$ and $\pi(m,2)$ be the expected percentage of solvent and insolvent firms that are correctly classified using method m, as defined in Table 3, then a simulation run is described as follows:

- A firm is randomly extracted from the original data set (without resampling).
- Based on the income I of the extracted firm (measured in terms of total sales), a loan L is generated as $L = k \cdot I$, with k uniformly distributed in the range $[0.01; 0.1]$.
- If the firm is labelled as 'solvent', then the bank grants the loan with a probability equal to $\pi(m,1)$, whereas it denies the loan with probability $(1 - \pi(m,1))$. Similarly, if the firm is labelled as 'insolvent', the loan is granted or denied with probability $(1 - \pi(m,2))$ and $\pi(m,2)$, respectively.
- If the loan is granted, the NPV of this operation is computed as $\mathrm{NPV} = -\mathrm{L} + \sum_{i=1}^{n} \frac{R_i}{(1+d)^n}$, where d is the discount factor; and n is the number of instalments R paid by the firm. Clearly, if the firm is solvent, then n equals the total number N of instalments needed to repay the loan and so the bank makes a profit. Conversely, if the firm is insolvent (i.e., misclassification case), n coincides with the last instalment paid by the firm before its bankruptcy, so $n < N$ and the bank registers a loss. Also note that if a solvent firm is erroneously classified as insolvent, the loan is not granted and so the potential revenue is lost.
- The process is iterated by randomly extracting another firm until a maximum number of firms F has been extracted, or a budget B (the sum of all the loans granted) has reached a certain threshold level.
- The sum of all the NPVs thus generated (that as explained could be either positive or negative) quantifies in monetary terms the performance, say the profit, of the bank.

We calculate the average profit for each method, repeating the simulation M times. All the operational parameters used in the simulation are detailed in Table A5 in the supplemental data online. As indicated, we assume that if the company goes bankrupt, the bankruptcy always takes place in the same year in which the loan has been granted, and before the first instalment R_1 has been paid. Therefore, because $n = 0$, the NPV is negative and coincides with the loan, that is, $NPV = -L$. This is the worst-case scenario because all the capital is lost by the bank.

The simulation results are displayed in Figure 3, which shows the expected profit (expressed in percentage terms relative to the best alternative). For instance, XGB with financial and industrial variables is the method that ensures the maximum profit, whereas RF with financial variables

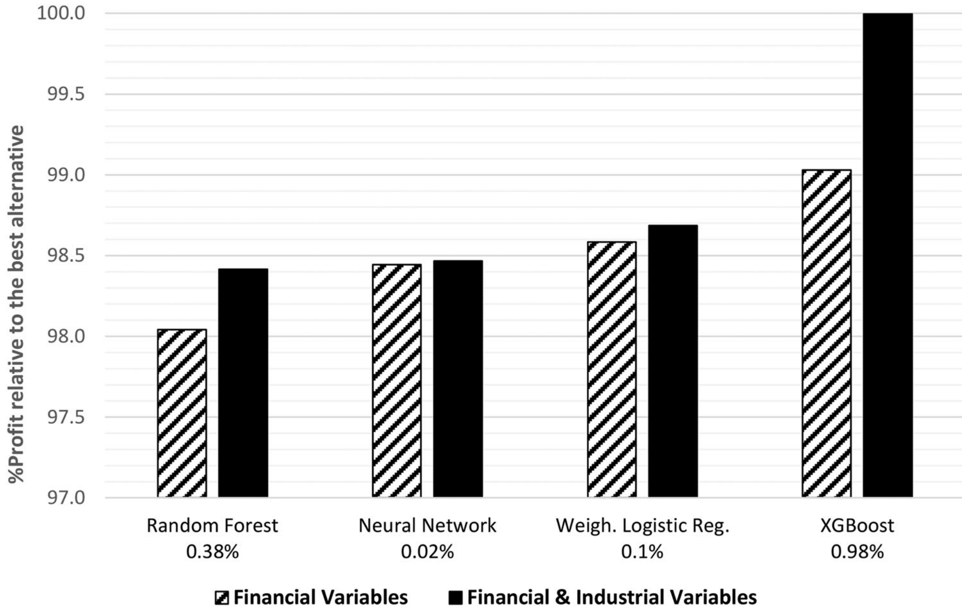

Figure 3. Simulation results.

only is the worst alternative. The percentage increase of profits that can be obtained adding the industrial variables is also displayed. Figure 3 shows that incorporating industrial variables is always beneficial, but especially for RF and XGB, with percentage increases of profits equal to 0.38% and 0.98%, respectively. Since both classifiers are particularly performing in correctly classifying insolvent companies, the simulation is a further demonstration of the benefit of the industrial variables, especially as a means to avoid wrongly granted loans. This is particularly important for banks of small and medium size that have a limited budget and are particularly sensitive to possible losses.

LR in-sample results

Given that industrial and regional variables seem to be important for firms' bankruptcy forecasting, we expect that these variables have a significant role in determining firms' probability of becoming insolvent. For this purpose, we report the average result of the logistic regression over the training sets (in-sample results) in order to check the sign and significance of the different variables.

Table 5 reports the average marginal effects and their significance. Table 6 reports the coefficients. The results show that indeed industrial variables have a significant impact on the probability of bankruptcy. In particular, belonging to an ID, having a high mark-up and a high market share diminish the probability of bankruptcy.

Regarding sectors and regional dummies results, we find that food has a lower probability of bankruptcy compared with other sectors, and that Central and Southern regions have a higher bankruptcy probability with respect to other regions.

The relevance of regional disparity is an expected result given the recent increasing dualism of the Italian economy. Also, the result on sectors is not surprising. The food industry has a stronger capacity, in comparison with the other sectors, to propose a differentiated product and thus increase its competitive advantage, crowding out foreign competitors. The first novel result of this paper concerns the positive relation between district membership and a firm's solvency. The vast empirical literature on ID is silent on this issue and has

Table 5. In sample results: weighted logistic average marginal effects – results over 200 training samples.

	S1		S2	
Financial ratios				
Net working capital/total assets	−0.1808***	(0.0087)	−0.1992***	(0.0089)
EbIT/total assets	−1.3752***	(0.0342)	−1.3149***	(0.0341)
Net worth/total debt	−0.3428***	(0.0079)	−0.3300***	(0.0077)
Total sales/total assets	−0.0983***	(0.0035)	−0.0922***	(0.0035)
Total assets growth	0.1200***	(0.0065)	0.1083***	(0.0064)
Total sales growth	0.0542***	(0.0041)	0.0556***	(0.0042)
RoE variation	0.0005*	(0.0002)	0.0005*	(0.0002)
Industrial variables				
Mark-up			−0.0292***	(0.0034)
Market share			−1.0665***	(0.1345)
District dummy			−0.0219***	(0.0041)
Macro-regions (baseline: North East)				
North West			0.0034	(0.0039)
Centre			0.0247***	(0.0048)
South			0.0846***	(0.0056)
Sectors (baseline: Food)				
Textile			0.0674***	(0.0080)
Leather			0.1193***	(0.0081)
Wood			0.0811***	(0.0079)
Glass, ceramic			0.0700***	(0.0135)
Metal products			0.0515***	(0.0055)
Machinery			0.0496***	(0.0060)
Observations	70,309		70,309	
Pseudo-R^2	0.3964		0.4061	

Note: S1 = only financial (baseline); S2 = financial + industrial (augmented); EbIT = earnings before interest and taxes; and RoE = return on equity. Standard errors are shown in parentheses. All specifications include a constant term. Significance levels: *10%; **5%; ***1%. The log-likelihood ratio test rejects H_0, that is, the augmented specification is a significant improvement over the baseline.

focused primarily on the benefits that agglomeration economies have on economic growth through local externalities. The result of the paper highlights a different advantage linked to ID membership, that is, bankruptcy reduction. A possible explanation could be related to the presence of social capital, which increases the level of trust among firms and institutions sharing the same territory. A higher level of trust might in turn translate, for example, into easier access to credit, which could be decisive in curbing the probability of going bankrupt. Also, the result concerning the positive relation between mark-up and solvency is worthy of note. It seems to suggest that a high mark-up is associated with an efficient use of the firms' large rent and/or a greater market power. Finally, the results show that the size of the single firm relative to its sector (market share) is also relevant to reduce the probability of going bankrupt.

Table 6. In sample results: weighted logistic coefficients – results over 200 training samples.

	S2	
Financial ratios		
Intercept	0.9389***	(0.0541)
Net working capital/total assets	−1.4627***	(0.0630)
EbIT/total assets	−9.6567***	(0.2268)
Net worth/total debt	−2.4252***	(0.0483)
Total sales/total assets	−0.6773***	(0.0244)
Total assets growth	0.7952***	(0.0461)
Total sales growth	0.4083***	(0.0302)
RoE variation	0.0035*	(0.0016)
Industrial variables		
Mark-up	−0.2145***	(0.0247)
Market share	−7.8302***	(0.9835)
District dummy	−0.1621***	(0.0308)
Macro-regions (baseline: North East)		
North West	0.0248	(0.0288)
Centre	0.1799***	(0.0351)
South	0.6038***	(0.0397)
Sectors (baseline: Food)		
Textile	0.4856***	(0.0572)
Leather	0.8507***	(0.0573)
Wood	0.5800***	(0.0558)
Glass, ceramic	0.5012***	(0.0953)
Metal products	0.3774***	(0.0402)
Machinery	0.3624***	(0.0439)
Observations	70,309	
Pseudo-R^2	0.4061	

Note: S2 = augmented model; EbIT = earnings before interest and taxes; and RoE = return on equity. Standard errors are shown in parentheses. Significance levels: *10%; **5%; ***1%.

CONCLUSIONS

We provide a predictive model specifically designed for the Italian economy with the aim of correctly classifying solvent and insolvent firms one year in advance.

Our results seem to suggest two different possible takeaways for economists and practitioners. The first is methodological. WLR and NN perform well in correctly classifying NB firms with the augmented model; and XGB is the best classifier of B firms when using industrial and regional variables. Moreover, the simulation exercise confirms that XGB beats all other methodologies in both specifications and even more in the augmented version.

The second takeaway is related to pinning down the set of variables with which to feed our bankruptcy forecasting models. For the Italian economy industrial and regional variables seem to be relevant not only in determining the probability of bankruptcy, but also in incrementing the forecasting performance of the models. It is important to account for sectoral and regional disparities, and to consider the industrial structure of the firms.

This result is also relevant for the literature on ID and mark-up given that belonging to a district and having a high mark-up increase the ability of firms to be solvent.

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NOTES

1 SMEs dominate the business landscape in Italy, accounting for nearly 80% of the industrial and service labour force, and generating about two-thirds of turnover and value added (see https://www.oecd-ilibrary.org/_nance-and-investment).

2 For different ways to calculate mark-up measures, see the European Central Bank's Competitiveness Research Network (CompNet) study (https://www.ecb.europa.eu/home/pdf/research/compnet/CompNet-database-userguide-round4.pdf).

3 That is, the set of norms and values that creates the fabric of society, glues individuals and institutions together, and constitutes a necessary link for its governance.

4 A part of the literature, differently from this view, highlights the inefficiencies stemming from high market power, that is, when industries are able to charge relatively high prices and benefit from large rents, they might have fewer incentives to improve their efficiency (Cette et al., 2016).

5 See the ISTAT website (https://www.istat.it/it/archivio/150320).

6 Even if this evaluation procedure might resent from the fact that future data may be used to predict past or present data, we choose to apply it since it allows us to construct 200 repeated samples which make our estimates more robust and equipped with additional information such as standard errors. A time-split procedure (a training set on the first years of the sample and a test set on the last years) would overcome the timing problem, but at the same time it would make the repeated random subsampling validation difficult to implement.

7 To compare the error T1 (or T2) provided by two specifications (baseline versus augmented), we apply the McNemar test, which can be used to test the differences between proportions in paired samples. The rationale behind this idea is based on the fact that T1 is a proportion of incorrectly classified as B over the number of observed NB firms. Hence, we are testing the differences between proportions evaluated with different techniques on the same sample. The McNemar test tests the following null hypotheses: (1) the percentage of B firms classified as bankrupted does not change using the augmented model instead of the baseline; and (2) the percentage of NB firms classified as not bankrupt does not change using the augmented model instead of the baseline. Note that the McNemar test is applied on the average confusion matrix obtained on the 200 repetitions.

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