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Thermal characterisation of Triple Concentric Tube Heat Exchangers by applying parameter estimation: direct problem implementation

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Abstract. Heat transfer enhancement in heat exchangers's design represents a key technological challenge because of the increase in the cost of energy and raw materials. A promising technology is represented by the triple tube heat exchangers, in which the heat transfer is enhanced in comparison with the traditional double tube heat exchangers, due to the larger heat transfer area per unit length. Among the different methodologies that can be adopted to assess the performance of the triple tube heat exchangers, parameter estimation procedure represents a promising tool, since it has been successfully applied in many disciplines of engineering. To apply this inverse technique, it is mandatory defining the direct problem, which for the issue here addressed allows evaluating the outlet temperatures of the fluids flowing in the heat exchanger. Since in a triple tube heat exchanger there are three fluids, the approach based on the evaluation of the logarithmic mean temperature difference is no longer valid and an alternative procedure has to be followed. In the present analysis a numerical model for the performance evaluation of triple tube heat exchangers is presented. The validation of the proposed numerical model, carried out by adopting the analytical model available in literature, highlights that the model can be considered accurate and reliable. Moreover, the computational time required to solve the set of equations is very limited.

1. Introduction

A Triple Tube Heat Exchanger (TTHE) is a modified version of a double tube heat exchanger (DTHE), which consists of three concentric tubes. It has three sections: inner tube, inner annulus and outer annulus. The fluid that has to be heated (or cooled), for instance the product in food processes, flows into the inner annulus, while the hot (or cold) service fluid flows both in the inner tube and in the outer annulus. Therefore, the advantages guaranteed by the use of a TTHE compared to a DTHE are represented by the larger heat transfer area per unit length.

Unlike the DTHE, in which the possible flow configurations are two, i.e. parallel flow and counter flow, in a TTHE there are three flows resulting in four configurations: counter-current, co-current, counter-current & co-current and co-current & counter-current [1].

TTHEs are widely used in food and pharmaceutical industries. In particular, in food industry this kind of heat exchanger is used for sterilization, pasteurization and cooling treatments; for instance, the liquid products like milk, cream, pulpy orange juice, etc. are pasteurized with the help of TTHE [2].

Despite the advantages and the wide use of TTHEs, only a few studies that rely on the analysis of the heat transfer phenomena in this kind of device, have been performed, as highlighted by Kumar and. Hariprasath in their recent review [3]. Particularly, the performances of TTHEs have been experimentally investigated by Gomaa et al. [1] who analysed the temperature distribution of the three fluids (chilled water in the inner tube, hot water in the inner annulus, and normal tap water in the outer annulus) along the length of the TTHE. Moreover, they performed a comparison between the performance of the TTHE and that of a DTHE, in terms of effectiveness and heat transfer per unit pumping power. Their findings revealed that the effectiveness of the TTHE was higher than that of the DTHE by about 50%. It has to be highlighted that to evaluate the heat transfer coefficient of the inner annulus they suggested the use of the average log-mean temperature difference between the three fluids, which was defined as the arithmetic mean between the log-mean temperature difference between hot and chilled water and the log-mean temperature difference between hot and normal tap water.

The performance of the TTHE, in terms of temperature variation of the three fluids (i.e. hot, normal tap and cold water) along the length of the heat exchanger, was also experimentally analysed by Quadir et al. [4], for different fluid flow rates and in co-current parallel flow configuration. By considering two flow arrangements, for normal tap and cold water, they found that the heat transfer between the three fluids was more effective when normal tap and cold water flowed in the inner pipe and in the outer annulus, respectively.

The performance analysis of a TTHE, in terms of overall heat transfer coefficient and effectiveness of the heat exchanger was carried out by Mohapatra et al. [5] who experimentally investigated an improved double pipe heat exchanger with a helical tube inserted between two concentric straight tubes. The experiments were carried out by considering hot water (flowing inside the helical tube), normal water (flowing outside the helical tube) and air. Convective heat transfer coefficients of normal and hot water were calculated by adopting "Wilson plots" method, which is one of the simplest methods to assess the average performance of heat exchangers [6]. By adopting this approach, the internal heat transfer coefficient can be indirectly evaluated from the experimental measurements of the overall thermal resistance. Since the technique proposed by Wilson presented some limitations, several different approaches have been proposed in the literature to overcome this limits [7]. Among these techniques, the parameter estimation procedure can be considered a promising tool, since this approach has been successfully adopted in many engineering applications [8-10]. Particularly, the parameter estimation procedure allows to estimate unknown parameters that play an important role in the design and optimization of heat transfer devices. This methodology, based on the assumption that both internal and external convective heat transfer coefficients can be expressed as a function of the Reynolds and Prandtl numbers, has been recently adopted to investigate the performance of DTHE [11].

Since the coefficients, which are usually unknown and cannot be directly measured, are estimated by minimizing the square errors of the prediction with respect to the experimentally measured values, the first step of the parameter estimation procedure is to define the direct problem, which allows to evaluate the predicted values, knowing all of the input parameters. More specifically, the application of this inverse procedure to the performance evaluation of the TTHE starts from the implementation of the direct problem that enables to evaluate the outlet temperatures of the three fluids.

Despite the DTHE, in which there are only two fluids, in a TTHE the energy of the fluid that flows in the inner annulus is exchanged in two opposite directions (to the fluids in the inner tube and outer annulus); therefore, the approach based on the evaluation of the logarithmic mean temperature difference is no longer valid and an alternative procedure has to be followed to evaluate the performance of a TTHE. Furthermore, despite the DTHE, in a THE overall heat transfer coefficient depends on the temperature differences.

Batmaz e Sandeep [12] proposed an analytical model that enables to evaluate the axial temperature distribution of fluids in a TTHE, but to solve the set of equations presented in [12] the use of a mathematical software that have the capability to solve systems of non-linear equations, is required. Furthermore, that analytical model was developed by considering many assumptions (i.e. fluid properties were constant, the heat exchanger was insulated from the surroundings, the system was at steady state, both fluids were incompressible and phase change did not occur at any point in the heat exchanger).

Therefore, in the present work, a numerical approach that allows to tackle the thermal problem in a TTHE is proposed. The numerical model adopted here, which will be used as direct problem in the parameter estimation procedure, was validated using the analytical model.

2. Heat exchanger model

In the present study a TTHE operating in a counterflow arrangement was considered, where the process fluid to be heated (e.g. sterilization of a food fluid) flowed into the inner annulus (i.e. the annulus formed between the pipes 1 and 2, as shown in Figure 1), and the hot service fluid flowed both in the inner tube and in the outer annulus (i.e. the annulus formed by the pipes 2 and 3).

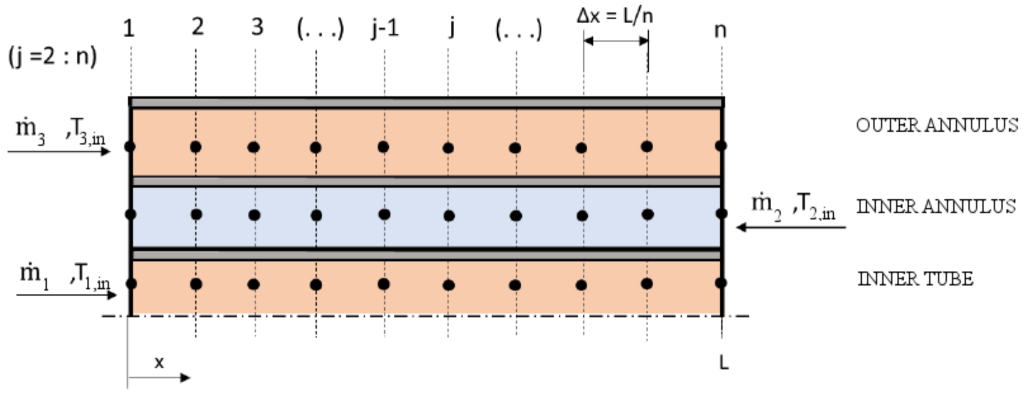

Figure 1. Schematic representation of the studied TTHE.

The three fluids that passed through the system were assumed to be single phase, incompressible, and with constant thermal properties. The inlet temperatures of three fluids were assumed to be known, and so were the three mass flow rates, as shown in Figure 1.

The heat exchanger was assumed to be in a steady-state regime and thermally insulated. Therefore, the heat transfer rates exchanged was obtained from the energy balance that for an infinitesimal fluid element red as follows:

$$dQ_1 = -\dot{m}_1 c_{p1} dT_1 \tag{1}$$
$$dQ_2 = -\dot{m}_2 c_{p2} dT_2 \tag{2}$$
$$dQ_3 = -\dot{m}_3 c_{p3} dT_3 \tag{3}$$

where $\dot{m}$ was the mass flow rate, c_p was the fluid specific heat at a constant pressure and T the fluid bulk temperature.

The subscript *2* indicated the fluid flowing in the inner annulus, while the subscripts *1* and *3* the fluids flowing in the inner tube and in the outer annulus, respectively.

The negative sign in the energy equations were due to the *x*-axis direction and to the counter-current flows configuration.

Being the heat exchanger thermally insulated, energy balance equation for the product side could be evaluated also by:

$$dQ_2 = dQ_1 + dQ_3 \tag{4}$$

By introducing the overall heat transfer coefficients U_{21} and U_{23} Eqs. (1) and (3) were rewritten as follows:

$$dQ_1 = U_{21} \Delta T_{21} dA_1 \tag{5}$$
$$dQ_3 = U_{23} \Delta T_{23} dA_2 \tag{6}$$

where U_{21} and U_{23} were the overall heat transfer coefficient between the product and the hot fluid 1 and between the product and the hot fluid 3, respectively.
ΔT_{21} and ΔT_{23} were the temperature difference between the product and the hot fluid 1 and between the product and the hot fluid 3, respectively, which were defined as follows:

$$\Delta T_{21} = \mathrm{T}_1 - \mathrm{T}_2 \tag{7}$$
$$\Delta T_{23} = \mathrm{T}_3 - \mathrm{T}_2 \tag{8}$$

dA_1 and dA_2 were the heat transfer area of the pipe 1 and 2, respectively, which could be expressed in terms of the perimeter of the pipes P_1 and P_2 (i.e. $dA = Pdx$).
Therefore, Eqs. (5) and (6) were rewritten as follows:

$$dQ_1 = U_{21}P_1\Delta T_{21}dx \tag{9}$$
$$dQ_3 = U_{23}P_2\Delta T_{23}dx \tag{10}$$

By substituting Eqs. (9) and (10) in Eq. (4)

$$dQ_2 = U_{21}P_1\Delta T_{21}dx + U_{23}P_2\Delta T_{23}dx \tag{11}$$

Therefore, the complete set of energy equations were:

$$\dot{m}_2\mathrm{c}_{p2}\mathrm{dT}_2 = -U_{21}P_1\Delta T_{21}dx - U_{23}P_2\Delta T_{23}dx \tag{12}$$
$$\dot{m}_1\mathrm{c}_{p1}d\mathrm{T}_1 = -U_{21}P_1\Delta T_{21}dx \tag{13}$$
$$\dot{m}_3\mathrm{c}_{p3}d\mathrm{T}_3 = -U_{23}P_2\Delta T_{23}dx \tag{14}$$

Energy equations were solved by applying the finite difference method:

$$\dot{m}_2\mathrm{c}_{p2}\left(\mathrm{T}_{2\,(j-1)} - \mathrm{T}_{2\,(j)}\right) = U_{21}P_1(\mathrm{T}_2 - \mathrm{T}_1)_{(j)}\Delta x + U_2P_2(\mathrm{T}_2 - \mathrm{T}_3)_{(j)}\Delta x \tag{15}$$
$$\dot{m}_1\mathrm{c}_{p1}\left(\mathrm{T}_{1\,(j)} - \mathrm{T}_{1\,(j-1)}\right) = -U_{21}P_1(\mathrm{T}_2 - \mathrm{T}_1)_{(j-1)}\Delta x \tag{16}$$
$$\dot{m}_3\mathrm{c}_{p3}\left(\mathrm{T}_{3\,(j)} - \mathrm{T}_{3\,(j-1)}\right) = -U_{23}P_2(\mathrm{T}_2 - \mathrm{T}_3)_{(j-1)}\Delta x \tag{17}$$

being $\Delta x = L/n$ the space step, as shown in Figure 1.
Equations (15)-(17) were solved by considering the following boundary conditions:

$$\mathrm{T}_{2\,(n)} = \mathrm{T}_{2,in} \tag{18}$$
$$\mathrm{T}_{1\,(1)} = \mathrm{T}_{1,in} \tag{19}$$
$$\mathrm{T}_{3\,(1)} = \mathrm{T}_{3,in} \tag{20}$$

where the subscript *in* refers to the inlet conditions.
Therefore, the outlet temperatures were evaluated as follows:

$$\mathrm{T}_{2\,(j-1)} = \mathrm{T}_{2\,(j)} + \frac{\Delta x}{\dot{m}_2\mathrm{c}_{p2}}\left[U_{21}P_1\left(\mathrm{T}_{1\,(j)} - \mathrm{T}_{2\,(j)}\right) + U_{23}P_2\left(\mathrm{T}_{3\,(j)} - \mathrm{T}_{2\,(j)}\right)\right] \tag{21}$$

$$\mathrm{T}_{1\,(j)} = \mathrm{T}_{1\,(j-1)} - \frac{U_{21}P_1\left(\mathrm{T}_{1\,(j-1)} - \mathrm{T}_{2\,(j-1)}\right)\Delta x}{\dot{m}_1\mathrm{c}_{p1}} \tag{22}$$

$$\mathrm{T}_{3\,(j)} = \mathrm{T}_{3\,(j-1)} - \frac{U_{23}P_2\left(\mathrm{T}_{3\,(j-1)} - \mathrm{T}_{2\,(j-1)}\right)\Delta x}{\dot{m}_3\mathrm{c}_{p3}} \tag{23}$$

The overall heat transfer coefficients, referring to the internal heat transfer surface areas, were related to the convective heat transfer coefficients by [13]:

$$\frac{1}{U_{21i}A_{1i}} = \frac{1}{h_1 A_{1i}} + R_{w12} + \frac{1}{h_2 A_{1o}} \tag{24}$$

$$\frac{1}{U_{23i}A_{2i}} = \frac{1}{h_2 A_{2i}} + R_{w23} + \frac{1}{h_3 A_{2o}} \tag{25}$$

where A_{1i} and A_{1o} were the internal and external heat transfer area of the inner pipe (i.e. pipe 1), respectively, while A_{2i} and A_{2o} were the internal and external heat transfer area of the inner annulus (i.e. pipe 2), respectively; U_{21i} was the internal overall heat transfer coefficient between pipe 1 and 2; h_{21i} and h_{21o} were the internal and external convective heat transfer coefficients between pipe 1 and 2, respectively; U_{23i} was the internal overall heat transfer coefficient between pipe 2 and 3; h_{23i} and h_{23o} were the internal and external convective heat transfer coefficients between pipe 2 and 3, respectively. R_{w12} and R_{w23}were the wall thermal resistances, which were approximated as [13]:

$$R_{w12} = \frac{\ln[(D_{1o})/D_{1i}]}{2\pi\lambda_w L} \tag{26}$$

$$R_{w23} = \frac{\ln[(D_{2o})/D_{2i}]}{2\pi\lambda_w L} \tag{27}$$

where D_{1i} and D_{1o} were the internal and external diameter of the pipe 1, while D_{2i} and D_{2o} were the internal and external diameter of the pipe 2 (see Figure 2), respectively; λ_w, and L were the wall thermal conductivity, and pipe length, respectively.

Figure 2. Scheme of the investigated heat exchanger.

The convective heat transfer coefficients were evaluated by the Nusselt numbers, which were expressed by the following equations:

$$\mathrm{Nu}_1 = \frac{h_1 D_{1i}}{\lambda_1} \tag{28}$$

$$\mathrm{Nu}_2 = \frac{h_2 D_{2h}}{\lambda_2} \tag{29}$$

$$\mathrm{Nu}_3 = \frac{h_3 D_{3h}}{\lambda_3} \tag{30}$$

where D_{2h} and D_{3h} were the hydraulic diameter of the pipe 2 and 3, respectively.

For the inner tube the Nusselt number in the fully developed region was expressed as follows [13]:

$$\mathrm{Nu}_1 = C_1 Re_1^{\alpha_1} Pr_1^{\beta_1} \tag{31}$$

On the other hand, for inner and outer annulus, the Nusselt numbers in the fully developed region were evaluated by [14]

$$\mathrm{Nu}_2 = \frac{C_2 \dfrac{D_{2i}}{D_{1e}}}{\left(\dfrac{D_{2i}}{D_{1e}} + 1\right)^{\gamma_2}} Re_2^{\alpha_2} Pr_2^{\beta_2} \tag{32}$$

$$\mathrm{Nu}_3 = \frac{C_3 \dfrac{D_{3i}}{D_{2e}}}{\left(\dfrac{D_{3i}}{D_{2e}} + 1\right)^{\gamma_3}} Re_3^{\alpha_3} Pr_3^{\beta_3} \tag{33}$$

where C_1, α_1, β_1, C_2, α_2, β_2, γ_2, C_3, α_3, β_3, and γ_3 were a set of characteristic coefficients of the heat exchanger under test. All of these coefficients are usually unknown and can be estimated by applying the parameter estimation procedure. It has to be highlighted that it is very important to know these parameters since they allow to develop generalized Nusselt number correlations, which play an important role in the design and optimization of heat transfer devices.

Reynolds and Prandtl numbers were defined as follows:

$$Re_1 = \frac{\rho_1 \bar{v}_1 D_{1i}}{\mu_1} \tag{34}$$

$$Re_2 = \frac{\rho_2 \bar{v}_2 D_{2h}}{\mu_2} \tag{35}$$

$$Re_3 = \frac{\rho_3 \bar{v}_3 D_{3h}}{\mu_3} \tag{36}$$

$$Pr_1 = \frac{\rho_1 c_{p1}}{\mu_1 \lambda_1} \tag{37}$$

$$Pr_2 = \frac{\rho_2 c_{p2}}{\mu_2 \lambda_2} \tag{38}$$

$$Pr_3 = \frac{\rho_3 c_{p3}}{\mu_3 \lambda_3} \tag{39}$$

where $\bar{v}$ was the mean fluid velocity over the specific cross-section, whereas ρ and μ are the density and dynamic viscosity of each fluid, respectively.

3. Results and discussion

The model for the direct problem defined in Eqs.(21) – (23) was implemented within the Matlab R2019a environment, by considering 1000 nodes (i.e. $\Delta x = L/n$=10.1/1000=0.0101).

Since the proposed method was iterative, the evaluation of the outlet temperatures was repeated several times until a proper tolerance was reached, starting from the output of the previous iteration. It was observed that for a number of iterations equal higher than 9000, the tolerance of $1e^{-6}$ could be reached, as shown in Figure 3. Therefore the results presented here were obtained by repeating the evaluation at least 9000 times.

The proposed model was validated by resorting to the iterative analytical model available in literature [12]. The validation was carried out by considering a TTHE operating in a counterflow arrangement with cold water in the inner tube and hot water in the annuli.

The geometrical and thermal characterizations of the considered heat exchanger are presented in Table 1. The heat exchanger was supposed to work in a range of Re_1, Re_2 and Re_3 from 3,000 to 10,000.

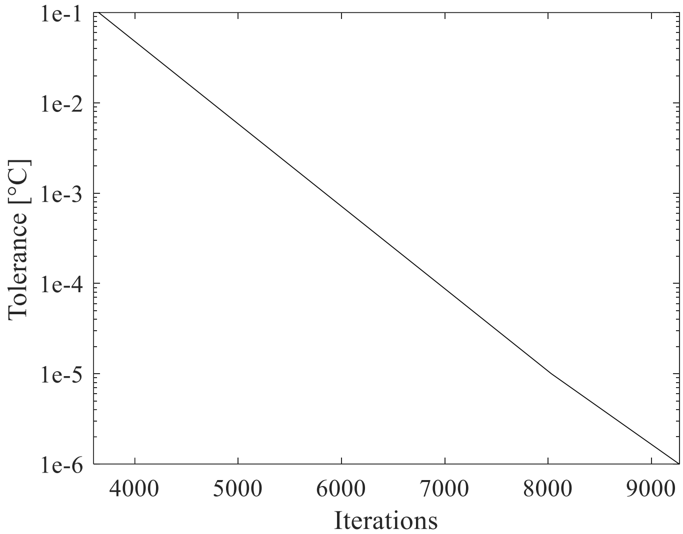

Figure 3. Convergence analysis.

The results presented here were obtained assuming the following values: 0.023, 0.80 and 0.40 for C_1, α_1, and β_1, respectively; 0.04, 0.80, 0.40 and 0.20 for C_2, α_2, β_2 and γ_2, respectively; and 0.04, 0.80, 0.40 and 0.2 for C_3, α_3, β_3, and γ_3, respectively.

Table 1. Geometrical data of the TTHE and thermal properties of the considered working fluids for validating the proposed model.

Parameter	Pipe 1	Pipe 2	Pipe 3
Internal diameter [mm]	40.94	66.93	83.80
External diameter [mm]	48.30	73.03	88.90
Length [mm]	10.10	10.10	10.10
Wall thermal conductivity [W/(m K)]	15	15	15
Fluid density [kg/m^3]	1000	1000	1000
Fluid dynamic viscosity [mPa s]	1.00	1.00	1.00
Fluid specific heat at a constant pressure [J/ (kg K)]	4180	4180	4180
Fluid thermal conductivity [W/ (m K)]	0.60	0.60	0.60

An excellent agreement was found for all of the Reynolds numbers considered in the present analysis, as shown in Table 2, where the difference between the temperatures estimated by applying the numerical model presented here and the results obtained by means of the analytical model are reported.

Table 2. Comparison between present results and data available in literature [12]

Parameter	$\lvert \Delta T \rvert$ [°C]			
	$Re_1 = Re_3$= 3000 and Re_2=3000	$Re_1 = Re_3$= 3000 and Re_2=10000	$Re_1 = Re_3$= 10000 and Re_2=3000	$Re_1 = Re_3$= 10000 and Re_2=10000
$T_{1,out}$	0.06	0.45	0.05	0.02
$T_{2,out}$	0.09	0.15	< 1e^{-6}	0.18
$T_{3,out}$	0.06	0.02	0.29	0.03

It can be observed that for all of the analysed Reynolds numbers, the error between the numerical and analytical results is comparable with the uncertainty in the temperature measurements. Therefore, the proposed model can be considered reliable and robust.
Moreover, it has to be highlighted that unlike the analytical model, the numerical model presented here allows potentially considering fluid with temperature-dependent thermophysical properties and can be easily updated to account for the heat losses through the environment as well.

4. Conclusions

A numerical model for the performance evaluation of the triple tube heat exchangers was developed and validated. Particularly, the validation, carried out by comparing the results of the proposed model to those obtained by means of an analytical model available in literature, reveals that the model is accurate and reliable. Therefore, the proposed model is suitable for the application of the parameter estimation procedure to the evaluation of the performance of the TTHE. More specifically, this model will be adopted for solving the direct problem embedded in the formulation of the inverse approach applied to the same issue, with the aim of estimating the relevant parameters for the accurate design of TTHE.

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