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Probabilistic assessment of footbridge response to single walkers

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Abstract Among the load scenarios considered for the serviceability assessment of human-induced footbridge vibration, is that of the transient action of a single pedestrian or a small group of pedestrians. Although such action is stochastic due to the variability of gait parameters, available Codes and Guidelines all assume it is deterministic and equal to that coming from the "worst pedestrian ever" for the given footbridge. This approach is sound from an engineering point of view but does not allow control of the probability of failure. The present work deals with a reliability-based procedure for the serviceability assessment of the footbridge peak characteristic accelerations due to pedestrian induced actions. Based on the results obtained incorporating the effects of the inter-subject variability of gait parameters and of the uncertainties in footbridge dynamic properties, a design response spectrum is proposed for both vertical and lateral vibrations. The proposed procedure lends itself for immediate Code implementation.

Keywords Vibration serviceability · Probabilistic assessment · Footbridges · Peak response · Human-induced vibrations · Single walker crossing

1 Introduction

If only static live loads are considered in the design of footbridges, these may turn out too flexible and insufficiently damped, therefore unable to meet serviceability requirements against vibrations. Indeed, the vibration serviceability assessment is central to the structural design of footbridges, and the accurate prediction of footbridge dynamic response under different pedestrian-induced load scenarios is required. Among these, there are those considering a single pedestrian, a group of pedestrians, and a more or less dense crowd.

The design of footbridges against pedestrian-induced vibrations requires knowledge of (*i*) the characteristics of pedestrian action, (*ii*) a response evaluation method, and (*iii*) a comfort criterion [1]. Standards and Guidelines have been developed over the years to help the designer in the evaluation of the vibration serviceability based on simplified loading models, simulating different possible scenarios [2,3]. Most of these contain procedures for vibration serviceability assessment assuming that the action is deterministic [4–7]. Yet this is known to be stochastic, and it would be desirable to incorporate its variability in the loading models [8]. Only Hivoss Guidelines [9] give a method in which the footbridge acceleration is evaluated from the spectral characteristic of the load, and the 95th fractile of the peak acceleration due to a stream of walkers is calculated applying an empirical peak factor to the RMS response due to N uncorrelated walkers.

The 95th fractile of vertical peak accelerations was derived by [10]. They studied the vertical vibration serviceability of footbridges, based on a probabilistic characterization of pedestrian-induced forces taking into account inter-subject variability and considering only one mode of vibration. Comparison of their procedure with similar methods contained in Standards and design Guidelines has pointed out that the latter are mainly

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conservative. Consequently, further investigations are required on the evaluation of the PDF of the maximum dynamic response.

Ricciardelli and Demartino [3] compared background hypotheses, fields of applicability, and results obtained of a number of different loading and response evaluation models. In particular, they compared single-walker models, multiple-walker models, interaction models (inter-walker and walker-structure), and instability models, together with current design procedures incorporated in available Standards and Guidelines. Similarly, [11] applied design procedures to a number of steel footbridges, and concluded that a critical revision of these is needed, as even though inspired by the same principles and applying the same rules, they bring rather different results and different reliability levels. An investigation into the reliability level of several Guidelines across different design conditions and load scenarios was developed by [12]; it was found that the considered Guidelines are not consistent among each other, for both design and rare reversible load scenarios (according to ISO 2934 [13]). The results of their study suggested that Guideline provisions should be calibrated to achieve a higher reliability index for design conditions, and to meet the minimum acceptable reliability index for rare events. They also recommended that comfort limits, depending on the occurrence frequency of the load and on the class of the bridge should be adopted to yield cost-effective design. Then, [14] presented a first attempt at calibrating partial factors to be used in conjunction with footbridge design Guidelines so to achieve a uniform target reliability level across different classes of bridges and under both design and rare traffic events.

A probabilistic method for the serviceability assessment of footbridge vibrations due to single and multiple walkers was proposed by [15]. The procedure complies with ISO 2394 [13] and EN1990 [16], and allows controlling the reliability level. In particular, the probability distribution of the footbridge peak acceleration was fitted to a generalized extreme value (GEV) distribution through the maximum likelihood estimates (MLE) method. However, this approach proved to potentially lead to an inaccurate estimation of the probability of failure due to a possible poor fitting of the tail of the empirical distribution.

In spite of this effort, a consistent probabilistic procedure for the serviceability assessment of footbridge vibrations due to a single walker, and a comparison with deterministic approaches is not yet available. Although the research interest is nowadays mainly oriented toward the multi-pedestrian case, the need for analyzing the single pedestrian case arises for three main reasons: (*i*) it may induce the largest acceleration, especially for short span and low-damped footbridges, (*ii*) many Standards and Guidelines refer to this load scenario, and (*iii*) vibration assessment procedures for multi-pedestrian loading are often derived from the single pedestrian case.

Within this topic, the present work aims to provide a response model for the reliability assessment of footbridge vibrations to single walkers. To this purpose, first a Standard Population (SP) of walkers is introduced and the effects of the inter-subject variability are evaluated in terms of characteristic footbridges acceleration. Then, uncertainties in footbridge dynamic properties are discussed and their consequences assessed. Finally, the results are used to develop Design Response Spectra to be used within the Partial Safety Factors Method incorporated in structural Codes. The proposed probabilistic approach is applied to a prototype footbridge and its response compared with the the results deriving from application of a deterministic approach.

2 Footbridge loading and response models

2.1 Serviceability assessment of footbridges

The serviceability of footbridges against pedestrian-induced vibrations can be evaluated through a two-stage process. Within the first stage, the structural frequency is restricted to fall outside critical ranges, which ensures that serviceability requirements are right satisfied. On the other hand, if the natural frequency falls within the critical frequency range, the footbridge accelerations need to be evaluated and compared with acceleration limits. Thus, a dynamic analysis is required and the serviceability limits assessed by:

$$a_{\max,N} \leq a_{\lim} \tag{1}$$

where $a_{\max,N}$ is the peak response of the footbridge under N walkers, and $a_{\lim}$ is the limit acceleration.

The maximum acceleration is usually given in the format:

$$a_{\max,N} = N_{\mathrm{e}} \cdot a_{\max} \cdot \Psi(f_{\mathrm{b}}) \tag{2}$$

where N_{e} is the equivalent number of walkers, $a_{\max}$ is the maximum transient acceleration due to one walker exciting the footbridge, and $\Psi(f_{\mathrm{b}})$ is a coefficient that reduces the response when the fundamental frequency of the footbridge, f_{b}, is away from the walking frequency.

The equivalent number of walkers, N_e, accounts for the partial level of synchronization of a larger group. It is equal to unity in the case of a single walker, and to 3.0 or 2.34 in the case of a group of 13 walkers for vertical and lateral vibrations, respectively [4]; it is related to the pedestrian density δ in the case of stream of walkers [9]. The acceleration $a_{\max}$ of a generic beam-like footbridge (i.e., multi-span or single span with different support conditions) is often described as function of the dynamic response of the equivalent simply supported footbridge and of a configuration factor [17,18]. For a more comprehensive description of N_e, $a_{\max}$, and $\Psi(f_b)$, see [2,3,11]. Following this approach, the acceleration induced by the crossing of a group or a stream of walkers can be evaluated from the maximum transient response of a simply supported footbridge due to single walkers.

Although probabilistic models to describe the vibration response induced by moving loads are available, the approach followed by Standards and Guidelines is usually deterministic. Thus, a probabilistic procedure for the serviceability assessment of footbridge vibrations has not yet been considered and the reliability level is not assessed.

2.2 Walking force modeling

Ground reaction forces (GRFs) are the forces exerted by pedestrians to the walking surface. They are characterized by a large level of randomness which is due to both inter- and intra-subject variability [19,20]. In addition, GRFs are modified by the interaction among walkers and of the walkers with the vibrating structure.

One possible approach to the modeling of GRFs is to neglect intra-subject variability, therefore assuming that a walker generates identical footfalls with constant frequency. According to the approach used by different Standards [5,21], the dynamic part of the GRF can be expressed as:

$$F(t) = W \cdot \sum_{k=1}^{n} \mathrm{DLF}_k \; [\sin(k \cdot \bar{\omega} \cdot t) - \psi_k] \tag{3}$$

where W is the weight of the walker; n is the number of load harmonics considered; DLF_k is the kth dynamic load factor (DLF); $\bar{\omega}$ the load fundamental circular frequency equal to $2\pi f_w$ for vertical direction and πf_w for lateral direction, f_w being the step frequency; ψ_k is the phase lag of the kth harmonic.

2.3 Transient response to a single crossing

A closed-form solution of the equation of a beam subjected to a transient crossing load is provided by different authors [22–24]. In this paper, the solution of [22] is used. Accordingly, the acceleration in the vertical or lateral direction can be expressed as [22]:

$$a(t) = -\frac{W}{mL} \cdot \sum_{k=1}^{n} \mathrm{DLF}_k \cdot g_k(t) \tag{4}$$

where L is the span length, m is the footbridge mass per unit length, n is the number of load harmonics considered, and:

$$g_k(t) = \left[C_{1k} \frac{\omega_{1k}^2}{\omega_k^2} \sin(\omega_{1k} t + \varphi_{1k}) - C_{2k} \frac{\omega_{2k}^2}{\omega_k^2} \sin(\omega_{2k} t + \varphi_{2k}) + s_k C_{Dk} e^{-\xi_k \omega_k t} \sin(\omega_{Dk} t + \varphi_{Dk} + \varphi_k) \right] \tag{5}$$

In Eq. (5), ω_D is the damped frequency of the beam in the first mode, and:

$$\omega_{ik} = \bar{\omega}\left(1 + (-1)^i \frac{k\pi v}{\bar{\omega} L}\right); \quad \tan\varphi_{ik} = \frac{\omega_k^2 - \omega_{ik}^2}{2\xi_k \omega_k \omega_{ik}} \qquad \text{for } i = 1, 2$$

$$C_{ik} = \frac{\omega_k^2}{\sqrt{\left(\omega_k^2 - \omega_{ik}^2\right)^2 + (2\xi_k \omega_k \omega_{ik})^2}}$$

$$C_{Dk} = \sqrt{A_k^2 + B_k^2}\,; \; \tan\varphi_{Dk} = \frac{B_k}{A_k}\,; \; s_k = -\mathrm{sgn}(A_k)$$

$$A_k = \frac{\xi_k}{\omega_k^2\sqrt{1-\xi_k^2}}\left[\left(\omega_k^2+\omega_{1k}^2\right)C_{1k}^2+\left(\omega_k^2+\omega_{2k}^2\right)C_{2k}^2\right]$$
$$B_k = \frac{1}{\omega_k^2}\left[\left(\omega_k^2-\omega_{1k}^2\right)C_{1k}^2+\left(\omega_k^2-\omega_{2k}^2\right)C_{2k}^2\right]$$

Notice that in Eq. (5), v is the walking speed and is the modal damping ratio of the footbridge. Moreover, the two frequencies ω_{1k} and ω_{2k}, and the corresponding phase angles φ_{1k} and φ_{2k}, are representative of the two harmonic components of the same amplitude $W \cdot \mathrm{DLF}_k$ in which the load is shown to be decomposed by [22].

From Eq. (4), it follows that:

$$a_{\max} = \max|a(t)| = \frac{W}{mL}\cdot\max\left(\sum_{k=1}^{n}\mathrm{DLF}_k\cdot g_k(t)\right) \tag{6}$$

In general, the maximum response of a structure to a multi-harmonic load is not equal to the sum of the maximum responses to each single harmonic load. Indeed, if t^* is the time at which the maximum acceleration occurs, then:

$$a_{\max} = \frac{W}{mL}\cdot\sum_{k=1}^{n}\mathrm{DLF}_k\cdot g_k(t^*) \tag{7}$$

On the other hand, when the footbridge frequency is close to the jth frequency of the load, then the sum:

$$a_{\max} = \sum_{k=1}^{n}a_{\max,k} = \frac{W}{mL}\cdot\sum_{k=1}^{n}\mathrm{DLF}_k\cdot\max|g_k(t)| \tag{8}$$

of the maximum effects $a_{\max,k}$ of the kth harmonic load can be considered a reasonable approximation of Eq. (7); this brings a slight error, which increases as the span length of footbridge decreases.

3 Probabilistic modeling of footbridge response to single walkers

3.1 Standard population

The force model of Eq. (3) is deterministic within the same pedestrian, as it neglects the intra-subject variability of gait parameters. Thus, in this work, the definition of a Standard Population (SP) of walkers aims at characterizing inter-subject variability, which allows building a footbridge probabilistic response model. On the other hand, some aspects of intra-subject variability (i.e., the variation of the GRF from one step to another) are implicitly considered in the values chosen for the DLFs. These are usually derived from a broad- or a narrow-band loading model, so to avoid the underestimation of the GRFs which would derive from the use of the perfectly periodic model of Eq. (3) [25].

The parameters governing GRFs vary with the physical characteristics of the walker (height, age, gender, etc.), cultural and racial differences, travel purpose, clothing and shoes as well as with the type of walking surface. The SP defined in this Section is based on research developed in European countries.

It is observed that humans can walk up to 4 m/s [26] but the speed of roughly 2.2 m/s represents a natural transition from walking to running [27,28]. The walking speed v is usually considered as normally distributed, and a large scatter in the mean value and Standard Deviation (STD) is found in the literature. Although the correlation between the walking speed and the step frequency is also high [25,29], many researchers have considered the step frequency as independent of walking speed and normally distributed [30]. The DLFs are usually derived from force measurements on instrumented floors and treadmills [31]. Vertical DLFs are found to be frequency-dependent [32], while lateral DLFs are found to be widely scattered and with a low to zero correlation with the walking frequency [25,31].

All the distribution parameters used in this work to characterize the European Standard Population are shown in Table 1. To each gait parameter in column 1, a random variable is associated with column 2, and the lowercase symbol is then used to indicate the value taken by that random variable.

Table 1 Standard population of walkers: distribution parameters and references

Gait parameter	Random variable	Unit	Probability distribution	References
v	A_1	m/s	$\mathcal{N}\,(1.41,\ 0.224)$	[33]
f_{w}	A_2	Hz	$\mathcal{N}\,(1.87,\ 0.186)$	[34]
W	A_4	N	$\mathcal{N}\,(744,\ 130)$	[9]
$\mathrm{DLF}_{V,1}$	A_3	–	$0.37(f_{\mathrm{w}} - 0.95) \le 0.5$	[35]
$\mathrm{DLF}_{V,2}$	A_3	–	$0.054 + 0.0044\,f_{\mathrm{w}}$	[35]
$\mathrm{DLF}_{V,3}$	A_3	–	$0.026 + 0.0050\,f_{\mathrm{w}}$	[35]
$\mathrm{DLF}_{V,4}$	A_3	–	$0.010 + 0.0051\,f_{\mathrm{w}}$	[35]
$\mathrm{DLF}_{L,1}$	A_3	–	$\mathcal{N}\,(0.042,\ 0.0144)$	[25,31]
$\mathrm{DLF}_{L,2}$	A_3	–	$\mathcal{N}\,(0.007,\ 0.0016)$	[25,31]
$\mathrm{DLF}_{L,3}$	A_3	–	$\mathcal{N}\,(0.022,\ 0.0061)$	[25,31]
$\mathrm{DLF}_{L,4}$	A_3	–	$\mathcal{N}\,(0.004,\ 0.0023)$	[25,31]

3.2 Analytical model

The response model of Eq. (8) suggests that the maximum value of the modal acceleration of Eq. (6) can be evaluated starting from the maximum footbridge acceleration to a single harmonic load:

$$a_{\max,k} = \frac{W}{mL} \cdot \mathrm{DLF}_k \cdot h_k(v,\, f_{\mathrm{w}}) \tag{9}$$

with $h_k(v,\ f_{\mathrm{w}}) = \max |g_k\,(t)\,|$, and with $g_k(t)$ given by Eq. (5) and evaluated for the relevant values of the velocity v and step frequency f_{w} of the walker. The input parameters to Eq. (9) are the characteristics of the walker, which determine the modal force, and the properties of the footbridge, which affect the dynamic response.

Setting:

$$y = mL \cdot a_{\max,k} = W \cdot \mathrm{DLF} \cdot h_k(v,\, f_{\mathrm{w}}) \tag{10}$$

for the kth harmonic response, the random variable Y can be expressed as the product function:

$$Y = A_3 \cdot A_4 \cdot U\,(A_1, A_2) \tag{11}$$

where $U(A_1, A_2)$ is the random variable associated with the function $h(\cdot)$, and the terms A_1, A_2, A_3, and A_4 are introduced to represent the variables associated with the pedestrian characteristics v, f_{w}, W, and DLF_k, respectively, so to make the next steps easier to read (see also Table 1). Equation (11) can be also written as:

$$Y = A_4 \cdot Z\,(A_1, A_2, A_3) \tag{12}$$

with:

$$Z(A_1, A_2, A_3) = A_3 \cdot U\,(A_1, A_2) \tag{13}$$

Probability theory is used to derive the PDF of the random variable Y which can be expressed as:

$$f_Y(y) = \int_{-\infty}^{+\infty} \frac{1}{|a_4|} \cdot f_{A_4}\,(a_4) \cdot f_Z\left(\frac{y}{a_4}\right) \mathrm{d}a_4, \tag{14}$$

while the distribution function of the random variable Z is

$$f_Z(z) = \int_{-\infty}^{+\infty} \frac{1}{|a_3|} \cdot f_{A_3}(a_3) \cdot f_U\left(\frac{z}{a_3}\right) \mathrm{d}a_3 \tag{15}$$

Furthermore, applying the definition of conditional probability to the pair of random variables $U(A_1, A_2)$ and A_2 , it follows that:

$$f_{U,A_2}(u, a_2) = f_{A_2}(a_2) \cdot f_{U|A_2}\,(u|a_2) \tag{16}$$

and the marginal distribution function of U is:

$$f_U(u) = \int f_{U,A_2}(u, a_2)\,\mathrm{d}a_2 = \int f_{A_2}(a_2) \cdot f_{U|A_2}\,(u|a_2)\;\mathrm{d}a_2 \tag{17}$$

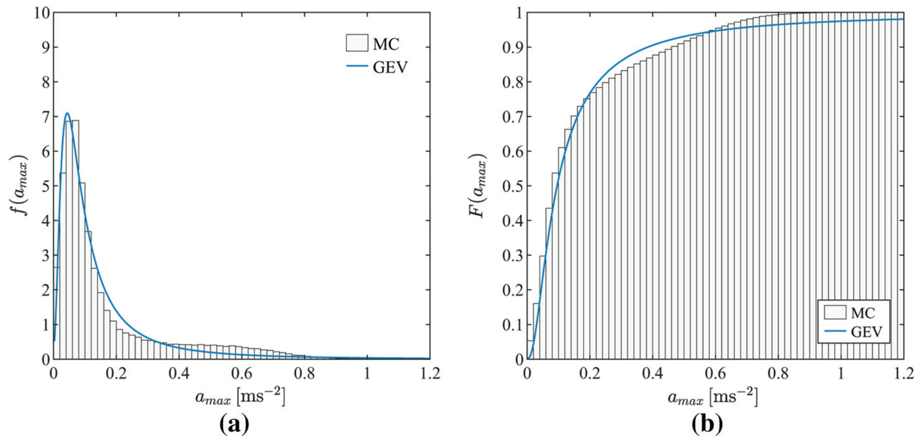


Fig. 1 PDF (**a**) and CDF (**b**) of the response of a footbridge with span length, $L = 50$ m, damping ratio $\xi = 0.5\%$, and distributed mass $m = 1000$ kg resonant with the first load harmonic ($f_b = 1.87$ Hz)

Finally, substitution of Eq. (17) into Eq. (15) and then into Eq. (14) leads to:

$$f_{A_k}(a_{\max,k}) = \iiint \frac{1}{|a_3 \cdot a_4|} \cdot f_{A_3}(a_3) \cdot f_{A_4}(a_4) \cdot f_{A_2}(a_2) \cdot f_{U|A_2}\left(\frac{mL}{a_3 \cdot a_4} \cdot a_{\max} \middle| a_2\right) \mathrm{d}a_2\, \mathrm{d}a_3\, \mathrm{d}a_4 \quad (18)$$

being $f_A(a_{\max}) = f_Y(mL \cdot a_{\max})$ because of the deterministic footbridge characteristics.

The term $f_{U|A_2}$ of Eq. (18) can be expressed as:

$$f_{U|A_2}(u|a_2) = f_{A_1}\left(h^{-1}(u|a_2)\right) \cdot \left[\left|\frac{\mathrm{d}h}{\mathrm{d}a_1}\left(h^{-1}(u|a_2)\right)\right|\right]^{-1} \quad (19)$$

It must be noted that the function $h(a_1, a_2)$ associated with the random variable $U(A_1, A_2)$ represents the acceleration frequency response function of the footbridge subjected to a moving harmonic force given the crossing velocity $a_1 = v$. Thus, the function $h(a_1|a_2)$ describes the variation of the modal response for a fixed value of $a_2 = f_w$.

In Eq. (18), the stochastic nature of all uncorrelated load parameters is taken into account. This condition is considered for modeling the PDF of maximum lateral acceleration, $f_{A_k}^L(a_{\max,k})$. On the other hand, the DLFs are assumed as frequency dependent for vertical vibrations. Thus, Eq. (18) must be rewritten as:

$$f_{A_k}^V(a_{\max,k}) = \iint \frac{1}{|a_3(a_2) \cdot a_4|} \cdot f_{A_2}(a_2) \cdot f_{A_4}(a_4) \cdot f_{U|A_2}\left(\frac{m \cdot L}{a_3(a_2) \cdot a_4} \cdot a_k \middle| a_2\right) \mathrm{d}a_2\, \mathrm{d}a_4 \quad (20)$$

3.3 Numerical solution

The probabilistic assessment of the maximum lateral or vertical accelerations requires integration of Eq. (18). However, a closed form solution cannot be found, also because a suitable expression for the function $h(v, f_w)$ and for its inverse $h^{-1}(v, f_w)$ is not available. Consequently, for the evaluation of the PDF of the maximum acceleration, an integration scheme is needed. In the following, the PDF of Eq. (18) is evaluated through Monte Carlo simulations.

An example of empirical PDF and CDF of the maximum acceleration, $f_A(a_{\max})$ and $F_A(a_{\max})$, is shown in Fig. 1 for vertical vibrations. They are derived for a simple supported footbridge with frequency $f_b = \bar{f}_w$ ($\bar{f}_w$ being the mean value of the SP step frequency, f_w), span length $L = 50$ m, uniformly distributed mass $m = 1000$ kg/m, and damping ratio $\xi = 0.5\%$. A sample of $5 \cdot 10^6$ walkers was used in the simulations, and the maximum footbridge acceleration response was evaluated. As suggested by [15], in Fig. 1, an attempt to model the maximum response to a single pedestrian using the generalized extreme value (GEV) distribution is also made. This, however, proves inappropriate as the underlying population of maxima is not identically distributed. In fact, even though the acceleration response comes from identically distributed walkers, application of a deterministic response model makes the response population not identically distributed, due to the nonlinear nature of the response spectrum.

In the next sections, Monte Carlo simulations are used to evaluate the empirical CDF of the footbridge accelerations. Also in this case, a sample of $5 \cdot 10^6$ walkers is considered, larger than $3/p_f$ that is generally required to achieve an acceptable level of accuracy, p_f being the target probability of failure [36]. Numerical simulations involved ideal simply supported footbridges, i.e., beam-like bridges which structural properties are assumed as deterministic and constant along the longitudinal axis. Fundamental frequencies up to 5 Hz for vertical vibrations and up to 4 Hz for lateral vibrations are considered. To cover the typical range of footbridges, the damping ratio and span length are set to be in the range 0.2 to 2.0% and 10 to 200 m, respectively [3]. Finally, Eq. (4) suggests that the results are not influenced by m, thus this quantity is set to one ($m = 1$ kg/m). To maximize acceleration, different combinations of responses to single harmonic loads are considered, i.e., the phase angle of each harmonic is set as 0° or 180°, according to the result of pedestrian force measurements (e.g., [37,38]).

4 Probabilistic assessment of footbridge accelerations

4.1 Probabilistic assessment of the acceleration demand

Structural reliability assessments require the evaluation of the probability of failure P_{fail}, expressed as:

$$P_{\text{fail}} = P(a_{\max} > a_{\lim}) = 1 - \int_{-\infty}^{+\infty} F_{A_{\max}}(a) \cdot f_{A_{\lim}}(a)\, da \tag{21}$$

It shall be verified that:

$$P_{\text{fail}} \leq P_D \tag{22}$$

where P_D is the design probability reported, e.g., in ISO 2394 [13] or in EN1995-2 [4] as function of the relative cost of safety measures and of the consequences of failure.

In this paper, a deterministic value of the limit acceleration, $a_{\lim}$, is considered. Thus, Eq. (21) becomes:

$$P_{\text{fail}} = 1 - F_{A_{\max}}(a_{\lim}) \tag{23}$$

Indeed, definition of $f_{A_{\lim}}$ would require modeling the physiological and psychological reaction of walkers to vibrations, which is beyond the scope of this research. On the other hand, the simplification introduced is justified by the large scatter of the available data (which in fact derive from measurements on buildings [39]).

Within a third-level probabilistic approach, the structural safety of a footbridge can be assessed by evaluating the probability of exceedance of the limit acceleration $a_{\lim}$ from the CDF of its maximum acceleration. Alternatively, the maximum acceleration $a_{\max}(P_D)$ associated with a given design probability P_D can be evaluated and then compared with the corresponding acceleration limit $a_{\lim}$, since Eq. (23) applies.

Third-level methods require the use of either numerical integration, or approximate analytical methods (such as first- and second-order reliability methods) or simulation methods [40]. Design Codes and Guidelines, on the other hand, are calibrated for the use of the Partial Safety Factors Method, thus requiring the characteristic values of both acceleration demand and capacity.

4.2 Frequency dependency of accelerations

For a footbridge having mass m, span length L and modal damping ratio ξ, the maximum transient acceleration due to one crossing can be expressed as a function of the frequency ratio $\alpha = f_w/f_b$, between the walking frequency and the fundamental frequency of the bridge. Within a deterministic approach, the largest among the maxima of the acceleration time series obtained with varying frequency ratio can be assumed to occur for $\alpha = 1$; indeed, the largest response occurs for values of α only very slightly smaller than 1, due to the transient behavior. On the other hand, within a probabilistic approach the frequency ratio is defined as the ratio $\bar{f}_w/f_b$ between the mean step frequency of the SP and the footbridge frequency. In this case, any chosen value of the frequency ratio gives rise to a probability distribution of the acceleration response maxima; from such distribution, a characteristic value can be extracted, associated with a prescribed fractile. It is observed

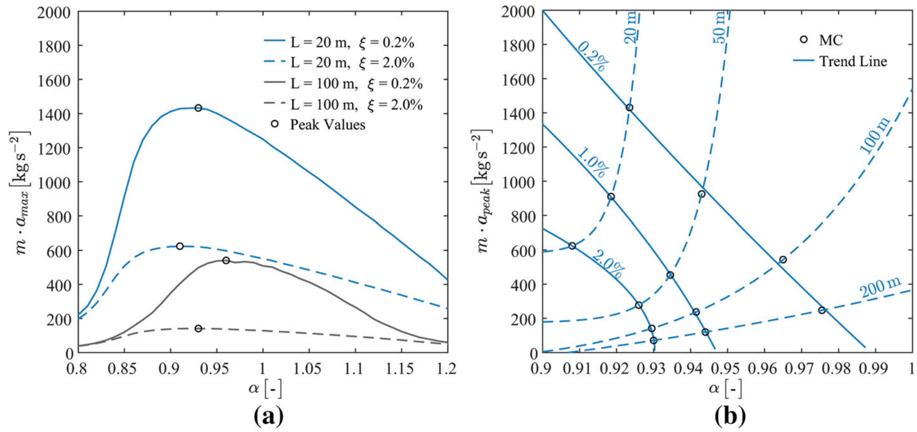


Fig. 2 95% characteristic acceleration spectrum (**a**) and corresponding peak accelerations (**b**) in the vertical direction as function of the frequency ratio

that the largest among such characteristic values with varying frequency ratio is obtained for $0.85 < \alpha < 1.0$, independently of the chosen fractile.

In Fig. 2a, the 95% characteristic value of the maximum vertical accelerations, $a_{\max}$, multiplied by the distributed mass m is shown as a function of the frequency ratio, for two values of the span length and two values of the modal damping ratio; these curves have the meaning of a response spectrum. It can be observed that the shorter the span and the smaller the damping ratio, the higher the characteristic acceleration. A peak acceleration is then defined as the largest value within each spectrum.

In Fig. 2b, the vertical peak acceleration is shown as a function of the frequency ratio for different values of the span length and of the damping ratio, together with approximated trend lines. The figure shows that for the largest damping ratio considered in the analyses ($\xi = 2\%$) the peak acceleration occurs for a frequency ratio between 0.91 and 0.93. Instead, for the smallest damping ratio ($\xi = 0.2$) the peak acceleration occurs for a frequency ratio between 0.92 and 0.98. Independently of the damping value, shorter spans bring smaller values of the frequency ratio of the peak response.

Outside the peak range, the response decays quickly. As an example, in Fig. 3, the response of a footbridge of length $L = 50$ m and 0.5% of damping ratio is shown in terms of frequency response factor (solid blue line). This is defined as the ratio between the maximum acceleration and the quantity P_k/mL , $P_k = (W \cdot \mathrm{DLF}_1)_k$ being the characteristic value of the product between the weight of the walkers and the dynamic load factor for the first load harmonic. Its availability allows direct calculation of the peak acceleration of the particular footbridge for which it has been derived. For Code implementation, however, one would need a universal curve, applying to a whole class of footbridges, and such aspect will be discussed in Sect. 4.4.

4.3 Uncertainty on structural parameters

The results shown in the previous section involve ideal simply-supported footbridges subject to a single pedestrian action. Moreover, the response model of Eq. (4) disregards pedestrian–structure interaction. Indeed, both mechanical and geometrical properties are not deterministic, and the effects of interaction between the pedestrian and the structure could be not negligible. Thus, some observations are in order.

The accurate evaluation of modal characteristics is the first step for an accurate assessment of footbridge acceleration. Modal frequencies can be related to the structural properties as:

$$2\pi f_{b,i} = \left(\frac{\beta_i}{L^2}\right)\sqrt{\frac{EI}{m}} \tag{24}$$

β_i being a coefficient depending on the support conditions and on the chosen mode; it is derived from the solution of the undamped free vibration Equation of Motion. Equation 24 indicates that uncertainties in the distributed mass, m, in Young's modulus of the material, E, and in the cross-section moment of inertia, I, translates into an uncertainty in the footbridge frequency. Quantification of these uncertainties can be summarized in a coefficient of variation (CoV) of 10% for the footbridge frequency [14,41]. Considering that value for the

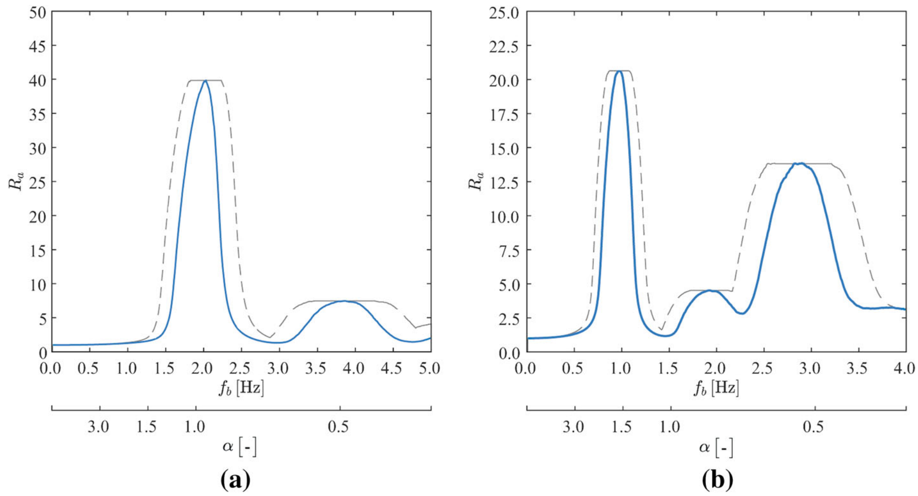


Fig. 3 Characteristic frequency response factor, R_a, for vertical (**a**) and lateral (**b**) vibrations of footbridges with span-length $L = 50$ m and damping ratio $\xi = 0.5\%$ (solid blue line), and effect of the uncertainty on the structural properties (dashed grey line). (Color figure online)

CoV, the response can be evaluated in a frequency range of ±10% around the mean value, that is, from $0.9 f_b$ to $1.1 f_b$, f_b being the mean value of the modal frequency. As shown in Fig. 3b, this effect leads to a translation of the frequency response factor (dashed grey line).

The choice of the modal damping ratio is also an important step in the evaluation of the dynamic response, and its overestimation leads to the underestimation of the footbridge acceleration and, thus, to a misleading assessment of the structural reliability. The interaction between the pedestrian and the footbridge can be also a source of change for the modal parameters of footbridges. It is well-known that the use of a perfect periodic force model allows a conservative assessment of vertical acceleration as it neglects the effect of walker-structure interaction and the potential increase in the apparent damping [41]. On the other hand, when the walking surface moves in lateral direction, the pedestrian-induced lateral force can be such to produce a reduction in the apparent damping[42]. All the aspects associated with damping variability are not accounted for in the proposed procedure.

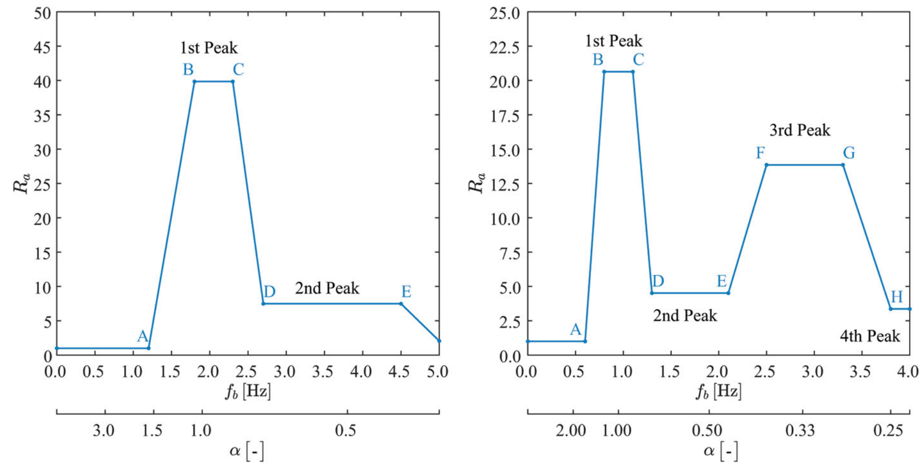
4.4 Design response spectra

Aiming at the definition of a probabilistic procedure for the assessment of footbridge accelerations, simplified Design Response Spectra can be derived as envelopes of the characteristic frequency response factors; these give the peak acceleration as a function of the footbridge frequency, f_b, or of the frequency ratio, α. In their calibration, the uncertainties of both the pedestrian load and the footbridge properties are considered. Examples of design response spectra are given in Fig. 4, again for a span-length $L = 50$ m and a damping ratio $\xi = 0.5\%$

The first peak (plateau from point B to point C) and the second peak (plateau from point D to point E) are defined in the range of frequency of 1.8 to 2.3 Hz and of 2.7 to 4.5 Hz for vertical vibrations, respectively. They are defined in the frequency range of 0.8 to 1.1 Hz and of 1.3 to 2.1 Hz for lateral vibration, respectively. In addition, the lateral acceleration has a third peak in the range of frequency of 2.5 to 3.5 Hz (plateau from point F to point G) and a fourth peak for $f_b > 3.7$ Hz (point H). The coordinates of points A through E in Fig. 4a and of points A through H in Fig. 4b depend on the length and on the damping ratio of the footbridge.

The values of the two (characteristic) peak accelerations for vertical vibrations and of the four (characteristic) peak accelerations for lateral vibrations are given in Fig. 5 for footbridges having span-length ranging from 10 to 70 m and damping ratios ranging from 0.2 to 2%. On the other hand, it is noticed that when f_b is small (i.e., α is large) the frequency response factor is slightly larger than 1 and independent of damping. Thus, in the design response spectra the values to the left of point A can be set equal to 1, and the maximum acceleration equal to:

$$a_{\max} = \frac{(W \cdot \mathrm{DLF}_1)_k}{mL} \tag{25}$$



(a) **(b)**

Fig. 4 Design response spectra for vertical (**a**) and lateral (**b**) vibrations of footbridges with span-length $L = 50$ and damping ratio $\xi = 0.5\%$

where the characteristic value $(W \cdot \mathrm{DLF}_1)_k$ turns out to be 785 N and 150 N in the case of vertical and lateral vibrations, respectively. Finally, it is noticed that for vertical vibrations twice the value of Eq. (25) can be taken for the design response spectrum at 5.0 Hz.

5 Application

5.1 Footbridge prototype

As an example of the application of the proposed procedure, a prototype steel footbridge with 25 m span length was analysed. First, its reliability assessment is made according to the results of Sect. 4. Then, the results are compared with the approach of EN1995-2 [4]. Finally, some conclusions on the acceleration limit are drawn.

The footbridge properties are designed according to the Eurocodes, applying an equivalent static vertical uniform live load of $5\,\mathrm{kN/m^2}$. The structure is made of a simple supported, single-cell steel box girder, with a uniform trapezoidal section. The cross-sectional geometry was selected by setting the maximum deflection under live loads to equal to $1/400$ of the span length.

Grade S355 steel was used in the design. Assuming Class 4 sections, safety checks were carried out using effective geometric properties, according to EN1993 [43]. Finally, the natural frequencies were calculated assuming simple support end conditions, in both vertical and lateral directions. A damping ratio of 0.4% was assumed, in agreement with the values suggested by the relevant literature [15]. The geometric and dynamic properties of the prototype footbridge are shown in Fig. 6.

5.2 Reliability assessment

The evaluation of the characteristic values of vertical and lateral accelerations within a first-level probabilistic approach was carried out. In particular, the design response spectra were evaluated as in Sect. 4.4 by using the peak accelerations of Fig. 5. The linear interpolation of values corresponding to span lengths of 20 m and 30 m leads to the accelerations of Fig. 7.

Maximum accelerations of $0.037\,\mathrm{ms^{-2}}$ and $0.099\,\mathrm{ms^{-2}}$ were found for modal frequencies of 0.862 Hz and 1.12 Hz in vertical and lateral directions, respectively. The latter correspond to frequency ratios of 2.2 for vertical vibrations and of 0.83 for lateral vibrations. Comparison between the maximum accelerations and the limiting values given by EN1995-2 [4] of $a_{\mathrm{lim}} = 0.7\,\mathrm{ms^{-2}}$ in the vertical direction and $a_{\mathrm{lim}} = 0.2\,\mathrm{ms^{-2}}$ in the lateral direction indicates that the target reliability level is achieved for single pedestrian action.

According to Eq. (23), within the third-level probabilistic approach, the probability of failure is evaluated from the empirical CDF of the acceleration response of the prototype footbridge, assuming the deterministic

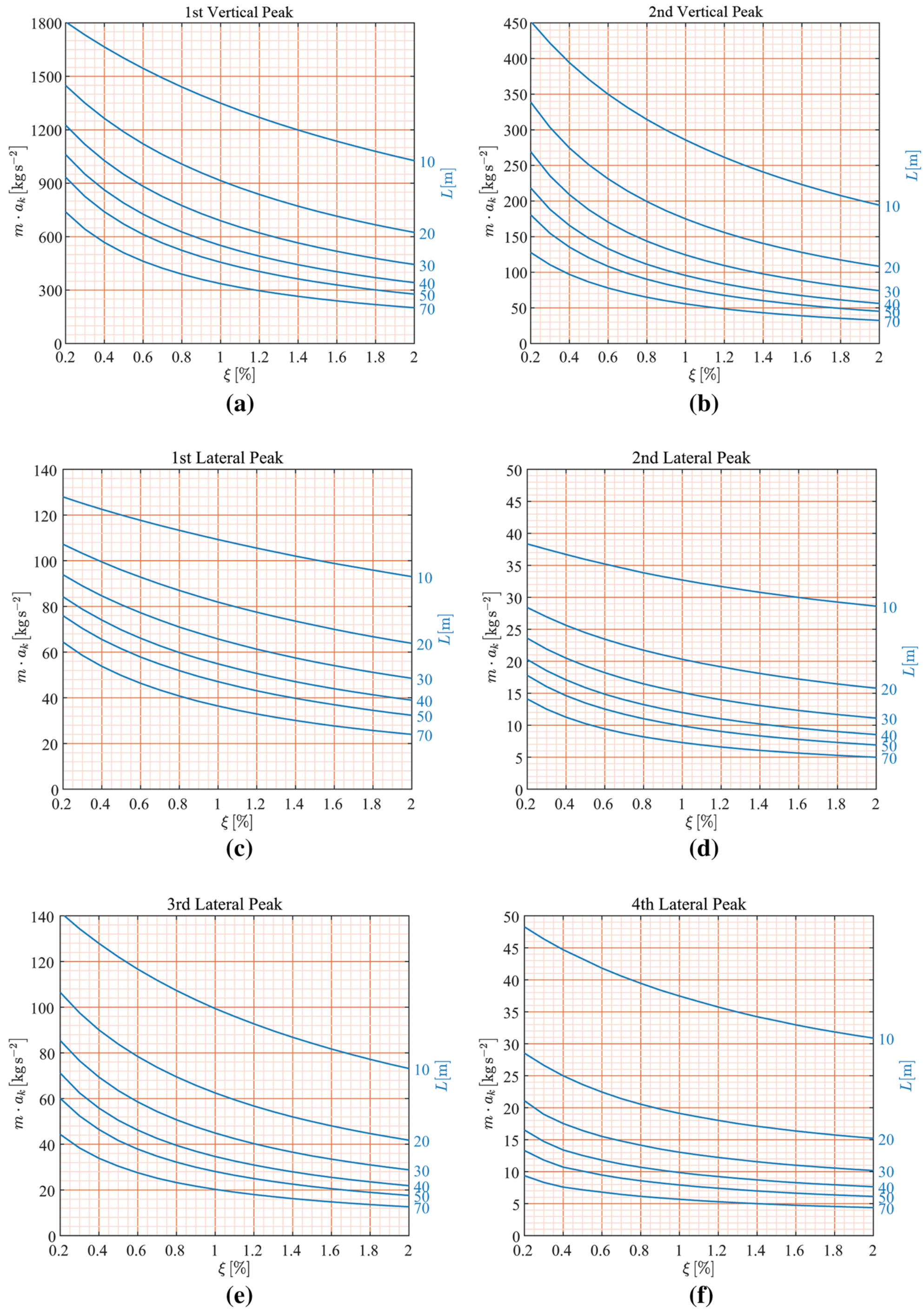
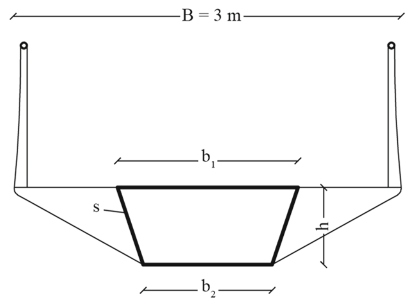


Fig. 5 Characteristic peak accelerations a_k of simple supported footbridges for vertical and lateral direction

acceleration limit values provided in EN1990 [16]. A probability of failure lower than 10^{-5} is found for vertical vibrations, and equal to $3.2 \cdot 10^{-4}$ for lateral vibrations. Both values comply with the limit $P_{\mathrm{D}} = 10^{-2}$ suggested by EN1990 for reversible Limit States.



h [mm]	b_1 [mm]	b_2 [mm]	s [mm]	m [kg/m]	f_V [Hz]	f_L [Hz]
600	1400	1000	10	857	0.862	1.12

Fig. 6 Cross section of the prototype footbridge

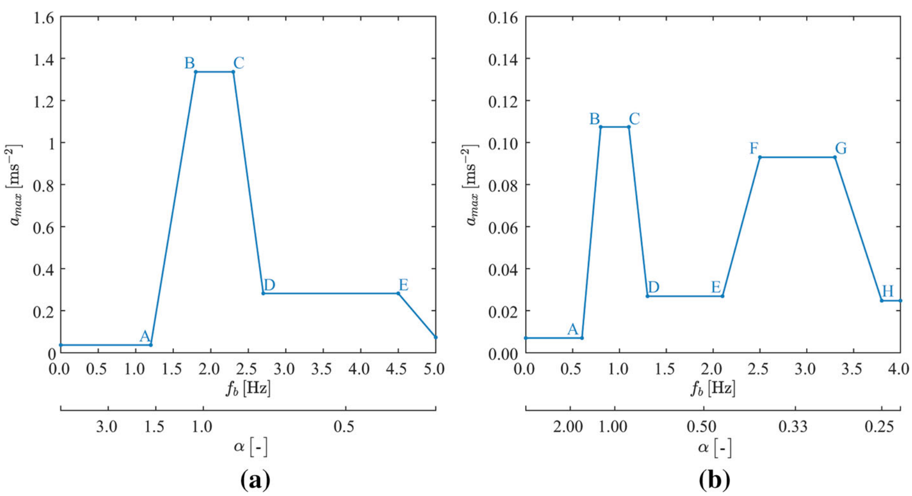


Fig. 7 Simplified spectra of the characteristic vertical (**a**) and lateral (**b**) accelerations of the prototype footbridge

5.3 Comparison with EN1995-2 representative acceleration

A comparison between the characteristic values of the maximum acceleration and the corresponding deterministic acceleration calculated in accordance with EN1995-2 [4] is made in the current section. The characteristic values of the maximum acceleration evaluated in Sect. 5.2 can be compared with the deterministic acceleration calculated in accordance with EN1995-2 [4]. The latter are:

$$a_V = \frac{200}{M\xi} = 2.3\,\text{m/s}^2 \tag{26}$$

$$a_L = \frac{50}{M\xi} = 0.58\,\text{m/s}^2 \tag{27}$$

where $M = mL$ is the total mass of the footbridge.

It is found that the Eurocode deterministic approach brings a much larger value of the maximum vertical acceleration with respect to the characteristic value calculated in Sect. 5.2, and a difference of about six times in lateral direction. The lower scatter between the two results for the lateral direction with respect to those for the vertical direction is mainly due to the fact that the footbridge frequency, f_b, falls outside the critical range for the vertical direction, and falls within it for the lateral one. In the latter case, a larger magnification factor is associated. It should also be noted that the deterministic approach suggested by EN1995-2 [4] takes into account the maximum stationary acceleration due to one walker resonant with the lowest harmonic.

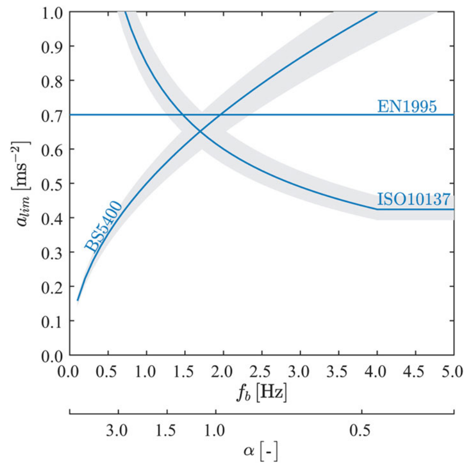


Fig. 8 Mean limiting acceleration according to the different codes (continuous line) and stochastic values in the range of 5 to 95% of exceedance probability (filled area)

5.4 Variability of limiting acceleration

In the evaluation of footbridge response, according to the model of Sect. 4.4, the uncertainties in the structural properties are accounted for assessing by the maximum value of characteristic accelerations in the range of footbridge frequencies of $0.9\,f_\mathrm{b}$ to $1.1\,f_\mathrm{b}$. Although the limit acceleration takes a constant value according to EN1990 [16], it is modeled as frequency dependent in BS5400 [18] and ISO10137 [21] for the common range of vertical footbridge frequencies (Fig. 8). This leads the limiting acceleration to be stochastic since the modal bridge frequency is.

If a coefficient of variations (CoV) of 10% is considered for the footbridge frequencies, the limiting acceleration can be modeled as normal distributed variable which CoV is 5%. Thus, the footbridge reliability can be assessed within the third-level probabilistic method according to Eq. (21). It is found that the characteristic value of the vertical acceleration limit (i.e., acceleration limit having 95% of exceedance probability) is lower by 8.6% with respect the mean value in the case of BS5400 and by 7.3% in the case of ISO10137. Thus, a characteristic value of limit acceleration $a_{\mathrm{lim},k} = 0.92\,a_{\mathrm{lim}}$ can be assumed within the first-level probabilistic method.

As a consequence, within the third-level reliability method a probability of failure $P_\mathrm{f} < 10^{-5}$ is evaluated for vertical accelerations, and the corresponding reliability level is satisfied ($P_\mathrm{D} = 10^{-2}$). Moreover, the characteristic maximum acceleration, $a_{\mathrm{max}} = 0.037\,\mathrm{ms}^{-2}$ is found to be lower than the corresponding acceleration limit within the first-level probabilistic approach ($a_{\mathrm{lim},k} = 0.7\,\mathrm{ms}^{-2}$ [16], $a_{\mathrm{lim},k} = 0.43\,\mathrm{ms}^{-2}$ [18], $a_{\mathrm{lim},k} = 0.91\,\mathrm{ms}^{-2}$ [21]).

Although a frequency dependency model of the lateral acceleration limit is not defined for footbridges, a stochastic model similar to the vertical one can be assumed in the case of lateral vibrations. Accordingly, the probability of failure $P_\mathrm{f} = 2.9 \cdot 10^{-4}$ is calculated for the prototype footbridge within the third-level probabilistic approach, and (21) is satisfied. Moreover, within the first-level reliability method, the characteristic maximum acceleration $0.099\,\mathrm{ms}^{-2}$ is found to be lower than the characteristic lateral acceleration limit $a_{\mathrm{lim},k} = 0.18\,\mathrm{ms}^{-2}$.

It should be noted that within the first-level reliability method, the stochastic effects on the limit acceleration are lower than the differences between the acceleration limits given by the different Codes. However, although modeling the bridge using the mean structural parameters is adequate in the case of resonant single pedestrian response ($\alpha = 1$) within a first-level reliability method, some differences were found in the application of the current work within a third-level reliability method, in the case of a moving load away the resonant condition ($\alpha = 0.82$ in lateral direction).

6 Conclusions

Many Codes and Standards assume walking induced forces as deterministic; yet they are stochastic, and it is reasonable to incorporate their variability into the response model. In this paper, the acceleration of footbridges due to single pedestrian crossings is analyzed, incorporating both the inter-subject variability of gait parameters and the uncertainty in footbridge dynamic parameters. It is shown that:

- The largest accelerations occur for frequency ratios $0.85 < \bar{f}_w/f_b < 1.00$, in contrast with the deterministic approach which considers the resonant (i.e., $\bar{f}_w/f_b = 1$) as the worst case.
- The amplitude of the vertical vibrations is larger when the footbridge frequency falls in the range of the first load harmonic and decreases for higher harmonics. On the other hand, the response of footbridges whose fundamental frequency in the lateral direction falls in the range of the third load harmonic is also shown to be relevant.
- As an effect of uncertainties in the footbridge dynamic properties, a modification of the response spectrum is observed. In particular, uncertainty in the fundamental frequency leads to a broadening of the response spectrum without changes in peak accelerations; this can be incorporated by considering the maximum value of the acceleration occurring in a range of $0.9\ f_b$ to $1.1\ f_b$.
- A "Design Response Spectrum" is defined starting from the values of the maximum acceleration with a non-exceedance probability of 0.95. Characteristic peak accelerations are given for both vertical and lateral directions, for footbridges with span-length of 10 to 70 m, and for damping ratios of 0.2 to 2%.
- An application to a 25 m span footbridges is presented, as an example of implementation of the proposed approach.

Therefore, the main aim of current work is to provide an acceleration response model for simple supported footbridges due to single pedestrian crossings. This represents the first step in the development of a probabilistic approach for the assessment of footbridges reliability to be coded. Possible extension of the work could be the investigation of different support conditions, e.g., multi-span footbridges, and the relationship between single- and multi-pedestrian probabilistic response in a comprehensive method.

Declarations

Conflict of interest The authors declare that there are no known conflicts of interest associated with this work. Moreover, the authors have no financial or proprietary interests in any material discussed in this article. The work is original research that has not been published previously and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the final version of the manuscript.

References

1. Wheeler, J.: Prediction and control of pedestrian-induced vibration in footbridges. J. Struct. Div. **108**, 2045–2065 (1982)
2. Van Nimmen, K., Lombaert, G., De Roeck, G., Van den Broeck, P.: Vibration serviceability of footbridges: evaluation of the current codes of practice. Eng. Struct. **59**, 448–461 (2014)
3. Ricciardelli, F., Demartino, C.: Design of footbridges against pedestrian-induced vibrations. J. Bridge Eng. C **4015003**, 1–13 (2016)
4. CEN, Eurocode 5: Design of timber structures, Comité Européen de Normalisation (2004)
5. BSI, U.K.: National annex to eurocode 1 (2008)
6. FIB, Guidelines for the design of footbridges, Fédération Internationale du Béton, Lausanne, Switzerland (2005)
7. Sétra, Technical guide—footbridges: Assessment of vibrational behaviour of footbridges under pedestrian loading (2006)
8. Pedersen, L., Frier, C.: Sensitivity of footbridge vibrations to stochastic walking parameters. J. Sound Vib. **329**, 2683–2701 (2010)

9. Heinemeyer, C., Butz, C., Keil, A., Schlaich, M., Goldack, A., Lukić, M., Chabrolin, B., Lemaire, A., Martin, P., Cunha, A., Caetano, E.: Design of lightweight footbridges for human induced vibrations—background document in support to the implementation, harmonization and further development of the eurocodes (2009)
10. Piccardo, G., Tubino, F.: Simplified procedures for vibration serviceability analysis of footbridges subjected to realistic walking loads. Comput. Struct. **87**, 890–903 (2009)
11. Avossa, A., Demartino, C., Ricciardelli, F.: Design procedures for footbridges subjected to walking loads: comparison and remarks. Baltic J. Road Bridge Eng. **12**, 94–105 (2017)
12. Dey, P., Narasimhan, S., Walbridge, S.: Evaluation of design guidelines for the serviceability assessment of aluminum pedestrian bridges. J. Bridge Eng. **22**, 04016109 (2017)
13. ISO, 2394 general principles on reliability for structures (1998)
14. Dey, P., Narasimhan, S., Walbridge, S.: Calibrating pedestrian–bridge standards for vibration serviceability. J. Bridge Eng. **23**, 04018072 (2018)
15. Demartino, C, Avossa, A.M., Ricciardelli, F.: Deterministic and probabilistic serviceability assessment of footbridge vibrations due to a single walker crossing. Shock Vib., 1917629 (2018)
16. CEN, Eurocode 0: Basis of structural design, Comité Européen de Normalisation (2002)
17. Blanchard, J., Davies, B., Smith, J.: Design criteria and analysis for dynamic loading of footbridges. In: Proceeding of a Symposium on Dynamic Behaviour of Bridges at the Transport and Road Research Laboratory, Crowthorne, Berkshire, England, May 19, 1977. TRRL Rpt. 275 Proceeding (1977)
18. BS5400, 5400, part 2: Steel, concrete and composite bridges. Specification for loads (1978)
19. Ohlsson, S.: Floor vibrations and human discomfort. Chalmers University of Technology, Division of Steel and Timber Structures (1982)
20. Eriksson, P.: Vibration of low-frequency floors-dynamic forces and response prediction, Chalmers University of Technology (1994)
21. ISO, ISO 10137: Bases for design of structures—serviceability of buildings and walkways against vibration (2007)
22. Ricciardelli, F., Briatico, C.: Transient response of supported beams to moving forces with sinusoidal time variation. J. Eng. Mech. **137**, 422–430 (2011)
23. Abu-Hilal, M., Mohsen, M.: Vibration of beams with general boundary conditions due to a moving harmonic load. J. Sound Vib. **232**, 703–717 (2000)
24. Piccardo, G., Tubino, F.: Dynamic response of Euler–Bernoulli beams to resonant harmonic moving loads. Struct. Eng. Mech. **44**, 681–704 (2012)
25. Ingólfsson, E., Georgakis, C., Ricciardelli, F., Jönsson, J.: Experimental identification of pedestrian-induced lateral forces on footbridges. J. Sound Vib. **330**, 1265–1284 (2011)
26. Borghese, N., Bianchi, L., Lacquaniti, F.: Kinematic determinants of human locomotion. J. Physiol. **494**, 863–879 (1996)
27. Li, L., van den Bogert, E., Caldwell, G.E., van Emmerik, R., Hamill, J.: Coordination patterns of walking and running at similar speed and stride frequency. Hum. Mov. Sci. **18**, 67–85 (1999)
28. Lelas, J., Merriman, G., Riley, P., Kerrigan, D.: Predicting peak kinematic and kinetic parameters from gait speed. Gait Posture **17**, 106–112 (2003)
29. Pachi, A., Ji, T.: Frequency and velocity of people walking. Struct. Eng. **83**, 36–40 (2005)
30. Matsumoto, Y., Nishioka, T., Shiojiri, H., Matsuzaki, K.: Dynamic design of footbridges. In: IABSE Proceeding (1978)
31. Ricciardelli, F., Pizzimenti, A.: Lateral walking-induced forces on footbridges. J. Bridge Eng. **12**, 677–688 (2007)
32. Yoneda, M.: A simplified method to evaluate pedestrian-induced maximum response of cable-supported pedestrian bridges. In: Proceedings of the International Conference on the Design and Dynamic Behaviour of footbridges (2002)
33. Ricciardelli, F., Briatico, C., Ingólfsson, E., Georgakis, C.: Experimental validation and calibration of pedestrian loading models for footbridges. In: Experimental Vibration Analysis for Civil Engineering Structures (2006)
34. Venuti, F., Bruno, L.: Crowd–structure interaction in lively footbridges under synchronous lateral excitation: a literature review. Phys. Life Rev. **6**, 176–206 (2009)
35. Young, P: Improved floor vibration prediction methodologies. In: ARUP Vibration Seminar, vol. 4 (2001)
36. Melchers, R.E.: Structural Reliability Analysis and Prediction, vol. 4, pp. 294–295. Ellis Horwood, Chichester. Distributed by John Wiley & Sons Ltd. (1987)
37. Bachmann, H., Ammann, W.: Vibrations in Structures: Induced by Man and Machines, vol. 3. Iabse (1987)
38. Ingólfsson, E.: Pedestrian-induced lateral vibrations of footbridge. Ph.D. thesis, Ph. D. thesis, Department of Civil Engineering, Technical University of Denmark (2011)
39. Bashor, R., Kijewski-Correa, T., Kareem, A.: On the wind-induced response of tall buildings: the effect of uncertainties in dynamic properties and human comfort thresholds. In: Proceedings of Americas Conference on Wind Engineering, Baton Rouge, LA, vol. 31 (2005)
40. Vrouwenvelder, A. C. W. M., Karadeniz, H.: Overview of structural reliability methods. In: Safety and reliability of industrial products, systems and structures, pp. 181–189. CRC Press / Balkema - Taylor & Francis Group. (2010)
41. Tubino, F., Pagnini, L., Piccardo, G.: Human-induced vibrations on two lively footbridges in Milan. J. Bridge Eng. **21**, C4015002 (2016)
42. Ricciardelli, F., Mafrici, M., Ingólfsson, E.: Lateral pedestrian-induced vibrations of footbridges: characteristics of walking forces. J. Bridge Eng. **19**, 04014035 (2014)
43. CEN, Eurocode 3: Design of steel structures., Comité Européen de Normalisation (2005)