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A Paris Law Based Mesh Independent Numerical Methodology for the Simulation of Fatigue Driven Delamination in Composites

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ABSTRACT

Delamination evolution under cyclic loading is one of the most important research topics for the application of composite materials to aerospace, naval, automotive and, in general, transportation fields. Large experimental campaigns are needed to assess the fatigue behavior of Carbon Fiber Reinforced Polymers (CFRPs), which may result extremely time and cost consuming. Nevertheless, composite materials design needs to take into account the evolution of fatigue driven damage. Subsequently, the development of efficient and robust computational finite element methodologies to evaluate progression of delamination in composite structural components subjected to cyclic loading conditions has become relevant.

In this paper, a numerical finite element procedure able to simulate the fatigue driven delamination growth is introduced. A Paris-law based cycle jump strategy, combined with the Virtual Crack Closure Technique (VCCT) approach, has been implemented in the commercial Finite Element Code ANSYS MECHANICAL via the Ansys Parametric Design Language (APDL). The main advantages of the proposed numerical procedure, named FT-SMXB, are related to its independence on the time step and element size in the frame of incremental analyses. The procedure has been preliminary validated, in this research study, at coupon level, by comparing the numerical results to literature experimental data on a unidirectional graphite/epoxy Double Cantilever Beam (DCB) specimen. The significant agreement between the obtained numerical results and the literature experimental benchmark data confirms the accuracy and the potential of the proposed methodology.

INTRODUCTION

The increasing adoption of Carbon Fiber Reinforced Polymers (CFRPs) in naval, aerospace, automotive and, in general, transportation applications have led to a considerable improvement in the need of experimental and numerical assessment of the damage mechanisms that characterize these innovative materials [1,2].

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Among the others, interlaminar failure mechanisms, such as delamination [3,4] and debonding [5,6], represent a challenging task for the design of damage tolerant structures [7], which are often over-dimensioned with a consequent loss in terms of the promised weight reduction and high load-bearing performance. Indeed, the knowledge in simulating the delamination phenomenon due to quasi-static loading conditions has significantly increased. Furthermore, experimental procedures to evaluate the fracture toughness at various opening modes have been standardized [8-10] and advanced numerical models for the simulation of static delamination growth have been developed and implemented in the main commercial Finite Elements (FE) codes.

On the other hand, the knowledge and the simulation capabilities for delaminations under cyclic loading condition is still in an embryonal stage and relies mainly on models developed and assessed for metallic materials. Actually, there is a growing need of robust and efficient computational methodologies able to simulate the total cumulative life of composite materials under cyclical loads (including delaminations) to which the structure is subjected during its operational life. These methodologies would be of a relevant added value especially during the preliminary design phases, and would allow to decrease the extensive and costly number of fatigue tests needed to assess the fatigue behavior of CFRPs and evaluate the effect of different cyclic loading spectrum on the structural safety.

The available delamination onset and growth numerical models have demonstrated to be more or less excessively sensitive to user-defined input parameters, in particular load increment and mesh refinement, which can be correctly set only with the support of a massive amount of experimental data. To avoid this issue, as an example, in [11], a method to correctly select the input parameters to be employed in the constitutive equations of Cohesive Zone Model (CZM) for the simulation of delamination by means of decohesion finite elements, is presented. Even if such procedure allows the use of a coarse meshed models, less accurate results are obtained with respect to finer meshes. This is confirmed in [12] where the interfacial strength has been demonstrated to largely affect the stress concentration at the crack tip, which is extremely sensitive to the mesh size. Actually, the simulation of interlaminar crack evolution using CZM [13], which can be considered the less mesh sensitive technique to simulate the delamination growth, shows difficulties in the definition of a proper cohesive stiffness. Hence, with CZM, particularly fine mesh is required for modelling the delaminated area, which leads to high computational costs and convergence problems. Another well-known method to simulate delamination propagation is the Virtual Crack Closure Technique (VCCT) [14]. However, even if it offers excellent capabilities to predict composite delamination and easy definition of the experimental parameters (critical energy release rates) the right mesh size and time step minimum increment need to be selected in FE models to obtain an acceptable accuracy of the numerical predictions, as demonstrated in [15].

The described difficulties, characteristic of the static delamination growth approaches, extends to fatigue driven delamination growth when the strain energy release rate has to be computed and the delamination propagation on irregular delamination fronts has to be simulated. Indeed, the fatigue numerical simulation can be seen as a sequence of static simulations of the load/displacement application at key cycles. The power law function, proposed by Paris et al. [16,17] for metallic alloys, have been extended to assess the delamination propagation under fatigue loading conditions in CFRPs. More in detail, the Paris Law calculates the crack growth rate, and consequently the propagation at key cycles, taking into account the Energy Release Rate (ERR) at crack front, which can be calculated through numerical approach derived from fracture mechanics, such as VCCT. Different studies can be found in literature which relate VCCT-based numerical procedures and Paris-Law function [18-20]. Carvalho and Krueger in [21,22] introduced a method to model the initiation and propagation of interlaminar cracks in carbon/epoxy composites under fatigue loads. They presented a User Defined Subroutine, implemented in Abaqus/Standard, based on the Floating Node Method (FNM) combined with VCCT approach and Paris Law. In [23], the VCCT approach has been used in combination with the XFEM methodologies for the analysis of fatigue delamination propagation. Magi et al. [24] proposed the use of the VCCT-Paris Law approach for the analysis of vibration fatigue.

In this paper, a numerical procedure to simulate the fatigue driven delamination growth, based on the Paris Law, is introduced. The presented numerical approach can be considered as an enhancement of the VCCT-based "SMart-Time XB" (SMXB) numerical mesh and time step independent delamination evolution tool, presented in [25-27]. The introduced procedure has been preliminary validated, in the frame of this research study, at coupon level by considering unidirectional graphite/epoxy Double Cantilever Beam (DCB) specimen. The obtained numerical results have been compared with literature experimental data from literature [28,29].

First, the theoretical background of the proposed fatigue approach is described, highlighting the SMXB main characteristics. Then, Double Cantilever Beam (DCB) specimen is presented, together with the results of the quasi-static and fatigue analyses, and is compared with experimental literature data, to validate the proposed numerical procedure.

THE FATIGUE SMXB APPROACH: THEORETICAL BACKGROUND

The simulation of delamination growth by using the standard VCCT (or modified MVCCT) is usually applied in conjunction with the fail-release (FR) approach. Indeed, delamination is typically modelled as two sub-laminates with coincident nodes, connected by means of contact elements. When delamination propagation condition is satisfied, at one or more nodes of the delamination front, the contacts are released, and the delaminated area is increased. In [25], the VCCT has been demonstrated strongly dependent from the model discretization and the load step size used in the frame of the geometrically nonlinear analyses. Indeed, an under/overestimation of the delaminated area is usually obtained, even if a very small load step size is considered, when the increase in area requested to satisfy the delamination growth criterion is smaller/bigger with respect to the adopted element size.

The SMXB approach allows to overcome the difficulties related to the VCCT-FR approach, by providing a robust prediction of the growth phenomenon whatever the mesh and load step size. The procedure, schematized in Figure 1, has been validated by Authors in [25-27], considering different complex CFRP structural components under various loading conditions.

Proposed Approach For Fatigue Delamination Simulation

The real accomplishment of this work is the enhancement of the SMXB numerical tool with the ability to simulate fatigue driven delamination. Indeed, all the SMXB features has been enhanced with the definition of a fatigue modulus. The resulting new tool, named FT-SMXB (FaTigue SMart Time Xie Bigger), can simulate delamination growth under any fatigue loading condition (at any level of acting stress), can mimic static delamination propagation within a fatigue cycle, can correctly evaluate the splitting of the SERR among the different failure modes for any delamination front shape and it is completely mesh and time step independent.

As already mentioned, a Paris Law based modulus has been introduced in the SMXB numerical tool for the simulation of the fatigue driven delamination. The Paris Law used to characterize the rate of delamination growth can be represented by Equation (1), where *C* and *m* can be evaluated by fitting experimental data and ΔG is the variation of the ERR.

$$\frac{dA}{dN} = C\Delta G^m = C(G_{max} - G_{min})^m \tag{1}$$

The values of G_{max} and G_{min} can be determined using the VCCT equations when the model is subjected to the maximum and minimum load, respectively. A load spectrum of a single fatigue cycle needs to be defined. If constant amplitude fatigue is considered the load cycle is kept constant up to the number of cycles to failure or the user-defined total number of cycles are reached. The solution of each load cycle is calculated by means of the static nonlinear algorithm of the ANSYS MECHANICAL solver. All the implementations have been performed by adopting the Ansys Parametric Design Language (APDL).

The delamination growth initiation cycle can be found through a power law which coefficients have been determined by fitting experimentally determined G-N curve [30]. Such procedure allowed to determine constant C_1 and C_2 reported in Equation (2).

$$N_{ONSET} = C_1(\Delta G)^{C_2} \tag{2}$$

The components of ERR can be evaluated by using the VCCT equations [14]. The propagation condition in a quasi-static analysis can be evaluated by adopting different criteria. The criterion for the delamination growth adopted in the frame of the SMXB procedure is the linear Power Law criterion. This criterion is represented by Equation (3), where G_{ic} (i=I,II,III) are the fracture toughness of the material and G_i (i=I,II,III) are the Mode I, Mode II and Mode III SERR components, respectively.

$$E_d = \left(\frac{G_I}{G_{Ic}}\right) + \left(\frac{G_{II}}{G_{IIc}}\right) + \left(\frac{G_{III}}{G_{IIIc}}\right) \geq 1 \tag{3}$$

The key cycle to be simulated, corresponding to the fatigue cycle at which delamination advances, can be calculated reversing the Paris Law, as defined in Equation (4), where c3 and c4 are again experimental determined fitting parameters. The variation of the ERR is carried out by means of nonlinear analyses at each load application (k).

$$\Delta N_i^{(k)} = \frac{\Delta A_i^{(k)}}{C_3(\Delta G)^{C_4}} \quad (4)$$

The delamination growth is modelled by releasing the node pairs between the surfaces at the crack tip. The node j with the minimum value of the cycles to growth is released, as expressed by Equation (5), taking into account the local damage accumulation. Indeed, a damage coefficient (D) is defined for each node of the delamination front (i=1…n), as described in Equations (6) and (7).

$$\Delta N_j^{(k)}(1 - D_j^{(k-1)}) \quad (5)$$

$$\Delta D_i^{(k)} = \frac{\Delta N_j^{(k)}}{\Delta N_i^{(k)}}\left(1 - D_j^{(k-1)}\right) \quad (6)$$

$$D_i^{(k)} = D_i^{(k-1)} + \Delta D_i^{(k)} \quad (7)$$

Once the fatigue key cycle has been determined, the node with the highest value of damage accumulation factor is released and the new delamination front is updated for the subsequent nonlinear analysis step. The static solution phase is restarted from the last converged one, considering the new updated crack front. Equations (8) and (9) describe the number of fatigue cycles and the total delaminated area to be considered for the next application of the Paris Law - based fatigue modulus.

$$N_i^{(k)} = N_i^{(k-1)} + \Delta N_j^{(k)}(1 - D_j^{(k-1)}) \quad (8)$$

$$A^{(k)} = A^{(k-1)} + \Delta A_j \quad (9)$$

The presence of the SMART-TIME and the XIE-BIGGER modules, detailed described in [25], in the SMXB methodology, allows to obtain real mesh independent results by correctly predicting the propagated delaminated area for the exact verification of the criterion and by correctly determining the components of Energy Release rate for irregular delamination front during propagation.

Additionally, the capabilities to combine static and cyclic delamination growth has been implemented. Indeed, when static propagation arises within a single fatigue cycle (before reaching the maximum load application) the SMXB tool search for the correct equivalence between the aera effectively released and the area that should release to satisfy exactly the propagation criterion. Hence, the Paris Law modulus is neglected until the energy release rate lower the threshold value, in all the nodes of the delamination front.

A schematic representation of the FT-SMXB procedure is given in Figure 1.

VALIDATION OF THE PROPOSED PROCEDURE

The proposed numerical procedure has been preliminary validated, in the frame of this research, at coupon level. The Double Cantilever Beam test benchmark developed by Kruger has been considered for this purpose [29]. The specimen has been investigated under quasi-static delamination growth and constant amplitude fatigue propagation.

The finite elements model of the investigated DCB sample has been developed according to the geometrical specifications in [29]. Graphite/epoxy material system with unidirectional 0 fiber-oriented layup has been considered. The elastic material properties and the fatigue parameters, taken from literature [29], are given in Table I. The specimen has been modelled in the commercial FE code ANSYS MECHANICAL and discretized by using twenty-node solid layered elements. The Finite Element model consists of two sub-laminates, representing the specimen arms, and characterized by a pre-existing crack. The sub-laminates have been connected by 3-D 8-Node Surface-to-Surface contacts elements with birth and death feature, that allows to change the elements status from “existing”, in the undamaged phase, to “non existing”, when the delamination propagates. One element in the thickness direction has been used for each sub-laminates. Whit the aim to apply the DCB test typical boundary conditions, the nodes of one edge of the specimen (the one with pre-existing crack) have been subjected to opening displacement, while the other edge of the sample has been fully constrained.

Quasi-Static Numerical Analysis

First, quasi-static numerical analysis has been performed. A coarse mesh, characterized by elements of 2 mm size, and a finer mesh, with 1 mm elements size, have been considered to assess the SMXB features in terms of load step and mesh independency. The load as a function of the applied opening displacement ($\delta/2$) has been plotted, along with the experimental Krueger specimen [28], in Figure 2. Linear response can be observed until delamination starts to propagate. An excellent agreement has been found between the numerical results and the literature benchmark data, both in terms of stiffness and maximum load.

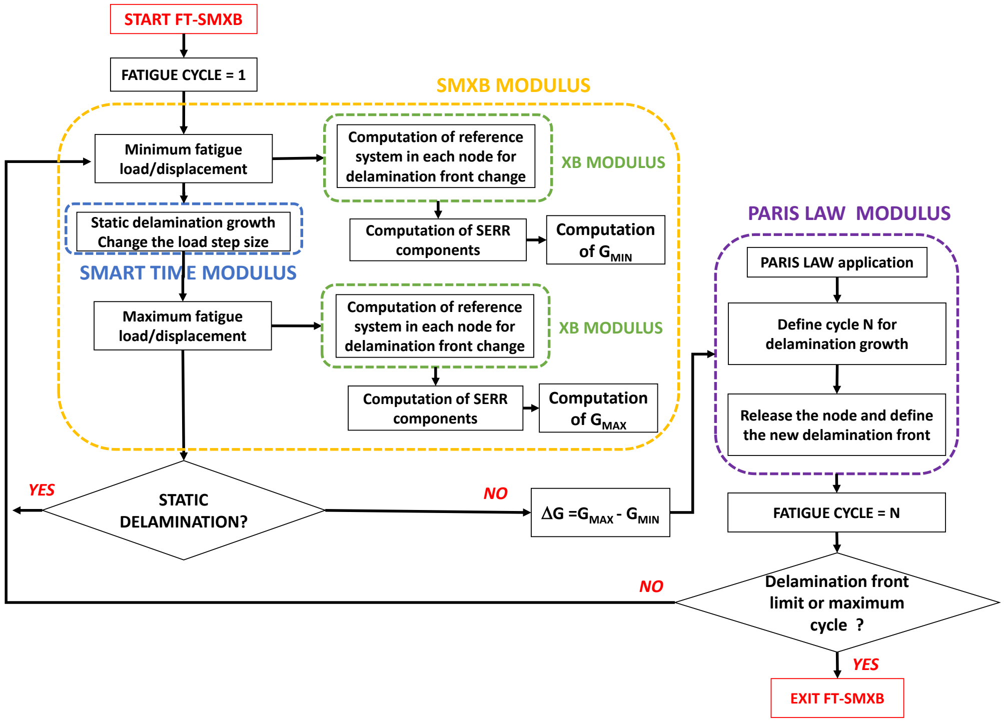

Figure 1. FT-SMXB schematic representation.

TABLE I. MATERIAL PROPERTIES

Mechanical properties		Fatigue properties	
Property	Value	Property	Value
E1	139400 [MPa]	C1	2.8461E-9
E2=E3	10160 [MPa]	C2	−12.415
G12=G13	4600 [MPa]	C3	2.44E6
G23	3540 [MPa]	C4	10.61
ν12= ν13	0.3 [-]	δmax/2	0.67
ν23	0.436 [-]	δmin/2	0.067
GIc	0.17 [kJ/m2]	R	0.1
GIIc	0.49 [kJ/m2]		

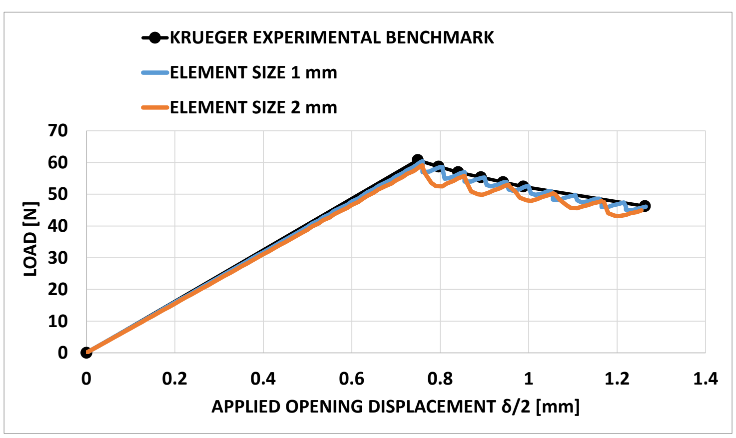

Figure 2. Static numerical analysis: load versus applied displacement.

According to Figure 2, the use of larger elements has shown an increase in the magnitude of the sawtooth pattern (which is an artifact of the VCCT implementation as detailed explained in [28]). However, the peak values, for both the considered mesh sizes, lie on the benchmark curve.

Numerically evaluated delaminated area as a function of the applied displacement is shown Figure 3, where the outstanding performance of the SMXB tool in terms of mesh and time step independency are outlined. Furthermore, the deformed configurations, obtained with the different considered mesh size, are very similar in terms of deformed shape and minimum and maximum values of the out of plane displacements, as it can be observed in Figure 4.

The numerically predicted delamination propagation is shown in Figure 5 at crack initiation, corresponding to 0.755 mm of the applied displacement, intermediate debonding state, corresponding to 2.1 and 2.44 mm of the applied displacement, and debonding state at the maximum value of the applied displacement (3 mm). Figure 6, where the final debonding state is shown for the two analyzed mesh density, confirms the mesh independency of the SMXB approach. The delamination shows a slightly rounded delamination front during the evolution, as expected [28].

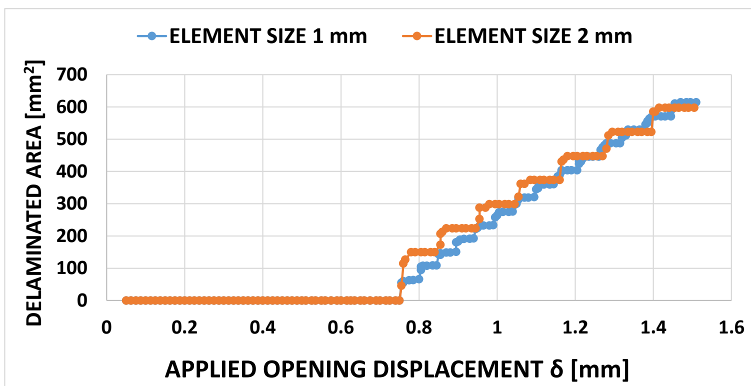

Figure 3. Static numerical analysis: delaminated area versus applied displacement.

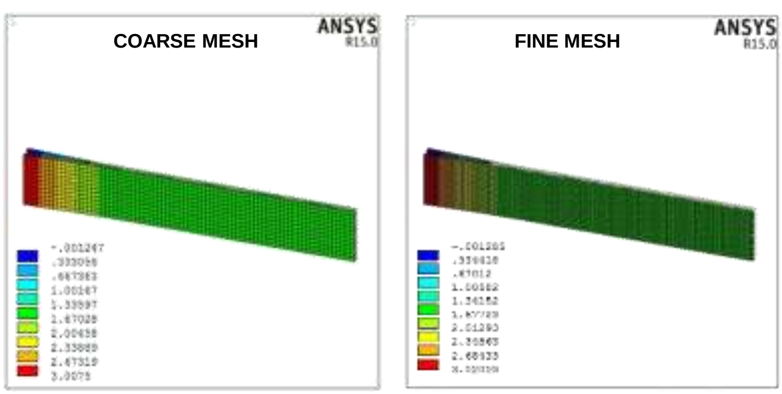

Figure 4. Static numerical analysis: delaminated area versus applied displacement.

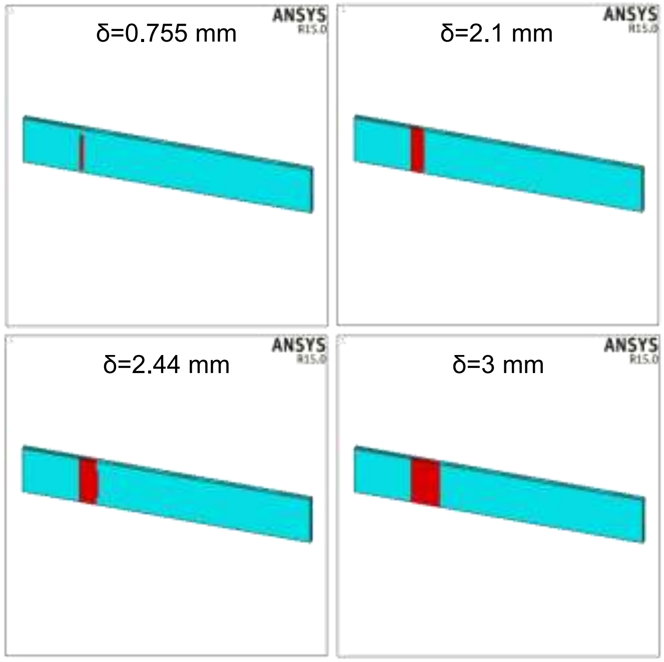

Figure 5. Static numerical analysis: delaminated area versus applied displacement.

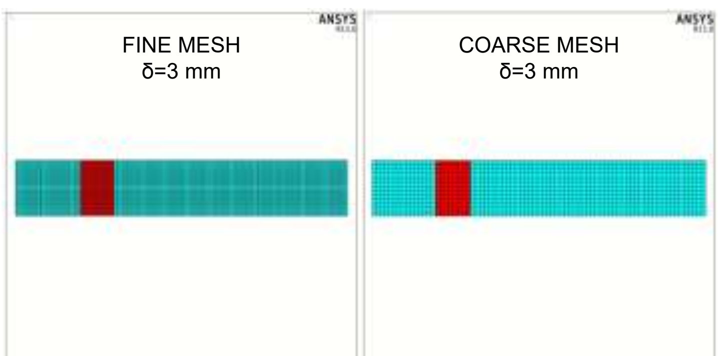

Figure 6. Static numerical analysis: delaminated area versus applied displacement.

Fatigue Numerical Analysis

For validation purpose of the implemented Paris Law based fatigue (FT) modulus in the SMXB, the numerical results obtained for the same DCB specimen under constant amplitude fatigue loading conditions have been compared to the literature experimental data provided in [29].

The fatigue numerical simulation can be seen as a sequence of static non-linear analyses where the displacement oscillates from the minimum (δmin) to the maximum displacement (δmax), as schematically shown in Figure 7. A pre-load phase has been considered to increase the displacement up to δmin.

For the analyses, a base size of the elements of 1 mm has been selected. Finer and coarser mesh with elements of 2 and 0.5 mm, have also been generated.

The excellent performance of the FT-SMXB finite elements procedure in terms of mesh and time step independency are highlighted by comparisons with Krueger experimental results [29] in terms of delamination length over cycles. It is worth to highlight that Krueger in [29] explicitly shows the mesh and time step dependency of the standard VCCT based approach.

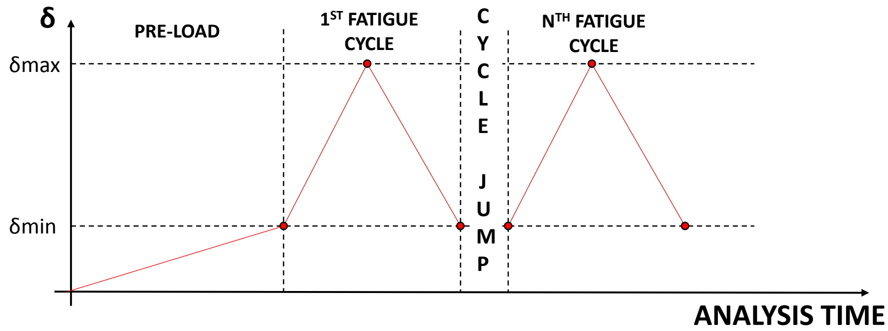

Figure 7. Fatigue loading pattern.

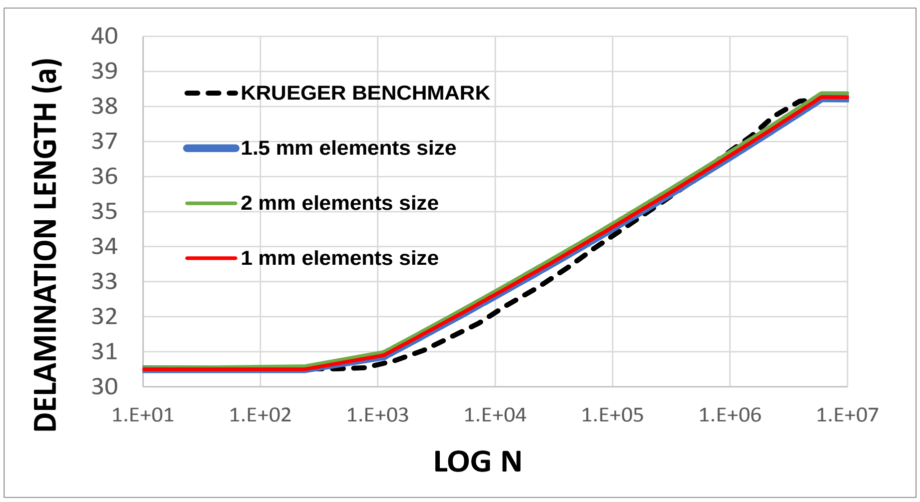

Figure 7. Fatigue delamination propagation, comparison against literature experimental results [29].

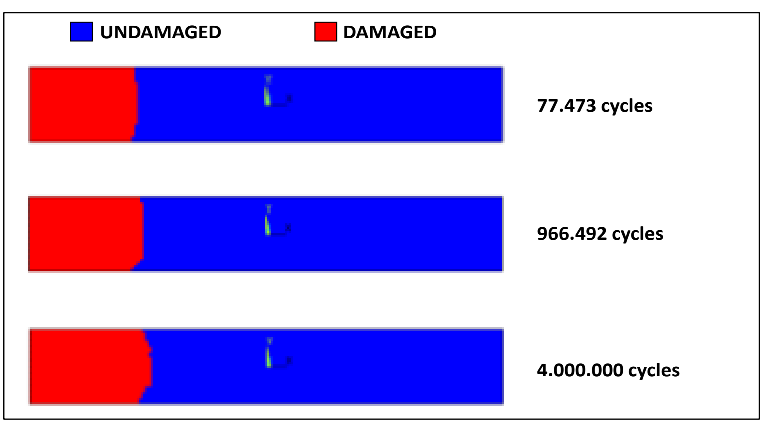

Figure 8. Delamination propagation at different fatigue cycles.

Indeed, Figure 8 shows the delamination extend as a function of the number of cycles for the different considered crack tip elements length. The increase in delamination length has been measured at the center of the specimen, regression curves have been considered for comparison purpose.

The SMXB features have been used in the frame of the fatigue delamination growth simulation tool to correctly simulate the crack evolution under fatigue loading conditions leading to results fully mesh and time step independent.

The delamination front at different fatigue cycles is shown in Figure 8. The same delamination evolution shape observed for the quasi-static analysis can be appreciated.

CONCLUSIVE REMARKS

In this work, a novel approach, named FT-SMXB, the fatigue driven delamination simulation, implemented in the FEM code ANSYS by means of the Ansys Parametric Design Language (APDL), has been presented. A Paris Law based approach has been used in conjunction with a Virtual Crack Closure based Technique characterized by an improved definition of the delaminated area increments and by an improved methodology for the correct evaluation of the Energy Release Rate components on irregular delamination fronts.

Double Cantilever Beam specimen has been analyzed, considering different mesh refinements, and the results have been compared against experimental literature benchmark [28,29]. The proposed numerical model has shown and excellent agreement with experimental results in terms of crack length as function of the number of cycles compared to literature data from Kruger [29]. The numerical results have been found to be really independent from mesh and time step size.

The effectiveness of the proposed procedure has been demonstrated at coupon level. However, in the future, composite structures with more complex geometry and loading conditions will be numerically analysed and experimentally tested to prove the effectiveness of the proposed numerical approach and its reliability to predict the fatigue delamination growth.

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