

The majolica dome of *Santa Maria della Sanità* in Naples. Geometric configuration analysis and stability studies

La cúpula de mayólica de Santa Maria della Sanità en Nápoles. Configuración geométrica y estudios de estabilidad

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ABSTRACT

This article deals with the survey and representation of the dome of *Santa Maria della Sanità* in Naples (Italy). The survey led to a geometric analysis of the spatial configuration of the dome, the drawing of its majolica decoration and its structural behavior. The overall objectives of this research can be briefly outlined as follows. First, the study focuses on defining the correct geometry of the dome obtained through accurate surveys. Finally, the study performs a simplified structural analysis of the compound dome-but-tress system set within the theoretical framework of the Limit Analysis through the graphical statics.

Keywords: Dome; geometric analysis and 3D modeling; masonry domes; equilibrium analysis.

RESUMEN

Este artículo examina el proceso de medición, la ejecución de levantamientos y la representación de la cúpula de Santa Maria della Sanità en Nápoles (Italia). El estudio produjo un análisis geométrico de la configuración espacial de la cúpula, el diseño de su decoración de mayólica y el estudio de su comportamiento estructural. Los objetivos generales de esta investigación pueden resumirse brevemente como sigue. En primer lugar, la investigación se centra en la definición de la geometría correcta de la cúpula obtenida mediante levantamientos detallados. Por último, el presente trabajo realiza un análisis estructural simplificado del conjunto cúpula-sistema de contrarresto dentro del marco teórico del Análisis Límite a través de la estática gráfica.

Palabras clave: Cúpula; análisis geométrico y modelación 3D; cúpulas de mampostería; análisis de equilibrio.

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1. INTRODUCTION: THE DOME OF THE BASILICA OF SANTA MARIA DELLA SANITÀ IN NAPLES

The church of *Santa Maria della Sanità* in Naples is part of a religious complex designed by the Dominican friar Giuseppe Nuvoletto between 1602-13. Its central plan (1), according to a polycentric scheme which, derived from the examples of *San Pietro Church* by Bramante and Michelangelo, integrated “a singular recovery, in an ideal sense, of the five-nave typology of Constantine’s great basilicas complex” (2), and from which the friar seems to learn creative ability (Figure 1, a).

In addition to the church plan design, the Nuvoletto’s inventiveness is expressed in the characteristic structural system of the dome (accessible from the roof) and its tiled decoration of the extrados with strong pictorial value. The use of this kind of roof tiles is a very common practice in the seventeenth century, evidence of Spanish influences in the Neapolitan context. In fact, it should be considered that majolica is an ancient craft that was developed along with the Middle Ages by Muslims and Christians in Spain, and later exported to Italy. In addition, the considerable elevation of the horizontal plane which it lies makes this dome one of the most impressive and attractive on the Neapolitan landscape (3). These peculiarities let the dome hire a double role of reference, spiritual and visual (Figure 1, b-c).

The double shell dome of *Santa Maria della Sanità* (which can be inspected inside) recalls the masterful example made by Filippo Brunelleschi (1377-1446) in Florence in the Church of *Santa Maria del Fiore* (Fig. 2, a), where the architect, rather than referring to the single-shell Roman construction method, or the medieval one with the use of ribs, decides to raise the dome without centering, devising a new technique based on calculation and balancing it with a self-supporting double shell system, by means of a frame of vertical and horizontal elements (the main ones visible on the outside). Furthermore, for a better stability, he uses various precautions, among which, bricks in a herringbone pattern (4).

Even if their similar conceptual and structural genesis, the dome of *Santa Maria della Sanità* presents dimensional and geometric differences compared to *Santa Maria del Fiore*. The Neapolitan dome is smaller than Brunelleschi’s one and the space in the cavity between the two caps (about 1,5 m) does not allow rapid use of the space. In geometric terms, however, the dome of *Santa Maria della Sanità* is not set on an octagonal drum but a cylindrical one. This last feature is analogous to the dome of the *San Pietro Church* in Rome designed by Michelangelo (1475-1564) (Figure 2, b), from whose drum protruding double columns alternating to the windows empty spaces, with a remarkable plastic value. In *Santa Maria della Sanità*, the double-columns are replaced by buttresses which, although interrupted by the large overhanging cornice of the drum, continue its path also on the dome extrados.

Up to now, that of *Santa Maria della Sanità* seems to be the first example of a double shell dome in the city of Naples, unless future discoveries (5). Chosen as a case study for all these peculiarities, through numerous inspections and the direct architectural survey, it was possible to carry



Figure 1. A, the complex of *Santa Maria della Sanità*: original project by Giuseppe Nuvoletto in a drawing by Angelico Majorino (1714). Retrieved from Alfredo Buccaro (Ed.), 1991, *Il Borgo dei Vergini. Storia e struttura di un ambito urbano*, p. 127; B-C, the dome of *Santa Maria della Sanità* with a detail on the tiled structure (picture by I. Todisco, 2019).

out a geometric analysis on the dome spatial configuration and the tiled design decoration, as well as a stability study.

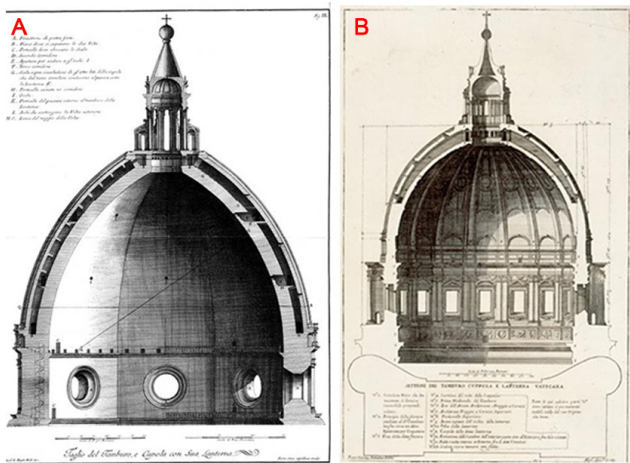


Figure 2. A, Dome of Santa Maria del Fiore Church by Filippo Brunelleschi, engraving by Sansone Sgrilli on survey by G. Battista Nelli, from *Descrizione e studj dell'insigne fabbrica di S. Maria del Fiore...* (1733); B, Dome of San Pietro Church by Alex. Spec. Sculp.

2. THE DOME ARCHITECTURE

The dome of *Santa Maria della Sanità* has a circular base. Leaning on four pillars placed at the square vertices, the connection between the dome impost floor and the square in which it is inscribed consists of four pendentives. To ensure greater momentum, the dome rests on a cylindrical windowed drum. The intrados of the dome is a composed vault, consisting of a simple vault generated by the rotation around the vertical axis of a curve and intersected by eight nails located above the windows (6-7). The keystone of the dome is replaced by a hole (*opaion*) surmounted by a lantern with a hemispherical cap.

Access to the dome can be gained consisting in a small spiral staircase that, carved into a side pillar of the church, leads to the dome's impost floor. Through a cylindrical tunnel in the drum, one its cavity. The space between the double shells is not equidistant from each other, and walls connect them. A ladder has been created in the cavity which, leaning against the intrados line, leads to the base of the lantern. The intrados of the dome has eight-ribs radial design with a decorative and structural role. Eight masonry walls connect the two shells in continuity of the inner ribs.

The extrados of the dome is a simple vault generated by the rotation of the curve around the vertical axis and on the outside, it has a radial tiling system with eight green ribs. Both the dome and the small dome (*cupolino*) have a two-color tiled pattern (yellow and green). The chromatic alternation of the tiles gives a rhomboid pattern. In the drum there are volute buttresses, originally double and then joined with an alternating rhythm (apparently for stability needs). Finally, the lantern is the logical conclusion of the dome, with the same structures below synthetic aesthetic, in which the eight ribs are occupied by as many coupled arches, support the final dome and represent the point of maximum gaze of the observer (8).

2.1. The survey methodology

The survey of the dome of *Santa Maria della Sanità* (by Pasquale Galdiero and scientifically coordinated by Ornella Zerlenga and the writer) was carried out both with direct methodology using Laser rangefinder, and with photographic adjustment at the survey campaign level. This latter was edited by Igor Todisco, who remotely supported the dome graphic restitution especially in the presence of physical obstacles to data acquisition (9). The choice to adopt a direct survey methodology is given –despite the good physical accessibility to the architectural artefact– by the presence narrow, cramped and dimly lit spaces that would not have guaranteed the transport and/or the use of advanced instrumentation, especially in the area between the double shell that barely contained an operator in its interior in complete safety.

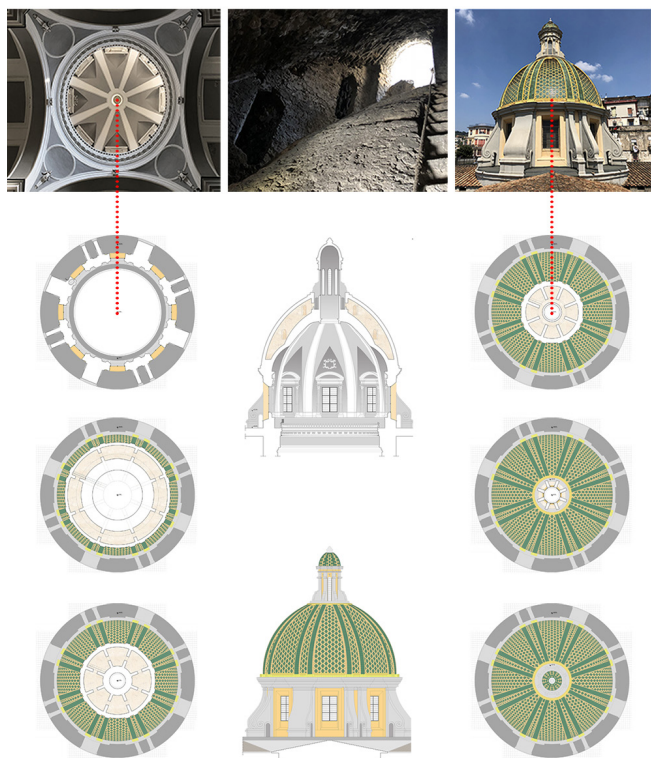


Figure 3. The dome of *Santa Maria della Sanità* Church in Naples built by Giuseppe Nuvolo: plans of the dome at different levels, cross-section and front (architectural survey by P. Galdiero, 2019).

Therefore, the direct survey methodology used required a fundamental critical process of designing horizontal and vertical cross-section planes from which to extrapolate the metric information. The data return took place in the form of planimetric and elevation representations, that allowed a graphic visualization of the various altimetric levels of which the dome is composed (Figure 3).

Then, this survey documentation provided subsequent thematizations for the realization of 3D geometric models through the dome configurative and geometric-spatial genesis definition (Figures 4-5) (10-11). Finally, a geometric study of the ornamental design of the tile roofing system covering the extrados of the dome was conducted (figure 6). In this sense, the survey and three-dimensional modeling with a feedback operation with the existing icono-

graphic documentation allowed to constantly monitor the results and integrate the methodologies of knowledge between tradition and innovation (11).

2.2. Graphical analysis, 3D modeling and tiled pattern of the dome

The graphical analysis and three-dimensional modeling of the dome of *Santa Maria della Sanità* enabled us to represent the intrados and extrados geometric-configurative matrices (Figure 4, a-d) and give a synthesis of spatial visualization (12-13). The dome geometric configuration is regulated by a symmetry of order eight; therefore, the 3D modeling (for intrados and extrados) was done by iterating the base module eight times (Figure 4, a, b). In particular, the intrados base module was configured according to the dome model with eight ribs and eight nails. Instead, the extrados base module was configured according to the dome model with eight double ribs (Fig. 4, c, d). The 3D view of the dome was made by completing the model with the addition of the drum on which the dome lies, and the lantern, which stands on the *opaion*.

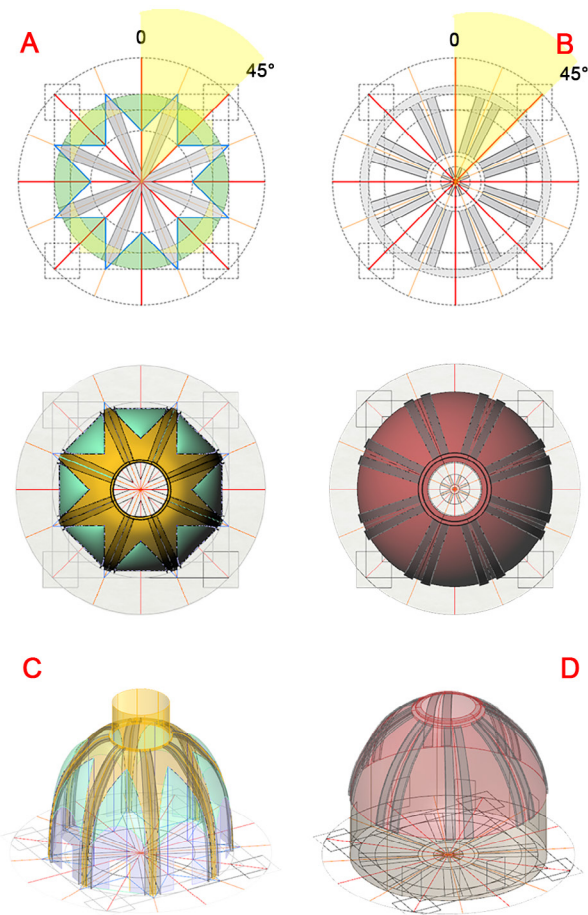


Figure 4. Geometric-configuration of the intrados and extrados of the dome (a-b) and 3D modeling (c-d) (drawing by V. Cirillo).

The geometric model of the drum is a straight cylinder with a circular base in which eight windows are placed. The large windows are interspersed with buttresses, simple and double (alternated), to which eight twin ribs correspond to the extrados of the dome. The lantern is also geometrically regulated by a radial symmetry of order eight. Therefore, the dome three-di-

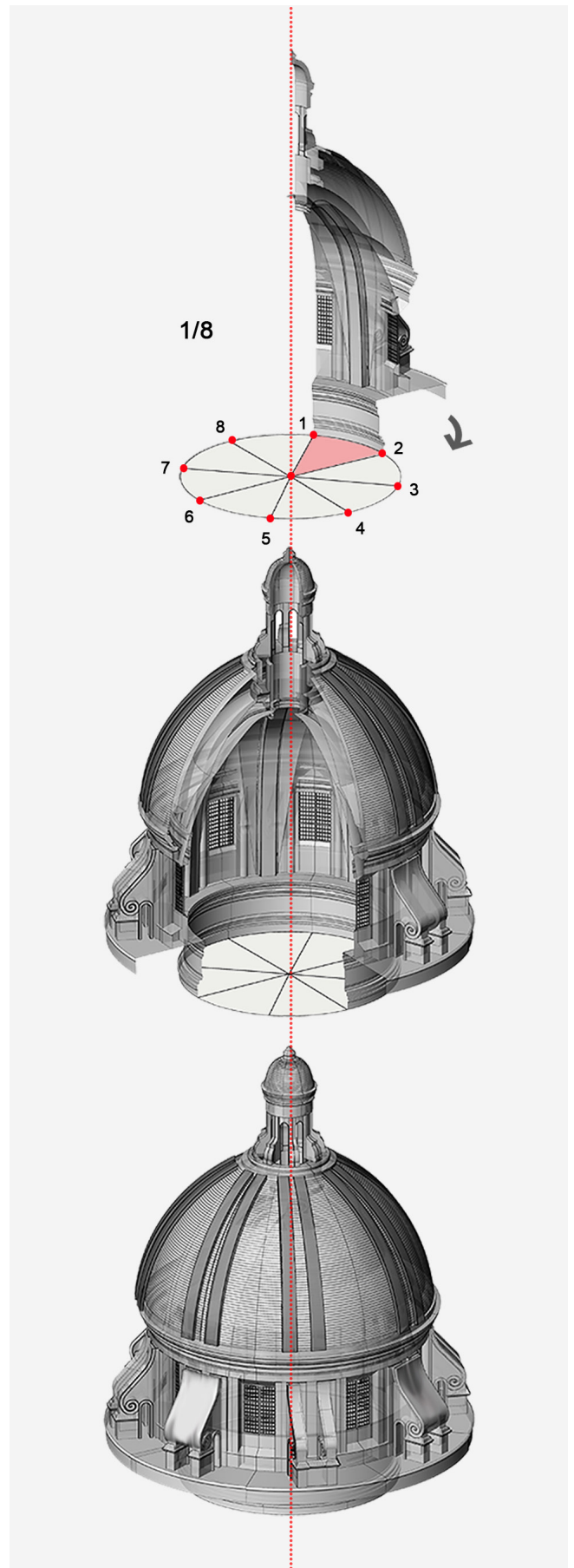


Figure 5. 3D model of the dome carried out iterating eight times the base module drum-dome-lantern (drawing by V. Cirillo).

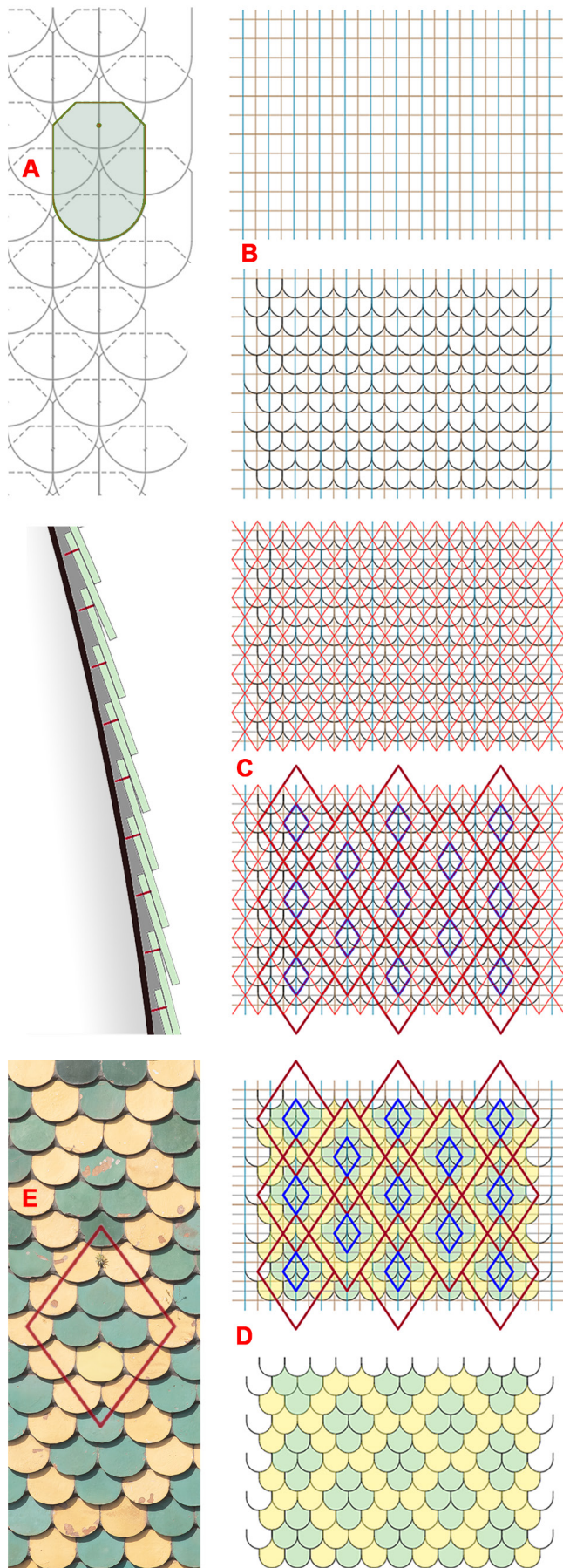


Figure 6. The geometric ornamental pattern genesis of the tiled dome (by Vincenzo Cirillo; picture of the dome by Igor Todisco).

dimensional model was made by iterating eight times the base module (drum, dome, lantern), bearing in mind the alternation of buttresses placed at the height of the drum (Figure 5).

Of great geometric interest is the ornamental solution to the dome extrados obtained by the application of majolica tiles in the characteristic Neapolitan yellow and green colors (14). The roof tiles used have an elongated shape along the vertical axis and end in the lower part with a semi-circumference. Specifically, from the roof tiles architectural survey it appears that the vertical measurement corresponds to 30 cm while the width to 20 cm. To be fixed with nails on the dome mantle, the roof tiles have a small hole along the vertical axis (Figure 6, a). From the geometrical point of view, the roof tiles installation is regulated by a rectangular grid where the rows of tiles along the horizontal direction are alternately arranged (Figure 6, b) (15). From the ornamental point of view, the roof tiles returned a rhomboid pattern obtained with the use of yellow and green colors.

From the geometric point of view, this ornamental design is obtained by superimposing another rhomboid grid on the rectangular ones (Figure 6, c). This rhomboid grid allows the identification of the directions in which the yellow tiles are placed and the consequent fields in which to collocate the green tiles (Figure 6, d). In this sense, the set of the two ordering grids (rectangular and rhomboid) perceptually returns an aggregation of tiles characterized by yellow ochre, for the tiles arranged along the perimeter of the rhombuses and by the green “ramina” for those placed in the rhombuses center (Figure 6, e). This geometric arrangement adapted to the dome extrados follows the radial sectors course and therefore in the dome planimetric and altimetric development it is progressively reduced towards the lantern.

3. STRUCTURAL STUDIES

3.1. Limit state analysis of masonry

In this research work, the limit state analysis of masonry has been applied. This theory was originally introduced for the evaluation of the collapse load of elastic-perfectly-plastic structures with unlimited ductility, such as those made of steel, and later extended to the study of the behavior of masonry constructions, as has already been demonstrated by Kooharian (17) and Heyman (18).

As known, the basic criteria for the analysis of a masonry structures concern the material properties. Masonry construction is built with a material that fulfills three conditions. Firstly, it works in compression: the tensions are very low and this implies the possibility to assume that the compressive strength is infinite. In almost all cases, there is no problem with the compressive strength of the material. Secondly, the tensile strength is zero, and this is in favor of security whereas there is always a certain adherence to the mortar. Finally, no sliding occurs. Under these conditions, the material is standard, and the principles of Limit Analysis can be demonstrated for masonry (18). The hypotheses made, allow schematizing the masonry construction as a set of rigid and mono-dimensional blocks held together by compressive forces. According to this approach, the collapse occurs for the formation of non-dissipative hinges. When the number of hinges is high enough to convert the structure into a mechanism, the collapse takes place.

Since the Church dome of *Santa Maria della Sanità* is a masonry structure, the three key assumptions on material properties of *no tensile strength (i)*, *infinite compressive strength (ii)*, *no yield through sliding (iii)* formulated by Heyman (18) for the simply voussoir arch, but applicable to any masonry structural forms, have been taken into account. By applying the Safe Theorem, static graphical analysis has been carried out in the point 4.

3.2. Dome behavior and graphic statics

A dome is a three-dimensional element of revolution obtained by rotating a semi-arch around an axis, although octagonal domes also exist. Any complex shapes can be studied starting from the behavior of the arch. In fact, a dome, however complex it may be, can always be traced back to as a set of arches. It can be studied with the slicing technique, a quite ancient method that consists in imagining the dome cut for meridian planes: every two-sliced obtained, form an arch that rests on a common key. For the Safe Theorem, if a thrust line fits within the thickness of the structure, the arch is stable and the dome will be too (19-20).

The thrust line theory and graphic statics arrived during the 19th c. Graphic statics supplies a practical method to assess the stability of ancient constructions, and it systematically based on the catenary principle introduced by Robert Hooke in 1676. Hooke's inversion law stating that "as hangs the flexible line, so but inverted, will stand the rigid arch" refers to the ideal shape of a stone arch whose equilibrium is that of the inverted catenary curve traced by a chain subjected to the same weight distribution (21). In other words, the idea is that if something hangs under a certain loading condition under pure tension, if you only look at static equilibrium, you can flip this geometry and this geometry will be in perfect compression (22). This paragraph quotes Figure 7, shown on the right.

This is a very important concept. You could explain the stability of a masonry structure by making hanging models, as ancient architects and engineers did. Alternatively, more simply, by using graphic statics as a paper version of hanging models. Diagrams using force vectors and closed force polygons represent nothing more than the equilibrium in a hanging system (Figure 8).

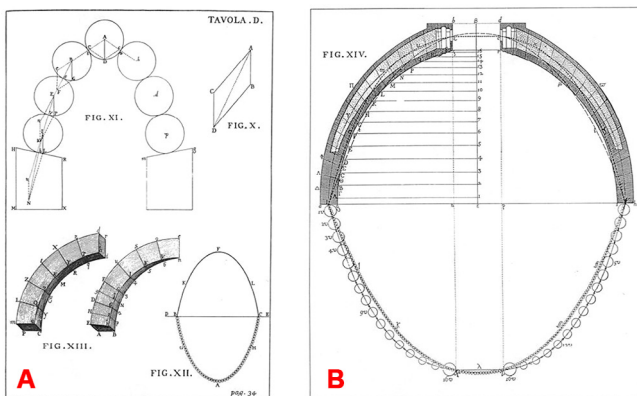


Figure 7. (a) Hooke's analogy between hanging chains and arches. Drawings by Poleni [1748]. (b) Poleni hanging model built to assess the stability of St.-Peter's Dome in Rome (24).

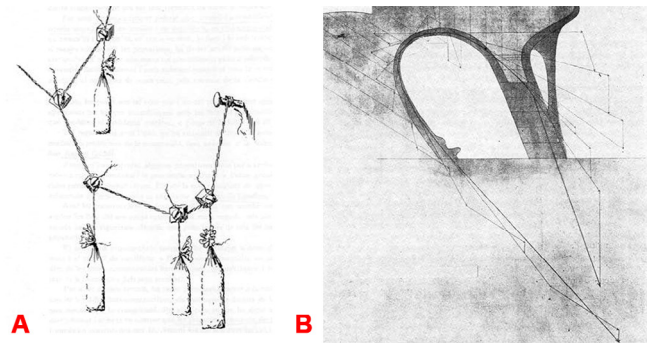


Figure 8. (a) Use of sachets to apply loads to the hanging model (24). Graphical design of the retaining walls of the Parque Güell in Barcelona (25).

Equilibrium in a masonry arch, and thus of a dome, can be visualized using a line of thrust, the trajectory of the resultant of the compressive forces within the structure. This line is nothing but the inverted catenary.

4. STABILITY ANALYSIS

4.1. Assumptions on the geometrical parameters used and on the applied loads

Geometry is the most important factor in determining the structural behavior of a dome. Stability methods consider only the geometry of the structure, and feasibility is assessed based on conditions of equilibrium; material failure is not considered (26).

The main dome of the Church is a double-shelled dome that spans approximately 16.42m, with a rise of 9.41m from the dome base. This structural analysis assumes that the dome is simply supported on the vertical cylinder wall of the drum, whose continuous support is only provided in

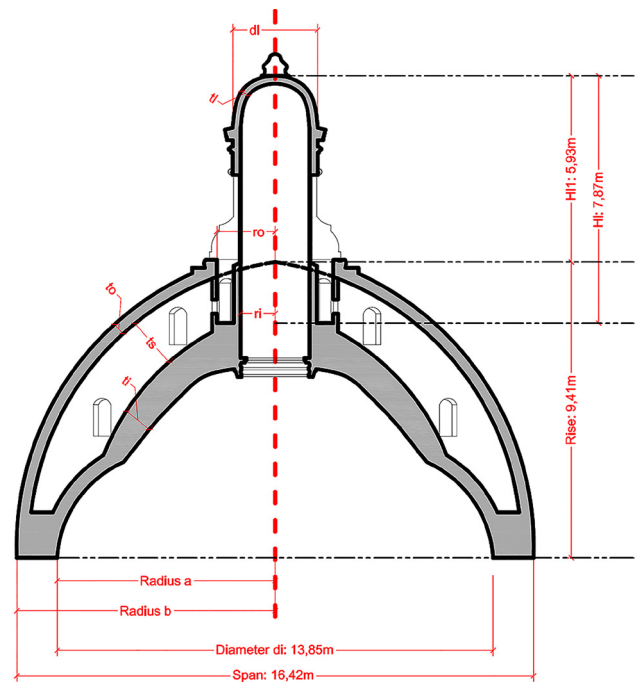


Figure 9. Section of the dome cut in correspondence to the lunettes. Identification of the main parameters of the dome.

the vertical direction with no transfer of bending forces. The internal radius of the outer shell, a , measures 8.21m while the radius of the inner shell, b , measures 6.93m. The thickness of the external shell is 0.40m, constant for all the section, and the thickness of the internal shell grows from 0.40m at the springing to about 1.30m to the crown. Figure 9 and Table 1 show the main parameters for the dome.

Table 1. Summary of the main metric data concerning the dome.

Dome geometry		
Parameter	Value	Symbol
Span (m)	16,42	
Rise (m)	9,41	
<i>Outer shell</i>		
Dome radius (m)	8,21	a
Dome thickness (m)	0,40	t_o
Skylight radius (m)	1,85	r_o
Radius to thickness ratio	20,5	a/t_o
<i>Inner shell</i>		
Dome radius (m)	6,93	b
Dome thickness (m)	0,90-1,40	t_i
Skylight radius (m)	1,10	r_i
Diameter (m)	13,85	d_i
<i>Lantern</i>		
Diameter (m)	2,7	d_l
Elevation (m)	7,87	H_l
Lantern thickness (m)	0,26	t_l

Figure 10 shows the studies on the geometry of the dome, which have been conducted to determine the shape and curvature of both outer and inner shells.

Once identified the geometry, a preliminary structural analysis of the dome has been performed to assess its stability. The structure has been disassembled by elements whose weight has been individually calculated. On this basis, the dome has been sliced into spherical sectors having an angle of 45° in plan, and a generic cross-section of the structure has been studied.

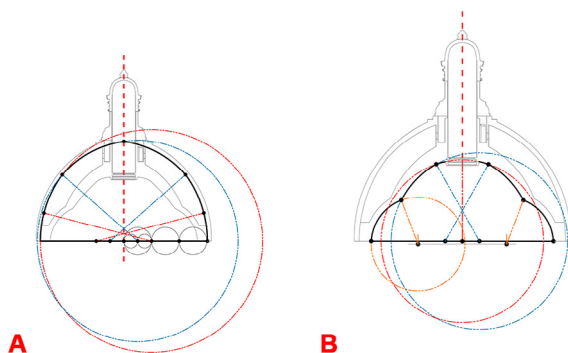


Figure 10. Studies on geometry: (a) geometric construction of the intrados line of the outer shell; (b) geometric construction of the intrados line of the inner shell.

Each slice obtained has been then subdivided into 6 ideal voussoirs to discretize the calculation of the load due to self-weight, to estimate the final thrust of the dome transmitted to the supports, and to define the possible trajectories of the pressure line inside the masonry. A specific weight of the masonry equal to 17 kN/m^3 has been considered.

Firstly, the weight of the lantern has been calculated taking into account that the lantern is composed of 8 windows and that it mostly stands on the inner shell; only the volute buttresses of the lantern rest on the outer shell. The estimated total weight of the lantern is equal to $W_{l_{tot}} = 334.22 \text{ kN}$ and the weight of the lantern buttresses is equal to $W_{l_b} = 76.16 \text{ kN}$.

Secondly, the weight of the 8 masonry walls pierced by windows that connect the two shells has been calculated, obtaining a value equal to $W_p = 720.08 \text{ kN}$.

Subsequently, the weight of the dome has been calculated as follows. The weight of the outer shell estimated taking into account the own weight of the structure, the tiled covering and the contribution of the lantern buttresses is equal to $W_{e_{tot}} = 2961.14 \text{ kN}$ (Figure 11, a). The weight of the inner shell calculated considering the weight of the lantern and the weight of masonry partitions is equal to $W_{i_{tot}} = 3734.06 \text{ kN}$ (Figure 11, b). Summing the two values, the total weight of the dome equals to $W_{tot} = 6695.20 \text{ kN}$ (Figure 11, c) is obtained. Figure 11 shows the graphical computation of the total weight of the dome with lantern. Notice that only the self-weight has been considered in the analysis.

4.2. Thrust of the dome

This section focuses on calculating the total thrust of the dome from the slicing method while the next section centers on checking the stability of the abutments.

Based on the slicing illustrated in Figure 11c, for each of the voussoirs the area and the horizontal distance to the symmetry axis have been determined.

Considering the weights of the voussoirs on the entire surface of revolution, the value obtained for the horizontal thrust is $H = 1987.60 \text{ kN}$. At the base of the dome, the vertical reaction, calculated directly from the weight of the dome equals to $V = 6695.20 \text{ kN}$.

From the application of the loads to their vector graphic expression it has been observed that the thrust line is contained within the masonry (Figure 12), and the dome is stable, as postulated by the Fundamental Theorem of the Limit Analysis (Safe Theorem).

The power of this Theorem is that the thrust line, namely the equilibrium condition, can be freely chosen. In fact, considering that the line of thrust is nothing else than the graphical representation of the equilibrium equations, the structural analysis via graphic statics illustrated in Figure 12, represents one of the possible equilibrium states in compression for the dome.

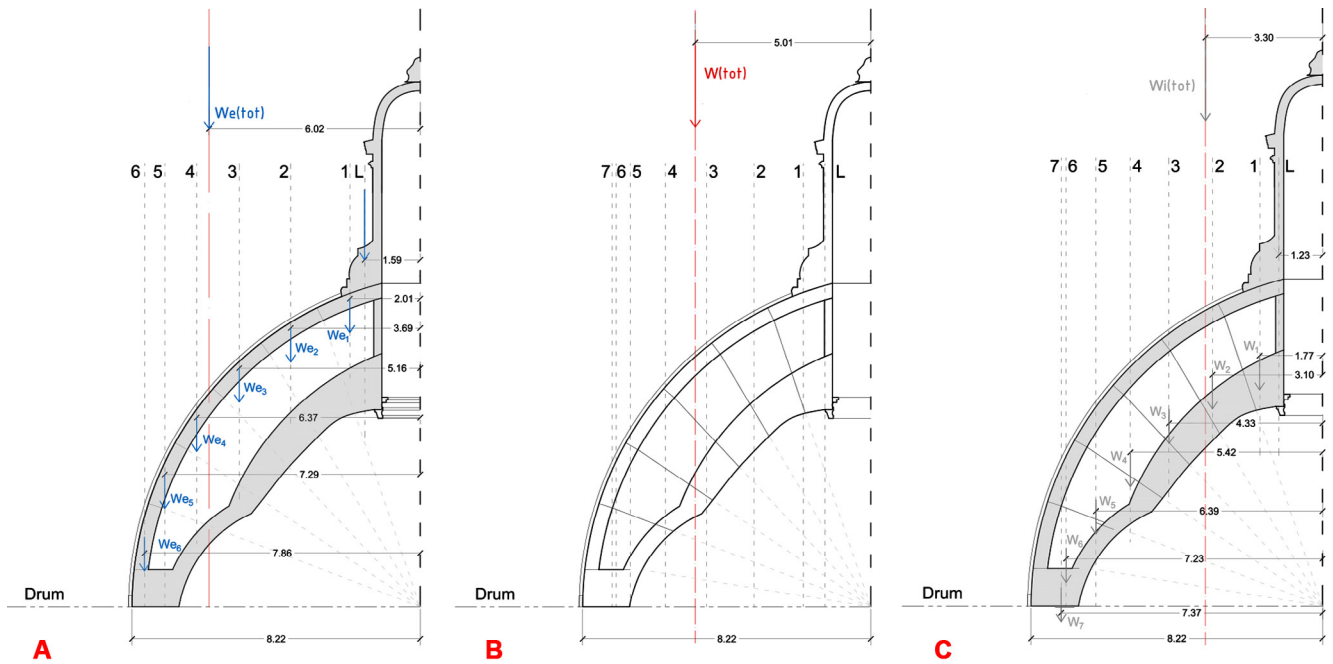


Figure 11. Analyzed transverse section of the dome: (a) graphic computation of the total weight of the outer shell with lantern buttresses $W_{e_{tot}}$; (b) graphic computation of the total weight of the inner shell with lantern $W_{i_{tot}}$; (c) dome total weight W_{tot} .

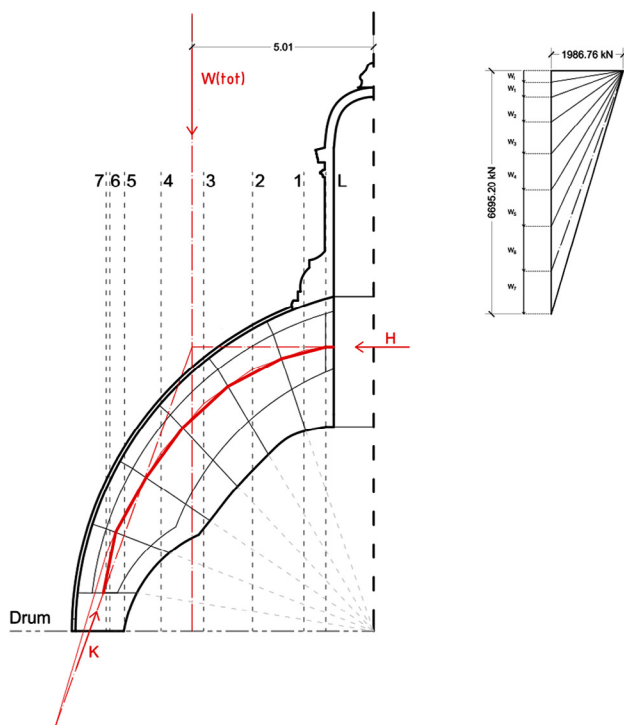


Figure 12. Graphical equilibrium analysis of the dome.

4.3. Safety assessment of the dome-buttress system

The support system of the dome consists of the drum that stands on a circular base and of the four central pillars of the Greek cross plan of the Church, sustaining the tambour.

After obtaining the thrust values of the dome, the horizontal (H) and vertical (V) components have been applied to the support system. The thrust dome combines with the total

weight of the drum and the abutment. On the basis of the thrust values and weights shown in Figure 13a, by using the overturning moment, the distance d of the reaction X from the wall boundary is calculated. Table 2 below shows, in the summary, the computation of the loads and thrusts in the analysis of the dome-buttress system.

Table 2. Analysis of the dome-buttress system.

	weight [kN]	X_{gi} [m]	$W_i \cdot x_{gi}$ [kNm]	d [m]	x_g [m]	s.c.s.
W8	1575.73					
W9	5698,74					
Wc	29097,88	2,31	86985,54			
Wb	1285,20	4,14	6885,65			
Wd	17436,43	5,59	126137,18			
dome	6695,20	5,58	58275,68			
X	54514,71					
sum			278284,05			
					3,92	
H	1987,60	34,13	105707,44			
x_d			172576,62	2,56		
s.c.s.						2,88

The analysis carried out graphically in Figure 13b shows that the diaphragm of the support system conveniently provides the necessary counterweight. A geometric safety coefficient of the structure equals 2,88 is obtained.

The analysis carried out shows that the structure is stable and that the geometric safety coefficient calculated is a good value

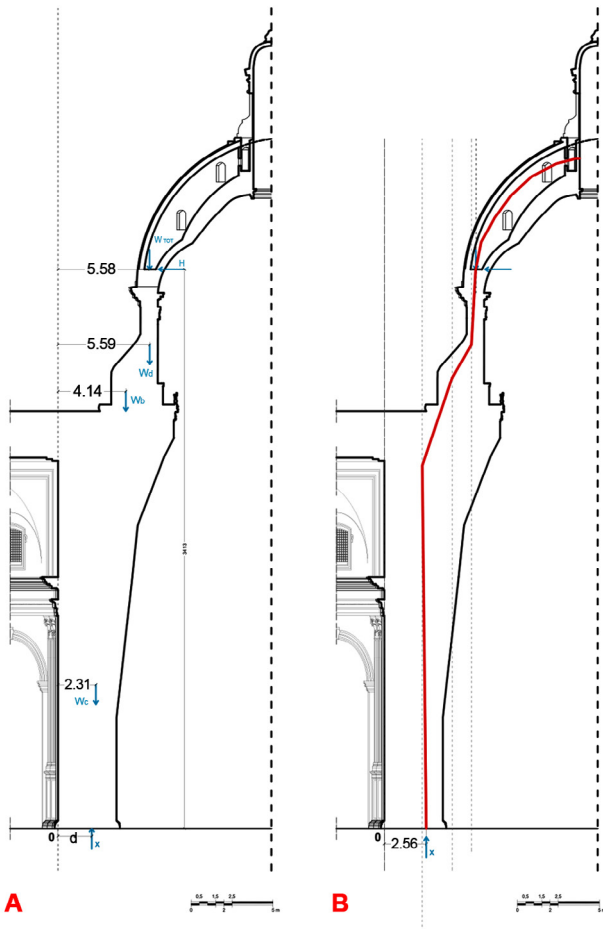


Figure 13. (a) Thrust and weights acting on the pillar. (b) Equilibrium analysis of the dome-buttress system represented by thrust line.

for these kinds of constructions (27). Even if the geometry of the dome is peculiar, the global equilibrium is guaranteed. Although at this stage, the role and the behavior of the internal walls have not been studied.

As already explained above, only the stability of the dome has been evaluated, and no vulnerability analysis of the dome has been performed, on which topic two of the authors have provided relevant scientific contributions (28-31).

5. CONCLUSIONS

The geometric analysis carried out on the architectural survey of the church of *Santa Maria della Sanità* dome in Na-

NOTES

- (1) This contribution is the result of a multidisciplinary team. The chapter 1 was written by Ornella Zerlenga; the chapter 2 by Vincenzo Cirillo; the chapters 3 and 4 by Claudia Cennamo and Concetta Cusano. The conclusions were written by Ornella Zerlenga and Claudia Cennamo.

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ples allowed a study on its spatial configuration, majolica tiles decoration and stability¹.

Regarding the geometric analysis, the dome, a surface of revolution, has the following characteristics: different curvature for the intrados and extrados, the structure is regulated by a symmetry of order eight; the double bands of ribs delimit eight surfaces sectors. The distribution of modular roof tiles requires a support distribution network consisting of two groups of curves which are mutually orthogonal; the roof tiles ornamental design is an adaptation of prefabricated modules.

From the structural studies illustrated to point 3 and from the analysis performed to point 4, some remarks can be outlined.

Safety in masonry constructions is a matter of geometry (32). As deeply discussed, the material imposes that the thrust line, namely the locus of the position of the resultant of the compressive forces, must be contained within the thickness of the structure as it appears.

The equilibrium approach, which comes directly from the Safe Theorem of Limit Analysis, has demonstrated to be the most adequate for the analysis of masonry structures (33), also considering that the geometrical design of such ancient buildings embeds rules used by old master builders over centuries. Some of the authors emphasized the importance of the use of compressive thrust line analysis, via graphic statics, to explore the range of possible equilibrium states both for 2d and 3D structures (34-39).

This work contributes, on the one hand, to explore unsolved matters concerning the examined case study of the Neapolitan dome of *Santa Maria della Sanità*. On the other hand, from a wider perspective, it provides an even more important contribution to a potential way of approaching the study of historical constructions (40). Therefore, in conclusion, a multidisciplinary approach that includes in-depth studies on the geometry and construction aspects of historical buildings is a topic of great relevance when dealing with architectural heritage (41), both to assess its stability and to prevent inappropriate or, in most cases, unnecessary interventions.

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