Propositions

belonging to the thesis

Variations on the method of Chabauty and Coleman

by

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1. There are families of curves over $\mathbb{Q}$ of each genus $g \geq 2$, such that if a curve in such a family satisfies Chabauty's rank condition, then it has the maximal possible number of rational points allowed by Coleman's bound.

2. Siksek's symmetric Chabauty method can be generalised by merging Siksek's two criteria into one.

3. Expanding the relevant differentials further than in Siksek's approach leads to a criterion which is sometimes applicable when the previous proposition is not.

4. Using the previous two propositions, one can compute cubic points on some modular curves having infinitely many quadratic points. Explicit examples of these propositions are shown in Chapter 4 of this thesis.

5. Computing the local p-adic height above p on hyperelliptic curves can be reduced to Balakrishnan's algorithm for Coleman integration.

6. Let p be a prime number. The probability that a random polynomial in $\mathbb{Z}_p[x]$ has a root in $\mathbb{Q}_p$ is an explicitly computable rational function in p satisfying interesting symmetry properties.

7. Number theory has applications in topological quantum field theory; for example, Zsigmondy's Theorem implies that a certain 2-dimensional topological quantum field theory is faithful.

8. Let $S = \{a^n \colon a \in \mathbb{Z}, n \geq 2\} \subseteq \mathbb{Z}$ be the set of perfect powers. Let $T \subseteq S$ be a given finite subset. Using Mihăilescu's theorem (Catalan's conjecture), one can construct a polynomial $f \in \mathbb{Z}[x]$ such that $f(\mathbb{Z}) \cap S = T$.

9. A feature of certain algorithms is that the description of the algorithm is less involved than the precision analysis needed to verify its output.

10. Sometimes it is easier to create a method than to find an instance where this method leads to a satisfactory answer.