

Partially Observable Monte Carlo Planning with State Variable Constraints for Mobile Robot Navigation

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Abstract

Autonomous mobile robots employed in industrial applications often operate in complex and uncertain environments. In this paper we propose an approach based on an extension of Partially Observable Monte Carlo Planning (POMCP) for robot velocity regulation in industrial-like environments characterized by uncertain motion difficulties. The velocity selected by POMCP is used by a standard engine controller which deals with path planning. This two-layer approach allows POMCP to exploit prior knowledge on the relationships between task similarities to improve performance in terms of time spent to traverse a path with obstacles. We also propose three measures to support human-understanding of the strategy used by POMCP to improve the performance. The overall architecture is tested on a Turtlebot3 in two environments, a rectangular path and a realistic production line in a research lab. Tests performed on a C++ simulator confirm the capability of the proposed approach to profitably use prior knowledge, achieving a performance improvement from 0.7% to 3.1% depending on the complexity of the path. Experiments on a Unity simulator show that the proposed two-layer approach outperforms also single-layer approaches based only on the engine controller (i.e., without the POMCP layer). In this case the performance improvement is up to 37% comparing to a state-of-the-art deep reinforcement learning engine controller, and up to 51% comparing to the standard ROS engine controller. Finally, experiments in a real-world testing arena confirm the possibility to run the approach on real robots.

Keywords: Planning under uncertainty, POMDP, POMCP, Mobile robot planning, Industry 4.0, Explainable planning

1. Introduction

2 Planning under uncertainty is a key task for long-term robot auton-
3 omy. Despite recent advances of intelligent robotic systems in several con-
4 texts, such as, industrial robots, service robots, patrolling, search and res-
5 cue (Farinelli et al., 2017; Parker et al., 2016; Bevacqua et al., 2015; Orlan-
6 dini et al., 2013; Basilico et al., 2012), the results of recent robot challenges
7 (Krotkov et al., 2017; Correll et al., 2018) show that improvements are still
8 needed to achieve a reliable management of uncertainty in unstructured en-
9 vironments. The risk introduced by uncertainty in these contexts can, in
10 fact, make traditional planning methods impractical (Lanighan and Grupen,
11 2019). Novel approaches are needed to overcome these issues enabling robots
12 to make effective decisions while managing risk (Wang et al., 2018; Laroche
13 et al., 2019; Simao and Spaan, 2019).

14 In several application domains robots are required to execute series of
15 tasks having similar properties. A typical example in Industry 4.0 concerns
16 mobile robots involved in warehouse pick-and-place operations (Caccavale
17 and Finzi, 2019). They traverse aisles possibly populated by people, other
18 robots and obstacles with the goal of moving objects from one place of the
19 warehouse to another. Tasks are represented by movements across aisles
20 and a task property of interest is the traffic level in each aisle, that char-
21 acterizes the difficulty of the aisle and of the entire path traversed by the
22 robot, which affects the movement capability of the robot. Interestingly,
23 some aisles have similar physical properties that make their difficulties also
24 similar to each other. Another example is represented by flying drones in-
25 volved in autonomous package delivery (Grippa et al., 2019). Their goal is to
26 deliver as many packages as possible in the shortest possible time and using a
27 fixed amount of energy due to battery limitations (Chen et al., 2019). Tasks
28 in this context are movements through flight sections and a task property is
29 the energy requirement of each flight section, which could depend on wind
30 direction, presence of buildings and other features. Also in this case, simi-
31 larities between pairs of flight sections can be identified, which characterize
32 the similarities between the energy demand of these flight sections. Simi-
33 lar applications concern exploration and surveillance using unmanned aerial
34 vehicles (UAV), where control strategies based on model predictive control
35 (Altan and Hocolu, 2020), metaheuristic optimization (Altan and Hocolu,

36 2020) and other approaches (e.g., neural networks) have been integrated in
37 the flight control for path planning. We instead focus on the integration of a
38 probabilistic planner based on Markov decision processes. Another domain
39 concerns aquatic drones involved in water monitoring, that traverse path seg-
40 ments with properties depending on water flow and waves (Castellini et al.,
41 2020a).

42 The similarity structure of tasks involved in robot planning can provide
43 useful information for improving planning performance. However, in the
44 majority of cases this structure is only partially known in advance. In the
45 warehouse pick-and-place application domain, for instance, it is typical to
46 know in advance that two aisles have similar degree of difficulty (due to their
47 position in the warehouse and physical properties) but this information could
48 be available only for a subset of aisles or it could be uncertain, namely, only a
49 probability that two aisles have the same degree of difficulty could be known.

50 In this paper we investigate the impact of prior knowledge about task sim-
51 ilarity structure on planning performance of real mobile robotic platforms.
52 We focus, in particular, on a problem concerning velocity regulation of a
53 mobile robot following a pre-specified path in an environment with uncer-
54 tain obstacle densities. The robot has to reach the end of the path in the
55 shortest possible time and to avoid collisions with obstacles in the path to
56 guarantee safety. This problem is used to show that the proposed approach
57 can be applied to real robotic platforms. Similar problems were proposed in
58 the literature to test planning methods in mobile robots (Yang et al., 2014).
59 Real-world applications of this case study concern, for instance, safety man-
60 agement in Industry 4.0.

61 In our case study the path that has to be traveled is divided into seg-
62 ments and subsegments, and every segment is characterized by a *difficulty*
63 that depends on the density of obstacles in the segment. A time penalty is
64 imposed to the robot each time it collides. The real difficulty of segments
65 is unknown to the robot in advance, and the robot has to reach the end
66 of the path as quickly as possible, hence it should move slowly in difficult
67 segments to avoid collisions, and fast in simple segments to minimize the
68 traveling time. Since it is known in advance that some pairs of segments
69 can (probabilistically) have the same difficulty (e.g., because they have simi-
70 lar properties), the information about (unknown) segment difficulties can be
71 collected as the robot moves and then propagated to subsequent segments
72 to improve planning performance. The problem has therefore a sequential
73 structure, in which difficulties of previously traveled segments are used to

74 infer the difficulty of subsequent segments known to have similar difficulty.

75 We use *Partially Observable Markov Decision Processes (POMDP)* (Rus-

76 sell and Norvig, 2003; Kaelbling et al., 1998) to formalize the problem. This

77 framework allows to model dynamical processes in uncertain environments

78 and to synthesize optimal policies in this context. To overcome scalability is-

79 sues (Papadimitriou and Tsitsiklis, 1987), we use *Partially Observable Monte*

80 *Carlo Planning (POMCP)* (Silver and Veness, 2010) as a solver. It is an ap-

81 proximate online algorithm (Ross et al., 2008) able to synthesize the planning

82 policy step-by-step and without representing the overall state space. Then,

83 we consider the POMCP extension proposed in (Castellini et al., 2019) to

84 introduce and exploit prior knowledge about relationships between segment

85 difficulties. In particular, we represent these relationships by probabilistic

86 state-variable constraints using Markov Random Fields (MRFs). This prior

87 knowledge yields a performance improvement in terms of expected return

88 that we show to be related to the improvement of two external measures,

89 namely, the distance between the real state and the belief, and the mu-

90 tual information between segment difficulty and action taken in the segment.

91 These two measures are defined and analyzed across the paper together with

92 other informative measures that support the interpretability of the approach

93 and the related results (Anjomshoae et al., 2019; Langley et al., 2017; Zhang

94 et al., 2017). The novel contribution we propose in this work is the integra-

95 tion of the POMCP-based planner presented in (Castellini et al., 2019) into

96 a real robotic task. This is not trivial and requires a specific formalization

97 of the problem. We provide a two-layer control architecture in which the

98 upper layer uses an extension of Partially Observable Monte Carlo Planning

99 (POMCP) for regulating the velocity of a mobile robot, and the lower layer

100 uses a standard engine controller for dealing with path planning. The inte-

101 gration of the two layers, with POMCP set on top of the engine controller and

102 used to control high-level aspects of robot motion, is also not trivial. The

103 proposed architecture allows to *i)* devise a probabilistically optimal strat-

104 egy based on POMDPs for regulating the robot velocity, *ii)* integrate prior

105 knowledge about the environment, *iii)* improve the efficiency of standard en-

106 gine controllers for path planning, e.g., we present results using a Deep RL

107 controller in the lower level of the architecture but a standard ROS controller

108 can also be used. The interaction between the two controllers is explained in

109 the following and the overall software/hardware architecture described. The

110 results on simulated and a real Turtlebot are fully documented and compar-

111 isons with state-of-the-art control approaches are performed.

112 The proposed methodology is evaluated in four ways. First, we compute
113 a statistical analysis of performance considering a large number of instances
114 of our problem having different configurations of segment difficulties. This
115 analysis is performed on two environments, namely, a rectangular path and
116 a path that reproduces a real industrial environment. Experiments are per-
117 formed using the standard C++ simulator provided by the POMCP soft-
118 ware¹. Such simulator does not consider the physical properties of the en-
119 vironment (e.g., frictions, etc.). Transition and observation models of the
120 POMDP are learned from tests performed on Unity (Juliani et al., 2018), a
121 simulation tool increasingly used in recent robotics works (Marchesini and
122 Farinelli, 2020a; Yoon et al., 2018), since it represents a viable and faster
123 alternative to other simulation tools such as Gazebo². The native simula-
124 tion time speed-up introduced by Unity (up to $\times 100$ times) enables quick
125 data collection. Our experiments show that the planning approach based on
126 prior knowledge (Castellini et al., 2019) outperforms the standard POMCP
127 approach (Silver and Veness, 2010). The improvement is small (i.e., 0.7%) in
128 the simple rectangular path and larger (i.e., 3.1%) in the more complex ICE
129 path. Second, we perform tests on Unity simulators of the rectangular and in-
130 dustrial environments using a *TurtleBot3*³ as an agent. In these experiments
131 the simulator considers the physical properties of the environment. We use a
132 state-of-the-art engine controller (Marchesini and Farinelli, 2020b) for path
133 planning and collision avoidance in single subsegments, a localization algo-
134 rithm for improving robot localization, inter-process communication between
135 planner and robot, and other tools explained in the next sections. These tests
136 show that our approach has good performance in physics-grounded simula-
137 tions (OpenAI et al., 2018) of real-world environments. Third, we actually
138 deploy our method in a real-world testing arena reproducing the rectangular
139 path environment and using a real *TurtleBot3* as an agent. As expected, tests
140 (displayed in an attached video) are faithful reproductions of the evaluations
141 performed on Unity simulators. They therefore confirm the results obtained
142 in simulations and the possibility to apply the two-layer planning approach
143 to real-world robotic platforms. As a fourth experimental test, we provide a
144 comparative analysis of performance between the proposed methodology (us-

¹<http://www0.cs.ucl.ac.uk/staff/D.Silver/web/Applications.html>

²<http://gazebosim.org/>

³<https://www.turtlebot.com/>

145 ing POMCP for velocity regulation and a deep reinforcement learning - DRL
146 - engine controller for path planning) and state-of-the-art engine controllers
147 (without the POMCP layer). This analysis confirms that the proposed ap-
148 proach achieves a performance improvement up to 37% comparing to the
149 DRL engine controller. The performance improvement reaches even 51% if
150 the standard ROS⁴ engine controller is used. In summary, analyzing advan-
151 tages and disadvantages of the proposed approach we observe that it has
152 an increased complexity than standard controllers, because it introduces a
153 new POMCP-based layer for velocity regulation, but it also achieves higher
154 planning performance exploiting prior knowledge about task relationships.

155 The main contributions of this paper to the state-of-the-art can be then
156 summarized in the following four points:

- 157 • we formalize a problem of mobile robot velocity regulation in industrial-
158 like environments, considering uncertain motion difficulties (e.g., due
159 to clutterness or presence of moving obstacles) and prior knowledge
160 about similarities in segment difficulties;
- 161 • we propose a two-layer approach in which an extended version of POMCP
162 regulates the robot velocity considering prior knowledge about segment
163 similarities, and a standard engine controller performs path planning
164 and operates the robot considering the velocity selected by POMCP;
- 165 • we deploy the methodology on a real Turtlebot and on a Unity sim-
166 ulator of the Turtlebot, showing that it outperforms state-of-the-art
167 controllers when prior knowledge about segment similarity is consid-
168 ered by POMCP;
- 169 • we introduce three measures that support the explainability of results
170 achieved by the extended POMCP, providing insight about the rela-
171 tionship between prior knowledge and performance improvement.

172 The rest of the paper is organized as follows. Section 2 presents related
173 work. In Section 3 we define our problem and in Section 4 formalize it as a
174 POMDP. Section 5 describes the three versions of POMCP used to synthesize
175 the policy and Section 6 defines three useful measures for explaining perfor-
176 mance differences among policies. In Section 7 we provide full details on the

⁴<https://www.ros.org/>

177 experimental setup of simulated and real-world tests, and describe the main
178 elements of the proposed architecture. Then, in Section 8 the three evaluation
179 methods are described and results are analyzed providing in-depth interpre-
180 tation of the internal mechanisms that allow the performance improvement.
181 Finally, Section 9 draws conclusions and indicates future directions.

182 2. Related work

183 Planning under uncertainty dates back to the seventies (Feldman and
184 Sproull, 1977; Russell and Norvig, 2003) when aspects of mathematical deci-
185 sion theory started to be incorporated into the predominant symbolic problem-
186 solving techniques. The interest in this topic subsequently grew (Ratering
187 and Gini, 1995; Kaelbling et al., 1998; Boutilier et al., 1999; Zhang et al.,
188 2015; Godoy et al., 2016), since planning under uncertainty is a critical task
189 for autonomous and intelligent agents based on currently available data-
190 driven technologies. The most recent developments mainly concern the use of
191 point-based value iteration (Spaan and Spaan, 2004; Spaan and Vlassis, 2005;
192 Wang et al., 2019), Monte Carlo Tree Search (MCTS) based solvers (Coulom,
193 2006; Kocsis and Szepesvári, 2006; Browne et al., 2012; Beretta et al., 2019)
194 and Deep Reinforcement Learning (DRL) methods (Silver et al., 2016, 2017;
195 Sutton and Barto, 2018; Leonetti et al., 2016). The first two approaches aim
196 to deal with very large state spaces and the third to learn the policy only
197 from observations and without using a model of the environment dynamics.

198 Prior knowledge, in different forms, is used in the literature to improve
199 robot navigation (Luperto et al., 2019). However, we have found only a few
200 approximate (Hauskrecht, 2000) and online (Ross et al., 2008) planning ap-
201 proaches based on POMCP, such as (Amato and Oliehoek, 2015; Lee et al.,
202 2018), in which prior knowledge about the domain is used to improve plan-
203 ning performance and scale to large problem instances. The main differences
204 between those approaches and our work are threefold. First, we use a differ-
205 ent method to introduce prior knowledge (Castellini et al., 2019). Namely,
206 in our method constraints on the state space are used to refine the belief
207 space and increase performance (in terms of shorter execution time), while
208 in (Amato and Oliehoek, 2015) the multiagent structure of a problem is used
209 to decompose the value function into a set of overlapping factors that en-
210 able scalability and performance improvements in POMCP. In (Lee et al.,
211 2018) cost-constraints are used to solve problems with multiple objectives.
212 Our method to define prior knowledge is also different to that of factored

213 POMDPs (Boutilier and Poole, 1996), since we constrain states instead of
214 the transition model, hence the prior knowledge is expressed in a different
215 way. Second, we focus on an original problem related to obstacle avoidance
216 in industrial-like environments. Our problem has a strong sequential nature
217 in the way in which the robot explores the environment and transfers
218 the acquired knowledge to future exploration (see problem formalization in
219 Sections 3 and 4). In particular, our goal is only velocity regulation, a high-
220 level planning problem, and we delegate the low-level navigation (i.e., path
221 planning) to a state-of-the-art controller (Marchesini and Farinelli, 2020b)
222 realized with a DRL approach (i.e., Rainbow (Hessel et al., 2018)) using
223 standard setup for navigation problems (Marchesini et al., 2021; Tai et al.,
224 2017). This controller is independent to the POMCP planner used for ve-
225 locity regulation and it was selected for its simplicity to directly control and
226 modify the linear speed of the Turtlebot. The literature on obstacle avoid-
227 ance for general robotic applications is very wide (Steccanella et al., 2020;
228 Kumar and Kumar, 2018; Correll et al., 2018) but, to the best of our knowl-
229 edge, it does not contain techniques as that proposed in this work. Third,
230 one of our main goals is to actually deploy the proposed approach on a real
231 robotic platform, therefore we provide in-depth technical details about how
232 the planner is integrated with the engine controller, the localization module
233 and all other modules to make the approach work in practice. A preliminary
234 work towards the integration of the methodology introduced in (Castellini
235 et al., 2019) into robotic platforms is (Castellini et al., 2020b) where the
236 POMCP extension based on MRF prior knowledge is applied to a real-world
237 implementation of the rocksample problem. Here we significantly extend
238 that preliminary work providing an improved methodology and a thorough
239 experimental setting with several tests performed also on an industrial-like
240 environment.

241 Robotics literature provides extensions of motion planning techniques for
242 velocity regulation (Huang, 2009; Zhong et al., 2014; Gopalakrishnan et al.,
243 2017), but these methods have completely different assumptions than our
244 method. In particular, these approaches use the position and velocity of
245 the robot in the 2D environment as an input, and aim to plan the overall
246 movement of the robot (e.g., direction, velocity and acceleration) considering
247 also positions and velocities of moving obstacles in the 2D environment. To
248 solve this problem they use potential field methods (Koren and Borenstein,
249 1991), velocity obstacles (Fiorini and Shiller, 1998) and other standard mo-
250 tion planning methods. We instead assume a hierarchical organization of

251 the problem where the (low-level) motion planning is solved by a standard
252 controller and the (high-level) velocity regulation is solved by POMCP. The
253 baseline controllers for motion planning to which we compare are the DRL
254 controller presented in (Marchesini and Farinelli, 2020b) and the standard
255 ROS navigation stack⁵. Velocity regulation is, however, only an example of
256 possible high-level decisions that can be taken by our approach. Other exam-
257 ples include, for instance, alerting operators, blocking the robot movement,
258 and other high-level decisions that can affect motion planning. We also notice
259 that the hierarchical organization of the controller can be applied to other
260 problems to which POMCP has been recently applied, such as, active visual
261 search (Wang et al., 2020; Wandzel et al., 2019; Lauri and Ritala, 2016).

262 Explainable planning (XAIP) (Miller, 2019; Fox et al., 2017; Langley
263 et al., 2017) is a branch of eXplainable Artificial Intelligence (XAI) (Gun-
264 ning and Aha, 2019) which is related to our work. Three main challenges of
265 XAI are the development of methods for learning more explainable models,
266 the designation of effective explanation interfaces, and the understanding
267 of psychologic requirements for effective explanations (Gunning and Aha,
268 2019). The aim of XAIP is to create artificial intelligence systems whose
269 models and decisions can be understood and trusted by end users. XAIP
270 has a strong impact on safety-critical applications, such as industrial robotic
271 ones (Anjomshoae et al., 2019; Sridharan and Meadows, 2019; Zhang et al.,
272 2017), wherein people accountable to authorize the execution of a plan need
273 complete understanding of the plan itself. First approaches of XAIP (Smith,
274 2012) focus on human-aware planning and model reconciliation and on data
275 visualization (Chakraborti et al., 2018). The two measures defined in Sec-
276 tion 6 and their analysis performed in Section 8 are the first steps towards
277 the interpretability of decisions taken by POMCP-based planners.

278 3. Problem definition

279 Assume to have a pre-defined path in an industrial environment which
280 must be traversed by a mobile robot. The path (a possible instance is dis-
281 played in Figure 1) is made of segments $s_i, i = 1, \dots, m$, which are then split
282 into subsegments s_{ij} . Each segment (and related subsegments) is character-
283 ized by a difficulty $f_i \in F$, related to the average density of obstacles, the type

⁵<http://wiki.ros.org/navigation>

284 of terrain (e.g., stairs, slopes, etc.) or luminosity (which makes perception
285 more difficult). The robot has to reach the end of the path in the shortest
286 possible time, regulating the speed in each subsegment to avoid obstacles,
287 since the probability of collision depends on speed and segment difficulty, and
288 each collision generates a time penalty. The robot cannot directly observe
289 segment difficulties, which are hidden state variables, but it can only infer
290 their values in two ways, namely, by observing the occupancy of the segment
291 (detected by lasers on top of the robot), and sensing the angular velocity
292 kept in the segment. The generality of the concept of difficulty lends itself
293 to different contexts in which a mapping exists between some properties of
294 the environment and the difficulty the agent experiences when it executes its
295 tasks. Our method can work also if this mapping is only coarsely defined.

296 In this work we test our planning approach on two specific paths depicted
297 in Figure 3. The first path has a *rectangular* shape with short sides of
298 three meters and long sides of five meters. An external and an internal wall
299 delimit the area accessible by the robot and some obstacles are arranged
300 along the way. The second path is developed in a real environment for
301 industrial research, namely, the Industrial Computer Engineering (ICE) lab⁶
302 of the Verona University (Italy), which is a laboratory for Industry 4.0 with
303 a modern production line, extended with equipment for augmented reality
304 and digital production. Technical details about the two paths are reported
305 in Section 7. These paths are only two possible instances of several possible
306 environments that can be dealt with by our approach. Also the problem
307 formulation can be extended to more complex scenarios considering larger
308 sets of actions and observations, or more realistic tasks but here we aim to
309 show the applicability of the proposed approach to real robotic platforms.

310 4. POMDP representation of the problem

311 A Partially Observable Markov Decision Process (POMDP) (Kaelbling
312 et al., 1998) is defined as a tuple $(S, A, O, T, Z, R, \gamma)$, where S is a finite set
313 of partially observable *states*, A is a finite set of *actions*, Z is a finite set of
314 *observations*, $T: S \times A \rightarrow \Pi(S)$ is the *state-transition model* (where $\Pi(S)$ is
315 the power set of S), $O: S \times A \rightarrow \Pi(Z)$ is the *observation model*, $R: S \times A \rightarrow \mathbb{R}$
316 is the *reward function* and $\gamma \in [0, 1)$ is a *discount factor*. The goal of an

⁶<http://www.di.univr.it/?ent=progetto&id=4935&lang=en>

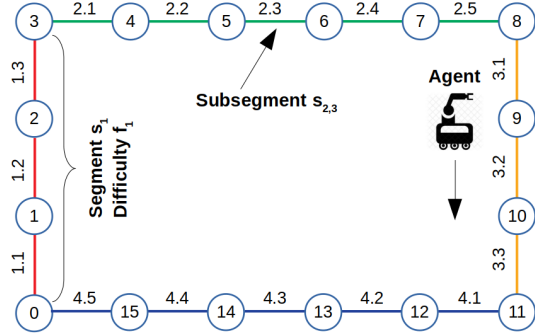


Figure 1: Problem definition: path travelled by the agent. Nodes are subsegment starting points.

317 agent operating a POMDP, is to maximize its expected total discounted
 318 reward (also called *discounted return*) $E[\sum_{t=0}^{\infty} \gamma^t R(q_t, a_t)]$, by choosing the
 319 best action a_t in each state q_t at time t ; γ is used to reduce the weight of
 320 distant rewards and ensure the (infinite) sum's convergence. As mentioned
 321 above, the partial observability of the state is dealt with by considering at
 322 each time-step a probability distribution over states, called *belief*. The belief
 323 space is here represented by symbol B . POMDP *solvers* are algorithms that
 324 compute, in an exact or approximate way, a *policy* for POMDPs, namely a
 325 function $\pi: B \rightarrow A$ that provides an optimal action for each belief.

326 The problem described in the previous section can be formalized as a
 327 POMDP. The *state* contains *i*) the true configuration of segment difficulties
 328 (f_1, \dots, f_m) , which is hidden, *ii*) the position $p = (i, j)$ of the robot in the
 329 path, where i is the index of the segment and j the index of the subsegment
 330 (notice that, saying that the agent is in position (i, j) we mean that it is at the
 331 beginning of subsegment $s_{i,j}$), *iii*) t is the time elapsed from the beginning of
 332 the path. *Actions* correspond to the speed the robot keeps in a subsegment,
 333 which may have three possible values, namely low (L), medium (M) or high
 334 (H).

335 *Observations* are related to subsegment *occupancy* and integral of robot
 336 *angular velocity*. The occupancy oc of a subsegment is computed from a
 337 laser rotating on top of the robot. Laser values in front of the robot (with
 338 a 30° angle) are averaged and thresholded obtaining a binary value where
 339 0 means that the laser detects no obstacles in the next subsegment, and 1
 340 that it detects some obstacle. The *occupancy model* provides the probability
 341 of occupancy given segment difficulties, namely $p(oc | f)$. The integral of

342 robot *angular velocity* av is computed from the output signals (i.e., direction
 343 angle) of the low-level engine controller. In particular, the controller has 5
 344 outputs meaning to go straight or to turn right/left with angular velocity
 345 of 45° or 90° deg/sec. In each subsegment the controller provides several
 346 low-level actions to the robot. We count the actions corresponding to turn
 347 right/left and threshold this count obtaining a binary signal av which is 0 if
 348 the robot performs few curves in the subsegment and 1 if it performs several
 349 curves. This method to gather information about segment difficulty based on
 350 angular velocity works because the low-level engine controller that drives the
 351 robot inside subsegments is completely independent from the POMCP-based
 352 planner that regulates the speed. The *angular velocity model* provides the
 353 probability of angular velocity given segment difficulties, namely $p(av | f)$.
 354 Notice that the discretization of occupancy and angular velocity used in our
 355 tests is coarse-grained because this degree of precision is enough for our tests
 356 but more precise discretizations can be used.

357 The final observation is a coding of both variables oc and av computed
 358 as $o = av + 2 \cdot oc$. Namely $o = 0$ if $av = 0$ and $oc = 0$, $o = 1$ if $av = 1$ and
 359 $oc = 0$, $o = 2$ if $av = 0$ and $oc = 1$, and $o = 3$ if $av = 1$ and $oc = 1$. The
 360 *observation model* provides the probability of observations given segment
 361 difficulties, namely $p(o | f)$. The parameters of the occupancy and angular
 362 velocity models for the rectangular and the ICE paths are displayed, respec-
 363 tively, in Tables 1.a,b (rectangular path) and Tables 2.a,b (ICE path). The
 364 methodology used to derive such parameters from real-world environments
 365 is described in Section 7.4.

366 The state transition model deals with the update of robot position and
 367 current time at each step. Position update is performed in a deterministic
 368 way since at each step the robot is assumed to reach the beginning of the next
 369 subsegment in the path. The current time is instead updated depending on
 370 both the action performed by the agent and (possibly) the collision penalty.
 371 The time spent to traverse a subsegment depends on the action performed
 372 by the robot, which is discretized as $t = \ell$ (where ℓ is the subsegment length)
 373 if action is H (namely the robot spends ℓ time units if the action is *high*
 374 *speed*), $t = 2\ell$ if action is M , and $t = 3\ell$ if action is L . The time penalty
 375 due to collision has been set to 40 time units in the rectangular path and
 376 20 time units in the ICE path. The collision probability is governed by the
 377 *collision model* $p(c | f, a)$. Collision models of rectangular and ICE paths are
 378 displayed, respectively, in Table 1.c and Table 2.c., where $c = 0$ means no
 379 collision and $c = 1$ that a collision occurs. Notice, that the probability of not

380 making a collision is one minus the probability of making a collision, since
 381 the collision value is binary. The reward function here used is $R = -(t_1 + t_2)$,
 382 where t_1 is the time depending on agent’s action and t_2 is the penalty due to
 383 collisions. Finally the discount factor we used is $\gamma = 1$ because the paths have
 384 limited length and this allowed us to consider it the same way the time spent
 385 to move in different sections of the path. However, the proposed approach is
 386 general to problems with infinite horizon, that can be tackled using $\gamma < 1$.

Table 1: Probabilistic model for the rectangular path. (a) Occupancy model $p(oc | f)$. (b) Angular velocity model $p(av | f)$. (c) Collision model $p(c | f, a)$.

f	$p(oc = 1 f)$	f	a	$p(c = 1 f, a)$
L	0.600	L	L	0.000
M	0.690	L	M	0.033
H	0.940	L	H	0.033
(a)		M	L	0.000
f	$p(av = 1 f)$	M	M	0.033
L	0.170	M	H	0.067
M	0.240	H	L	0.000
H	0.530	H	M	0.067
(b)		H	H	0.100
		(c)		

387 5. POMCP-based planners

388 Three planning strategies are used in our tests. The original implemen-
 389 tation of POMCP, named *STD* in the following, is used as a baseline. An
 390 extended version of POMCP allowing the definition of state-variable con-
 391 straints by Markov Random Fields (Castellini et al., 2019), named *MRF* in
 392 the following, is used to introduce prior knowledge about segment difficulty
 393 relationships. For example, in an instance of our problem we could know that
 394 the probability that segment s_0 and segment s_1 have same difficulty is 0.9.
 395 Planner *MRF* can use this information to improve the policy it generates
 396 and, consequently, the planning performance. Finally, we consider an oracle
 397 planner, named *ORC* in the following, in which perfect knowledge of seg-
 398 ment difficulties is used. This planner performs the POMCP strategy using

Table 2: Probabilistic model for the ICE path. (a) Occupancy model $p(oc | f)$. (b) Angular velocity model $p(av | f)$. (c) Collision model $p(c | f, a)$.

f	$p(oc = 1 f)$	f	a	$p(c = 1 f, a)$
L	0.65	L	L	0.0
M	0.83	L	M	0.0
H	0.93	L	H	0.0
(a)		M	L	0.0
f	$p(av = 1 f)$	M	M	0.056
L	0.083	M	H	0.14
M	0.3	H	L	0.028
H	0.3	H	M	0.11
(b)		H	H	0.25
		(c)		

399 only a unique particle corresponding to the true state (i.e., configuration of
400 segment difficulties) hence it has exact and complete prior knowledge. In the
401 following the standard POMCP algorithm (Silver and Veness, 2010) and its
402 extension based on MRF (Castellini et al., 2019) are briefly described.

403 5.1. Standard POMCP

404 *Partially Observable Monte Carlo Planning (POMCP)* (Silver and Veness,
405 2010) is an online Monte-Carlo based algorithm for solving POMDPs. It uses
406 *Monte-Carlo Tree Search* (MCTS) for selecting optimal actions at each time-
407 step. The main elements of POMCP are a *particle filter*, which represents
408 the belief state, and the *Upper Confidence Bound for Trees* (UCT) (Kocsis
409 and Szepesvári, 2006) search strategy, that allows to select actions from the
410 Monte Carlo tree. The particle filter contains, at each time-step, a sampling
411 of the agent’s belief at that step (the belief evolves over time). In particular,
412 it contains k particles, each representing a specific state. At the beginning
413 the particle filter is usually initialized following a uniform random distribu-
414 tion over states, if no prior knowledge is available about the initial state.
415 Then, at each time-step the Monte Carlo tree is generated performing $nSim$
416 simulations from the current belief. In other words, for $nSim$ times a parti-
417 cle is randomly chosen from the particle filter and the related state is used
418 as initial state to perform a simulation. Each simulation is a sequence of

419 action-observation pairs that collect, altogether, a final return, where each
420 action and observation brings to a new node in the tree. Rewards are then
421 propagated upwards in the tree obtaining, for each action of the root node,
422 an expected (approximated) value of the cumulative reward that this action
423 can bring. The UCT strategy is then used to select actions considering both
424 their expected cumulative reward and the necessity to explore new actions
425 from time to time. The belief is finally updated, after performing the selected
426 action a and getting a related observation o , by considering only the parti-
427 cles (i.e., states) in the new node. New particles can be generated through a
428 *particle reinvigoration* procedure based on local transformation of available
429 states, if the particle filter gets empty. A big advantage of POMCP is that
430 it does not require a complete matrix-based definition of transition model,
431 observation model and reward, but it only needs a black-box simulator of the
432 environment.

433 5.2. *Extended POMCP*

434 The methodology we use to introduce prior knowledge in POMCP (Castellini
435 et al., 2019) allows to define probabilistic relationships of equality between
436 pairs of state-variables by means of Markov Random Fields (MRF). State
437 variables in our application domain are segment difficulties and a relation-
438 ship says that two segments have a certain relative *compatibility* to have the
439 same difficulty. The MRF approach then allows to factorize the joint prob-
440 ability function of state-variable configurations and this probability is used
441 to constrain the state space. In our application domain the state space is
442 the space of all possible segment difficulty configurations and the constraints
443 introduced by the MRF allow to (probabilistically) reduce the chance to
444 explore states that have small probability to be the true state. The inte-
445 gration of MRF-based prior knowledge into POMCP is mainly performed
446 in the particle filter initialization and in the reinvigoration phase (Castellini
447 et al., 2019), where the constraints are used to optimize the management
448 of the particle filter representing the agent belief. In this work the MRF is
449 manually generated using expert knowledge about the application domain.

450

451 5.3. *Complexity analysis*

452 The complexity of the extended POMCP is the same as that of the stan-
453 dard POMCP (described in Section 5.1). Being Monte-Carlo methods, they
454 have a sample complexity determined only by the underlying difficulty of

455 the POMDP, rather than the size of the state space or observation space
456 (Silver and Veness, 2010). This sample complexity, namely the complexity
457 to perform a Monte-Carlo simulation in the known POMDP environment, is
458 multiplied by the number of simulations, which is a constant parameter called
459 $nSim$ in this paper. At the beginning of each subsegment of the path our
460 approach performs $nSim$ simulations (in the POMCP context) to select the
461 velocity, and it runs the standard engine controller to perform path planning
462 in the next subsegment. The increase in complexity introduced by our ap-
463 proach compared to using a standard engine controller is therefore a constant,
464 namely the time to perform $nSim$ simulations, for each subsegment. Since
465 POMCP is an anytime algorithm, the time allotted for its execution can be
466 also defined in advance to satisfy the requirements of specific applications.

467 6. Measures for policy explanation

468 To quantify the influence of prior knowledge on policy performance we
469 introduce three measures, namely, the *belief-state distance*, the *mutual in-*
470 *formation between difficulty and action* and the *expected time to traverse the*
471 *segment* (Castellini et al., 2020c). They strongly contribute to explain the
472 mechanisms that affect the performance improvement and to improve the
473 interpretability of results, as shown in Section 8. Moreover they represent
474 a fundamental tool for explaining and improving planner performance since
475 they allow to precisely identify undesired behaviours of the planner.

476 6.1. Belief-state distance

We define the belief-state distance at a certain instant as the weighted averaged Manhattan distance between the configuration of segment difficulties in the true (hidden) state and the configurations of segment difficulties in the belief at that instant. Mathematically, if we define the configuration of segment difficulties in the true state as $\mathbf{f} = (f_1, \dots, f_m)$, where m is the number of segments, and we define the set of k possible configurations of segment difficulties in the belief as $\{(f_1^i, \dots, f_m^i), i \in 1, \dots, k\}$, where the probability of each difficulty configuration (f_1^i, \dots, f_m^i) in the belief is p_B^i (computed step-by-step by POMCP), then the belief-state distance is

$$d_{SB} = \sum_{i=1}^k (p_B^i \cdot \sum_{j=1}^m |f_j - f_j^i|). \quad (1)$$

477 since the belief is updated at each time-step, this measure can be computed
 478 at each time-step as well. It allows to quantify the discrepancy between
 479 what the agent believes about the real state of the environment and the real
 480 state itself, hence the addition of prior knowledge about segment difficulty
 481 relationships is expected to decrease this distance.

482 6.2. Mutual information (MI) between segment difficulty and action

483 In our problem the agent is expected to take actions that minimize both
 484 the time to reach the end of the path and the risk of collision. The quality
 485 of actions strongly depends on the degree of knowledge the agent has about
 486 the true configuration of segment difficulties. In fact, analyzing the collision
 487 model in Table 2.c, for instance, we observe that high speed (i.e., $a = H$)
 488 should be selected in segments with low difficulty (i.e., $f = L$) because the
 489 collision probability is always 0.0 in those segments, hence high speed should
 490 be preferred to reach earlier the end of the path. On the other hand, in
 491 segments with high difficulty (i.e., $f = H$) the collision probability is low
 492 (i.e., 0.028) if low speed (i.e., $a = L$) is kept, while the probability grows to
 493 0.11 and 0.25, respectively, if medium or high speed (i.e., $a = M$ or $a = H$)
 494 is kept. For this reason low speed should be preferred in these cases.

To check if the POMCP policy effectively generates actions related to
 segment difficulties we compute the mutual information between all actions
 taken in a run and the corresponding segment difficulties. In other words,
 given a run we consider the sequence of actions $\mathcal{A} = (a_{i,j})$, where i is the
 index of a segment and j is the index of a subsegment, and the sequence of
 related subsegment difficulties $\mathcal{F} = (f_{i,j})$. The mutual information (Bishop,
 2006) between the two sequences, treated as random variables, is

$$I(\mathcal{A}, \mathcal{F}) = \sum_{a \in \mathcal{A}} \sum_{f \in \mathcal{F}} p_{(\mathcal{A}, \mathcal{F})}(a, f) \log \left(\frac{p_{(\mathcal{A}, \mathcal{F})}(a, f)}{p_{\mathcal{A}}(a) p_{\mathcal{F}}(f)} \right), \quad (2)$$

495 where $p_{(\mathcal{A}, \mathcal{F})}(a, f)$ is the joint probability mass function of \mathcal{A} and \mathcal{F} , and $p_{\mathcal{A}}$
 496 and $p_{\mathcal{F}}$ are the marginal probability mass functions of \mathcal{A} and \mathcal{F} , respectively.
 497 Average MI values are computed on sets of runs. We notice that selecting ac-
 498 tions with high difficulty-action MI is not trivial since the true configuration
 499 of segment difficulties is hidden. In Section 8 we experimentally analyze the
 500 trend of this measure depending on the prior knowledge provided in different
 501 planners.

502 *6.3. Expected time to traverse a subsegment*

The expected time to traverse a subsegment of unitary length and having difficulty $f \in F$ by performing action $a \in A$ is

$$\mathbb{E}[t]_{f,a} = t_a + p(c \mid f, a) \cdot w(c), \quad (3)$$

503 where t_a is the time due to the action (i.e., L, M or H) without considering any
504 collision (see time t_1 in Section 4), $p(c \mid f, a)$ is the collision probability given
505 action a and difficulty f , and $w(c)$ is the collision penalty, which depends on
506 the collision value.

507 This measure is very useful to analyze and compare planner performance
508 on different configurations of segment difficulties (see Section 8.1). In par-
509 ticular, in those experiments we want to compare the average reward (i.e.,
510 the opposite of time spent to traverse the path) of two planners where the
511 average of each planner is computed on a set of runs having different diffi-
512 culty configurations. The problem in this case is that the expected time to
513 traverse the overall path is affected by strong randomness. One source of
514 this randomness is the chance to get a collision penalty in each subsegment.
515 Namely, two runs performed on the same configuration of difficulties, using
516 the same actions, could get very different reward if a different number of
517 collisions occurs, because the penalty of a single collision is much larger than
518 the time needed to traverse a segment without collisions. The only way to
519 get a statistically significant average performance in this case is to run a huge
520 large number of tests, which is computationally infeasible.

521 The expected time to traverse a subsegment $\mathbb{E}[t]_{f,a}$ removes the random-
522 ness due to collisions because, for each subsegment, it considers the average
523 time (over infinite tests) to traverse it, which is the sum of the time due
524 to the selected action (without considering collisions) plus the fraction of
525 penalty related to performing the action a in segment with difficulty f (i.e.,
526 $p(c \mid f, a) \cdot w(c)$). Notice that the randomness is removed by using the
527 knowledge about the true segment difficulty, which is instead unknown by
528 the planner.

529 **7. Experimental setup**

530 We describe here the experimental setup for our work. We first provide an
531 overview of the complete architecture and then describe in detail all modules.

532 7.1. Overall architecture

533 Our experimental setup mainly involves the elements shown in Figure 2.
 534 The POMCP planner, explained in Section 5, is a C++ module that computes
 535 step-by-step the speed the robot must keep in each subsegment. This
 536 speed, which depends on the estimated difficulty configuration, is commu-
 537 nicated to a low level *engine controller*, namely, the low-level engine controller
 538 that deals with path planning and obstacle avoidance in each subsegment. In
 539 particular, it allows the robot to reach the other side of the subsegment while
 540 keeping the linear velocity defined by the POMCP planner. The communica-
 541 tion between planner and engine controller is managed by a communication
 542 layer described in Section 7.2.

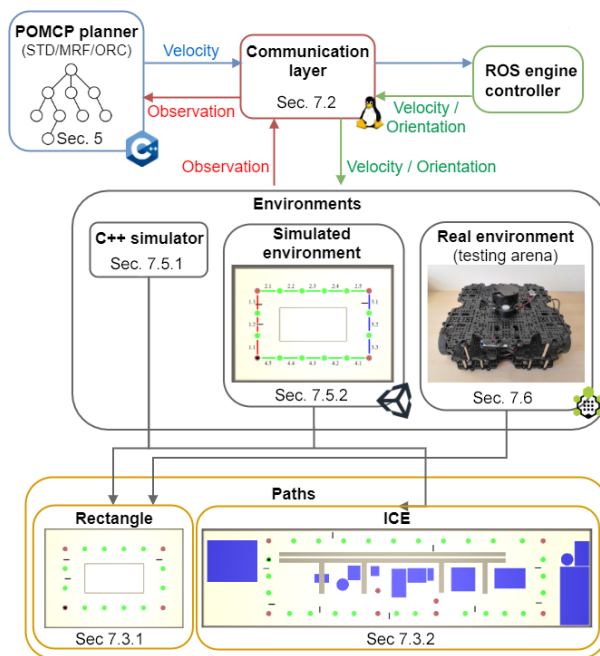


Figure 2: System overview: POMCP planner communicates actions to the Turtlebot in a simulated (Unity) or real (testing arena) environment. The environment returns observations and rewards to the planner which updates (online) its belief and policy.

543 The approach is then tested on two kinds of simulator and in a real-world
 544 testing arena. The first simulator is a C++ module (see Section 7.5.1) which
 545 does not consider physical properties of the environment, the second simu-
 546 lator is designed with Unity (see Section 7.5.2) which relies on the Havok

547 physics engine⁷ to realize realistic simulations and allows to use low-level
548 engine controllers. The real-world testing arena (see Section 7.6) is a real
549 environment wherein a Turtlebot3 was used as an agent to test the planning
550 approach. As shown in the bottom of Figure 2, experiments are performed
551 on two paths, described respectively in Section 7.3.1 and 7.3.2, namely, a
552 rectangular path and a path defined in the ICE industrial research labora-
553 tory. Finally, we highlight that the transition and observation models of the
554 POMDPs are trained on the Unity simulator following an approach described
555 in Section 7.4.

556 7.2. Communication layer

557 The communication between the POMCP planner and the engine con-
558 troller was implemented by inter-process communication via Unix named
559 pipes. This is a common feature for both the Unity simulations and the
560 experiments on the testing arena. Furthermore, Unity natively supports the
561 communication between the low-level controller model (i.e., the engine con-
562 troller) and the Unity environments. Communication among the controller
563 and the robot is instead managed by a ROS node in the real scenario (i.e.,
564 the testing arena).

565 7.3. Paths

566 The two paths on which we developed our architecture are described in
567 the following.

568 7.3.1. Synthetic rectangular path

569 The first path is a rectangle with short sides of 3 meters and long sides
570 of 5 meters. We assume the robot performs two laps of the path, hence the
571 total number of segments in this path is 8, the total number of subsegments
572 is 32 and the total length is 32 meters. Figure 3.a shows the map of the path
573 with also measures of passage widths and an example of obstacle disposal.
574 Subsegments are all 1 meter long. Obstacles are objects with width of 2 cen-
575 timeters and a length of 50 centimeters, that are randomly placed along the
576 robot path. Their density depends on segment difficulty. This path was first
577 implemented in Unity and then in a real world environment, considering the
578 obstacle configuration and difficulties presented in Section 8.2.1.

⁷<https://www.havok.com/products/havok-physics/>

579 *7.3.2. Industrial computer engineering (ICE) lab*

580 The second path is located in the industrial computer engineering lab-
 581 oratory of the Verona University. As shown in Figure 3.b, the entire room
 582 is 4.55 meters large and 18.45 meters long. On the left hand wall there is
 583 a vertical warehouse from which raw materials can be taken and finished
 584 products stocked. Then a production line spreads over the whole room, from
 585 left to right, with a conveyor belt that transports items across the processing
 586 stations. The currently available stations are (from left to right) a milling
 587 machine, a 3D printer, an assembly/disassembly station, and a quality con-
 588 trol station. The path contains 8 segments, the total number of subsegments
 589 is 36 and the total length 36.3 meters. The robot performs one lap starting
 590 and ending at the top-left corner. Subsegment lengths range from 0.6m to
 591 1.4m. In Figure 3.b a possible arrangement of obstacles is shown.

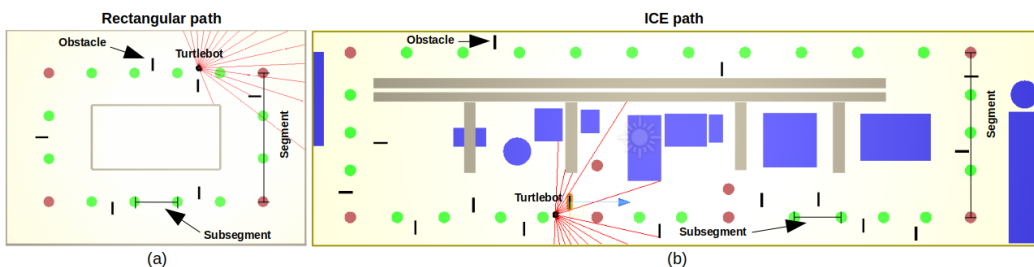


Figure 3: Unity environments. (a) Rectangular path. (b) ICE path. Red circles represent segment extremes, green circles represent subsegment extremes, small red lines represent obstacles, the black circle represents the Turtlebot, red lines are laser scans.

592 *7.4. POMDP model generation*

593 A data collection phase is initially performed on both the Unity environ-
 594 nments, to compute the POMDP models described in Section 4. In particular,
 595 for each environment and action (i.e., velocity) we perform a complete run
 596 of the path keeping the action fixed in all subsegments. The low speed (i.e.,
 597 $a = L$) corresponds to 0.07 m/s, medium speed (i.e., $a = M$) to 0.14 m/s
 598 and high speed (i.e., $a = H$) to 0.21 m/s. For each subsegment we collect:
 599 a binary value for marking collisions occurrence, five values with counts of
 600 actions (i.e., go straight, turn left/right with angular velocity of 45° or 90°
 601 deg/sec) taken by the engine controller along the subsegment, and five values
 602 for the frontal laser scans of the robot normalized in range (0, 1), where 1

603 corresponds to the presence of obstacles at a distance of 3.5 m and 0 cor-
604 responds to the presence of obstacles at a distance of 0 m . Values are then
605 grouped by segment difficulty and action and probabilities are computed to
606 make the POMCP collision and observation models. For instance, the colli-
607 sion probability for each difficulty-action pair is computed from the binary
608 value accounting for collisions. Similar procedures are applied for occupancy
609 and angular velocity models, using, respectively, the counts of laser scan and
610 those of the controller actions.

611 We notice that POMCP assumes a full knowledge of the environment
612 parameters. Since we have estimated these parameters from simulations
613 an error in these parameters could bring a performance decrease as in all
614 model-based planning methods. However, our tests showed that in prac-
615 tice also approximated model parameters can yield performance improve-
616 ment. Bayesian reinforcement learning approaches, such as Bayesian Adap-
617 tive POMCP (Katt et al., 2017) have been recently introduced but their
618 complexity is currently too high to be applied to real robotic planning tasks,
619 hence we will consider them in future work.

620 7.5. Simulators

621 The two simulators used to test our approach are described in the follow-
622 ing.

623 7.5.1. C++ simulations

624 The C++ simulator was developed by following the guidelines provided
625 by the standard POMCP package⁸. Interfaces were implemented together
626 with transition and observation models, and other specific features of the
627 paths, such as number of segments, number of subsegments, subsegment
628 lengths, and so on. The POMCP code was also extended to save to file,
629 step-by-step, details of the simulation progress used to compute statistics
630 and to plot charts shown in the result section.

631 7.5.2. Unity simulations

632 Our simulated Unity environment, displayed in Figure 3, is realized using
633 primitive 3D objects of the Unity engine (e.g., cubes and cylinders) for walls
634 and the occupancy of the processing stations of the ICE environment. For our

⁸<http://www0.cs.ucl.ac.uk/staff/d.silver/web/Applications.html>

635 robotic agent we use the manufacturer model of the Turtlebot3, simulating
636 the laser sensor using Raycasts and the motors using Hinge joints. In detail,
637 the former is used to create rays that detect collisions and the latter connects
638 the 3D model of the wheels with the main robot components, simulating the
639 rotation of the Turtlebot3 motors (previous work (Marchesini and Farinelli,
640 2020a) demonstrates that this is a realistic simulation of the behavior of the
641 robot). The position, required by our controller, is returned by Unity as
642 polar coordinates with respect to the modeled environment. Finally, default
643 physics parameters are considered to simulate gravity and frictions.

644 *7.6. Testing arena*

645 We reproduced the rectangular path in Figure 3.a, considering the same
646 setup presented in Section 7.3.1, to provide an explanatory evaluation of our
647 POMCP planners (i.e., ORC, STD, MRF) in a real scenario. Specifically,
648 the robot travels the path (a rectangle $3 \times 5 m$) two times, hence the path
649 has 8 segments, 32 subsegments and its length is of 32 m . Figure 7 shows an
650 overview of our scenario, where obstacles are wood panels $50 \times 25 \times 1.5 cm$,
651 subsegments are represented with green marks and segments with red marks.
652 The obstacle configuration and difficulties are the same presented in Section
653 8.2.1. The attached video provides full details about experiments performed
654 in this real environment.

655 **8. Results**

656 We perform four kinds of test to evaluate our approach. First, we compare
657 the average performance of the three planners on several difficulty configura-
658 tions (Section 8.1). This test is performed on the C++ simulator described in
659 Section 7.5.1 which enables massive testing since it does not consider any in-
660 teraction with the physical (real or simulated) environment. Then, in Section
661 8.2 we select a specific configuration of difficulties and show how the three
662 planners perform on the Unity simulator described in Section 7.5.2. The aim
663 of this test is to show that our planner can be used to control the robot
664 in a physical (although simulated) environment. Moreover, this experiment
665 shows how the three planners make decisions, in a specific run, according
666 to their knowledge of the environment, and how this knowledge influences
667 the decisions. In Section 8.3, we provide results achieved in a real testing
668 arena where a Turtlebot is used as an agent. This experiment shows that our
669 architecture (i.e., planner, robot, communication layer, localization module,

670 etc.) is able to work also in real world environments, with results very similar
 671 to those achieved in Unity simulations. Finally, in Section 8.4 we perform a
 672 comparative analysis of the proposed method against state-of-the-art engine
 673 controllers, showing that our method outperforms these controllers in terms
 674 of time spent by the robot to traverse the path. The time needed to per-
 675 form a single run increases from the C++ simulator, to the Unity simulator,
 676 and from the Unity simulator to the real environment, therefore we perform
 677 statistical analyses of performance (that need several runs) using the C++
 678 simulator, then we introduce the physical environment by Unity and make
 679 tests on a subset of runs, and finally display technical details of functioning
 680 in the real-world for a small number of (time consuming) tests.

681 8.1. Statistical analysis of planner performance

682 In this experiment we run each planner (i.e., ORC, STD and MRF) on 100
 683 different configurations of segment difficulty. For each run $n \in \{1, \dots, 100\}$
 684 we collect five parameters, namely, the discounted return $r_n \in \mathbb{R}$ of the run,
 685 the number of collisions $c_n \in \mathbb{N}$ of the run, the average action $\bar{a}_n \in \mathbb{R}^+$ of
 686 the run (where the average is computed over the actions taken in the run),
 687 the final belief-real state distance $d_{SB}^n \in \mathbb{R}^+$ of the run, and the normalized
 688 MI between difficulty and action $I(\mathcal{A}, \mathcal{F})_n \in \mathbb{R}^+$ in the run. Moreover, for
 689 each segment s_i of run n , we collect the discounted return $r_{ni} \in \mathbb{R}$ in the
 690 segment, the average action $\bar{a}_{ni} \in \mathbb{R}^+$ in the segment, the average belief-real
 691 state distance $d_{SB}^{ni} \in \mathbb{R}^+$ in the segment, and the expected time $\mathbb{E}[t]_{ni} \in \mathbb{R}^+$
 692 to traverse the segment (see definition in Section 6.3). These parameters
 693 are then averaged over runs and compared between different planners to
 694 generalize on their performance.

695 Figure 4 summarizes the results of the overall analysis performed on both
 696 the rectangular path (sub-figures *a*, *b*, and *c*, analyzed in detail in Subsection
 697 8.1.1) and the ICE path (sub-figures *d*, *e*, and *f*, analyzed in detail in Subsec-
 698 tion 8.1.2). To have a fair comparison on how the different planners exploit
 699 the knowledge on the environment we compare their performance by fixing
 700 a difficulty configuration and a series of observations and running the three
 701 planners with this input. We repeat this process for 100 times. In this way
 702 we remove two sources of randomness, namely that coming from difficulty
 703 configuration and that coming from observations, and keep only two sources
 704 of randomness that cannot be removed, namely, that coming from policy
 705 generation (which is intrinsically related to the POMCP strategy) and that

706 related to collision events (which depends on the actions selected by each
 707 planner).

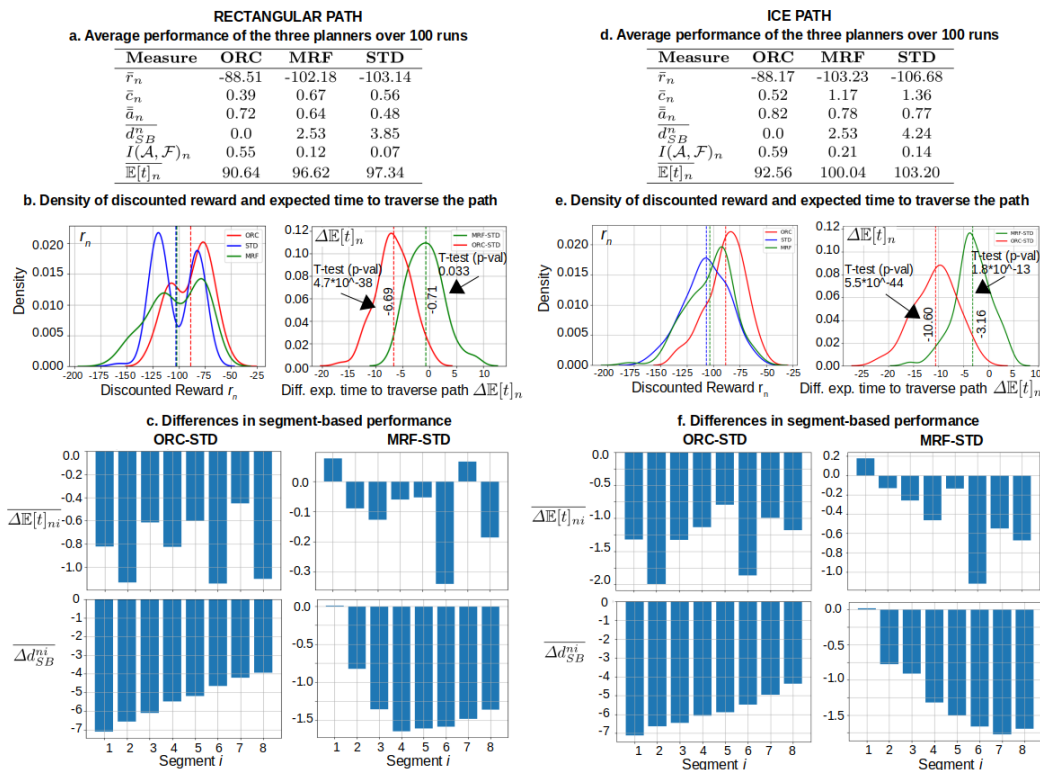


Figure 4: Average performance and properties of runs executed on the rectangular path (left) and the ICE path (right).

708 Furthermore, we provide run-based performance differences between *i*)
 709 ORC and STD, *ii*) MRF and STD. These differences are computed on a run
 710 basis and subsequently averaged. For instance, the difference of discounted
 711 return r_n in run n is computed as $\Delta r_n = r_n - r_n$. This value is then averaged
 712 over runs to obtain the *average difference of discounted return* $\overline{\Delta r_n}$. The
 713 same approach is used to compute the average difference of average action
 714 $\overline{\Delta \bar{a}_n}$, the average difference of final belief-state distance $\overline{\Delta d_{SB}^n}$, the difference
 715 of normalized MI between difficulty and action $\overline{I(\mathcal{A}, \mathcal{F})_n}$, and the average
 716 difference of expected time to traverse the path $\overline{\Delta \mathbb{E}[t]_n}$. The last measure is
 717 particularly stable because it removes also the randomness due to collisions,
 718 since it considers, for each subsegment, only the fraction of time penalty due
 719 to the collision probability instead of real collision events. All measures de-

720 scribed above are computed for both the entire path and single segments. In
 721 the second case we add subscript i to identify segment s_i obtaining, respec-
 722 tively, symbols $\overline{\Delta r_{ni}}$, $\overline{\Delta \bar{a}_{ni}}$, $\overline{\Delta d_{SB}^{ni}}$, and $\overline{\Delta \mathbb{E}[t]_{ni}}$. These measures are analyzed
 723 in the following to compare planner performance. In both cases we used a
 724 number of simulations per step $nSim = 10^{15}$.

725 8.1.1. Discussion of results on rectangular path

726 Average performance of the three planners over 100 runs in the rectan-
 727 gular path are displayed in Figure 4.a. ORC has the best average return
 728 \bar{r}_n (i.e., -88.51), followed by MRF (i.e., -102.18) and then by STD (i.e.,
 729 -103.14). These differences are generated by different decision strategies
 730 that are affected by the level of prior knowledge provided to the planners,
 731 which has direct effect on i) the average distance between real state and
 732 belief $\overline{\Delta d_{SB}^n}$, whose values at the end of the path are 0.0 for ORC, 2.53 for
 733 MRF and 3.85 for STD; ii) the MI between (hidden) segment difficulty and
 734 action $I(\mathcal{A}, \mathcal{F})_n$ (i.e., 0.55 for ORC, 0.12 for MRF and 0.07 for STD), which
 735 is higher for planners able to understand earlier the difficulty of the segment
 736 and to act accordingly. Finally, the average expected time to traverse the
 737 path $\overline{\mathbb{E}[t]_n}$ confirms the ranking of planners (respectively ORC, MRF, and
 738 STD) with a larger difference between ORC and MRF, and a smaller one
 739 between MRF and STD. Figure 4.b shows, on the left, the density of dis-
 740 counted return r_n for the three planners. The multi-modality of these curves
 741 are related to runs without and with collisions. As expected, these densities
 742 have large variance due to the strong randomness of the process generating
 743 the stochastic variable r_n .

744 The methodology exploiting run-based performance differences, described
 745 above, allows to remove some sources of randomness and point out the real
 746 statistically significant differences between the performance of the three plan-
 747 ners in terms of average difference of expected time to traverse the path
 748 $\overline{\Delta \mathbb{E}[t]_n}$, which is our main result. The density plot on the right of Figure 4.b
 749 shows two charts, namely, the distribution of *differences* between expected
 750 time to traverse the path for ORC and STD (in red), and that between MRF
 751 and STD (in green). The two charts show that both ORC and MRF out-
 752 perform STD, and their performance improvement is statistically significant.
 753 Focusing on the difference between ORC and STD (red line), the mean is
 754 -6.69 , meaning that ORC needs on average 6.69 time units less than STD
 755 to reach the end of the path (average time to traverse the path is 97.34 for
 756 STD - see Figure 4.a - hence the improvement is of about 6.8%). Performing

757 a *t-test* for the null hypothesis that the expected value (mean) of this distri-
758 bution a is equal to zero we obtain a p-value $p = 4.7 \cdot 10^{-38}$, which proves
759 the statistical significance of the result (since it is less than 0.05). A similar
760 result is achieved by comparing MRF and STD. Their mean difference is
761 lower (i.e., -0.71, meaning that MRF needs on average 0.71 time units less
762 than STD to reach the end of the path, with an improvement of 0.7%) but
763 still statistically significant, with a p-value $p = 0.033$.

764 Further details about differences in planner behaviors on specific path
765 segments are displayed in Figure 4.c. From top to the bottom, we show the
766 average difference of discounted return $\overline{\Delta r_{ni}}$ in each segment s_i (the segment
767 index is in the x-axis in all the charts), the average difference of expected
768 time $\overline{\Delta \mathbb{E}[t]_{ni}}$ to traverse the segment, the average difference of average ac-
769 tion $\overline{\Delta \bar{a}_{ni}}$ in each segment, and the average difference of average belief-state
770 distance $\overline{\Delta d_{SB}^{ni}}$ in each segment. The bar plots on the left show differences
771 between ORC and STD, while those on the right display differences between
772 MRF and STD. Interestingly, the average difference of expected time $\overline{\Delta \mathbb{E}[t]_{ni}}$
773 shows in which segments ORC and MRF outperform STD. In particular, the
774 improvement of MRF with respect to STD in the expected time to traverse
775 a segment increases in the second lap of the rectangle (see higher negative
776 bars mainly in segments 6 and 8), which coincides with a smaller difference
777 in average belief-state distance of MRF than STD (see negative $\overline{\Delta d_{SB}^{ni}}$). In
778 other words, MRF and STD have almost the same behavior in the first seg-
779 ment, then MRF’s belief gets closer to the real state than STD’s belief (due
780 to the use of prior knowledge to infer future segment difficulties) allowing
781 MRF to outperform STD.

782 8.1.2. Discussion of results on ICE path

783 Tests performed on the ICE path, displayed in Figures 4.d, 4.e and 4.f,
784 show even better performance. Focusing on discounted reward r_n (first row
785 of the table in Figure 4.d and density function in Figure 4.e) we see that
786 ORC has the best average performance (i.e., -88.17), followed by MRF (i.e.,
787 -103.23) and then by STD (i.e., -106.68). The average difference of expected
788 time to traverse the path $\overline{\Delta \mathbb{E}[t]_n}$, shown on the right of Figure 4.e, are re-
789 spectively -10.60 between ORC and STD, and -3.16 between MRF and STD.
790 This means that ORC takes on average 10.60 time units (i.e., 10.2%) less
791 than STD to traverse the path, and MRF takes on average 3.16 time unit
792 (i.e., 3.1%) less than STD to traverse the path. In both cases the *t-test* for
793 the null hypothesis that the expected value (mean) of this distribution is

794 equal to zero is very close to zero (namely $p = 5.5 \cdot 10^{-44}$ for ORC-STD and
 795 $p = 1.8 \cdot 10^{-13}$ for MRF-STD), which proves the statistical significance of the
 796 performance difference. This difference can also increase for more complex
 797 and repeated paths.

798 The analysis of performance over single segments, displayed in Figure
 799 4.f, enables to draw similar conclusions to those drawn for the rectangular
 800 path. A large part of performance improvement (in terms of both discounted
 801 reward and expected time to traverse the path) come from segments in the
 802 second part of the path, where MRF has already collected information about
 803 its difficulty from previous segments (see the negative difference of distance
 804 between real state and belief Δd_{SB}^{mi} which shows that MRF has a more precise
 805 belief than STD in the second part of the path).

806 8.2. Performance on Unity simulations

807 Tests on Unity aim at evaluating the performance of our approach on
 808 a simulated environment considering the physical properties of the environ-
 809 ment, which are not considered by the C++ simulator analyzed in the pre-
 810 vious subsection. The experimental setup for these tests is explained in
 811 Section 7.5.2.

812 8.2.1. Discussion of results on the rectangular path

813 We analyze here a single run performed on a Unity simulation of the
 814 rectangular path with difficulty configuration (H, L, M, L, H, L, M, L) . The
 815 path is displayed in Figure 5.a, where segments with low difficulty (L) are
 816 colored in green, segments with medium difficulty (M) are colored in blue,
 817 and segments with high difficulty (H) are colored in red. The agent travels
 818 the rectangle twice and segment difficulties are fixed in the two laps.

819 The ORC planner, in Figure 5.b, performs an optimal strategy (in terms
 820 of expected reward), since it selects action L in subsegments with high or
 821 medium difficulty and action H in subsegments with low difficulty (green
 822 subsegments in Figure 5.b represent action L and red subsegments represent
 823 action H). The time to traverse the path (i.e., the opposite of the discounted
 824 return) is 56 time units, and no collision occurs. Under the rectangular path
 825 we display a heatmap of robot’s belief per subsegment. Columns represent
 826 subsegments, rows represent difficulties, and the color of a general cell i, j
 827 represents the probability (from the current belief) that subsegment j has
 828 difficulty i . For instance, the robot’s belief in all subsegments of the first
 829 segment has high probability (i.e., 1.0) for high difficulty (see the red cells

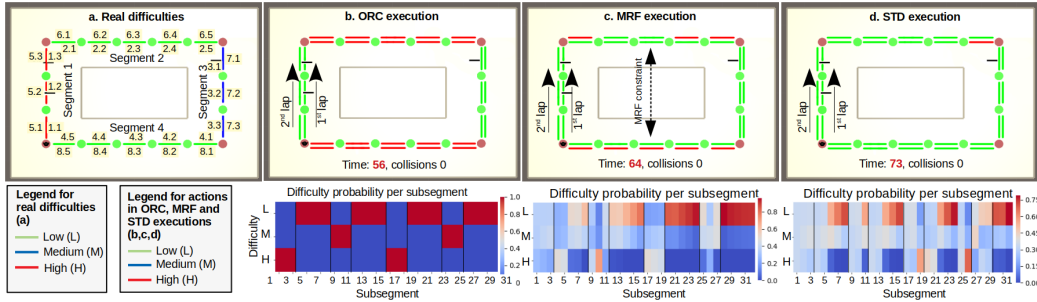


Figure 5: Performance comparison between the three planners on a specific configuration of difficulties in the rectangular path.

830 in the first three columns of the heatmap) and low probability (i.e., 0.0) for
 831 low and medium difficulty.

832 Comparing the plan generated by MRF (see Figure 5.c) with that gener-
 833 erated by STD (see Figure 5.d) we observe that in the first lap they both
 834 select always action *L* (apart from a single action *H* chosen by STD at the
 835 end of the second segment). This is because their confidence to be in a
 836 segment with low difficulty is not high enough to choose action *H* (see the
 837 related heatmaps of belief probabilities), even in segments 2 and 4, where
 838 the difficulty is actually low. As expected, things change in the second lap,
 839 where MRF takes advantage of the knowledge about segment difficulties ac-
 840 quired in the first lap and it transfers this knowledge to subsequent seg-
 841 ments connected by relationships of similarity between difficulties. For in-
 842 stance, the MRF edges (with probability 0.9) that generate a connected com-
 843 ponent containing segments 2, 4 (first lap), and 6, 8 (second lap) clearly im-
 844 prove the belief of the agent in segments 6 and 8. This is proved by the higher
 845 probabilities of low difficulty in segments 6 and 8 of the MRF heatmap (in
 846 the bottom of Figure 5.c) with respect to those of the same segments in the
 847 STD heatmap (in the bottom of Figure 5.d). Consequently, the MRF plan
 848 has action *H* in three subsegments of segment 6 and five subsegments of
 849 segment 8 (see red lines in the path of Figure 5.d), which improve the per-
 850 formance from the 73 time units required by STD to the 64 time units required
 851 by MRF. In both cases no collision occurs. In summary, the analysis shows
 852 that the MRF planner is able to propagate the knowledge about the hidden
 853 state acquired in initial segments to subsequent segments, with a consequent
 854 improvement of performance.

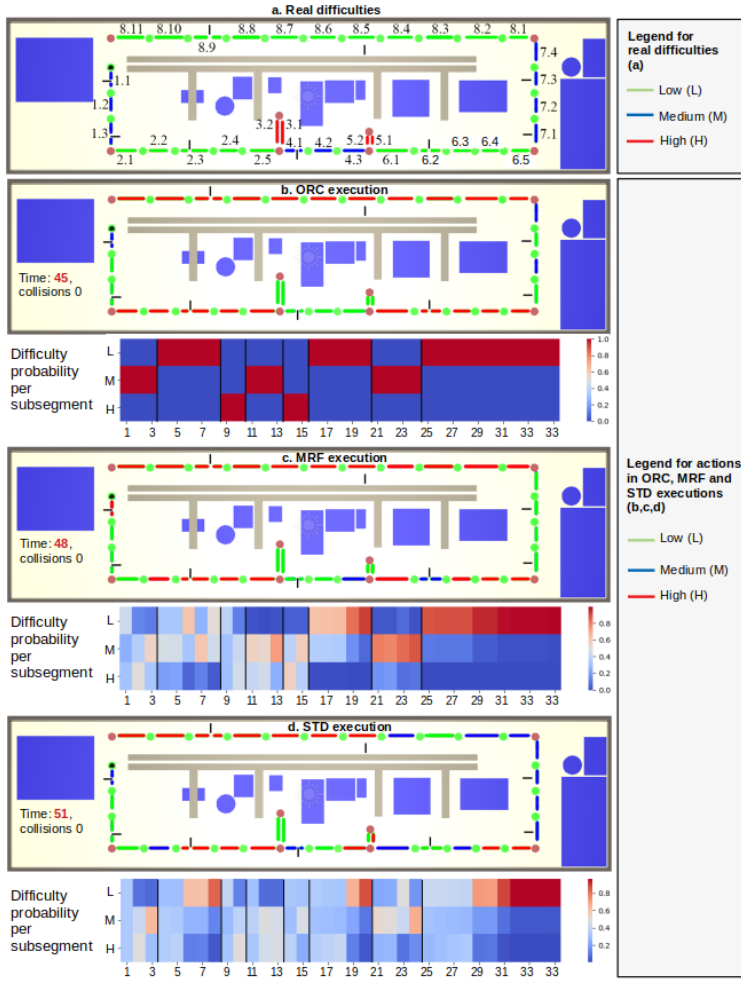


Figure 6: Performance comparison between the three planners on a specific configuration of difficulties in the ICE path.

855 *8.2.2. Discussion of results on the ICE path*

856 The test performed in the ICE path considers the difficulty configuration
 857 (M, L, H, M, H, L, M, L) (see Figure 6.a). The analyzed plans generated by
 858 ORC, MRF and STD take, respectively, 45, 48 and 51 time units, confirming
 859 the planner ranking observed in previous tests on the rectangular path.

860 The ORC planner selects almost always optimal actions (in terms of expected
 861 reward), namely, L in segments with high difficulty, and H in seg-
 862 ments with low difficulty, and L or M in segments with medium difficulty

863 (we separately checked the optimality of this relationship between difficulty
864 and action). More interestingly, the comparison between MRF and STD
865 confirms that MRF is able to get in advance higher confidence on the true
866 configuration of difficulties with respect to STD and to use this knowledge to
867 choose better actions. This capability is mainly shown in the second part of
868 the path (from the fourth segment). For instance, in segment 6, MRF selects
869 the optimal action (i.e., H) in four subsegments out of five while STD selects
870 it in only two subsegments. The analysis of the related heatmaps shows that
871 the reason of this difference is that MRF has a high confidence on the (true)
872 low difficulty of segment 6 from the beginning of the segment itself (see red
873 cells in difficulty L for segment 6 in the MRF heatmap) while STD reaches
874 this confidence level only in the last two subsegments of segment 6 (see red
875 cells in difficulty L for segment 6 in the STD heatmap). Accordingly, STD
876 selects the optimal action only in the last two subsegments, while MRF se-
877 lects them from the beginning. MRF has gathered this knowledge about the
878 difficulty of segment 6 from segment 2, which has the same difficulty and it
879 is connected to segment 6 by an edge in the MRF. A similar behaviour is
880 observed in the (long) segment 8, where again MRF performs the optimal
881 action (i.e., H) in all eleven subsegments while STD performs it only in the
882 last seven subsegments.

883 *8.3. Performance on the real world environment: testing arena*

884 The explanatory run of our three planners in the real environment (testing
885 arena) proves that the movements of the real robot have close correspondence
886 to movements of the agent in the Unity simulation environment of the rect-
887 angular path. Figure 7 shows the starting point of a test performed in the
888 arena. The video of the entire experiment is attached. It shows the Turtlebot
889 moving in the real environment in three cases, namely, using the ORC, the
890 MRF and the STD planner, respectively, with difficulty configuration shown
891 in Figure 6.a. The video displays the evolution of the agent’s belief, the se-
892 quence of actions selected by the planners and the time required to complete
893 the path. Crucially, we were able to obtain similar results to those obtained in
894 the Unity simulation, showing that a porting of our POMCP-based planners
895 from the Unity simulation to the real robot is possible.

896

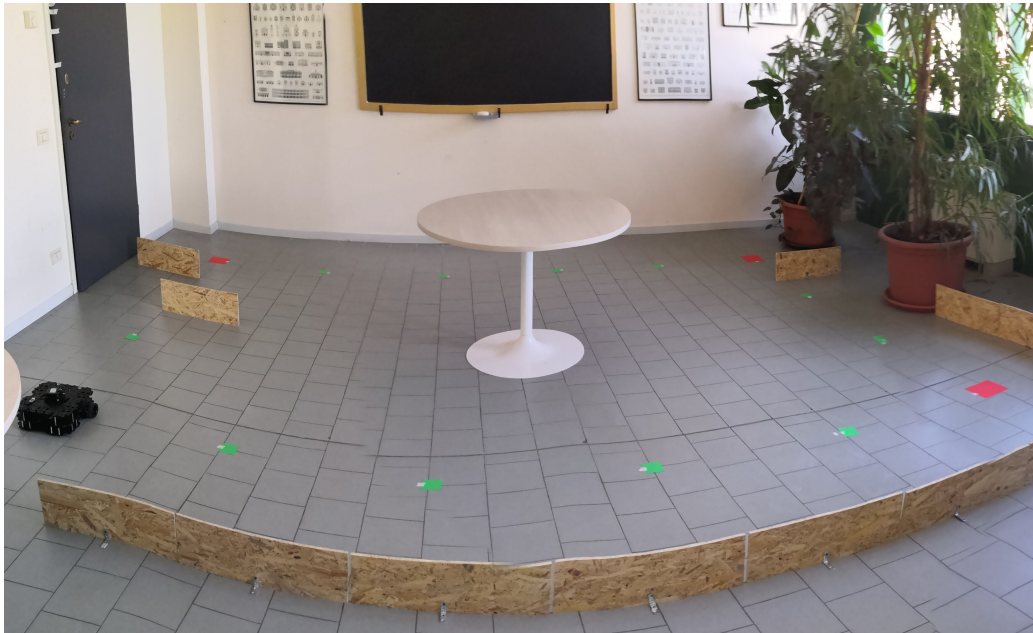


Figure 7: Overview of the testing arena.

897 *8.4. Comparative analysis*

898 To further investigate the proposed approach and highlight its benefits,
899 we compare its performance with that of some state-of-the-art controllers
900 for mobile robots. Our goal is to show that by introducing the POMCP-
901 based planner with MRF on top of the engine controller we actually get a
902 performance improvement over using only the engine controller. We perform
903 this test using two state-of-the-art controllers as a baseline, namely, the DRL
904 engine controller presented in (Marchesini and Farinelli, 2020b) and the ROS
905 navigation stack⁹. For a fair comparison, we separately tuned the parameters
906 for the navigation stack (e.g., for localization and sensing of the environment)
907 in both the rectangular and ICE path. In detail, to balance the trade-off
908 between performance and navigation, the difficulty of each segment was set
909 to medium, and the default settings for the navigation stack provided good
910 performance in both environments.

911 Table 3 shows the results of the comparative analysis. Tests were per-
912 formed on the experimental setting described in Section 8.2, using Unity

⁹<http://wiki.ros.org/navigation>

913 simulators of the rectangular path (left column) and the ICE path (right
 914 column). We call our planner POMCP[MRF]+DRL as it uses the POMCP
 915 and the MRF to regulate velocity and the DRL controller for path planning.
 916 On the rectangular path, our planner requires 64 time units to complete the
 917 path, while the DRL engine controller alone requires 70 time units (about
 918 9% more), showing that the velocity regulation performed by POMCP with
 919 MRF is important to improve navigation performance. On the other hand,
 920 the standard ROS controller manages to complete the path in 63 time units,
 921 one less than our approach. This difference is negligible, hence we consider
 922 the two controllers as having almost the same performance in this environ-
 923 ment. The motivation for this result is that the environment is quite simple
 924 and there is no room to improve performance beyond that of ROS controller.
 925 On the ICE path instead POMCP[MRF]+DRL requires 48 time units to
 926 complete the path, the DRL engine controller alone requires 65.9 time units
 927 (about 37% more) and the ROS controller requires 72.3 time units (about
 928 51% more). In this case our approach strongly outperforms both the DRL
 929 controller and the ROS controller. The reason for this good result is clearly
 930 related to the introduction of POMCP for velocity regulation, since also
 931 the standard planner POMCP[STD]+DRL outperforms both the DRL con-
 932 troller and the ROS controller with 51 time units required to complete the
 933 path. However, the introduction of prior knowledge about segment diffi-
 934 culty relationships via the MRF yields a further performance improvement
 935 of 6%, showing the importance of the MRF in the proposed approach. In
 936 a longer and more articulated environment, as the ICE path, our approach
 937 shows all its capability to outperform standard controllers. We finally no-
 938 tice that the performance improvement, in terms of time units, between
 939 POMCP[STD]+DRL and POMCP[MRF]+DRL in Table 3 is higher than
 940 the performance improvement, in terms of expected time to traverse the path
 941 $\overline{\mathbb{E}[t]_n}$, shown in Figure 4 for C++ simulations. This is because the expected
 942 time to traverse the path $\overline{\mathbb{E}[t]_n}$ considers also the expected time due to possi-
 943 ble collisions (see Equation 3) while in the specific experiments considered in
 944 Table 3 no collision took place. Since the time related to collisions is much
 945 larger than that for traversing path segments the total times in Table 4 are
 946 smaller and the percentage of improvement increases. Furthermore, consid-
 947 ering a single test the results of Table 3 also do not consider the variability
 948 over all possible configurations of segment difficulties.

949

Controller	Time units	
	Rectangle	ICE
POMCP[MRF]+DRL	64	48
POMCP[STD]+DRL	73 (+14%)	51 (+6%)
DRL (Hessel et al., 2018)	70 (+9%)	65.9 (+37%)
ROS ¹⁰	63 (-2%)	72.3 (+51%)

Table 3: Performance comparison between the controller based on POMCP+DRL (proposed in this paper), the controller based only on DRL, and a standard ROS controller.

950 *8.5. Final remarks*

951 The experiments presented in this paper show that the extension of
952 POMCP presented in (Castellini et al., 2019) can be applied to real robotic
953 platforms. We used the extended POMCP for regulating mobile robot velocity
954 in an industrial-like environment characterized by uncertain motion diffi-
955 culties. The velocity selected by the planner was communicated to an engine
956 controller which performs path planning. This two-layer control architecture
957 was tested in four experimental settings always showing good performance.
958 In particular, extensive tests performed on the C++ simulator shown that
959 the extension of POMCP, which integrates prior knowledge on the environ-
960 ment, outperforms the standard POMCP from 0.7% to 3.1% depending on
961 the complexity of the path. Experiments performed on a more realistic Unity
962 simulator shown that also considering physical details of the environment
963 and the (Turtlebot) agent the overall architecture properly works. Then,
964 experiments in a real testing arena further confirmed the applicability of
965 the approach to real robotic platforms. The last test concerned performance
966 comparison between our (two-layer) approach and state-of-the-art (one-layer)
967 approaches using only the engine controller for both velocity regulation and
968 path planning. This test highlighted the contribution of the POMCP-based
969 regulation of the velocity. The performance improvement was up to 37%
970 against a DRL engine controller and up to 51% against a standard ROS
971 engine controller.

972 All tests show that the performance improvement of the extended POMCP
973 over the standard POMCP is due to the ability of the extended POMCP to
974 propagate the knowledge about segment difficulty acquired in the first part
975 of the path to subsequent segments via the MRF. This mechanism allows the
976 robot to have better confidence in the estimation of the difficulties for the
977 segments in the second half of the path. This allows to achieve a significant

978 performance improvement, specifically in that part. The proposed measures
979 show that the belief of the robot in the second half of the path is closer to
980 the ground truth (in terms of belief-state distance) for the extended POMCP
981 than for the standard POMCP. The improvement in the estimation of the
982 ground truth is the cause of the planning performance improvement. We
983 observed an increase of this effect in longer and more articulated paths be-
984 cause they give our approach more chance to exploit the similarity structure
985 among segments. This happens even more if the robot repeats the same path
986 several times.

987 **9. Conclusion and future work**

988 We presented an approach based on POMCP for improving the time per-
989 formance of a mobile robot navigating in a path when prior knowledge is
990 available about the similarity of path segments. Our results demonstrate the
991 possibility to apply this planning approach to a real-world industrial-like en-
992 vironment characterized by uncertain motion difficulties. Moreover, we show
993 that an extended version of POMCP considering prior knowledge about the
994 environment is able to outperform the standard POMCP by improving the
995 belief about the true state of the environment. This effect is obtained by
996 propagating the knowledge on the environment acquired in the initial parts
997 of the navigation to parts of the environment explored subsequently. Fur-
998 thermore, we have shown that the addition of the POMCP-based layer for
999 velocity regulation yields a consistent performance improvement compared
1000 to state-of-the-art engine controllers, such as DRL and ROS standard con-
1001 trollers. The approach can be applied to several domains in which series of
1002 tasks having similar properties are executed sequentially and some knowl-
1003 edge about task similarity is available. Performance measures introduced in
1004 the paper allow also to show that the belief improvement is transformed by
1005 the planner to improved action selection.

1006 This work takes a first important step towards the use of advanced plan-
1007 ning methods for mobile robots in industrial applications. This paves the
1008 way towards several interesting research directions. Specifically, our future
1009 work in this area includes three main topics: *i*) the development of methods
1010 for learning the MRF online, which is useful when the knowledge about task
1011 similarities is not available a-priori but it can be learnt; *ii*) the development
1012 of safe transfer learning methods for POMCP to allow imperfect transition
1013 and observation models trained on simulators (such as the Unity simulator

1014 we used in our experiments) to be used in real environments; *iii*) the devel-
1015 opment of new methods for improving the explainability of POMCP-based
1016 policies to allow their use in environments with safety requirements (and re-
1017 lated human responsibilities). Some example applications regarding the last
1018 point can be found in Industry 4.0 where the recent request for a stronger
1019 human-robot interaction needs reliable and comprehensible policies to guar-
1020 antee the safety. Some first results on this line have been recently published
1021 (Mazzi et al., accepted, 2021, 2020) and current work is focused on applying
1022 those results also to real robotic platform to further extend the work here
1023 presented.

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