Robust optimisation of dry port network design in the container shipping industry under uncertainty

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Declaration

This statement and the accompanying publications have not previously been submitted by the candidate for a degree in this or any other university.

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Abstract

The concept of dry port has attracted the attention of many researchers in the field of containerised transport industry over the past few decades. Previous research on dry port container network design has dealt with decision-making at different levels in an isolated manner. The purpose of this research is to develop a decision-making tool based on mathematical programming models to integrate strategic level decisions with operational level decisions. In this context, the strategic level decision making comprises the number and location of dry ports, the allocation of customers demand, and the provision of arcs between dry ports and customers within the network. On the other hand, the operational level decision making consists of containers flow, the selection of transportation modes, empty container repositioning, and empty containers inventory control. The containers flow decision involves the forward and backward flow of both laden and empty containers. Several mathematical models are developed for the optimal design of dry port networks while integrating all these decisions.

One of the key aspects that has been incorporated in this study is the inherent uncertainty of container demands from end customers. Besides, a dynamic setting has to be adopted to consider the inevitable periodic fluctuation of demands. In order to incorporate the abovementioned decision-making integration with uncertain demands, several models are developed based on two-stage stochastic programming approach. In the developed models, the strategic decisions are made in the first stage while the second-stage deals with operational decisions. The models are then solved through a robust sample average approximation approach, which is improved with the Benders Decomposition method. Moreover, several acceleration algorithms including multi-cut framework, knapsack inequalities, and Pareto-optimal cut scheme are applied to enhance the solution computational time.

The proposed models are applied to a hypothetical case of dry port container network design in North Carolina, USA. Extensive numerical experiments are conducted to validate the dry port network design models. A large number of problem instances are employed in the numerical experiments to certify the capability of models. The quality of generated solutions is examined via a statistical validation procedure. The results reveal that the proposed approach can produce a reliable dry port container network under uncertain environment. Moreover, the experimental results underline the sensitivity of the configuration of the network to the inventory holding costs and the value of coefficients relating to model robustness and solution robustness. In addition, a number of managerial insights are provided that may be widely used in container shipping industry: that the optimal number of dry ports is inversely proportional to the empty container holding costs; that multiple sourcing is preferable when there are high levels of uncertainty; that rail tends to be better for transporting laden containers directly from seaports to customers with road being used for empty container repositioning; service level and fill rate improve when the design targets more robust solutions; and inventory turnover increases with high levels of holding cost; and inventory turnover decreases with increasing robustness.

List of Publications

- **Sarmadi, K.**, Amiri-Aref, M., Dong, J.-X. and Hicks, C. (2020) 'Integrated strategic and operational planning of dry port container networks in a stochastic environment', *Transportation Research Part B: Methodological*, 139, pp. 132-164. (ABS/AJG 4*)

This publication is relating to Chapters 4-6 of this thesis.

- Li, JX., Liu, B., Sun, Q., **Sarmadi, K.**, Gao, Y., Dang, Y., Dong, JX. 'Load shifting of a supplier-based demand response of multi-class subscribers in smart grid', International *Journal of Industrial and Systems Engineering 2019*, Forthcoming. In Press. (ABS/AJG 1*)

- **Sarmadi, K.**, Amiri-Aref, M., Dong, J.-X. (2019), 'Robust Optimisation of Dry Port Location in Container Shipping Industry', *Presented in the Manufacturing and Service Operations Management (MSOM) Conference*. 2019, National University of Singapore (NUS).

This publication is relating to Chapter 4.

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Chapter 1. Introduction

1.1. Background

International transport plays a substantial role for the supply chain globalisation. Global trade is mainly facilitated by seaborn transportation as 75% of cargo by volume and 60% of cargo by value are moved by sea (Lee and Song, 2017). Within the seaborne transport, container shipping contributes to 52% of cargo by value. Containerisation started in the 1960s, and volumes increased moderately in the first three decades and significantly over the last two decades (Lee and Song, 2017). With an annual growth rate of 9.3% from 1990 to 2013, the volume of container transport has grown from 85 million twenty-foot equivalent unit (TEUs) to 651 million TEUs. The containerised transport industry has enabled the globalisation of supply chains by providing cost-effective transport, where the transport cost accounts for only 1% of shelf price of consumer goods (Lee and Song, 2017). Container shipping is a substantial component of globalised trade (Crist, 2003), which makes an important contribution to maintaining and improving the quality of life since the majority of consumer goods including clothing, consumer electronics, appliances, furniture, automobile parts are carried in containers (Rodrigue et al., 2013). Improving the efficiency of the container shipping supply chain can reduce the shipping industry's operational costs and increase its sustainability by reducing energy consumption and greenhouse gas emission. It can also help end customers acquire their ordered products in a more cost-effective and timely way in predetermined due time (Crainic and Kim, 2007).

A typical container shipping supply chain consists of a consignor, a consignee, an ocean carrier, a freight forwarder and a terminal operator (Lee and Song, 2017). Such a supply chain does not necessarily originate or terminate at seaports. In practice, the origin and destination (O/D) points are mostly inland points. Therefore, the transportation of goods cannot be solely carried out by ships. In other words, different transportation modes are needed to transport cargo through both seaborne and inland networks. In the inland network, cargos are mainly transported from an inland origin to a seaport using rail or road. Then the mode of transportation is changed to maritime transport at the seaport. Provided the ultimate destination is an inland point, further transfer in transportation mode will be required at the destination seaport, that is the containers will be delivered through rail or road. Thus, *a multimodal* transportation is an essential need for such a global transport chain. Multimodal transportation is defined as the *"transport of goods by at least* *two different modes of transport (Commission, 2010, p. 157)"*. *"The unit of transportation can be a box, a container, a swap body, a road/rail vehicle, or a vessel. As such, the regular and express delivery system on a regional or national scale, and long-distance pickup and delivery services are also examples of multimodal transportation (SteadieSeifi et al., 2014, p. 2)"*. Multimodal transport can establish a containerised transport network with higher efficiency, reliability and flexibility to deliver goods (SteadieSeifi *et al.*, 2014). Since containers have standard dimensions, they can be easily transported from the origin to the destination using different transportation modes (air, road, sea) without being opened. Hence, containerised transport services are an essential element of intermodal transportation and international trade (Crainic and Kim, 2007). Intermodal transportation is a particular type of multimodal transport (Commission, 2010), which is defined as *"Multimodal transport of goods, in one and the same intermodal transport unit by successive modes of transport without handling of the goods themselves when changing modes (van Riessen, 2013, p. 4)"*.

Even though the standardisation achieved by the containerisation can facilitate the integration of global supply chains, containerised transport chains are fragmented in practice. In comparison to other transport sectors like air transport, the container shipping industry has barely benefited from operations management tools and techniques due to sea transport's special features (Lee and Song, 2017). These special characteristics can differentiate the container shipping industry from the air industry. According to Lee and Song (2017), there are several differences between air transport and sea transport settings. Firstly, the air industry includes passenger networks which enables scholars and practitioners to adopt revenue management techniques broadly compared with the container shipping industry. Secondly, competition in the sea transport industry is generally based on costs due to the fact that service differentiation is less important compared to the air industry. This cost-based competition brings about establishing alliances and integrations in container shipping context. Thirdly, the consolidation and dominance of companies in container shipping industry is quite higher than the air industry due to the air traffic rights. The shipping liners have expanded their operations to include hinterland networks by consolidating with inland transport operators. Finally, slow steaming, which is the practice that a vessel is planned to sail at a speed significantly less than its designed speeds to reduce fuel consumption, may be applied in maritime container shipping operations, while it cannot be adopted in the air transport since there is a lower limit for the speed of aircrafts (Lee and Song, 2017).

Figure 1.1 provides a projection of the global market for shipping containers by value. The worldwide container shipping market has been increased steadily from 2016 to 2020 and this growth is predicted to continue until 2025. More specifically, the global shipping containers market was worth about 4.6 billion U.S. dollars in 2016, and its size is expected to reach 11 billion U.S. dollars by 2025 (Statista, 2020).

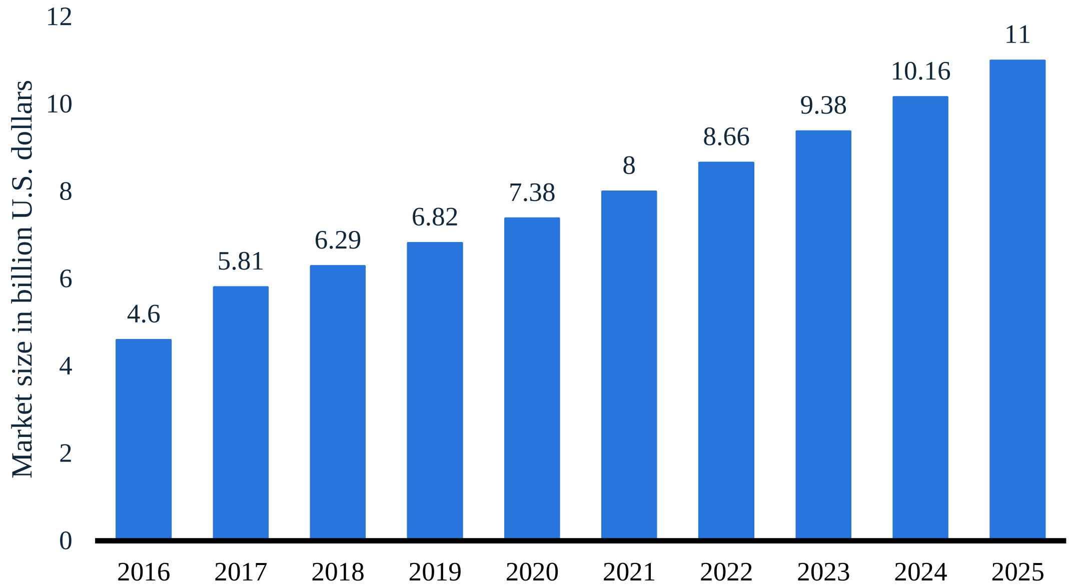


Figure 1.1. Projected size of the global market for shipping containers 2016-2025 (Statista, 2020).

1.1.1. Empty container repositioning

It is clear that the predicted ever-increasing market size shown in Figure 1.1 will lead to increased volume of container movements if it occurs. Accordingly, various problems are likely to be faced by shipping liners and other transport operators in container shipping industry. One of the main challenges in this context is the Empty Container Repositioning (ECR) problem. *"The empty container repositioning problem concerns arranging the storage and movements of empty containers in the shipping networks in order to better position the movable resources to better satisfy customer demands (Song and Dong, 2015a, p. 5)".* Containerised transport networks normally consist of two directions including the forward flow of laden containers and the backward flow of empty containers (Song and Dong, 2012b). The transportation-related operations associated with laden and empty containers are carried out in the same shipping network using the same resources, which indicates that the laden container flow and empty container movements are interconnected and should be addressed jointly (Song and Dong, 2012b). The most noticeable element that causes the empty container repositioning problem within containerised transport chains is the imbalance in global trade. More precisely, in the Trans-Pacific and Europe-Asia trade routes, there is surplus number of empty containers at American and European ports, whereas Asian ports are suffering from lack in empty containers (Song and Dong, 2015a). It is estimated that 20% of ocean container flows is related to the movement of empty containers (Drewry, 2006). It is also reported by Asariotis *et al.* (2010) that the flow of containers from Asia to Europe is at least twice as much as that from Europe to Asia. It implies that at least half of the accumulated containers at western ports should be sent back empty.

The quantity of empty container transported in inland networks is considerable as empty containers are normally stored at seaports or depots which are located far from demand zones (Song and Dong, 2015a). This has been verified by various scholars and research (see Crainic *et al.*, 1993; Konings, 2005; Braekers *et al.*, 2011) . The repositioning of empty containers comprises various costs, including the inland and seaborn transportation, storage and maintenance of containers at depots, handling and transhipment of containers at different network facilities (Song and Dong, 2015a). These cost components can cause huge expenditures due to empty container repositioning. In 2001, the cost incurred due to container management inefficiencies reached to US$17 billion (Boilé and Aboobaker, 2006; Theofanis and Boile, 2009). Furthermore, Veenstra (2005) reported US$20 billion of total costs was associated with the ECR problem. These figures suggest that the costs relating to ECR problem in container shipping industry is extremely high and this problem greatly affects the effectiveness and feasibility of shipping lines' business operations. Additionally, ECR can have environmental and sustainability impact as optimal repositioning of empty containers can reduce container flows percentage, which leads to reduction in emissions throughout the container shipping network (Song and Carter, 2009).

1.1.2. Container shipping uncertainty

The container shipping industry is highly impacted by the uncertain environment. In this business context, the uncertainty originates from various sources. The first and most important element of uncertainty in container shipping is fluctuations in container demand. Container demand varies on a seasonal basis, which stems from both long-term contractual demand and short-term spot-market demand (Lee and Song, 2017). In practice, container demand has a periodic impact. More specifically, the demand for containers in one period can affect the demand in other periods. In other words, the current container demand is reliant on the demand in previous periods (Meng *et al.*, 2015).

The uncertainty of customers’ demand for containers can have further impact. The quantity of empty container requests fluctuates with uncertainty in the timing which can cause unpredictability leading to the rescheduling or abandonment of orders (Francesco *et al.*, 2013). The optimal number of empty containers that should be repositioned is influenced by these uncertainties. Furthermore, empty containers might be accumulated in some unnecessary locations due to the uncertainty in container demand as well as container request times (Francesco *et al.*, 2013). Moreover, there are other uncertainties that are related to the transportation of containers. In reality, the distance between different locations (e.g. ports, depots, and customer zones) in a containerised transport network is considerable. This creates uncertainties for container transportation throughout the network due to various factors, including weather conditions and vehicle failures (Xie and Song, 2018). Hence, the dynamic and uncertain nature of container demand is an indispensable aspect of container shipping industry that requires to be embedded in research studies.

1.1.3. Dry port development in container shipping

The increasing trend of containerised transport is predicted to continue in the future. Figure 1.2 illustrates the global container over the recent decades and provides a projection for the years to come. It indicates that containerised transport has increased steadily since 2012 and is expected to increase in future. The worldwide container flow rate reached 802 million TEUs in 2019 which was a 2.3% increase with regard to the previous year (Drewry, 2020).

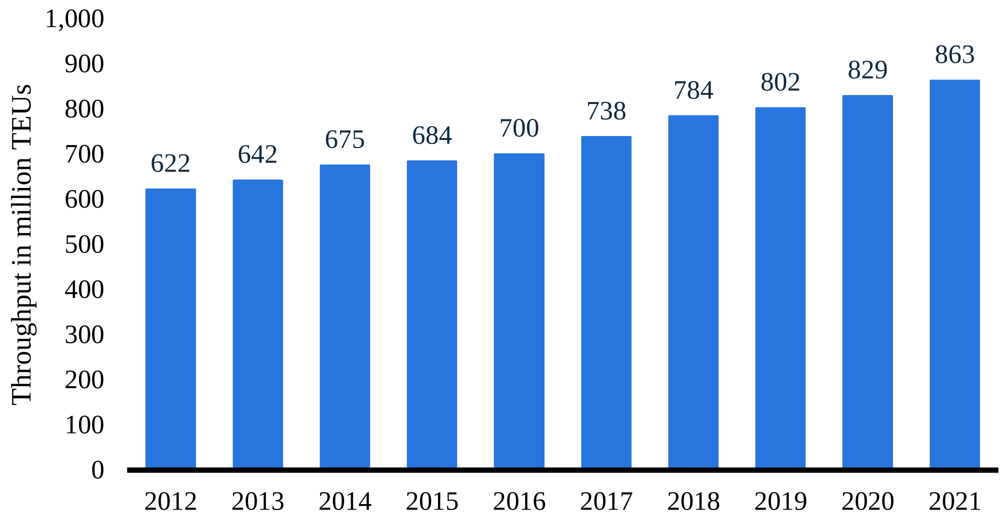


Figure 1.2. Container throughput worldwide with a forecast for 2020 and 2021(Drewry, 2020).

To handle the significant increase in flow of containerised transport chains, larger ships are used in the seaborne part of the transportation network which has led to a reduction in the transportation costs per TEU due to economies of scale (Cullinane and Khanna, 2000). Gigantic vessels which can accommodate 23,000 TEUs were unveiled in 2020 (World Cargo News, 2020). Yet, the economies of scale will not be completely achieved unless the hinterland operations are enhanced comparably (McCalla, 2007).

Although significant investments have been allocated to container terminal developments, seaport operations are facing excessive pressure as a result of larger ships developments and increasing container flows (Mourão *et al.*, 2002; McCalla, 2007). This issue can be resolved by the physical expansion of existing seaport capacity (McCalla, 1999), yet this requires huge expenditure and work (Pellegram, 2001). Alternatively, technological improvements as well as additional equipment can be used to improve seaport productivity (Ballis *et al.*, 1997). Furthermore, the seaports' hinterland transport operations, i.e. *"the interior region served by the port (Van Klink and van Den Berg, 1998, p. 1)"*, are adversely affected by the booming container flows. The ever-increasing container use, as well as inherent uncertainties that exist in container shipping leads to serious inefficiencies, including congestion and bottlenecks in seaports and their hinterland. It is clear that the seaports' hinterland accessibility infrastructures and transport capacities lags behind the maritime rapid developments (Cullinane *et al.*, 2012). Considering the fact that the container shipping industry is predicted to grow in the future (see Figure 1.2), it is crucial to incorporate a feasible solution to combat issues relating to capacity expansion, congestion, environmental concerns and hinterland accessibility. The Dry Port (DP) concept is an emerging solution from both seaport and hinterland perspective (Roso *et al.*, 2009). A Dry Port can be defined as an intermodal terminal which connects seaports directly to the inland shippers through rail networks (Roso, 2007). The dry port concept can be traced back to 1982. Beresford and Dubey (1990) considered dry ports to be inland terminals that could issue bills of loading for shipping lines. More recently, a dry port has been considered to be an inland facility that is able to offer the fundamental functions of a seaport (Cullinane and Wilmsmeier, 2011). The dry port notion and its regional coverage role in seaport hinterland operations has been investigated over the time by various scholars (including Heaver *et al.*, 2000; Heaver *et al.*, 2001; Notteboom and Winkelmans, 2001; Notteboom, 2002; Robinson, 2002). A more specific definition of dry port was provided by Roso *et al.* (2009): *"an inland intermodal terminal directly connected to seaport(s) with high capacity transport mean(s), where customers can leave/pick up their standardised units as if directly to a seaport (Roso et al., 2009, p. 341)"*. It is argued that the establishment of the dry port can improve seaports' efficiency and goods handling, resolve seaports' hinterland congestion, and decrease environmental impact by facilitating transport mode shifting to more energy efficient shipping modes (Roso *et al.*, 2009). Furthermore, dry port utilisation can smooth out seaports' storage space burden and provide more cost-effective transportation services to shippers (Padilha and Ng, 2012).

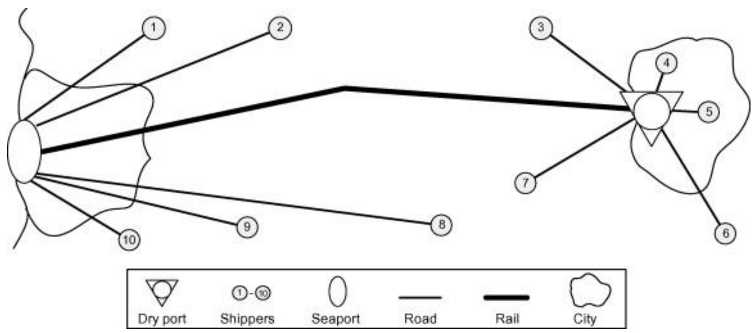
1.2. Statement of problem

As discussed in the above section, despite the benefits that are achieved through containerised transportation, various issues and challenges arise in the container shipping industry. More precisely, the hinterland part of seaports in containerised supply chains requires more attention and investigation so that it can respond to the fast-paced growth of seaborne network capacity. In this regard, container transportation companies, including shipping lines and inland transport operators need to address a multitude decision-making problems. This has been become even more complicated with the development of dry ports in the inland part of the container shipping supply chains. In the following, these decision-making problems and their significance are considered.

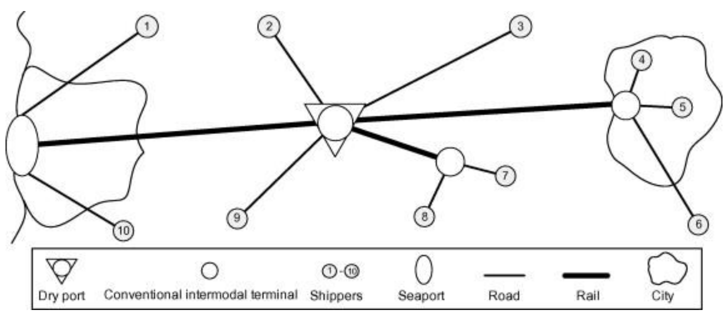
One of the main issues in this business context is the design of the container shipping network and tactical/operational level decision making (Lee and Song, 2017). The network includes different nodes (e.g. seaports, inland terminals, and customer zones) which are linked to each other by arcs (i.e. transport service routes). Containerised transport operators should identify the quantity of containers that are shipped throughout these service routes. More specifically, the optimal number of containers from an origin node to a destination node should be identified in order to efficiently meet customers' container demand. This also requires the decision to select the best route to deliver containers. This problem, which is known as the *"container cargo routing",* has been elaborated in the maritime shipping industry literature (Wang *et al.*, 2013; Brouer *et al.*, 2014b; Wang, 2014). There is a large body of literature that has considered container network design and routing in the seaborne shipping context. This problem needs to be investigated in the hinterland part.

The other crucial challenge that should be studied is related to the empty container repositioning problem. As discussed earlier, inefficient empty container management generates huge cost burdens for shipping lines and transport companies. Therefore, ECR decisions are needed to be made in an optimal manner. Similar to laden containers, the optimal quantity of empty containers that are transported throughout the network should be optimally identified. This problem contains various decisions that should be made to ensure the appropriate number of empty containers at the location that is required at the right time. The allocation of empty containers can be made through three different alternatives: allocating empty containers from overseas seaports; allocating empty containers from other inland nodes (e.g. terminals and depots); or leasing empty containers from other companies and lessors (Dang *et al.*, 2012). Accordingly, the optimal number of empty containers that are imported from other seaports, transported from other inland locations, and leased from lessors should be identified. It should be noted that the imported and leased empty containers should be sent back to the seaports and lessors in the future periods as appropriate. Furthermore, the availability of empty containers throughout the network requires effective inventory management. Hence, the inventory planning of empty containers is another problem which needs to be addressed in this study.

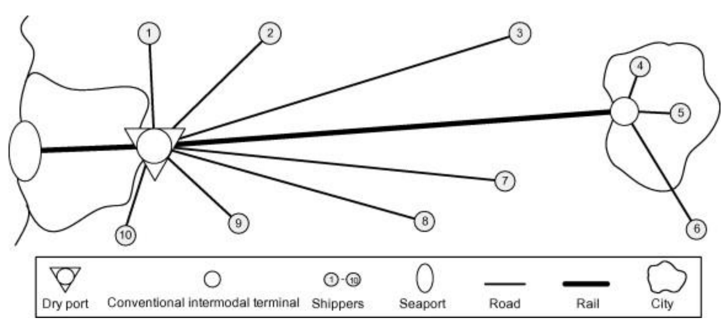
The next problem that needs to be taken into consideration is related to the dry port employment in the hinterland network. All the above-mentioned decisions regarding container network design and ECR problem are impacted by the dry port development. The key decision that reinforces this impact is the location of dry port in the hinterland network. Dry ports can be classified according to their location and the distance from the seaport as distant, midrange, and close (Roso *et al.*, 2009). Figure 1.3 illustrates a seaport hinterland with the utilisation of dry ports. Distant dry ports (see Figure 1.3.a) enable the hinterland network to benefit from rail mode's economies of scale for faraway customers requiring high volume container flow (Roso *et al.*, 2009). The midrange dry ports are normally set up at a distance from the seaport that can be served by road mode as shown in Figure 1.3.b. This type of dry ports is mainly used as a consolidation point for various rail services (Roso *et al.*, 2009). Finally, close dry ports (see Figure 1.3.c) are used to deal with the congestion problems in the local area of the seaport.



(a) A distant dry port network.



(b) A midrange dry port network.



(c) A close dry port network.

Figure 1.3. A seaport hinterland with three types of dry ports (Roso et al., 2009).

This shows that the location of dry port is a crucial decision that can define its role as well as the topology of the network. Furthermore, the number and location of dry ports in the hinterland of the seaport can affect other decisions in the container shipping network including both laden and empty container flow, empty container inventory level, and ECR problem. Therefore, the optimal number and location of dry ports should be integrated with container shipping network design and ECR problems.

It is worth mentioning that dry ports provide intermodal transport services for the inland movement of containers (Roso and Lumsden, 2009). The inland transport network can utilise rail, road, and waterway transportation modes for delivering containers depending upon geographical and infrastructural conditions. Different transportation modes provide different levels of flexibility with different levels of cost. Transport modality decisions should therefore be considered in the design of dry port container networks. Moreover, the uncertain and dynamic nature of the container shipping context impacts the previously mentioned decisions. It implies that the study should consider uncertain environments in the optimisation of decisions. This thesis, therefore, aims to study decision-making problems associated with hinterland container network design and dry port establishment under uncertainty. In the following sections, the proposed approach and research objectives are discussed further.

1.3. Proposed approach

The decision problems related to dry port container network design has been studied widely in a disjoint manner. More specifically, container network design (Shintani *et al.*, 2007; Tran, 2011; Mulder and Dekker, 2014; Wang and Meng, 2014), container cargo routing (Brouer *et al.*, 2014b; Plum *et al.*, 2014b; Wang, 2014; Wang *et al.*, 2015), empty container flow (Erera *et al.*, 2005; Brouer *et al.*, 2011; Epstein *et al.*, 2012; Chao and Chen, 2015), empty container inventory (Song, 2007; Dang *et al.*, 2012; Dang *et al.*, 2013; Zhang *et al.*, 2014; Xie and Song, 2018) and dry port location (Ka, 2011; Feng *et al.*, 2013; Ambrosino and Sciomachen, 2014; Wang *et al.*, 2018b) problems have been analysed separately. The strategies and methods that were proposed in these studies have dealt with different decisions in an isolated manner. The effects of dry port location on hinterland container flow, ECR, and inventory decisions have not been taken into consideration in the relevant literature. Furthermore, the majority of studies have assumed a deterministic environment, in which uncertainties of container shipping were neglected. These issues were investigated in this research.

In order to provide decision-making tools, as well as managerial insights for decision makers and practitioners in the container shipping industry, this thesis develops relevant mathematical programming models to address the outlined problems. More precisely, a stochastic programming approach is proposed to optimise the described decisions with uncertain container demands. It should be pointed out dry port is a strategic level decision since the dry port will operate over many years after being established. This decision is made before uncertain container demand occurs. On the other hand, decisions related to both laden and empty container intermodal transportation, empty container repositioning, empty container leasing, and empty container inventory planning are made at operational level. These operational level decisions should be made and reviewed over the planning horizon, when container demand is realised. It implies that the dry port container network design problem that this thesis aims to study possess a temporal hierarchical decision-making structure. To cope with this problem, a mathematical model based on two-stage stochastic programming was developed. The first-stage of the model focuses on optimising the number and location of dry ports as well as customers allocation to established dry ports. Then in the second stage, the periodic and operational decisions related to containers intermodal transportation and ECR problem are optimised according to the uncertain container demand realisation.

It should be noted that both container network design and facility location are *NP-hard* problems as proved by Brouer *et al.* (2014a) and Kariv and Hakimi (1979), respectively. The proposed two-stage stochastic programming model integrates these problems which leads to even higher complexity. Therefore, more advanced solution algorithms than standard solution methods were needed to be developed to solve the model efficiently. To handle this challenge, a Sample Average Approximation (Kleywegt *et al.*, 2002; Santoso *et al.*, 2005) method was employed to cope with uncertainty of the model. Then, a Robust Optimisation (Mulvey *et al.*, 1995) approach was adopted in order to improve the quality of solutions. Finally, a Benders Decomposition approach was utilised to enrich the solution procedure in terms of computational time for large scale instances. The Benders Decomposition method was further enhanced by proposing three different acceleration methods including multi-cut framework, knapsack inequalities, and Pareto-optimal cut scheme. This solution framework was then used to solve large instances of the developed robust dry port container network design.

A comprehensive computational study based on a hypothetical case study was developed to demonstrate the applicability of the proposed two-stage stochastic model and the efficiency of the proposed solution strategy. The model was validated by providing various analyses on obtained solutions to test network configuration, container flow decisions, ECR, and transportation modality. Additionally, different performance indicators including service level, fill rate, and inventory turnover were tested to provide further managerial insights into hinterland dry port container network design problem.

1.4. Research objectives

The aim of this research was to formulate mathematical models to integrate strategic and operational decisions for dry port container networks under an uncertain environment. More specifically, the strategic decision of dry port location-allocation was optimised jointly with operational decisions relating to the intermodal transportation of containers, empty container repositioning, and empty container inventory level. Therefore, the research objectives were:

1. To comprehensively review and obtain knowledge about container network design, empty container repositioning, dry port development and location decision-making;
2. To formulate mathematical models to identify the optimal number and location of dry ports in the seaport hinterland container network and the allocation of customers to established dry ports under periodic and uncertain container demand;
3. To determine the optimal decisions related to laden container flow, empty container flow, empty container leasing, and empty container inventory throughout the dry port container network;
4. To develop efficient solution methods in order to handle the complexity of the proposed models and to obtain high quality and robust solutions for the integrated dry port container network design problem;
5. To provide key performance indicators for shipping lines to improve their service level, fill rate, and inventory turnover in dry port container networks.

1.5.Thesis structure

This thesis is made up of seven different chapters. This thesis is organised as follows:

- **Chapter 1: Introduction.** The first chapter introduces the research. It involves a background of the related topics including containerised transport industry, empty container repositioning problem, container shipping uncertainty, and dry port description in container shipping. The statement of the problem, the proposed approach, and the research objectives are described in the chapter.
- **Chapter 2: Literature Review.** This chapter reviews relevant studies related to dry port container network design. The review contains previous research in container network design and container routing problems, empty container repositioning, dry port development, and facility location problem. Moreover, the identified research gaps based on the literature review are provided in the chapter.
- **Chapter 3: Research Methodology.** The third chapter describes the research methodology which is employed to accomplish research objectives. Firstly, an overview of modelling and solution methods in the relevant literature is presented. Then, the two-stage stochastic programming, which is the basis of this research modelling approach, is discussed in detail. Furthermore, the sample average approximation, robust optimisation, and Benders Decomposition concepts are clarified, which are used to design the solution strategy of this research.
- **Chapter 4: Mathematical Model.** This chapter is dedicated to the development of the mathematical model which integrates strategic and operational decisions associated with dry port container network design. The problem description related to the hinterland dry port network is outlined. A two-stage stochastic programming model is proposed to formulate the described problem under the uncertainty of container demand, in which the firs stage optimises strategic decisions while the second stage deals with the operational decisions.
- **Chapter 5. Solution Procedure.** This chapter aims to present a solution procedure for solving the proposed model and handling its complexity. A Sample Average Approximation is described and applied to the model to deal with the difficulty in obtaining the expected objective value. This solution method is then evaluated with a comprehensive statistical validation analysis. Moreover, a robust counterpart model is developed to increase the quality of solutions and to validate their accuracy with regard to uncertain input data. Finally, a

Benders Decomposition algorithm, accelerated with three different acceleration methods, is adopted to improve the solution procedure efficiency especially for large size problems.

- **Chapter 6. Computational Study.** In this chapter the proposed model and solution procedure are applied to a hypothetical case study. Various key performance indicators as well as sensitivity analyses are discussed to present managerial insights to container shipping companies. Considering 80 different problem instances, solutions related to the network configuration, container flow decisions, and key performance indicators are assessed. Finally, the performance of the robust optimisation approach and the computational efficiency of proposed accelerated Benders Decomposition are evaluated.
- **Chapter 7. Conclusions and Future Research Directions.** Finally, in the last chapter, the conclusions obtained associated with the research objectives are stated. Additionally, some recommendations and possible directions for further studies in this area is provided.

Chapter 2. Literature Review

2.1. Introduction

In this chapter, previous research relating to dry port container network design is reviewed. In Section 2.2 container network design and container routing problems are presented. This involves the mathematical models that have been designed to address container network design problems including port selection, transport routes, fleet management, customer demand allocation, and container cargo routing. Section 2.3 reviews empty container repositioning, which is one of the most important sub-problems associated with container network design. The research that has considered the empty container repositioning problem is further divided into two broad modelling approaches network flow models and inventory control models. Section 2.4 presents previous work related to dry port development and its role in the inland container network design. Finally, 2.5 considers the literature relating to the facility location problem, including the classic versions of the facility location problem as well as more recent advanced extensions.

2.2. Container network design and routing

2.2.1. Container network design

"The container network design problem aims to select ports, construct service routes and deploy a fleet of vessels so that service requests/customer demands can be served effectively (Lee and Song, 2017, p. 453)". There are many studies in the literature that have investigated the container network design problem. Agarwal and Ergun (2008) developed a mixed-integer linear programming model to design service routes among a set of ports. Álvarez (2009) proposed a model for the joint optimisation of container routing and fleet deployment for a global liner company. Reinhardt and Pisinger (2012) addressed the network design and fleet assignment in a liner shipping to minimise the total cost. Furthermore, Mulder and Dekker (2014) integrated container routing, ship scheduling and fleet design problems considering limited availability of ships in liner shipping. A profit-maximisation model based on service flows was formulated by Plum *et al.* (2014b) to handle the liner shipping network design problem. Wang and Meng (2014) considered delivery deadlines in container shipping network design problems. They utilised a column generation technique (Desaulniers *et al.*, 2006) to solve the proposed NP-hard non-linear model. These studies reveal that container network design contains various sub-problems

including container fleet management, vessel assignment, service route design, and container cargo routing. These sub-problems have been mainly studied disjointly or integrated by simplifying the service network design problem (Lee and Song, 2017).

One of the main decisions in the container network design is the definition of service route structures for meeting customer demands. Some studies have optimised and generated service routes from a given set of ports (e.g. Plum *et al.*, 2014). Shintani *et al.* (2007) used a dynamic stochastic model for a network design problem with a two-ports two-voyages structure. Tran (2011) dealt with the port selection problem in liner shipping to minimise the total cost, including inventory cost and inland transport cost. Song and Dong (2013) studied a route design problem, in which route structure design, ship deployment and empty container repositioning were addressed to minimise the total cost. They developed a three-stage optimisation model for a simplified route structure design problem. Brouer *et al.* (2014b) developed an integer programming model to decide container shipping routes for a capacitated cargo transport network. The objective was to maximise profit taking into consideration the revenue obtained from cargo transport and the cost incurred by network operations.

Another set of studies defined the service route structures by generating service routes from a set of pre-specified service routes (e.g. Mulder and Dekker, 2014; Wang and Meng, 2014). Brouer *et al.* (2014a) proved that the shipping network design is an NP-hard problem. They presented an integer programming mathematical model with the objective of maximising the liner shipping profit. In addition, Liu *et al.* (2014) examined the network design problem for a global liner shipping network. In this study, they incorporated the inland origin-destination pairs utilising an intermodal shipping system. Furthermore, a two-stage stochastic programming model was presented by Dong *et al.* (2015) to cope with the joint problem of service capacity planning as well as dynamic container routing. In the first stage, the optimal service capacity was decided. The second stage was dedicated to the stochastic optimisation of container routing using the service capacity identified by the first stage. Table 2.1 summarises the literature relating to the container network design which demonstrates: problem; considered network, problem environment, decisions, modelling approach; and solution method

Table 2.1. A summary of literature relating to container network design.

References	Problem	Network		Environment		Decision-making level		Decisions	Modelling approach	Solution method
		Seaborn	Inland	Deterministic	Uncertain	Strategic	Operational			
Agarwal and Ergun (2008)	Cargo routing	✓	-	✓	-	-	✓	Route assignment, container flow	Mixed integer linear program	Column generation
Álvarez (2009)	Routing and fleet deployment	✓	-	✓	-	-	✓	Container flow, fleet assignment	Mixed integer linear program	Tabu search
Reinhardt and Pisinger (2012)	Container network design	✓	-	✓	-	-	✓	Route assignment, transhipment	Mixed integer linear program	Branch and cut
Mulder and Dekker (2014)	Fleet design, cargo routing	✓	-	✓	-	-	✓	Cargo flow, transhipment	Linear programming	Heuristic
Plum *et al.* (2014b)	Liner shipping network design	✓	-	✓	-	-	✓	Container flow, service assignment	Mixed integer linear program	Heuristic algorithm
Wang and Meng (2014)	Liner shipping network design	✓	-	✓	-	-	✓	Route assignment, transit time	Mixed integer nonlinear programming	Column generation
Plum *et al.* (2014a)	Liner shipping service design	✓	-	✓	-	-	✓	Container flow, service assignment	Linear programming	Branch and cut
Shintani *et al.* (2007)	Container network design, empty container repositioning	✓	-	✓	-	-	✓	Container routing, cruising speed	Knapsack problem	Genetic Algorithm
Tran (2011)	Port selection	✓	-	✓	-	-	✓	Container routing and inventory	Nonlinear programming	Brute-force algorithm
Song and Dong (2013)	Service route design, empty container repositioning	✓	-	✓	-	-	✓	Route structure design, ECR	Linear programming	Heuristic algorithm
Brouer *et al.* (2014b)	Liner shipping network design	✓	-	✓	-	-	✓	Port assignment, vessel assignment	Integer programming	Matheuristic
Brouer *et al.* (2014a)	Liner shipping network design	✓	-	✓	-	-	✓	Container flow, vessel assignment	Integer programming	Metaheuristic
Liu *et al.* (2014)	Global intermodal liner shipping network design	✓	✓	✓	-	-	✓	Port allocation, container flow	Integer linear programming	Heuristic
Dong *et al.* (2015)	Service capacity planning, container routing	✓	-	-	✓	-	✓	Shipping service capacity, container routing	Two stage stochastic programming	SAA, PHA, APHA*

* SAA: Sample Average Approximation; PHA: Progressive Hedging Algorithm; APHA: Adapted Progressive Hedging Algorithm

In the container shipping industry, a customer's demand is satisfied when the laden containers are transported from a specified origin to a specified destination. The feasibility of the designed service network is interconnected with the decisions of choosing routes on which containers are delivered. In the following section, a review of previous studies relating to the container routing decision is provided.

2.2.2. Container cargo routing

Container cargo routing aims to allocate customer demands over a shipping network with minimum operational costs. This problem deals with the selection of routes for transporting containers. Therefore, it can be considered as a sub-problem of service network design (Lee and Song, 2017).

There are many studies in the literature that have considered the container cargo routing problem. Song *et al.* (2005) presented a model for the cost-efficient allocation of containers in global shipping networks to generate total costs, incomes, and container shipping patterns. Wang (2014) employed link-based multi-commodity network flow models to deal with the container routing problem in a shipping network. Additionally, Wang *et al.* (2013) included origin-destination transit time in the multi-commodity network flow for the container routing problem. Also, some scholars have considered the container routing problem together with empty container repositioning. Brouer *et al.* (2011) employed a multi-commodity flow model to integrate container routing and ECR problems. Bell *et al.* (2011) developed a frequency-based model for the container routing problem that minimised container shipment and dwell times. Song and Dong (2012a) also, studied the joint problem of cargo routing and ECR in multiple shipping service routes. Bell *et al.* (2013) used a cost-based linear programming model to optimise both laden and empty container assignments in a shipping system to minimise total cost of handling, inventory, and leasing. Huang *et al.* (2015) applied a mixed integer linear program model to formulate the cargo routing and ECR problems with the objective of minimising total cost. Wang *et al.* (2015) presented a profit-based model for maritime container assignment that considered elastic demand which depended on the freight rate. In the following section, a broader review of the empty container repositioning problem is presented.

The container shipping network design and container routing problem is complex. Agarwal and Ergun (2008) reduced the container shipping network design problem to a Knapsack problem to demonstrate the NP-hardness of the problem. With NP-hard problems the computational time required to solve problems increases exponentially with problem size (Van Leeuwen and Leeuwen, 1990). In addition, it was proved by that the container shipping network design problem is NP-hard by reducing it to a travelling salesman problem by Plum *et al.* (2014a). When the set of service routes is pre-specified, then the network design problem aims to select service routes to serve demands. Brouer *et al.* (2014a) stated that the container shipping network design problem can be proved to be NP-hard by reducing the problem into a set-covering problem. Table 2.2 summarises the literature relating to container cargo routing which specifies various features.

Table 2.2. A summary of literature relating to container cargo routing.

References	Problem	Network		Environment		Decision-making level		Decisions	Modelling approach	Solution method
		Seaborn	Inland	Deterministic	Uncertain	Strategic	Operational			
Song *et al.* (2005)	Container shipping network design	✓	-	✓	-	-	✓	Container routing, transhipment	Nonlinear programming	Heuristic
Wang (2014)	Container routing	✓	-	✓	-	-	✓	Container flow, transhipment	Linear programming	Cplex LP* solver
Wang *et al.* (2013)	Container routing	✓	-	✓	-	-	✓	Container flow, transhipment	Integer linear programming	Cplex ILP* solver
Brouer *et al.* (2011)	Cargo allocation	✓	-	✓	-	-	✓	Container flow, empty container repositioning	Linear programming	Column generation
Bell *et al.* (2011)	Maritime container assignment	✓	-	✓	-	-	✓	Empty container repositioning	Linear programming	Excel LP solver
Song and Dong (2012a)	Cargo routing and empty container repositioning	✓	-	✓	-	-	✓	Container routing, transhipment, inventory	Integer programming	Heuristics
Bell *et al.* (2013)	Container assignment	✓	-	✓	-	-	✓	Container flow, empty container repositioning	Linear programming	MATLAB LP solver
Huang *et al.* (2015)	Liner service network design	✓	-	✓	-	-	✓	Fleet deployment, empty container repositioning	Mixed integer linear program	Cplex MILP* solver
Wang *et al.* (2015)	Container assignment	✓	-	✓	-	-	✓	Container flow	Nonlinear programming	Trial-and-error
Agarwal and Ergun (2008)	Cargo routing	✓	-	✓	-	-	✓	Route assignment, container flow	Mixed integer linear program	Column generation
Plum *et al.* (2014a)	Liner shipping service design	✓	-	✓	-	-	✓	Container flow, service assignment	Linear programming	Branch and cut
Brouer *et al.* (2014a)	Liner shipping network design	✓	-	✓	-	-	✓	Container flow, vessel assignment	Integer programming	Metaheuristic

* LP: Linear Programming; ILP: Integer linear programming; MILP: Mixed Integer Programming

2.3.Empty container repositioning

The empty container repositioning problem can be considered as a sub-problem of the container service network design problem since both laden and empty containers are transported through the same shipping network. With the fast-paced growth of container shipping and the noticeable imbalance of trade demands over the last two decades, the ECR problem has been a critical issue for the shipping industry (Lee and Song, 2017). The economic burden of empty container transportation has been investigated by a number of researchers. For instance, Rodrigue (2016) reported that the cost of repositioning empty containers for shipping companies was US$16 billion per year which was 15% of the total cost. Furthermore, the problem could give rise to environmental and social issues including extra emissions and congestion in the inland network. Nevertheless, shipping companies hardly focus on utilising operational models or tools to deal with ECR decisions (Lee and Song, 2017). In the following, we review the research that has been dedicated to empty repositioning problem modelling.

2.3.1. ECR problem classification

According to Lee and Song (2017), the ECR problem can be classified into two general types according to the objective relating to quantity decisions or cost estimation. In the former, carriers should determine the quantity of empty containers that are stored at each port, and the time and quantity that should be transported from one port to another. The latter suggests that the movement of empty containers would be economically justified only if the containers were loaded with shippers' goods. Thus, the objective is to determine the additional cost which is incurred for repositioning of empty containers so that they would be ready for the subsequent delivery.

The quantity-based ECR models can be categorised into two broad groups with regards to the modelling approach and solution strategy (Song and Dong, 2015a). The first group utilises network flow models adopting arc-based mathematical programming which identifies the number of empty containers to be transported on the arcs of the network. These models are built based on the concept of flow balancing which guarantees the flow conservation of empty containers through a node. The second group utilises inventory control models in order to find the number of empty containers that should be dynamically repositioned at a node by developing decision-making rules. It should be noted that quantity decision models attempt to cope with uncontrollable demands, while cost estimation models try to actively associate flow balancing with shipping contracts as described by Zhou and Lee (2009).

2.3.2. ECR network flow models

Trade imbalances are the major reason that causes the empty container repositioning problem. Accordingly, network flow models are used to balance the container flows in shipping networks which could produce tactical decision plans (Lee and Song, 2017). Shipping companies may not be able to apply these tactical plans at the operational level due to the dynamic and uncertain nature of the problem. However, simplified operational rules can be designed to enable shipping companies to adopt generated tactical plans in their operations.

"Major shipping lines operate global service networks consisting of multiple shipping service routes (Lee and Song, 2017, p. 464)". Researchers have proposed network flow models to deal with the ECR problem for these multiple service routes. Erera *et al.* (2005) formulated a multi-commodity flow problem using a time-discretized network to address the ECR problems for tank container operators in the chemical industry. They aimed to minimise the total operating costs by combining container routing and ECR decisions in the model. Brouer *et al.* (2011) adopted a multi-commodity flow problem to address the cargo allocation problem. *"A cargo allocation model is a strategic tool that, given a schedule and a fleet over time, evaluates network profitability concerned with routing profitable cargo in a fixed network incorporating the overall empty repositioning cost (Brouer et al., 2011, p. 109)"*. They incorporated the repositioning and leasing of empty containers into the model's constraints to tackle the empty container repositioning problem. A decomposition approach was applied to the arc-flow model which led to a path-flow formulation. They used a delayed column generation algorithm to solve the resultant model. Song and Dong (2012a) integrated empty container repositioning problem with laden containers routing at the operational level. The objective was to minimise total operational costs over the planning horizon. Epstein *et al.* (2012) proposed a multi-commodity, multi-period model for empty containers repositioning decisions for one of the world's largest shipping companies. An inventory model was used to compute the safety stock required at each location in order to maintain high service levels for an uncertain environment. Chao and Yu (2012) developed a multi-commodity network model for the ECR problem for East and North China ports. Chao and Chen (2015) also formulated a time-space model to deal with the repositioning of reefer containers.

The ECR problem has also been studied under uncertain conditions. Cheung and Chen (1998) adopted a two-stage stochastic programming to formulate the ECR problem under an uncertain environment. They argued that the considered uncertainty came from randomness in supplies, demands and ship capacities. These parameters were considered deterministically in the first stage, with random variables used in the second stage. The ECR problem was optimised to minimise the total cost of the first stage plus the expected total cost of second stage. Furthermore, a robust optimisation model was proposed by Erera *et al.* (2009) using time-space networks for the ECR problem with uncertain supply and demand. Di Francesco *et al.* (2009) built a scenario-based optimisation model to tackle the maritime ECR problem with uncertainties. Di Francesco *et al.* (2013a) extended the previous work by considering possible port disruption in their stochastic programming procedure. A profit-based container assignment model was adopted by Wang *et al.* (2015) to optimise the empty container routing problem in liner shipping networks. Zheng *et al.* (2015) analysed the ECR problem with coordination amongst liner carriers using a two-stage programming model. In the first stage, a centralised strategy was proposed, while in the second-stage empty containers exchange costs were determined. These costs were allocated to liner carriers as an incentive for following the obtained centralise solution.

The mentioned ECR studies were mainly conducted at a global level. There is literature dedicated to the repositioning of empty containers at the regional level, i.e. the movement of containers between port terminals and inland facilities. Braekers *et al.* (2013) examined drayage operations including empty containers movements by developing a vehicle routing mathematical program. Furió *et al.* (2013b) defined two different mathematical models for two different container patterns to optimise the ECR problem amongst shippers, consignees, terminals, and depots in the hinterland of Valencia. Moreover, Olivo *et al.* (2015) proposed a time-extended optimization model for the inland ECR problem considering container leasing decisions. Sterzik *et al.* (2015) stated that container sharing among transportation companies could reduce costs significantly based on the analysis of a scenario-based model for both laden and empty container movements at the regional level. Furthermore, Shintani *et al.* (2007) dealt with the ECR problem and container shipping service design jointly by developing a two-stage model which was solved using a Genetic Algorithm approach. Meng and Wang (2011) also worked on the joint optimisation of container shipping network design and ECR using a mixed-integer linear programming model.

Table 2.3. A summary of literature relating to ECR network flows.

References	Problem	Network		Environment		Decision-making level		Decisions	Modelling approach	Solution method
		Seaborn	Inland	Deterministic	Uncertain	Strategic	Operational			
Erera *et al.* (2005)	Asset management	✓	✓	✓	-	-	✓	Container flow, inventory	Integer programming	Cplex ILP* solver
Brouer *et al.* (2011)	Cargo allocation	✓	-	✓	-	-	✓	Container flow, empty container repositioning	Linear programming	Column generation
Song and Dong (2012a)	Cargo routing and empty container repositioning	✓	-	✓	-	-	✓	Container routing, transhipment, inventory	Integer programming	Heuristics
Epstein *et al.* (2012)	Empty container repositioning and stocking	✓	-	-	✓	-	✓	Container flow, inventory	Linear programming	Cplex LP* solver
Chao and Chen (2015)	Empty container repositioning	✓	-	-	✓	-	✓	Container flow, inventory	Linear programming	Cplex LP solver
Cheung and Chen (1998)	Dynamic empty container allocation	✓	-	-	✓	-	✓	Container flow, leasing	Two-stage stochastic programming	Stochastic hybrid approximation
Erera *et al.* (2009)	Dynamic empty container repositioning	-	✓	-	✓	-	✓	Container flow, inventory	Integer programming	Cplex ILP solver
Di Francesco *et al.* (2009)	Container repositioning problem	✓	-	-	✓	-	✓	Container flow, inventory	Integer programming	ILP standard solver
Di Francesco *et al.* (2013a)	Container repositioning problem	✓	-	-	✓	-	✓	Container flow, inventory	Stochastic programming	Cplex MILP* solver
Wang *et al.* (2015)	Container assignment	✓	-	✓	-	-	✓	Container flow	Nonlinear programming	Trial-and-error
Zheng *et al.* (2015)	Empty container repositioning	✓	-	✓	-	-	✓	Empty container flow and exchange cost	Two-stage optimization method	Cplex LP solver
(Braekers *et al.*, 2013)	Drayage operations	-	✓	✓	-	-	✓	Container routing	Travelling salesman problem	Deterministic annealing algorithm
Furió *et al.* (2013b)	Container assignment	✓	-	✓	-	-	✓	Container flow	Integer programming	LP solver
Sterzik *et al.* (2015)	Container exchange	-	✓	✓	-	-	✓	Empty container repositioning, vehicle routing	Mixed integer programming	Heuristic
Shintani *et al.* (2007)	Container network design, empty container repositioning	✓	-	✓	-	-	✓	Container routing, cruising speed	Knapsack problem	Genetic Algorithm
Meng and Wang (2011)	Liner shipping service network design	✓	-	✓	-	-	✓	Empty container repositioning, transhipment	Mixed integer programming	Cplex MILP solver

* LP: Linear Programming; ILP: Integer Linear Programming; MILP: Mixed Integer Linear Programming

Table 2.3 summarises the ECR network flow models in the relevant literature according to specific features including the problem, network, environment, decisions level, modelling approach and solution method.

As can be seen, there is a large number of studies that have dealt with container shipping service design and empty container repositioning. However, the majority of work has considered the seaborne part (see Tables 2.1−2.3). The hinterland network plays a key role in the container shipping industry. Therefore, considering inland networks, including road and rail transportation should be further investigated.

2.3.3. ECR inventory control models

In this section, we review studies that cope with the ECR problem from the inventory control point of view. Du and Hall (1997) suggested a decentralised inventory control policy to reposition empty equipment in a hub-and-spoke logistics network. Li *et al.* (2004) proposed a two-threshold inventory control policy for the ECR problem in a port with uncertain demand. Song and Zhang (2010) presented an optimal control policy to reposition empty containers in a port subject to uncertain demand using dynamic programming. Young Yun *et al.* (2011) proposed a simulation-based approach to find a near-optimal (s, S)-type inventory control policy for ECR decisions between customers and terminals under uncertain demand at the regional level. The (s, S)-type inventory policy was further extended by Dang *et al.* (2012) and Dang *et al.* (2013) and applied to the hinterland of a port to optimise three types of ECR decisions including positioning from other overseas ports, inland positioning between depots, and leasing. The threshold parameters were optimised through a simulation-based Genetic Algorithm.

At the global level, Song (2007) considered a periodic-review system to obtain an optimal stationary policy for ECR by minimizing the total cost including container leasing, inventory, and reposition. Lam *et al.* (2007) developed an approximate dynamic programming method to identify the optimum control policies for ECR in a two-ports two-voyages. Shi and Xu (2011) studied the optimisation of empty containers through a control policy in a two-port system. Song and Dong (2008) applied a three-phase threshold control policy to the ECR problem in order to minimise the system's total expected cost in a dynamic and stochastic environment. Li *et al.* (2007) optimised the inventory control policy for empty containers in a multi-port system. Zhang *et al.* (2014) addressed the multi-port ECR over multi-periods as an inventory control problem considering uncertain demand. Dong and Song (2009) studied the joint optimisation of ECR and container fleet sizing utilising a simulation-based approach to evaluate inventory control policies. Lee *et al.* (2012) employed a single-threshold policy to tackle ECR problem and optimise the flow and inventory of empty containers in a multi-depot system. Chou *et al.* (2010) studied the ECR problem for a single service route using a two-stage formulation. In the first stage, the model identified the optimal number of empty containers at a port by employing an inventory model with a fuzzy backorder quantity. Using the result obtained from stage one, the optimum number of empty containers that should be transported between two ports was determined via a network flow mathematical model. The proposed model was applied to a real-life case of a trans-Pacific liner route. Table 2.4 presents a summary of studies relating to ECR inventory control models.

Table 2.4. A summary of literature relating to ECR inventory control models.

References	Problem	Network		Environment		Decision-making level		Decisions	Modelling approach	Solution method
		Seaborn	Inland	Deterministic	Uncertain	Strategic	Operational			
Du and Hall (1997)	Fleet sizing and empty equipment redistribution	-	✓	✓	-	-	✓	Inventory control policy	Decentralised inventory policy	Decomposition technique
Li *et al.* (2004)	Empty container allocation	-	✓	-	✓	-	✓	Inventory control policy, containers import/export	Markov decision process	Heuristic
Song and Zhang (2010)	Empty container reposition	✓	-	-	✓	-	✓	Inventory control policy, containers import/export	Two-state Markov chain	Dynamic programming
Young Yun *et al.* (2011)	Empty container reposition	✓	-	✓	-	-	✓	Inventory control policy	Simulation	-
Dang *et al.* (2012)	Empty container reposition	-	✓	-	✓	-	✓	Inventory control policy, leasing	Genetic-based optimization	Heuristics
Song (2007)	Empty container reposition	-	✓	-	✓	-	✓	Inventory control policy, leasing	Markov decision process	-
Lam *et al.* (2007)	Relocation of empty containers	-	✓	-	✓	-	✓	Inventory control policy, leasing	Dynamic programming	Heuristic
Shi and Xu (2011)	Empty container reposition	✓	-	-	✓	-	✓	Optimal controlling policies	Markov decision process	Dynamic programming
Song and Dong (2008)	Empty container management	✓	-	-	✓	-	✓	Inventory control policy	Simulation	Heuristics
Li *et al.* (2007)	Empty container allocation	✓	-	-	✓	-	✓	Inventory control policy, containers import/export	Markov decision process	Heuristic
Zhang *et al.* (2014)	Empty container allocation	✓	-	-	✓	-	✓	Optimal controlling policies	Simulation	Heuristic
Dong and Song (2009)	Container fleet sizing, empty container repositioning	✓	-	-	✓	-	✓	Inventory control policy, container fleet	Simulation-based optimization	Genetic Algorithm
Lee *et al.* (2012)	Empty container repositioning, container fleet sizing	✓	-	✓	-	-	✓	Inventory control policy	Non-linear programming	Gradient search
Chou *et al.* (2010)	Empty container allocation	✓	-	-	✓	-	✓	Inventory control policy	Fuzzy decision making, optimisation programming	Graded mean Integration Representation

The literature reviewed in this section is mostly dedicated to the maritime container network design problems. The inland network is of focal significance to this thesis. In the following section, we will present a review of research that has focused on the inland container network design that has considered the dry port concept.

2.4. Dry port development

As discussed in previous sections, there is a substantial body of research relating to the development of seaport and maritime networks. However, inland networks are increasingly changing due to transport development (Notteboom and Rodrigue, 2009). This requires further attention. There is a substantial literature which focus on multimodal transportation (Ambrosino and Sciomachen, 2014). This type of transportation delivers goods from origins to destinations through two or more different transportation modes (Hayuth, 1987). The aim of multimodal transportation is to move cargo through the entire transport chain from the shipper to the consignee in a cost-effective and timely manner.

Inland transportation has become more important to maritime shipping because of increasing volumes driven by the economic growth. Dry ports were introduced in many areas around the globe, particularly where cargo transportation increases in inland distribution networks (Bentaleb *et al.*, 2015). Congestion is a restricting factor for economic growth and a cause of pollution (Mussone *et al.*, 2015). The significant increase of container flows and the growth of international multimodal transport (Mabrouki *et al.*, 2014), has created problems related to limited space and accessibility for seaports. As a result, dry ports have been developed to cope with the rising congestion in the hinterland of seaports. There is growing interest in the dry port concept in the literature that has focused on improving seaport operations and inland transport networks. Parola and Sciomachen (2005) simulated the worldwide augmentation of container flow and found that congestion increases proportionally with the container flow growth. Notteboom and Rodrigue (2005) emphasised that a port regionalisation phase should be introduced to port system development. They classified existing dry ports from geographical and decision-making points of view. In the following we present an overview of studies on the concept of dry port in the container shipping industry.

2.4.1. Inland container network design using dry port

Inland transport systems play a key role in the future of containerisation. It has been shown that the efficiency of seaport transport networks could be boosted by the development of dry port facilities (Notteboom and Rodrigue, 2008). Concurrently, seaport development and management has become more complicated (Mabrouki *et al.*, 2014). As mentioned before, the congestion level in seaports increases as the flow of containers grows. For the majority of seaports, the storage capacity is the weakest link in the multimodal transport network. Besides, congested roads and insufficient rail links leads to higher transportation costs and delivery delays (Parola and Sciomachen, 2005). There has been increasing attention and work to tackle the seaport space problem through the provision of new ports and terminals (Rytköonen, 1999). Dry ports have been introduced to lessen the containerised transport network costs and capitalise on added value (Paixão Ana and Bernard Marlow, 2003). Moreover, Bichou and Gray (2004) stated that the underlying aim of dry port development is to reduce seaport congestion. Cullinane and Wilmsmeier (2011) suggested that the incorporation of dry ports can assist seaports growth and maturity. They also believed that dry ports could expand and reinforce the hinterland of a container seaport. Jaržemskis and Vasiliauskas (2007) defined the concept of the dry port as an approach to deal with seaport space shortage by pushing intermodal terminals more towards the hinterland from the seaport.

Slack (1990) firstly analysed inland load centres to show their role in enhancing intermodal transportation networks. Later, Slack (1999) emphasised the impact that the inland part can have in reducing the environmental impact of transport networks. Notteboom and Rodrigue (2009) looked at the role of dry ports in addressing global supply chain capacity and efficiency issues. They considered the complexity of modern freight distribution, the increased focus on intermodal transport solutions and capacity issues to be the main drivers of dry port development.

Rodrigue *et al.* (2010) stated that there is no absolute agreement on how inland facilities should be referred to. In their view, “inland port” was more suitable term as important logistic activities could be conducted in the vicinity of inland terminals. The study by Beresford and Dubey (1990) was one of the first to focus on the dry port concept. Their definition was mainly related to inland clearance depots. This definition could reflect the properties and specific services, such as customs provided by dry ports. However, their definition neglected the important characteristic of having a direct link to the seaport. Leveque and Roso (2002) defined a dry port as an interior multimodal terminal directly linked to the seaport with different transport capacity, where clients are able to pick up their containers and leave as if it was in a seaport. Roso (2007) analysed the implementation of dry port from an environmental perspective. They used simulation to confirm that incorporating dry ports in a transport network could reduce CO_2 emissions, congestion, and waiting time. Roso and Lumsden (2009) analysed two different scenarios for a transport system at a seaport terminal. They compared physical flows in the system with and without a dry port. They concluded that incorporating a dry port in the hinterland of a seaport can resolve the problem caused by limited space.

According to Roso *et al.* (2009), dry ports provide additional services compared to traditional inland terminals. These additional services include maintenance and repair of containers, storage of empty containers, consolidation and customs clearance. They also indicated that dry port development can move cargo from road to more energy efficient transportation modes which have lower environmental impact. Cullinane *et al.* (2012), furthermore, proposed a dry port concept based on the "extended gate" concept as a solution for combating containerised transport congestion, capacity, and environmental issues.

Dry ports are classified depending on their location in the hinterland relative to the seaport. Roso *et al.* (2009) defined three different types of dry port as distant, mid-range and close dry ports, based on their services and location relative to the seaport. Distant dry ports are located at least 500 kilometres from the seaport (Henttu *et al.*, 2010). The main plus point of a distant dry port is providing long distance transportation for service users with reduced cost due to the utilisation of the cheaper rail transportation mode. This can also facilitate the reduction of congestion and environmental impact through the modal shift from road to rail. Mid-range dry ports are normally established between close and distant dry ports, i.e. 100−500 kilometres from the seaport (Henttu *et al.*, 2010). This type of dry port provides a depot storage facility for customers. Finally, close dry ports are situated within 100 kilometres distance of the seaport (Henttu *et al.*, 2010). Close dry ports provide a depot facility, increased terminal capacity and consolidation for the road mode to and from the seaport (Roso *et al.*, 2009).

2.4.2. Decision-making in dry port development

The container shipping decision-making problems can be categorised into three different levels relating to strategic, tactical, and operational decisions. Strategic level decisions are made for a long period of time, typically more than 10 years. This level includes decisions related to the location and capacity of facilities (e.g. dry ports) (Ivanov, 2019). Tactical level decisions are associated with the medium term horizon (5 to 10 years) containing transportation and inventory planning which defines the role of the dry port (Bentaleb *et al.*, 2015). Operational level decisions are made over short-term periods that aim to obtain the most efficient operations and services for a dry port. This level includes the empty container repositioning and inventory control decisions for dry port networks.

It should be noted that all decision-making levels are interconnected and have a significant impact on each other (Chopra and Meindl, 2007). However, in the dry port development context, most researchers have focused on the strategic level and have neglected the tactical and operational levels of the problem (Bentaleb *et al.*, 2015). The determination of the number and location of dry ports is a strategic decision that has been studied by various researchers in the literature (e.g. Ka, 2011; Feng *et al.*, 2013; Ambrosino and Sciomachen, 2014; Wang *et al.*, 2018). However, these studies ignored the impact of dry port location on tactical and operational decisions. In other words, dry ports location can define the context of transportation, empty container repositioning, and inventory decisions within the hinterland network (Lee and Song, 2017).

Although the impact of location decisions on other levels has not been studied in the context of dry port development, there are various researchers that have considered this interconnection in other contexts including production (Bhutta et al., 2003), distribution (Liao et al., 2011) and the parcel service (Wasner and Zäpfel, 2004). The approaches and techniques that have been developed in other industries can be adopted in the dry port context to integrate different decision levels. In the following we further discuss and review the facility location problem and its possible extensions that have been developed in the literature.

2.5. Facility location problem

This section reviews facility location models in the network and supply chain design context. The most-commonly used location problems in the literature are discrete models which are formulated as integer linear programs. Discrete models of small size can be solved using available commercial solvers.

2.5.1. Discrete models

Classic discrete location models were developed by Daskin (1997) and Salhi and Drezner (1996) for deterministic problems. These problems involve covering problems, centre and median problems, and fixed-charge location problems. The covering location problems, which are originally devised by Church and ReVelle (1974) and Christofides (1975), seek to determine the number and location of facilities to ensure no demand point will be farther than a distance limit (known as the maximal service distance). The centre and median problems utilise the concepts of 'absolute centre' and 'absolute median', respectively, to find the optimal location of facilities within a weighted graph (Hakimi, 1964). Fixed-charge location problems are developed in different contexts to optimise the number and location of facilities by considering a fixed cost for setting up these facilities (Gerard *et al.*, 1977; Mirzaian, 1985). In the following, these classic models are reviewed according to Daskin (1997).

Let the set of candidate locations for establishing facilities be denoted by $\mathcal{I}$. In all covering, centre, median, and fixed-charge location problems, customer demand is distributed in a set of nodes denoted by $\mathcal{J}$, where each customer $j \in \mathcal{J}$ generates D_j unit of demand.

The set covering problem attempts to minimise the number of facilities, while satisfying all customers' demand by optimising the location of facilities. This problem aims to meet (cover) a portion of demand (ReVelle *et al.*, 1976). Let a_{ij} represents the coverage relationship, where $a_{ij} = 1$ if the demand of customer $j \in \mathcal{J}$ can be served by facility $i \in \mathcal{I}$ and $a_{ij} = 0$, otherwise. The binary variable x_i is associated with the location decision of facility $i \in \mathcal{I}$; i.e. a facility is set up at potential location i if $x_i = 1$ and $x_i = 0$, otherwise. The objective of this problem is to minimise the total number of facilities, while ensuring a complete coverage in the network (Beasley and Chu, 1996). The set covering problem can be formulated as the following mathematical model:

Set covering problem

$$\min_{x} \sum_{i \in \mathcal{I}} x_i \tag{2.1}$$

subject to:

$$\sum_{i \in \mathcal{I}} a_{ij} x_i \geq 1 \qquad \forall j \in \mathcal{J} \tag{2.2}$$

$$x_i \in \{0,1\} \qquad \forall i \in \mathcal{I} \tag{2.3}$$

The objective function (2.1) minimises the total number of facilities, while each customer's demand should be met by the facilities as shown in constraint (2.2). Constraint (2.3) shows the standard binary decision variable. The model (2.1)−(2.3) would be transformed to the maximum covering problem if the total coverage constraint was relaxed and a budget restriction was imposed. In this case, the total number of established facilities are restricted as $|\mathcal{I}| \leq N$ due to the budgetary limit. In this problem, the binary variable y_j represents the demand coverage decision, where it equals to 1 if the demand of customer $j \in \mathcal{J}$ is met and it equals to 0, otherwise. Thus, the objective is the maximisation of the covered demand. This problem is formulated as follows:

Maximum covering problem

$$\max_{x,y} \sum_{j \in \mathcal{J}} D_j \, y_j \tag{2.4}$$

subject to:

$$\sum_{i \in \mathcal{I}} x_i \leq N \tag{2.5}$$

$$\sum_{i \in \mathcal{I}} a_{ij} x_i \geq y_j \qquad \forall j \in \mathcal{J} \tag{2.6}$$

$$x_i \in \{0,1\} \qquad \forall i \in \mathcal{I} \tag{2.7}$$

$$y_j \in \{0,1\} \qquad \forall j \in \mathcal{J} \tag{2.8}$$

The objective function (2.4) maximises the total satisfied demand over all customers. Constraint (2.5) indicates that the total number of opened facilities should not exceed the predetermined number of N. Constraint (2.6) ensures the demand coverage of customer j. Constraints (2.7)−(2.8) represent the binary decision variables. The model (2.4)−(2.8) can be adapted to various problems by considering the travel distance and cost.

The centre problem aims to identify the location of facilities in a manner that leads to the minimisation of a customer maximum travel distance, denoted by W. This problem is mainly appropriate to address location problems relating to the provision of public service facilities including hospitals and schools since their priorities service level and equity (Hakimi, 1964). The travel distance from facility $i \in \mathcal{I}$ to customer $j \in \mathcal{J}$ is denoted by d_{ij}. Accordingly, the binary variable y_{ij} equals to 1 if customer j is satisfied by facility i. Considering this notation, the centre problem is presented as follows:

Centre problem

$$\min_{x,y} \sum W \tag{2.9}$$

subject to:

$$\sum_{i \in \mathcal{I}} x_i \leq N \tag{2.10}$$

$$\sum_{i \in \mathcal{I}} y_{ij} = 1 \qquad \forall j \in \mathcal{J} \tag{2.11}$$

$$y_{ij} \leq x_i \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \tag{2.12}$$

$$W \geq \sum_{i \in \mathcal{I}} d_{ij} y_{ij} \qquad \forall j \in \mathcal{J} \tag{2.13}$$

$$x_i \in \{0,1\} \qquad \forall i \in \mathcal{I} \tag{2.14}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \tag{2.15}$$

In this model, the objective function (2.9) minimises the customers' maximum travel distance. Constraint (2.10) restricts the total number of opened facilities to N. Constraint (2.11) implies that each customer should be allocated to a single facility. Constraint (2.12) specifies that a customer can be served by a facility only if that facility is established. Constraint (2.13) calculates the maximum travel distance of customers. Finally, constraints (2.14)−(2.15) are standard binary variables.

In contrast to the centre problem which is applied to public services with social benefit and equity goals, the median problem is more suitable for private firms which provide transport and delivery services to generate profit. Therefore, the objective of the median problem is to minimise the total operating cost due to travelling between facilities and customers (Daskin and Maass, 2015) as shown in the objective function (2.16) below. The median problem can be formulated as follows:

Median problem

$$\min_{x,y} \sum_{i \in \mathcal{I}} \sum_{\forall j \in \mathcal{J}} D_j \, d_{ij} y_{ij} \tag{2.16}$$

subject to:

$$\sum_{i \in \mathcal{I}} x_i \le N \tag{2.17}$$

$$\sum_{i \in \mathcal{I}} y_{ij} = 1 \qquad \forall j \in \mathcal{J} \tag{2.18}$$

$$y_{ij} \le x_i \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \tag{2.19}$$

$$x_i \in \{0,1\} \qquad \forall i \in \mathcal{I} \tag{2.20}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \tag{2.21}$$

In addition to the operating cost, the fixed cost of opening the facilities should be considered as a major cost component when evaluating a location problem. In order to unify fixed opening costs and operating costs, the facility fixed opening cost can be distributed over the planning horizon or the operating cost can be aggregated at the strategic level. It should be noted that opening more facilities leads to a reduction in operating cost as the customers' accessibility would be improved. This implies the existence of a trade-off between facilities' fixed opening cost and the operating cost of deliveries. The fixed charge facility location problem focuses on determining the number and location of facilities to minimise the total cost of the system by balancing the mentioned trade-off (Daskin *et al.*, 2005). The one-time fixed opening cost of facility $i \in \mathcal{I}$ is denoted by f_i. The uncapacitated fixed charge facility location model can be formulated by considering the fixed opening cost to the objective function (2.16) and obtain the following objective function:

Uncapacitated fixed charge facility location problem

$$\min_{x,y} \sum_{i \in \mathcal{I}} f_i x_i + \sum_{i \in \mathcal{I}} \sum_{\forall j \in \mathcal{J}} D_j \, d_{ij} y_{ij} \tag{2.22}$$

subject to:

(2.17)−(2.21).

In the above model, we can relax constraint (2.17) if the budget allocation is considered in the minimisation objective function. In some contexts, the capacity of facilities limit the location design problem (Melkote and Daskin, 2001). This fact can be incorporated by adding the following constraint into the mathematical model:

Capacitated fixed charge facility location problem

$$\sum_{\forall j \in \mathcal{J}} D_j y_{ij} \leq Cap_i x_i \qquad \forall i \in \mathcal{I} \qquad (2.23)$$

where Cap_i shows the capacity of facility $i \in \mathcal{I}$. The model with constraint (2.23) is referred to as capacitated fixed charge location problem.

The above models form the basis of many facility location models within various public and private sector contexts. They can be further extended to cope with more realistic applications in different settings. One of the main factors that can help the location models to reflect real-world problems is the consideration of uncertainty within mathematical models (Snyder, 2006). In different applications relating to logistics and supply chain systems including containerised transport networks, taking the inventory of goods (e.g. containers) within the facilities is inevitable (Daskin *et al.*, 2002a). The following subsections review the related studies of facility location under uncertainty and location inventory problem.

2.5.2. Facility location under uncertainty

The decision associated with facility location has a long-lasting impact and it is difficult and costly to change after it has been made. After the design decisions relating to facility location have been made, each of the parameters in the problem including operating costs, customer demand and travel distances may fluctuate considerably over the operating horizon (Snyder, 2006). These parameters can be estimated using different methods; however, these estimations can lead to inaccuracies and poor measurements. The parameter fluctuations can be incorporated in the modelling development procedure using various approaches such as aggregating customers demand and using a distance norm (Snyder, 2006). Furthermore, the facility location problem maintains a two-stage nature, i.e. determining the locations first, before the future parameters' uncertainty is revealed for the decision maker (Klose, 2000). These problem characteristics have led to many scholars applying and developing approaches for decision making that take into account uncertainty when solving the facility location problem (Gao, 2012). In the following we review research that has addressed the facility location problem under uncertainty.

Research that has applied stochastic models for solving the facility location problem are presented in this section. These models have mostly used an objective function that aims to minimise the expected total cost or maximise the expected profits of the network or supply chain (Snyder, 2006). Depending on the nature of the problem and the structure of the model, the developed stochastic location models can be solved using general stochastic programming methods or specific customised algorithms designed by researchers (Ruszczyński, 1997). In developing stochastic programming models, a set of decisions should be made before the uncertain parameters are resolved, while other decisions are dealt with after the uncertainty has been realised. The former decisions are referred to as first-stage decisions and the latter are known as second-stage decisions. In the context of stochastic location problem, facility locations are first-stage decisions and operational tasks of customers travel and demand allocation are second-stage (recourse) decisions (Snyder, 2006).

- ***Minisum location problems***

In stochastic programming models, optimising the mean outcome of the system is the most-widely used objective by minimising the expected total cost or maximising the expected profit (Snyder, 2006). Cooper (1974) studied the location problem under the uncertainty of the demand points. In that study the demand locations were assumed to follow a bivariate normal distribution. The objective of their study was to find the optimal location of a single facility whilst minimising the expected demand weighted distance to demand zones. The convexity of the objective function was proved with regard to the chosen location. A scenario approach was applied to the facility location problem by Sheppard (1974). Mirchandani and Oudjit (1980) were among the first researchers to address the uncertain scenario-based location problem with the objective of minimising expected cost. This was done by proposing a tree-based model with discrete scenarios of stochastic edge lengths which minimise the expected demand-weighted distance. Weaver and Church (1983) used a Lagrangian relaxation algorithm to solve the stochastic median problem. They treated the scenario-based stochastic problem as a larger deterministic problem. More specifically, a problem with n customers and s scenarios was treated as a determinist problem with the overall number of ns customers. They solved the model via the standard Lagrangian relaxation method for the median problem proposed by Cornuejols *et al.* (1977).

Mirchandani *et al.* (1985) formulated the uncertain median problem as a deterministic model, where customer-scenario pairs were considered as a total of ns customers. They relaxed the constraint related to the required number of opened facilities and reduced the original problem to the uncapacitated fixed charge location problem. This enabled them to apply a Lagrangian relaxation, where the Lagrange multiplier was equivalent to the fixed cost of opening facilities. Louveaux (1986) studied the capacitated median problem as well as the capacitated fixed charge location problem under uncertain environment. In these problems, the uncertain parameters were related to customer demand, production costs, and selling prices. The objective of the model was to maximise the system profit by optimising the location of facilities, their capacities and their allocation to customers. The facilities locations and capacities were decided by the first stage which might be insufficient to meet all customers' demand in the second stage. Therefore, a penalty cost for unsatisfied customer demand was incorporated in the modelling procedure. Also, a budget restriction was included in the objective function to limit the maximum number of facilities. Later, Louveaux and Peeters (1992) proposed a dual-based procedure for the scenario-based capacitated fixed charge location problem, and Laporte *et al.* (1994) applied an exact solution strategy based on the L-shaped method (see Laporte and Louveaux, 1993) for the location problem with uncertain demands.

Ravi and Sinha (2006) designed an approximation algorithm for the two-stage stochastic location problem based on the Shmoys *et al.* (1997) rounding algorithm. The algorithm procedure developed by Ravi and Sinha (2006) allows the model to open facilities in either the first-stage or the second-stage with a different opening fixed costs in each.

Listeş and Dekker (2005) attempted to address the facility location problem in the context of a reverse logistics network of using sand from demolition sites in the Netherlands. The uncertainty modelled was due to the random sand supply and demand. Listeş and Dekker (2005) proposed a three-stage mixed-integer stochastic programming model which maximises the expected profit of the network. The decisions regarding initial facilities location, additional facilities location, and product flow are made in the first stage, second stage, and third stage, respectively.

Chan *et al.* (2001) developed a stochastic location-routing model for stochastically processed demands derived from a queuing system. In this study, a queuing process was used within an optimisation framework to estimate the demand probabilities. The objective of the model was to minimise the expected total cost. They applied a heuristic algorithm based on an extension of Benders decomposition to solve the model for a wartime medical evacuation problem. Ricciardi *et al.* (2002) considered the facility location problem with uncertain throughput costs at distribution centres. The objective was to minimise the total costs comprising determinist transportation cost among plants, distribution centres and customers, coupled with the expected throughput cost at distribution centres. The resultant model was a nonlinear integer program which was solved using a Lagrangian-based heuristic.

Daskin *et al.* (2002b) built a stochastic model to formulate the location as well as the inventory of facilities to minimise the expected total cost of location, transportation, and inventory under uncertain demand. They utilised a (Q, R) inventory policy for each facility which uses an Economic Order Quantity (EOQ) policy to approximate the inventory cost. The model was transformed to a deterministic problem by including the means and variances of the uncertain parameters in the objective function. Then the model was solved using the Lagrangian relaxation approach by Daskin *et al.* (2002b) and used column generation proposed by Shen *et al.* (2003). Furthermore, Snyder *et al.* (2007) extended the Daskin *et al.* (2002b) location-inventory model to consider stochastic costs, lead time and demand means and variances. They coped with the nonlinearity of the objective function by employing a Lagrangian relaxation–based exact algorithm.

- ***Multi-echelon facility location problems***

Multi-echelon location models, which are mostly known as supply chain network design models (Snyder, 2006), can be regarded as stochastic extensions of the original work of Geoffrion and Graves (1974). A two-stage stochastic programming model was formulated by MirHassani *et al.* (2000) to design a supply chain network with binary design variables in the first-stage and continuous variables in the second-stage. The uncertainties represented by the model were mainly related to customer demand and facilities capacity. They solved the stochastic model using a parallel implementation of Benders decomposition algorithm.

Tsiakis *et al.* (2001) developed a scenario-based stochastic model for multiproduct, multi-echelon supply chain networks. The decisions considered were the location and capacity of middle-echelon facilities, transportation arcs, and flows, in which transportation costs were piecewise linear concave. Alonso-Ayuso *et al.* (2003) proposed a two-stage stochastic model to optimise plant capacities, product mix, and sourcing decisions in the first-stage, and production, inventory, and transportation in the second-stage. The uncertainty arose from product demand and price, and the cost of raw materials and production.

Santoso *et al.* (2005) formulated a supply chain network design model at a global scale using two-stage stochastic programming. The model included uncertain costs, demands, and capacities with an a large number of scenarios. The objective of the model was to optimise the location of facilities and the machines within facilities in order to minimise the expected total cost. The first-stage of the model was associated with binary design variables, while the second-stage included continuous recourse variables. This structure enabled Santoso *et al.* (2005) to utilise a sample average approximation framework together with an accelerated Benders decomposition to solve the stochastic model. Moreover, Butler *et al.* (2003) developed a model for the strategic production and distribution planning for a new product taking uncertain costs, demands, and capacities into consideration. The objective was to maximise the profit of a large consumer electronics company.

2.5.3. Facility location-inventory problem

The facility location and capacity problems are the strategic decisions of supply chain network design which are normally made for a period of two to five years (Zokaee *et al.*, 2017). The different levels of decision-making in supply chain network design relating to strategic, tactical, and operational decisions have different nature, scope, and time horizons. This is the main reason that these decisions are made in a hierarchical sequence by researchers (Fahimnia *et al.*, 2013b). This modelling approach may give rises to contradictory and infeasible decisions which necessitates the need for designing integrated models (Fahimnia *et al.*, 2013a). Furthermore, in a business environment with high volatility, the strategic level decisions should be revisited at the tactical and operational levels to improve supply chain efficiency (Fahimnia *et al.*, 2012).

The literature reviewed in the previous subsection shows that cost is the main performance measure considered by location problems (Farahani and Hekmatfar, 2009). This cost component includes the trade-off between location, transportation and inventory costs (Farahani *et al.*, 2015). To cope with the complexity of facility location-inventory problems, the developed models are normally simplified by breaking a large problem into smaller sub-problems (Stadtler, 2008). Yet this method might lead to two optimal solutions that minimise location and inventory costs separately rather than identifying a global optimum. In contrast, joint location-inventory problems that can produce global optimal solutions are mainly large problems with high complexity which requires more advanced solution strategies.

Figure 2.1 shows a location-inventory problem that represents a three-layer supply chain network. The network is made up of a set of suppliers who provide products to distribution centres, which ultimately meet the customers' product demands. In the location-inventory problem the location of suppliers and customers is predetermined and known. The model decisions are then focused on identifying the optimal number and location of distribution centres, the allocation of customers to distribution centres and optimising the inventory service level at each distribution centre (Daskin *et al.*, 2002b). This model can be extended to be more realistic by integrating transportation and routing decisions into the location inventory problem. The work of Perl and Sirisoponsilp (1988) was one of the first examples of research that integrated location, inventory, and transportation decisions. In addition, Jayaraman (1998) attempted to design a distribution network that optimised location, inventory, and transportation decisions.

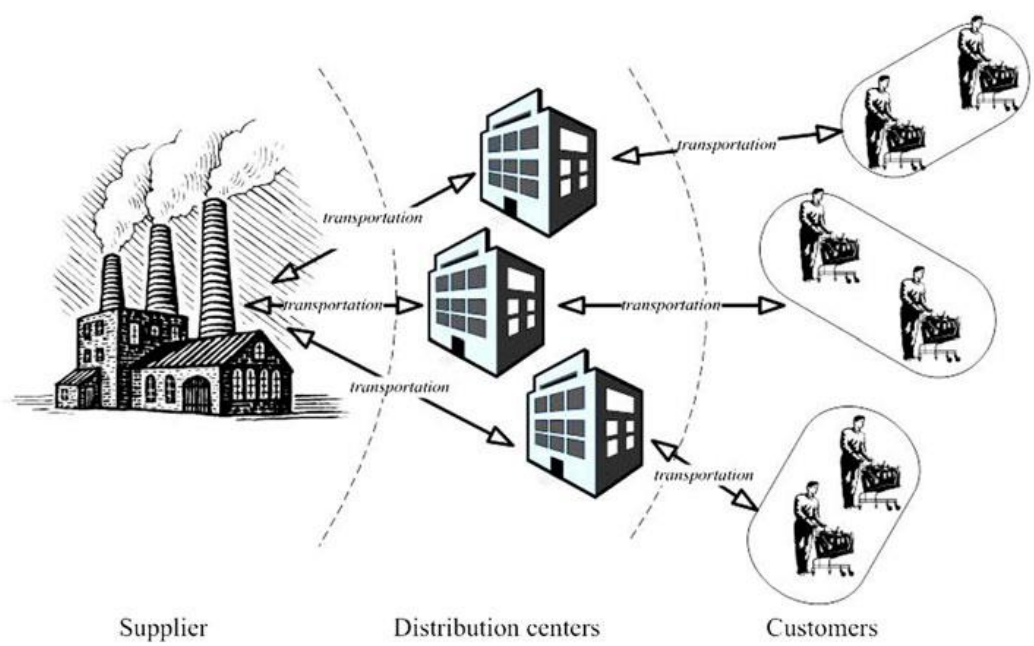


Figure 2.1. A three-layer location-inventory problem (Farahani et al., 2015).

In the following, we present a review of location-inventory problem modelling, mainly in the context of strategic and operational decision-making in supply chain network design.

2.5.3.1. Location-inventory problem modelling

Location-inventory problem models can be categorised into four types: the basic location-inventory problem, the dynamic location-inventory problem, the location-inventory routing problem, and the inventory-transportation problem (Farahani *et al.*, 2015). The basic location-inventory problem is the underlying model of all four mentioned problem types. Therefore, a mathematical model of the location-inventory problem is discussed in this section. This model was proposed by (Tanonkou *et al.*, 2005) for a three-echelon single commodity network which considered a single supplier, a set of decision centres, and multiple retailer. The problem involves determining the optimal location of distribution centres to serve the retailers in order to minimise the operational costs related to inventory and transportation as well as fixed opening cost of distribution centres. In addition, demand and delivery lead times are assumed as uncertain parameters that follow the normal distribution. Table 2.5 presents the parameters and decision variables used in the model formulation.

Table 2.5. Notation of sets, parameters, and variables used in location-inventory model

Sets	
$\mathcal{I}$	Set of retailers indexed by i.
$\mathcal{J}$	Set of distribution centres indexed by j.
Parameters	
A_j	The ordering cost at distribution centre $j \in \mathcal{J}$.
H_j	The unit holding cost at distribution centre $j \in \mathcal{J}$.
T_j	Total available product at distribution centre $j \in \mathcal{J}$.
Q_j	The order size at distribution centre $j \in \mathcal{J}$.
D_j	The annual demand at distribution centre $j \in \mathcal{J}$.
F_j	The fixed cost of locating distribution centre $j \in \mathcal{J}$.
l_j	The mean of delivery lead time from the supplier to distribution centre $j \in \mathcal{J}$.
δ_j^2	The variance of delivery lead time from the supplier to distribution centre $j \in \mathcal{J}$.
μ_i	The mean of demand at retailer $i \in \mathcal{I}$.
σ_i^2	The variance of demand at retailer $i \in \mathcal{I}$.
t_j	The unit transportation cost from the supplier to distribution centre $j \in \mathcal{J}$.
t_{ij}	The unit transportation cost from retailer $i \in \mathcal{I}$ to distribution centre $j \in \mathcal{J}$.
z_α	The standard normal deviate such that $P(Z \leq z_\alpha) = \alpha$.
α	The service level at distribution centre $j \in \mathcal{J}$.
ψ	Number of working days per year.
h_i^+	Cost of carrying inventory at retailer $i \in \mathcal{I}$.
h_i^-	Inventory shortage cost at retailer $i \in \mathcal{I}$.
β_i	Initial inventory level at retailer $i \in \mathcal{I}$.
w_i	Quantity delivered to retailer $i \in \mathcal{I}$.
$q_i(0)$	Inventory cost function at retailer $i \in \mathcal{I}$.
$C_i(0)$	Cumulative demand distribution function at retailer $i \in \mathcal{I}$.
Decision Variables	
X_j	Binary variable, equals 1 if retailer $j \in \mathcal{I}$ is considered as a distribution centre; 0 otherwise.
Y_{ij}	Binary variable associated with allocation decision of retailer i to distribution centre j.

Using the notation presented in Table 2.5, the basic location-inventory problem was presented as the following model:

Location-inventory problem

$$Min \sum_{j} F_j X_j + \psi \sum_{i}\sum_{j} t_j \mu_i Y_{ij} + \psi \sum_{i}\sum_{j} t_{ij} \mu_i Y_{ij} + \sum_{j} \left(A_j \frac{D_j}{Q_j} + H_j \frac{Q_j}{2} \right) + z_\alpha \sum_{j} H_j \sqrt{\sum_{i} l_j \, \sigma_i^2 Y_{ij}}$$

$$+ z_\alpha \sum_{j} H_j \sqrt{\sum_{i} \delta_j^2 (\mu_i Y_{ij})^2} \qquad (2.24)$$

subject to

$$\psi \sum_{i} \mu_i Y_{ij} = D_j \qquad \forall j \in \mathcal{J} \qquad (2.25)$$

$$\sum_{j} Y_{ij} = 1 \qquad \forall i \in \mathcal{I} \qquad (2.26)$$

$$Y_{ij} \leq X_j \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \qquad (2.27)$$

$$X_j, Y_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \qquad (2.28)$$

Equation (2.24) represents the objective function which calculates the total cost of the network. The firs term is the fixed cost of establishing distribution centres. The transportation costs from the supplier to distribution centres and from distribution centres to retailers are computed by the second and third terms, respectively. The fourth, fifth, and sixth terms calculate the holding inventory cost, safety stock cost, and inventory shortage cost, respectively. Constraint (2.25) computes the annual demand for each distribution centre. Constraints (2.26)−(2.27) guarantee that each retailer is allocated to an open distribution centre. Constraint (2.28) represents standard binary variables.

The basic location-inventory model (2.24)−(2.28) has been extended from different aspects to be used in different contexts and conditions. Eppen (1979) was one of the first researchers to compare centralised and decentralised inventory systems for a multilocation newsboy problem with normal demand at each location. Nozick and Turnquist (1998) evaluated the impact of inventory safety stock on location decision of distribution centres for designing an automobile logistic network. For this purpose, they developed a one-to-one inventory replacement model. A two-stage supply chain network was proposed by Sourirajan *et al.* (2007) to find the optimal location of distribution centres while making trade-off between safety stocks and lead times. They developed a

Lagrangian-based heuristic to obtain near-optimal solutions in a reasonable computational time. Ozsen *et al.* (2008) analysed the capacitated warehouse location model with risk pooling to model the interrelationship between the inventory and capacity issue within warehouses. The problem included one single plant that delivers products to multiple retailers with uncertain demands, in which warehouse locations, shipment size, safety stock levels, and retailers' allocation were optimised. The problem was modelled as a nonlinear integer program which was solved using a Lagrangian relaxation approach. Yang *et al.* (2010) examined the effect of distribution centre locations on the performance and profit of a supply chain with a single manufacturer, a single distribution centre, and multiple retailers. Schmitt (2011) worked on multiple strategies to protect the customer service-level when disruption occurs. Ağralı *et al.* (2012) studied the location-inventory problem for the supply chain network design with uncapacitated facilities that served a set of customers for a single product. They formulated the problem as a mixed-integer nonlinear programming model, which was solved utilising a Benders decomposition algorithm.

Various constraints and variables have been included in the location-inventory problem literature. Jayaraman (1998) explored the interconnection between facility location, inventory management, and transportation within a distribution network design problem. They incorporated the transhipment mode decision into the basic location-inventory model. Barahona and Jensen (1998) formulated an integer programming model for location-inventory problem. This model considered the distribution network for computer spare parts, which was solved through a decomposition framework.

Halvorsen-Weare and Fagerholt (2013), modelled a facility location-inventory problem for the Liquefied natural gas distribution network. Miranda and Garrido (2008) presented a location-inventory model for a three-echelon supply chain that considered stochastic demand. Also, a Lagrangian relaxation strategy improved with validity constraints was designed to solve the model. A four-echelon supply chain including a supplier, a central warehouse, several stores and demand nodes was designed by Mahar *et al.* (2009) to determine the optimal location of stores to fulfil customers stochastic demand considering. The costs considered in the objective function were associated with the location, transhipment (i.e. transfer of goods among facilities at the same echelon level), inventory holding and backordering. Firoozi *et al.* (2013) extended the location-inventory problem to consider both EOQ and quantity discount policies for distribution centre inventory control within the supply chain network. The model was solved by developing a two-stage heuristic algorithm. Silva and Gao (2013) investigated the location-inventory model by integrating the locational costs with inventory replenishment costs. van Wijk *et al.* (2012) described a multi-location inventory model that considered Poisson demands. Furthermore, Shavandi and Bozorgi (2012) introduced a nonlinear mixed integer programming model to formulate the location-inventory problem with Fuzzy demand and solved it using a Genetic Algorithm.

2.5.3.2. Dynamic location-inventory models

Dynamic location-inventory models have been developed to address the problem with fluctuations of allocation costs over the planning horizon (Khumawala and Clay Whybark, 1976). Erlenkotter (1981) investigated seven approximation strategies for a location problem in a dynamic setting to minimise total costs while serving customers increasing demands. Viswanadham and Srinivasa Raghavan (2000) evaluated procurement and delivery logistics in order to minimise the total cost of location, inventory carrying, and delays in a dynamic supply chain model.

Melo *et al.* (2006) presented a model to formulate the network design problem under a dynamic planning horizon that considered network configuration decisions and the relocation of facilities. Hinojosa *et al.* (2008) studied the location-inventory problem considering dynamic order size over the planning horizon. The decisions relating to opening new facilities and closing down old ones were incorporated in the model which was solved using a Lagrangian relaxation approach. Gebennini et al. (2009) built a cost-based and mixed-integer programming model for the dynamic location-allocation problem which aimed to optimise the number and location of facilities, the allocation of customer demand to facilities, inventory control, production rates and the service level.

2.5.3.3. Location-inventory transportation models

One of the initial studies that integrated vehicle routing and inventory control problems was carried out by Federgruen and Zipkin (1984). Liu and Lee (2003) formulated a model for the multi-depot location-routing problem together with inventory decisions. A two-stage heuristic method was developed to solve the model. Furthermore, Liu and Lin (2005) presented a heuristic strategy to solve the location-inventory routing model. Since this problem is a nonpolynomial (NP) problem. Ambrosino and Grazia Scutellà (2005) presented mathematical models to design distribution networks considering locational, transportation and inventory decisions as an extension to the work by Perl and Daskin (1985). Moreover, Ma and Davidrajuh (2005) designed a distribution network for an agile virtual environment as a location-inventory model. Max Shen and Qi (2007) presented a nonlinear integer programming model for a supply chain network design under uncertain demand, in which they incorporated routing and inventory costs in the strategic location problem. Ahmadi Javid and Azad (2010) formulated a mixed integer convex program to optimise location, allocation, capacity, inventory, and routing decisions jointly, where customers' stochastic demand follows the normal distribution. Mete and Zabinsky (2010) considered distribution network design with risk management. They proposed a stochastic programming model to optimise the storage locations of medical supplies and required inventory levels. Lieckens *et al.* (2013) studied the remanufacturing network design by optimising location, capacity, and inventory decisions to maintain a high service level for the supply of repairable service parts.

Rudi *et al.* (2001) proposed a joint-profit maximization model to determine the optimal inventory orders at each facility for a network with two locations considering transhipment decisions between them. Additionally, Gen and Syarif (2005) presented an optimisation model to identify the best facility location, distribution and inventory for a production/distribution problem. The impact of transhipment decisions related to ordering policies was analysed by Hu *et al.* (2005) to approximate the optimum (s, S) policies for multi-location inventory systems. Moreover, Özdemir *et al.* (2006) introduced a stochastic optimisation model to deal with stocking locations, inventory and transhipment among them under transportation capacity constraints. Wang *et al.* (2007), proposed a bi-level programming model to enable electronic markets in China to address the location-inventory problem. Decisions associated with location, production, inventory, and transportation was integrated using the continuous approximation approach proposed by Pujari *et al.* (2008). Additionally, a two-echelon logistics network consisting of a single central warehouse and multiple local warehouses were modelled as an integer non-linear program by Kutanoglu and Mahajan (2009). The aim of the study was to find the optimal location of warehouses and their inventory levels. Çapar *et al.* (2011) considered a location-inventory problem within a supply chain constructed from two distribution centres and two retailers under a periodic-review inventory policy. A simulation-based model was proposed by Hochmuth and Köchel (2012) to solve a multi-location-inventory system with lateral transhipment.

2.6. Research gaps

The literature discussed above illustrates a number of research gaps in the containerised transport concept. Firstly, the majority of studies in container shipping network design have related to the seaborn part of the transport chain. However, inland container movements have a significant effect on seaborne container movements (Dong and Song, 2012). Secondly, most of previous literature has treated model parameters as known and deterministic values. In the context of container shipping, there is high variability and fluctuations in the problem parameters such as the supply and demand of containers (Lee and Song, 2017). Thirdly, as discussed in the literature review, the strategic decisions relating to dry port location, and the operational decisions relating to transportation, ECR, and inventory have been examined separately. This overlooks the interdependency of different decision levels in the inland container network design which leads to non-optimal solutions.

Chapter 3. Research Methodology

3.1. Introduction

This chapter outlines the research methodology that was used to achieve the research objectives. In order to explain the appropriate approach to address the dry port network design problem, an overview of modelling and solution methods adopted in the existing literature is provided in Section 3.2. The structure and properties of the research problem under study and the two-stage stochastic programming procedure is elaborated in Section 3.3. Furthermore, Section 3.4 is dedicated to the sample average approximation technique which was used to handle the complexity of stochastic programming models. As mentioned earlier, uncertainties are an important characteristic of the container shipping industry which was thoroughly considered by this research. The robust optimisation concept which ensures the reliability of solutions obtained from two-stage stochastic programming is outlined in Section 3.5. Finally, the Benders decomposition strategy that was used to enhance the computational efficiency of the solution procedures is explained in Section 3.6.

3.2. Overview

In the transport network design uncertainty stems from both the supply and demand sides (Chen *et al.*, 2011). The sources of supply side uncertainty are due to factors such as weather conditions, traffic flow, congestion and construction activities. The uncertainty of demand side, on the other hand, is driven by seasonal effects, special occasions, and customers attributes (Chen *et al.*, 2011). These uncertainties and variations bring about supply and demand fluctuations. In the containerised transport networks, the uncertainty could be driven by seasonal variation in container flows. For instance, the quantity of container flow from Asia to Europe faces a noticeable rise in the last quarter of each year because of Christmas (Meng *et al.*, 2012b). The uncertainties inherent in the number of containers and their availability over time creates further complicates container planning and the ECR problem (Crainic *et al.*, 1993). Furthermore, when network design decisions are made, complete and accurate information is normally unavailable, which can cause inefficient operational planning and a significant increase in container transport costs. In other words, decision makers are not able to predetermine some of the parameters prior to the design stage (Hosseini and Sahlin, 2019). Hence, it is more realistic and important to consider the uncertainty associated with dry port container networks to achieve cost-effective designs with high performance. In the following section, decision making techniques and modelling procedures that have been applied to uncertain transport network design problems are discussed.

3.2.1. Network Design under Uncertainty

- *Expected value models*

The expected value model has been widely applied to transport network design in uncertain environments. The underlying objective is to minimise/maximise the expected value of total costs/profits subject to constrained decision variables (Chen *et al.*, 2011). A bilevel programming model was developed by Yin and Ieda (2002) to cope with uncertainty in transport network design that optimised the capacity and travel time within arcs of the network. Chen and Yang (2004) developed a stochastic model to minimise the expected total travel cost for network designs with uncertain demand. Ukkusuri and Patil (2009) proposed a bilevel stochastic mathematical programming model to formulate a multi-period network design problem under demand uncertainty. They dealt with the uncertainty by staging the investment decision over multiple periods. Furthermore, Chow and Regan (2011) studied the uncertain network design problem by developing network-based models to maximise the expected value of the investment in terms of net present value. Coslovich *et al.* (2006) applied a stochastic programming model to formulate the fleet management problem in a container transportation company. The model attempted to minimise the expected total cost, which comprised the routing costs, the resource assignment costs, and the container repositioning costs. Moreover, to tackle the empty container repositioning problem under uncertain parameters associated with supply, demand, and ship capacity, Long *et al.* (2012) adopted a two-stage stochastic programming approach. The objective of the model was to minimise the expected operational cost of ECR. They used a sample average approximation to estimate the expected objective value.

- *Mean-variance model*

The mean variance modelling approach was devised by Markowitz and Todd (2000) in the finance setting. The main idea of the model is to measure the risk through variance while maximising/minimising the return's expected value/variance. This modelling technique has been employed in the transportation context for designing robust networks under uncertainty. Chen *et* *al.* (2003) applied a stochastic bilevel programming model to deal with transportation network design with uncertain demands. The model aimed to maximise the mean of profit and minimise the risk (i.e. the variance profit). Karoonsoontawong and Waller (2007) formulated the network design problem with origin-destination demand uncertainty via a robust optimisation model. The model focused on minimising the expected total network travel time as well as the expected risk due to uncertain travel demand. Ng and Waller (2009) proposed a mean-variance model for the transportation network design in order to optimise the capacity expansion decisions. Yin *et al.* (2009) attempted to develop robust optimisation models for transport network design in a way that the system performance became less sensitive to the variation of demand. Shu and Song (2014) utilised a two-stage robust optimisation model to address laden container routing and ECR problems under supply and demand uncertainties. Zeng *et al.* (2010) adopted a robust optimisation model in the container shipping context to incorporate demand uncertainty in container resource allocation problems taking into account the risk preference of decision makers.

3.2.2. Decision making under uncertainty

According to the early work of Rosenhead *et al.* (1972), decision-making environments can be classified in terms of *certainty*, *risk*, and *uncertainty*. Under certain environment all parameters are known and deterministic, whilst some or all of the parameters are subject to randomness in risk and uncertain environments. More specifically, in risk environments the uncertain parameters are characterised by known probability distributions. However, in an uncertain environment, there is no information about probabilities of uncertain parameters. Problems associated with risk environments are known as stochastic optimisation problems with the objective of optimising the expected values. Problems within uncertain environments are known as robust optimisation problems with the main goal of optimising the worst-case scenario (Snyder, 2006).

In the modelling process, a performance measure based on the problem context is used to determine the performance quality of solutions. If the probability distribution of random parameters is known, the problem uncertainty is characterised by appropriate distribution parameters. However, in most of the cases this information is unavailable or incomplete; some prespecified intervals may be used to restrict continuous variables (Snyder, 2006).

The dry port container network design problems considered in this study were subject to uncertain environments. As described in this section, stochastic programming and robust optimisation modelling procedures can be employed to formulate the problem. For the purpose of this research, the two-stage stochastic programming method together with robust optimisation were applied. The estimation of the probabilities of uncertainties were addressed by using the sample average approximation method. Furthermore, the computational challenges inherent in the developed models were resolved by adopting the Benders decomposition technique. The mentioned modelling and solution methods are discussed further in the following sections.

3.3. Stochastic programming

In this thesis, the planning of dry port container network design is investigated at both the strategic and operational levels. The other crucial aspect of containerised transportation that has been incorporated in this study is the inherent uncertainty of this industry. The containers' supply and demand are subject to high uncertainty and periodicity which necessitates an approach to model the non-stationary and randomness of parameters. Therefore, deterministic modelling approaches that are employed in the majority of studies in the container shipping literature are unable to capture the uncertainty and periodicity of the problem, where the value of stochastic solutions is high (Birge and Louveaux, 2011). Furthermore, the simultaneous consideration of the strategic and operational levels leads to a temporal hierarchal structure which implies that a single-stage modelling approach cannot cope with the hierarchical decision-making structure of the problem. As a result, a two-stage stochastic programming approach was utilised to model hierarchical decision-making associated with the dry port container network design problem under a non-stationary and uncertain settings. The two-stage stochastic programming was initially introduced by Beale (1955) and Dantzig (1955). This modelling approach has been studied in detail by various scholars (see for example, Kall and Mayer, 1976; Birge and Louveaux, 2011; Prékopa, 2013). Below, the basic stochastic programming problem is explained based on the work of Shapiro (2008).

- *Single-stage stochastic programming*

As discussed earlier, in some business contexts such as container shipping, decisions should take into account uncertain conditions to obtain optimal solutions. Uncertainties can be modelled in various ways depending on the source of uncertainty. With mathematical modelling methods, an objective function f should be optimised (i.e. minimised or maximised) subject to a set of constraints associated with the problem. Let x denote the decision variables vector. The mathematical programming problem (say minimisation problem) can be formulated in terms of an objective function (3.1) and constraint (3.2):

$$\underset{x\in X}{Min} f(x) \tag{3.1}$$

subject to

$$g_i(x) \leq 0 \qquad \forall i \in I \tag{3.2}$$

where, I refers to the set of constraints indexed by i, and $g_i(x)$ denotes the constraints function. Clearly, the objective and constraint functions are dependent on problem parameters. For instance, in the container shipping context, customers' demand affects the total cost of system in the objective function and the flow of containers in the constraint function.

Let ξ denote the vector of parameters. Hence, the objective and constraint functions shown as $f(x,\xi)$ and $g_i(x,\xi)$ associated with the decision vector x and the parameter vector ξ. In almost all business contexts, the parameter vector ξ is uncertain and cannot hold a single value throughout time. One can fix parameters to a prespecified value such as $\xi = \xi^*$ and solve the resultant programming model. However, this could lead to a low-quality solution which cannot hold optimality over the entire planning horizon. The other option to model such a problem is stochastic optimisation. In this view, the uncertain parameter vector ξ is treated as a random vector with a probability distribution of π. The stochastic programming problem can the be formulated as follows:

$$\underset{x\in X}{Min}\ E[f(x,\xi)] \tag{3.3}$$

subject to

$$g_i(x,\xi) \leq 0 \qquad \forall i, \xi \tag{3.4}$$

where, $E[f(x,\xi)]$ is the expected value of the objective function according to the probability distribution π. The presented model is a single-stage stochastic programming model. Below, the two-stage stochastic programming is discussed.

- *Two-stage stochastic programming*

In a two-stage stochastic programming problem, there are two sets of decision variables. For a set of decisions corresponding to vector x, one should make the decision before the realisation of the random parameter ξ becomes available. This decision vector is referred to as *"here-and-now"* which should be made in the first stage as explained. For instance, the decision corresponding to the number and location of dry ports has to be made before the unknown container demands becomes known. The other set of decisions denoted by y, are made in the second stage after a realisation of uncertain parameters ξ becomes known. In the dry port container network design, the second stage decisions include container transportation, ECR, and container inventory control. The second-stage decisions y can be optimised by solving the following mathematical model:

$$\underset{y \in Y}{Min}\, F(x, y, \xi) \tag{3.5}$$

subject to:

$$Tx + Wy \leq h \tag{3.6}$$

As can be seen, the second-stage problem (3.5)−(3.6) depends on the firs-stage decisions x and the uncertain parameters ξ. Let $Q(x, \xi)$ denote the optimal objective solution of the above second-stage problem. The expected value of the second-stage problem is optimised at the firs-stage as the following optimisation problem:

$$Min\, E[Q(x, \xi)] \tag{3.7}$$

Considering the problem (3.5)−(3.7), the two-stage stochastic linear programming with recourse can be formulated as follows:

$$Min\, f(x) = c^T x + E[Q(x, \xi)] \tag{3.8}$$

subject to:

$$Ax + b \leq 0 \tag{3.9}$$

$$Tx + Wy \leq h \tag{3.10}$$

where c, A, and b are the first-stage vectors and matrices.

Solving this two-stage stochastic programming problem requires precise information about the probability of uncertain parameters ξ to compute the expected value $E[Q(x, \xi)]$. In situations with no information about the uncertain probability, the two-stage stochastic model (3.8)−(3.10) can be solved with a scenario-based approach by creating scenarios. More specifically, a set of scenarios $\omega = 1, \ldots, \Omega$ is generated, where each scenario corresponds to a realisation of the uncertain parameter, which is denoted by ξ_ω. Furthermore, a positive weight of π_ω is allocated to each ξ_ω such that $\sum_{\omega=1}^{\Omega} \pi_\omega = 1$. The generated set $\{\xi_1, \ldots, \xi_\omega\}$ of scenarios with the corresponding probabilities $\pi_1, \ldots, \pi_\omega$ are regarded as a representation of the underlying probability distribution. In this context, the expected value function $E[Q(x, \xi)]$ in (3.8) can be computed as the finite summation $E[Q(x, \xi)] = \sum_{\omega=1}^{\Omega} \pi_\omega \, Q(x, \xi_\omega)$. Using the scenario-based approach, the two-stage stochastic programming may be re-formulated as:

$$Min\, f(x) = c^T x + \sum_{\omega=1}^{\Omega} \pi_\omega \, Q(x, \xi_\omega) \tag{3.11}$$

subject to:

$$Ax + b \leq 0 \tag{3.12}$$

$$T_\omega x + W_\omega y_\omega \leq h_\omega \qquad \omega = 1, \ldots, \Omega \tag{3.13}$$

where $\xi_\omega = (T_\omega, W_\omega, h_\omega)$, $\omega = 1, \ldots, \Omega$ are the corresponding scenarios.

Solving the two-stage stochastic programming presented above is quite challenging. In order to solve the problem, a Sample Average Approximation (SAA) can be employed (Ahmed *et al.*, 2002). The solution difficulty as well as the SAA approach are clarified in the following section.

3.4. Sample Average Approximation

The two-stage stochastic programming problem specified by (3.11)−(3.13) is difficult to solve for two main reasons. Firstly, the computation of objective function (3.11) for a given first-stage solution x, requires evaluating the expectation of the linear programming value function $Q(x, \xi_\omega)$. The calculation of this expected value for continuous distributions requires taking multiple integrals which is computationally impractical. For discrete distributions, furthermore, evaluating the expected value involves solving a great number of linear programs (3.7) associated with uncertain parameters' scenarios. Secondly, even if the expected value (3.7) can be calculated, the optimisation of the problem is difficult since $E[Q(x, \xi)]$ is a non-linear function of x (Birge and Louveaux, 2011). Therefore, solving problem (3.11)−(3.13) with the implicit non-linearity is difficult. In order to deal with these difficulties, a Sample Average Approximation method (Mak *et al.*, 1999; Ahmed *et al.*, 2002; Kleywegt *et al.*, 2002) was developed as discussed below.

For adopting the SAA approach, a random sample of N scenarios are generated for the random parameter ξ. In other words, for the realisation of container demand a set of scenarios of size N is generated as $\Omega^N = \{\xi_1, \xi_2, \dots, \xi_N\}$. In this approach, the scenarios are produced with equal probability, i.e., $\pi_\omega = \frac{1}{N}$. Then, the expected value $E[Q(x, \xi_\omega)]$ is approximated by the sample average function $\frac{1}{N}\sum_{\omega=1}^{N} Q(x, \xi_\omega)$. Accordingly, the *"true"* problem (3.8)−(3.10) is approximated as the following problem:

$$Min\, f_N(x) = c^T x + \frac{1}{N}\sum_{\omega=1}^{N} Q(x, \xi_\omega) \tag{3.14}$$

The optimal objective value and the optimal solution vector of SAA problem (3.14) are denoted by v_N and $\widehat{\mathrm{X}}_N$, respectively. It should be emphasised that both v_N and $\widehat{\mathrm{X}}_N$ are random as they are functions of the randomly generated sample. However, for a given realisation of the random sample $\xi_1, \xi_2, \dots, \xi_N$, the SAA problem (3.14) can be solved deterministically using a suitable deterministic optimisation method.

It was shown by Kleywegt *et al.* (2002) that if the sample size N grows, the values of v_N and $\widehat{\mathrm{X}}_N$ converges to the values of the true problem with a probability of one. In addition, as the sample size N increases, the $\widehat{\mathrm{X}}_N$ converges to an optimal solution of the true problem. This implies that the SAA problem (3.14) with a moderate sample size generates a good approximation for the optimal solution of true problem (3.11)−(3.13). The quality of approximated solutions can be analysed in terms of statistical confidence intervals. In order to conduct such an analysis, the problem (3.14) should be solved repeatedly with independent samples. The procedure for providing statistical intervals is described in Section 5.

The statistical validation of the SAA approach was stated by (Norkin *et al.*, 1998a; Norkin *et al.*, 1998b). Then, Mak *et al.* (1999) developed the statistical evaluation further by proposing a sampling procedure. The SAA method was also utilised to deal with stochastic linear programs and stochastic routing problem by Linderoth *et al.* (2006) and Verweij *et al.* (2003), respectively.

In addition, the application of the SAA method in two-stage stochastic programming problems was studied in the work of Kleywegt *et al.* (2002).

3.5. Robust Optimisation

The development of a model for dry port container network design, similar to any other real-world problem, involves uncertain parameters including noisy, incomplete, or erroneous data (Mulvey *et al.*, 1995). The data uncertainty associated with optimisation problems can be driven by the following main reasons (Ben-Tal *et al.*, 2009).

Some data is subject to *prediction errors*. This mainly occurs when one uses forecasts within modelling procedure due to unavailability of data. Future demand and returns are the typical examples.

Some data is subject to *measurement errors* since they cannot be measured exactly. Some decision variables are subject to *implementation errors* as they cannot be applied to practice exactly as computed.

Data uncertainties can be found in different settings including business applications (e.g. returns of financial instruments, customers demand), social sciences (e.g. partial census surveys), physical sciences and engineering. In the mathematical programming context, models are mainly formulated based on “worst-case” or “mean-value” uncertain values (Mulvey *et al.*, 1995). However, Birge (1982) showed that mean value problems lead to large error bounds and worst-case models generate very conservative and expensive solutions.

One way to overcome the gap between real-life problems and the mathematical programming models is the use of *sensitivity analysis*. This post-optimality approach evaluates the effect of data variations on the obtained solutions of model. This implies that sensitivity analysis is a reactive approach as it only demonstrates the effect of uncertainties on generated solutions (Mulvey *et al.*, 1995). Hence, a proactive method is required to formulate the real-world problem that would be able to generate solutions, which are less sensitive to the input data. One approach that can be used is stochastic linear programming as described in the previous section. This approach heavily depends on the availability of historical data for the uncertain parameters to estimate their probability distribution. The scenario-based strategy explained in the previous section can relax the historical data challenge. Yet, uncertain parameters may not have the same or known distributions. This imperfect information as well as the need for the large number of scenarios, could lead to infeasible solutions to the stochastic programming (Neyshabouri and Berg, 2017).

The robust optimisation scheme is the other approach that is introduced to tackle the uncertainty of real-life problems in mathematical programming models which can proactively generate solutions which are optimal and feasible for all realisation scenarios (Ben-Tal *et al.*, 2009). This approach has been adopted for various applications. Paraskevopoulos *et al.* (1991) developed a capacity planning model in the plastic industry that employed a robust approach to address the problem's uncertainties. Sengupta (1991) studied the robustness concept for a stochastic linear programming problem. Escudero *et al.* (1993) adopted a robust optimisation model for outsourcing decisions in a manufacturing context and considered demand uncertainty by generating scenarios. Gutierrez and Kouvelis (1995) used a robust optimisation approach to formulate international production scheduling problems with uncertain exchange rates. In general, robust optimisation schemes can be divided into three approaches: robust scenario-based stochastic programming (Mulvey *et al.*, 1995), robust convex programming (Ben-Tal and Nemirovski, 1998; Ben-Tal and Nemirovski, 2000), and robust fuzzy programming (Pishvaee *et al.*, 2012; Pishvaee and Fazli Khalaf, 2016). The scenario-based robust optimisation approach proposed by Mulvey *et al.* (1995) is detailed in this section, which was adopted by this research.

The main aim of robust optimisation is to produce solutions which are less influenced by the model's uncertain data. This approach is obtained by combining the goal programming concept with a scenario-based description of model's data. The robust optimisation approach may be applied to models with two sets of variables: design variables and control variables. The former is associated with the structural component of the model with fixed input data. The latter, on the other hand, deals with the control component of the model which is subject to uncertain input data. Let x denote the vector of design decision variables whose optimal value is not restricted to the uncertain parameters' realisation. More specifically, when these decisions are made, their value cannot be changed once the data realisation is revealed. In addition, let y denote the vector of control decision variables which are modified once the uncertain parameters are realised. The optimal value of control variables depends on the uncertain parameters' realisation as well as the optimal value of design variables. In the current study, the design variables are related to the number and location of dry ports, which are fixed even when the uncertain containers' supply and demand are realised. Then, the control variables of the research problem are operational decisions associated with containers transportation, ECR and inventory control as their values depend on uncertain demand realisations. Considering the introduced notation, the model with two sets of variables is formulated as:

$$Min\ c^T x + d^T y \tag{3.15}$$

subject to:

$$Ax = b \tag{3.16}$$

$$Bx + Cy = e \tag{3.17}$$

$$x, y \geq 0 \tag{3.18}$$

Equation (3.16) represents the structural constraints, in which coefficients are fixed and known. Equation (3.17) represents the control constraints, where the coefficients are uncertain and dependant on input data.

In the robust optimisation problem, each scenario $\omega \in \Omega$ is associated with a realisation of control constraints' coefficients as the set of $\{d_\omega, B_\omega, C_\omega, e_\omega\}$, where $\sum_{\omega=1}^{\Omega} \pi_\omega = 1$. The mathematical programming model (3.15)−(3.18) "*will be robust with respect to optimality if it remains "close" to optimal for any realisation of the scenario $\omega \in \Omega$. It is then termed solution robust (Mulvey et al., 1995, p. 265)*". In other words, solution robustness concentrates on finding solutions which are optimal for all possible scenarios. Furthermore, the solution obtained from this model is "*robust with respect to feasibility if it remains "almost" feasible for any realisation of ω. It is then termed model robust (Mulvey et al., 1995, p. 265)*". More precisely, model robustness ensures the feasibility of generated solutions over all scenario realisations. Finding a solution for the above problem that can be both feasible and optimal for all scenarios of the set ω is impossible. Similarly, it is unlikely that a solution to the dry port network design problem can remain both feasible and optimal for all scenario realisations of container demand. Hence, a trade-off between the robustness of the solution and model robustness is required. In the following section, a robust optimisation model that can measure the trade-off is outlined.

Let $\{y_1, y_2, \dots, y_\omega\}$ be the set of control variables for each scenario $\omega \in \Omega$. Furthermore, the set $\{z_1, z_2, \dots, z_\omega\}$ was introduced to measure the control constraints' violation for each scenario. Then, a robust optimisation model is formulated as:

$$Min\ \sigma(x, y_1, \dots, y_\omega) + \vartheta\ \rho(z_1, \dots, z_\omega) \tag{3.19}$$

subject to:

$$Ax = b \tag{3.20}$$

$$B_\omega x + C_\omega y_\omega + z_\omega = e_\omega \qquad \forall \omega \in \Omega \tag{3.21}$$

$$x, y_\omega \geq 0 \qquad \forall \omega \in \Omega \tag{3.22}$$

The first term of the objective function (3.19) is the solution robustness, and the second term represents the model robustness weighted by ϑ. More specifically, the term $\rho(z_1, \dots, z_\omega)$ in the objective function is a feasibility penalty function which penalises violations of the control constraints over the scenario set. By considering the set of scenarios in the robust model above, the objective function $f = c^T x + d^T y$ becomes a random variable taking the value of $f_\omega = c^T x + {d_\omega}^T y_\omega$ with occurrence probability π_ω. The solution robustness $\sigma(x, y_1, \dots, y_\omega)$ in the objective function (3.19) was defined by Mulvey *et al.* (1995) as follows:

$$\sigma(z) = \sum_{\omega \in \Omega} \pi_\omega f_\omega + \lambda \sum_{\omega \in \Omega} \pi_\omega \left(f_\omega - \sum_{\omega' \in \Omega} \pi'_\omega f'_\omega \right)^2 \tag{3.23}$$

where λ determines the weight of the solution variance. However, the quadratic term in (3.23) creates a computational burden as it involves a large number of cross-products among variables (Yu and Li, 2000). This quadratic term is then replaced by an absolute deviation term as suggested by Yu and Li (2000) as follows:

$$\sigma(z) = \sum_{\omega \in \Omega} \pi_\omega f_\omega + \lambda \sum_{\omega \in \Omega} \pi_\omega \left| f_\omega - \sum_{\omega' \in \Omega} \pi'_\omega f'_\omega \right| \tag{3.24}$$

The absolute value in the above expression is linearised by the proposed procedure of Leung *et al.* (2007) as the following reformulation:

$$Min\ \phi(z) = \sum_{\omega\in\Omega} \pi_\omega f_\omega + \lambda \sum_{\omega\in\Omega} \pi_\omega \left[\left(f_\omega - \sum_{\omega'\in\Omega} \pi'_\omega f'_\omega \right) + 2\theta_\omega \right] \tag{3.25}$$

subject to:

$$f_\omega - \sum_{\omega'\in\Omega} \pi_{\omega'} f_{\omega'} + \theta_\omega \geq 0 \qquad \forall \omega \in \Omega \tag{3.26}$$

$$\theta_\omega \geq 0 \qquad \forall \omega \in \Omega \tag{3.27}$$

The linearisation method (3.25)−(3.27) can be proved considering two cases. In the first case, $f_\omega - \sum_{\omega'\in\Omega} \pi'_\omega f'_\omega \geq 0$. Then, the minimisation of (3.25) leads to $\theta_\omega = 0$. In this case, $\phi(z) = \sum_{\omega\in\Omega} \pi_\omega f_\omega + \lambda \sum_{\omega\in\Omega} \pi_\omega [(f_\omega - \sum_{\omega'\in\Omega} \pi_{\omega'} f_{\omega'})] = \sigma(z)$. In the second case, $f_\omega - \sum_{\omega'\in\Omega} \pi'_\omega f'_\omega < 0$. Then, the minimisation of (3.25) brings about $\theta_\omega = \sum_{\omega'\in\Omega} \pi'_\omega f'_\omega - f_\omega$. Therefore, $\phi(z) = \sum_{\omega\in\Omega} \pi_\omega f_\omega + \lambda \sum_{\omega\in\Omega} \pi_\omega [(\sum_{\omega'\in\Omega} \pi'_\omega f'_\omega - f_\omega)] = \sigma(z)$. The details of this linearisation approach can be found in Yu and Li (2000).

To guarantee the feasibility of the model over all scenarios (i.e., model robustness), any violation of control constraint (3.17) is penalised with a big penalty cost ϑ. Consequently, the objective function of the robust optimisation model given by (3.19) can be stated as:

$$Min \sum_{\omega\in\Omega} \pi_\omega f_\omega + \lambda \sum_{\omega\in\Omega} \pi_\omega \left[\left(f_\omega - \sum_{\omega'\in\Omega} \pi'_\omega f'_\omega \right) + 2\theta_\omega \right] + \vartheta \sum_{\omega\in\Omega} \pi_\omega z_\omega \tag{3.28}$$

The presented scenario-based robust model can be solved using the SAA method described above. In the following section, the Benders Decomposition method is outlined which improves the solution procedure in terms of computational time.

3.6. Benders Decomposition

The Benders Decomposition (BD) algorithm was proposed by Benders (1962) as an exact solution method to deal with mathematical models which contain complicating variables. The main goal of this approach is to cope with complex models through fixing complicating variables and solving the resulting simplified model. The Benders decomposition algorithm, which is also known as *variable partitioning* (Zaourar and Malick, 2014) and *outer linearisation* (Trukhanov *et al.*, 2010), is broadly applied to solve mathematical programming models by distributing the overall computational burden based on the models' structure. This approach has been employed successfully in various contexts including planning and scheduling (Hooker, 2007; Canto, 2008), energy and resource management (Cai *et al.*, 2001; Zhang and Ponnambalam, 2006), transportation and telecommunication (Costa, 2005), healthcare (Luong, 2015), and chemical process design (Zhu and Kuno, 2003).

The Benders decomposition technique includes a series of estimation, outer linearization, and relaxation (Geoffrion, 1970b; Geoffrion, 1970a). First, the complicating variables are fixed at a given value to generate a subspace of the model. Then, the dual problem of the resulted formulation is utilised to obtain extreme rays and extreme points. The former is used to ensure the feasibility of the model (feasibility cuts) and the latter is employed to guarantee the optimality of the model (optimality cuts). Thus, a corresponding formulation is generated by enumerating all the extreme points and extreme rays. Yet, this enumeration procedure and solving the resulting formulation is computationally impossible. To cope with this difficulty, the corresponding formulation is relaxed in the BD algorithm which results in a master problem and a subproblem. These problems are solved iteratively to generate feasibility and optimality cuts for directing the search process.

The BD algorithm is mainly applied to mixed integer linear programming (MILP) problems. The integer variables are the complicating variables in MILP models. By fixing the integer variables, the model would be transformed to a continuous linear program (LP) which can be dealt with through standard duality theory (Rahmaniani *et al.*, 2017). Two-stage stochastic programming models are mainly formulated as MILP problems. In the dry port network design model, the first stage location decisions are defined by binary variables, while second stage operational decisions are characterised by continuous variables. Based on this structure the Benders decomposition can be utilised to solve this research using the proposed two-stage stochastic programming approach. It is worth mentioning that this algorithm is referred to as the L-shaped decomposition method in the stochastic programming setting (Van Slyke and Wets, 1969). Furthermore, The SAA algorithm presented in Section 3.3 requires a repeated solution of the two-stage stochastic programming problem (3.14). Hence, Benders decomposition can be embedded with the SAA method to improve the solution efficiency (Santoso *et al.*, 2005). In the following, the Benders decomposition algorithm is outlined which will be later applied to solve the dry port network design problem.

The classic version of the BD algorithm proposed by Benders (1962) is presented.

Let x denote the vector of complicating (integer) variables, and y be the vector of continuous variables. The MILP model is given as:

$$Min\ f^T x + c^T y \tag{3.29}$$

subject to:

$$Ax = b \tag{3.30}$$

$$Bx + Dy = d \tag{3.31}$$

$$x \geq 0 \text{ and integer} \tag{3.32}$$

$$y \geq 0 \tag{3.33}$$

where positive integer variables x should meet constraint (3.30). Moreover, constraint (3.31) should be satisfied with both sets of variables x and y.

Let $\bar{x}$ be a given value of integer variables. Employing the value of $\bar{x}$, the model (3.29)−(3.33) can be rewritten as:

$$\underset{\bar{y} \in Y}{Min} \left\{ f^T \bar{x} + \min_{\bar{x} \geq 0} \{ c^T y : Dy = d - B\bar{x} \} \right\} \tag{3.34}$$

The inner minimisation in (3.34) is a continuous linear problem. Let ψ denote the dual variable associated with $Dy = d - B\bar{x}$. The dual formulation of the inner continuous problem is:

$$\underset{\psi}{Max} \{ \psi^T (d - B\bar{x}) : \ \psi^T D \leq c \} \tag{3.35}$$

The primal and dual problems can be interchanged as stated by the duality theory, which leads to the following equivalent formulation:

$$\underset{\bar{x} \in X}{Min} \left\{ f^T \bar{x} + \max_{\psi} \{ \psi^T (d - B\bar{x}) : \ \psi^T D \leq c \} \right\} \tag{3.36}$$

The inner maximisation problem's feasible space, i.e., $F = \{\psi \mid \psi^T D \leq c\}$, is independent of the value of $\bar{x}$. Accordingly, for any given value of $\bar{x}$, the inner problem generates either a feasible solution or unbounded solution. If the problem produces an unbounded solution, there is a direction of unboundedness r_q, $q \in Q$ for which $r_q^T (d - B\bar{x}) > 0$, where Q is the set of extreme rays of space F. In order to avoid the infeasibility driven by the unboundedness, the following cut should be added to the model:

$$r_q^T(d - B\bar{x}) \leq 0 \qquad \forall q \in Q \tag{3.37}$$

The inclusion of this cut restricts the movement in the unboundedness direction and therefore prevents the production of infeasible solutions. On the other hand, if the inner maximisation problem in (3.36) generates a feasible solution, it would be one of the extreme points ψ_e, $e \in E$, where E denotes the set of extreme point of space F. Hence, by adding the feasibility cuts (3.37), the inner problem generates one of its extreme points as the feasible solution. It implies that problem (3.36) can be restated as:

$$\underset{\bar{x}\in X}{Min}\, f^T\bar{x} + \max_{e\in E}\{\psi_e^T(d - B\bar{x})\} \tag{3.38}$$

subject to:

(3.37).

The maximisation function in the objective (3.38) leads to non-linearity in the problem. The problem can be linearised by introducing a continuous variable η. Then, the original problem (3.29)−(3.33) can be given as the following equivalent formulation, which is known as the Benders *Master Problem* (MP):

$$\underset{x,\eta}{Min}\, f^T x + \eta \tag{3.39}$$

subject to:

(3.37),

$$Ax = b \tag{3.40}$$

$$\eta \geq \psi_e^T(d - Bx) \qquad \forall e \in E \tag{3.41}$$

$$x \geq 0 \text{ and integer} \tag{3.42}$$

Constraints (3.37) and (3.41) are called feasibility cuts and optimality cuts, respectively. Since the complete enumeration of these cuts is computationally impractical, Benders (1962) introduced an iterative algorithm through a relaxation of the feasibility and optimality cuts. The BD algorithm solves the above MP problem with only a subset of cuts (3.37) and (3.41) to generate a value for $\bar{x}$. The obtained $\bar{x}$ is then used to solve *subproblem* (3.35). If the subproblem solution is feasible and bounded, an optimality cut (3.41) is generated. However, if the obtained solution from subproblem is unbounded (infeasible), a feasibility cut (3.37) would be generated. Then the current solution is tested through these cuts. If the current solution violates the cuts, they are included into the current MP and the procedure repeats until a stopping criterion is reached.

The algorithm repeats the interaction between the master problem and subproblem until an obtained solution converges to the optimal solution. This convergence is evaluated by calculating the optimality gap at each iteration. The optimality gap is computed using the lower bound and upper bound on the optimal objective solution. The lower bound is obtained from the objective value of the MP as it is a relaxation of the Benders reformulation. The upper bound is gained by solving the subproblem with the $\bar{x}$ obtained from the MP. The Benders decomposition procedure is illustrated in Figure 3.1.

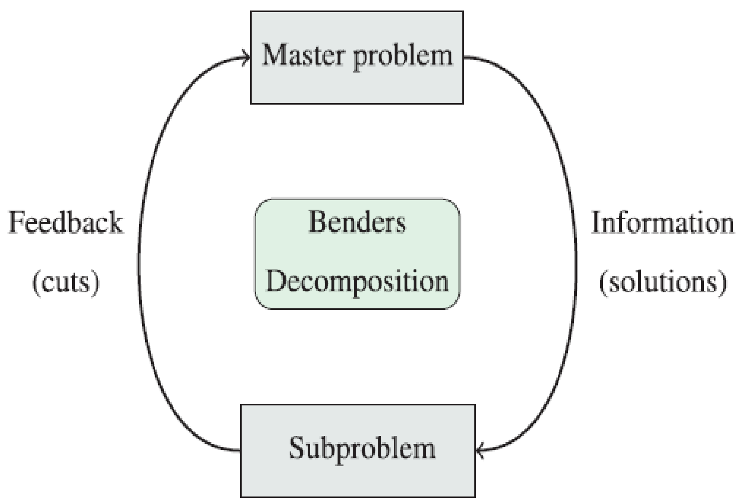


Figure 3.1. Benders decomposition procedure (Rahmaniani et al., 2017).

For the purpose of this research, the two-stage stochastic programming procedure was employed to formulate the dry port network design in the container shipping industry. The reliability of the solutions under the uncertain environment was then ensured by developing the robust counter part of the proposed models based on the concept of robust optimisation. The complexity of the models was handled by adopting the SAA method together with the Benders decomposition. The full problem description, modelling development, and solution strategies are detailed in the following chapters.

Chapter 4. Mathematical model

4.1. Introduction

In this chapter, a comprehensive mathematical model is presented that integrates the strategic and operational decisions associated with the design of a dry port network. The uncertainty in container demand is taken into consideration and has been addressed using a stochastic programming approach. In the following section the detailed research motivation and problem description are discussed.

Globalisation has led to increases in trade volume, cargo size and the number of ships (Yang, 2018). The shipping industry is subject to many short, medium and long-term risks and uncertainties arising from a range of factors including: technical faults, operational problems, finance, market conditions, competition and regulation (Thanopoulou and Strandenes, 2017). The customers of shipping lines typically need door-to-door transportation, which requires intermodal transportation. For them to be successful, it is necessary to achieve a high level performance in the management of their assets within the context of inland networks (Olivo *et al.*, 2013). The inland parts of container shipping networks involve complex distribution in a stochastic environment. The adoption of appropriate hinterland operations leads to higher efficiency and responsiveness which improves competitiveness (Yu *et al.*, 2018).

In this research, carrier haulage operations are incorporated, where the containers are under the responsibility of shipping line throughout the whole process (Yu *et al.*, 2018), which includes hinterland transportation and the storage of containers. Increasing container flows, driven by economic globalisation has given rise to the dry port concept as a joint seaport and hinterland approach (Crainic *et al.*, 2015). The shipping line invests in establishing dry ports in the hinterland network in order to enhance the mentioned container operations. A dry port differs from traditional inland depots by providing more services including: the storage and consolidation of laden containers; depot-storage of empty containers; maintenance and the repair of containers; and customs clearance. The significant advantage of a dry port is the provision of high capacity for different transportation modes with direct connection to seaports, which enables customers to drop off/pick up their containers using dry ports rather than directly using seaports (Roso *et al.*, 2009).

In order for a dry port to be competitive it needs to have enough volume and the operating costs need to be no more than a direct connection to the port (Lättilä *et al.*, 2013). The economic feasibility of an intermodal system may be evaluated in terms of the breakeven distance, which is the distance at which intermodal transport costs equal truck transport costs (Kim and Van Wee, 2011). Therefore, the shipping line should identify the location of dry ports optimally in a container shipping network taking into account the trade-offs between the costs and savings. In this context, it is important to optimise networks by making appropriate strategic and tactical decisions including: the number and location of new dry ports, the allocation of customers to dry ports, the transportation modes for moving containers, as well as empty container inventory and leasing decisions.

Several studies have considered the dry port location problem (e.g., Roso *et al.*, 2009; Ambrosino and Sciomachen, 2014; Wang *et al.*, 2018a) as discussed in Section 2.4. However, these studies neglected the effect of operational decisions and uncertainties associated with the location problem. Also, despite studies such as Wang *et al.* (2018b) applied a sensitivity analysis of model parameters, yet this post-optimality approach was a reactive way to investigate uncertainties and it could not yield robust solutions proactively. As mentioned earlier, in addition to providing transhipment services, dry ports offer a range of other services including: consolidation; the storage of laden and empty containers; maintenance and repair of containers; and customs clearance (Roso *et al.*, 2009 p.341). The provision of storage helps shipping line companies handle the empty container repositioning problem. In the literature the ECR problem has been considered independently from the dry port location problem. As discussed in Section 2.3, since the ECR problem is mainly due to international trade imbalances, most studies have applied network flow optimisation models (see for example, Brouer *et al.*, 2011; Song and Dong, 2012b) to address the problem. Jula *et al.* (2006), Deidda *et al.* (2008), and Furió *et al.* (2013a) analysed different policies for hinterland empty container repositioning within a deterministic environment. Erera *et al.* (2009) developed a robust optimisation framework for dynamic ECR that was modelled by time-space networks. Nonetheless, their approach was limited to the repositioning of empty containers without considering the strategic location decisions of ports or dry ports.

In reality, in a containerised cargo transport chain the demand for empty containers is driven by laden container flows. In addition, both laden and empty containers flows should occur in the same network (Song and Dong, 2015b). This implies that the flow of laden and empty containers should be integrated into the network design problem to capture real-life practice. Xie *et al.* (2017) and Vojdani *et al.* (2013) studied the repositioning of empty containers in hinterland intermodal networks. Although the collaboration of different carriers was studied by Vojdani *et al.* (2013), their model was restricted to a deterministic environment. The flows of laden containers were assumed to be fixed parameters rather than decision variables. This ignored the necessity to simultaneously consider decisions relating to the flow of the laden and empty containers in the optimisation model.

Strategic dry port location decisions should consider the impact of strategic and operational decisions since the deployment of dry ports, intermodal transportation planning and ECR are interdependent decisions (Lee and Song, 2017). For instance, empty container repositioning decisions involve empty containers' waiting time, the amount of movement, as well as destinations (e.g. a manufacturer or a seaport). However, ECR decisions depend on the number and location of dry ports which determines the inbound and outbound transportation times and storage capacity utilisation. There is a temporal hierarchical structure and periodicity between strategic and tactical/operational decisions (Amiri-Aref *et al.*, 2018). It is therefore important to integrate strategic level decisions with the tactical and operational level decisions in containerised transport chain and distribution network models. This research aims to address this problem.

Table 4.1. summarises the literature related to container shipping, which is classified according to: the modelling approach; the structure and periodicity; mode of transport; decision level; and model decisions. The mathematical modelling approach (second column) are mainly including linear programming, mixed integer linear programming, and game theory method. The parameters feature (third column) is relating to the structure and periodicity nature of the problem. The structure of the parameters shows the uncertainty of the research, and the periodicity of the parameters indicate the periodic fluctuations in the studied problem. The fourth column, i.e., mode of transport, specifies the modality of the previous works in container shipping. Next, the decision-making level is presented in fifth column which is divided into strategic, tactical, and operational levels. The tactical and operational levels are shown together as they are quite close to each other and cannot be separated in some applications. The last column is presented to illustrated decisions which have been taken into account in the relevant literature. These decisions involve the facility

location, containers flow or transportation, container inventory, and empty containers repositioning.

This research is positioned relative to the gap in the academic literature in the last row. In contrast to earlier studies on container shipping network design, this thesis considers dry port location, which is a strategic decision, together with the operational decisions relating to the intermodal container flow management in a dynamic and stochastic environment.

The rest of this chapter is organised as follows. Section 4.2 defines the problem and presents the proposed mathematical model in detail. In Section 4.2.1 the hinterland container network and its related decision making problem is presented. Section 4.2.2 provides a detailed two-stage stochastic programming model to address the described dry port container network design problem by integrating strategic and operational uncertain decisions.

Table 4.1. A classification of relevant articles in the literature.

References	Modelling approach	Parameters features		Mode of transport	Decisions Level		Model Decisions			
		Structure	Periodicity		Strategic	Tactical/Operational	Location	Transportation	Inventory	ECR
Jula *et al.* (2006)	LP	Deterministic	Multiple	Single modal	-	✓	-	✓	-	✓
Shintani *et al.* (2007)	LP	Deterministic	Single	Single modal	✓	✓	-	✓	-	✓
Deidda *et al.* (2008)	IP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Erera *et al.* (2009)	IP	Stochastic	Multiple	Single modal	-	✓	-	✓	✓	✓
Zhang *et al.* (2009)	MILP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Brouer *et al.* (2011)	LP	Deterministic	Single	Single modal	✓	-	-	✓	-	✓
Song and Dong (2012b)	IP	Deterministic	Multiple	Single modal	-	✓	-	✓	✓	✓
Meng *et al.* (2012a)	MIP	Deterministic	Single	Intermodal	-	✓	-	✓	-	✓
Furió *et al.* (2013a)	IP	Deterministic	Multiple	Single modal	-	✓	-	✓	✓	✓
Vojdani *et al.* (2013)	IP	Deterministic	Single	Intermodal	✓	-	-	-	✓	✓
Bell *et al.* (2013)	LP	Deterministic	Single	Single modal	-	✓	-	✓	-	✓
Di Francesco *et al.* (2013b)	SP	Stochastic	Single	Single modal	-	✓	-	✓	-	✓
Ambrosino and Sciomachen (2014)	MILP	Deterministic	Single	Intermodal	✓	-	✓	✓	-	-
Chang *et al.* (2015)	LP	Deterministic	Single	Intermodal	✓	-	✓	-	✓	-
Xie *et al.* (2017)	GM	Stochastic	Multiple	Intermodal	✓	-	-	-	✓	✓
Wang *et al.* (2018a)	IP	Deterministic	Single	Intermodal	✓	-	✓	✓	-	-
Yu *et al.* (2018)	GM	Deterministic	Single	Intermodal	-	✓	-	-	✓	✓
Current Study	**SP**	**Stochastic**	**Multiple**	**Intermodal**	✓	✓	✓	✓	✓	✓

LP: Linear Programming; MILP: Mixed-integer Linear Programming; IP: Integer Programming; MIP: Mixed Integer Programming; SP: Stochastic programming; GM: Game model

4.2. Problem definition and formulation

4.2.1. Problem description

A model is proposed for designing a container shipping network that includes the hierarchical decisions relating to dry port location and allocation and creation of arcs (links) between nodes in the network at the strategic level. The allocation of customers to facilities is considered as a strategic decision in previous research including the work by Amiri-Aref *et al.* (2018). At the operational level the model considers the intermodal transportation (flow) of laden and empty containers, as well as empty container repositioning and inventory planning. The model considers the uncertainties associated with containers' demand. It also establishes a hierarchy between different decision levels that uses a robust two-stage stochastic programming model in a multi-period setting. The purpose of the proposed model is to: optimise the location and allocation decisions relating to dry ports at the strategic level; and to minimise the costs associated with the container flow and empty container inventory at the operational level. This integrated approach will enhance the quality of the design and operation of hinterland container shipping networks. Figure 4.1 illustrates a typical hinterland container transport network. Figure 4.1a represents a traditional road and rail network that transports imported and exported raw materials, goods and empty containers between seaports and customers. A customer could be a producer of raw materials, a manufacturer, a distribution centre, freight forwarder, warehouse or retailer. An alternative is to add dry ports to the network as shown in Figure 4.1b. With this configuration there can still be direct flows to the seaport. The dry port can also be used to store empty containers. Hinterland empty container repositioning aims to satisfy the demand for empty containers whilst preventing unnecessary movements (Yu *et al.*, 2018).

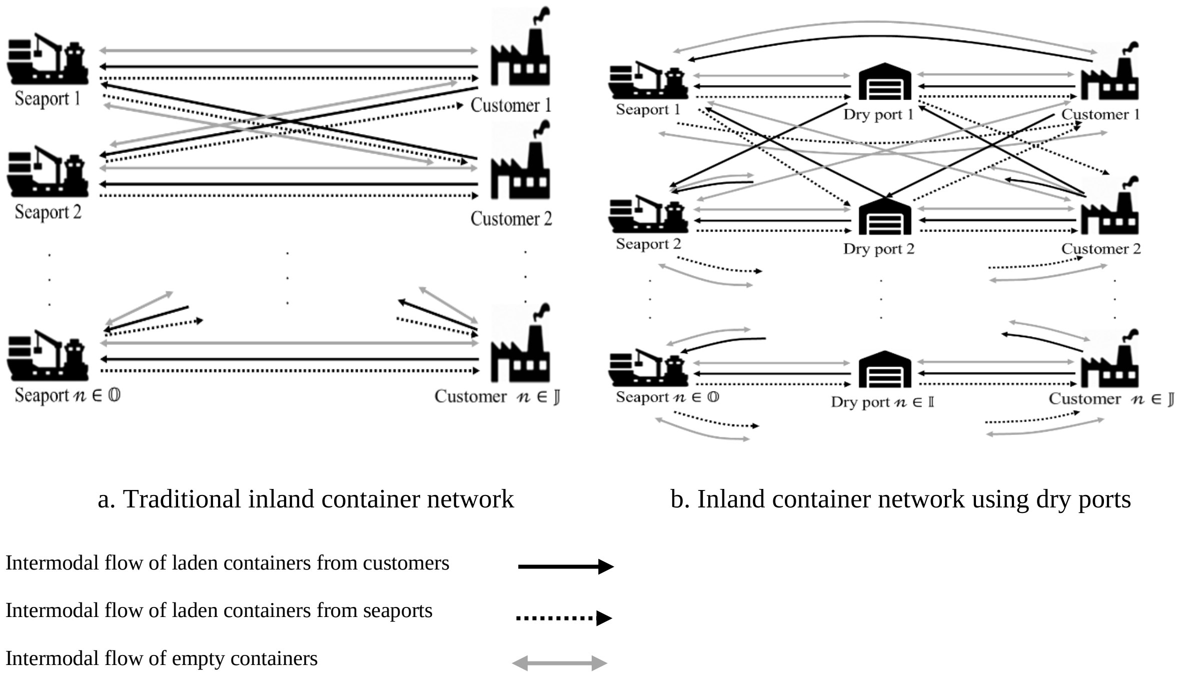


Figure 4.1. Hinterland container transport network

This thesis considers decisions associated with the transportation of empty containers throughout the network. The intermodal transportation of empty containers between the seaports, dry ports and customers are integrated with decisions related to the movement of laden containers throughout the network over the planning periods. The approach determines the optimal inventory levels of empty containers at the storage facilities located at the seaport, dry ports and customers for multiple periods. At the strategic level the model aims to determine the optimal location of dry ports from a given set of possible sites and decide the allocation of customers to dry ports. At the operational level it optimises: the intermodal transportation of laden and empty containers; and the inventory level of empty containers throughout the network. The model takes into account the uncertainties associated with the demand of containers.

4.2.2. Modelling approach

In this section, a robust two-stage stochastic programming model is proposed to cope with the hierarchal decision-making structure. Let $\mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J}$ denote the complete set of nodes in the network, where $\mathbb{O}$ represents the set of seaports, $\mathbb{I}$ represents the set of candidate dry ports, and $\mathbb{J}$ represents the set of customers. The available capacity for storing empty containers at each node is given by Cap_{n}, where $n \in \mathbb{N}$. $\mathcal{A} = \{(p, q): p, q \in \mathbb{N}\}$ represents the set of all arcs (links) in the network, where p, q represent a pair of nodes.

The model includes the abovementioned strategic decisions in the first stage and considers uncertainties in the second stage. The associated decision for the location of dry ports is denoted by the binary variable X_{p} with a fixed opening cost of f_{p}, where $p \in \mathbb{I}$. The binary variable Y_{pq} is defined as the decision associated with the allocation of node p to node q in the considered network with a cost of d_{pq}, where $(p, q) \in \mathcal{A}$. Similarly, Y_{qp} is the binary variable related to the allocation of node q to node p. It should be noted that p and q are used as indices to denote the nodes throughout the network ($i.e. n \in \mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J}$).

Uncertainty involves various causes. One of the main reasons of uncertainty is the lack of coordination between demand and supply, which are subject to high variability and volatility over the planning horizon. The uncertainty in the model is characterised by a set of plausible scenarios, denoted by ω, where each scenario $\omega \in \Omega$ can occur with a probability of $\pi(\omega)$.

Let $\mathbf{X} = \left(X_{p}, Y_{pq}\right)$ be the vector of all first-stage binary design variables relating to the network structure. The first-stage model optimises the strategic location-allocation decisions. The second-stage model optimises the expected operational planning costs. The first-stage objective function and constraints can be formulated as follows:

$$\min_{\mathbf{X}} \left\{ \sum_{\omega \in \Omega} \pi(\omega)\, \mathrm{q}(\mathbf{X}, \omega) + \sum_{p \in \mathbb{I}} f_{p} X_{p} + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} \right\} \tag{4.1}$$

$$Y_{pq} \leq X_{p}, \qquad \forall p \in \mathbb{I}, \forall q \in \mathbb{J} \tag{4.2}$$

$$\sum_{q \in \mathbb{O}} Y_{qp} \geq X_{p}, \qquad \forall p \in \mathbb{I} \tag{4.3}$$

$$\sum_{p \in \mathbb{O}} Y_{pq} + \sum_{p \in \mathbb{I}} Y_{pq} \geq 1, \qquad \forall q \in \mathbb{J} \tag{4.4}$$

$$X_{p}, Y_{pq} \in \{0,1\}, \qquad \forall p, q \in \mathbb{N} = \mathbb{O} \cup \mathbb{I} \cup \mathbb{J} \tag{4.5}$$

The objective function (4.1) includes three terms. The first term is the total expected operational costs of $q(\boldsymbol{X}, \omega)$ which are optimized at the second stage. The second term is the fixed cost of opening a dry port at a candidate location $\forall p \in \mathbb{I}$. The third term indicates the relevant transportation cost relating to the allocation of node p to node q in the network, where $(p, q) \in \mathcal{A}$. Constraint (4.2) ensures that the customers are allocated to opened dry ports. Constraint (4.3) ensures that when a dry port location is chosen, it should be allocated to at least one seaport. Constraint (4.4) ensures that each customer is connected to at least one seaport or one dry port. This constraint also ensures that a given customer can be supplied by both seaports and dry ports. Constraint (4.5) represent first-stage binary variables.

It should be noted that the location and allocation strategic decisions that determine the configuration of the container shipping network, have a significant impact on the second-stage operational decisions. In this study, the operational decisions relating to the flow of empty and laden containers and the inventory levels of empty containers throughout the network are made/revisited on a periodic basis $t \in T$. The set of containers are denoted by $K = \{\ell, e\}$, where ℓ and e are associated with laden and empty containers of Twenty Equivalent Unit (TEU) size, respectively. A FEU is considered as two TEUs (Song & Dong, 2012). All links which connect node p to node q can utilise transportation mode $m \in M$, where M is the set of available transportation modes.

Let c_{pqm}^{k} be the unit cost for transporting container type $k \in K$ on arc $(p, q) \in \mathcal{A}$ when transportation mode m is used. Note that the loading, unloading and operational costs of containers at each node are also included in c_{pqm}^{k}. The unit cost of holding an empty container at each node is denoted by h_n, for all $n \in \mathbb{N}$. Decisions relating to the leasing of empty containers is also considered as a part of inventory planning. If the inventory level of empty containers at a dry-port is not enough to satisfy the demand in a specific period, the shipping line can lease empty containers from lessors or other companies with the cost of g_n^{+}, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$, per unit of container. The shipping line returns the leased containers with a returning cost of g_n^{-}, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$, per unit of container. In addition, if the shipping line has a deficiency of empty containers at a given seaport, it could import its own empty containers from overseas seaports with the unit cost of v_n^{+}, where $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$. Accordingly, v_n^{-} shows the unit cost of returning (exporting) imported containers, where $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$. A unit backorder cost per container b_q is applied for backordered incoming or outgoing demand at a given customer $q \in \mathbb{J}$. In addition, ν denotes the unit cost per container for rejected incoming or outgoing demand from customers. The transportation lead-time is denoted by τ_{pqm}, where $(p, q) \in \mathcal{A}$ and $m \in M$. Moreover, θ denotes the container processing time associated with loading and unloading of finished goods/raw materials at customers sites.

Let $D^{\ell}_{qt}(\omega)$ and $S^{\ell}_{qt}(\omega)$ represent the realisation of the demand for incoming and outgoing laden containers at customer q in period t under scenario ω. Hereafter, we refer to the demand for outgoing laden containers (i.e. $S^{\ell}_{qt}(\omega)$) as "supply" for convenience. Let $F^{k}_{pqtm}(\omega)$ be the decision variable denoting the flow of container type k on arc $(p, q) \in \mathcal{A}$ in period t using transportation mode m under scenario ω. Let $I^{e}_{nt}(\omega)$ denote the inventory level of empty containers at a given node n in period t under scenario ω. Furthermore, let $L^{e+}_{nt}(\omega)$ and $L^{e-}_{nt}(\omega)$ be the number of leased and returned empty containers, respectively, at $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$ in period t under scenario ω. Due to the periodicity of decisions related to empty container leasing, it is important to consider the cost relating to the net number of leased containers at each period. Let us denote the net stock of leased empty containers by $L^{e}_{nt}(\omega) = L^{e}_{n,t-1}(\omega) + L^{e+}_{nt}(\omega) - L^{e-}_{nt}(\omega)$, with the corresponding cost of g_n, where $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$. Note that g^{+}_{n} denotes the fixed leasing cost per container, which is incurred only once when containers are leased, while g_n is the variable leasing cost per container per period for the net stock of leased empty containers remain in the network. Let $H^{e+}_{nt}(\omega)$ and $H^{e-}_{nt}(\omega)$ denote the number of empty containers of the shipping line which are imported and exported, respectively, at the seaport $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$ in period t under scenario ω. The backordered incoming and outgoing demand at a customer $q \in \mathbb{J}$ in period t under scenario ω is denoted by $U^{\ell}_{qt}(\omega)$ and $B^{\ell}_{qt}(\omega)$, respectively. Finally, $\gamma^{\ell}_{qt}(\omega)$ and $\delta^{\ell}_{qt}(\omega)$ represent the rejected incoming and outgoing demand at a customer $q \in \mathbb{J}$ in period t under scenario ω, respectively. The sets, parameters, and decision variables used in the proposed model are summarized in Table 4.2.

Table 4.2. Notation of sets, parameters, and decision variables

Sets	
$\mathbb{N}$	Set of nodes indexed by n.
$\mathbb{O}$	Set of seaports indexed by p, q.
$\mathbb{I}$	Set of candidate dry port locations indexed by p, q.
$\mathbb{J}$	Set of customers indexed by p, q.
$\mathcal{A}$	Set of arcs.
Ω	Set of scenarios indexed by ω.
T	Set of periods indexed by t.
K	Set of containers indexed by k, ℓ, e.
M	Set of available transportation modes indexed by m.
Parameters	
Cap_n	The storage capacity of node $n \in \mathbb{N}$
f_p	The fixed cost for opening a dry port at node $p \in \mathbb{I}$.
d_{pq}	The fixed cost for allocating node p to node q.
c_{pqm}^k	The unit cost of transporting container type k on arc $(p, q) \in \mathcal{A}$ using mode m .
h_n	The unit cost of holding an empty container at node $n \in \mathbb{N}$.
g_n^+/g_n^-	The unit cost of leasing/returning an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
g_n	The unit cost of leased empty containers' net stock per container per period at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
v_n^+/v_n^-	The unit importing/exporting cost of an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$.
b_q	The unit backorder cost per container at customer $q \in \mathbb{J}$.
ν_q	The unit cost of rejected demand per container at customer $q \in \mathbb{J}$.
τ_{pqm}	The transportation lead-time on arc $(p, q) \in \mathcal{A}$ using mode $m \in M$.
θ_q	The containers processing time associate with the loading and unloading of finished goods/raw materials at customers $q \in \mathbb{J}$.
$\pi(\omega)$	The occurrence probability of scenario ω.
$D_{qt}^{\ell}(\omega)$	The demand for incoming laden containers at customer q in period t under scenario ω.
$S_{qt}^{\ell}(\omega)$	The demand for outgoing laden containers at customer q in period t under scenario ω.
First-stage Decision Variables	
X_p	Binary variable associated with location of dry port p.
Y_{pq}	Binary variable associated with the demand allocation of an arc from node p to q.
Second Stage Decision Variables	
$F_{pqtm}^k(\omega)$	The flow of container type k on arc $(p, q) \in \mathcal{A}$ in period t using mode m under scenario ω.
$I_{nt}^e(\omega)$	The inventory level of empty containers at node $n \in \mathbb{N}$ in period t under scenario ω.
$L_{nt}^e(\omega)$	The net stock of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$L_{nt}^{e+}(\omega)$	The number of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.

$L_{nt}^{e-}(\omega)$	The number of returned empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$H_{nt}^{e+}(\omega)$	The number of imported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$H_{nt}^{e-}(\omega)$	The number of exported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$U_{qt}^{\ell}(\omega)$	The backordered incoming demand at a customer q in period t under scenario ω.
$B_{qt}^{\ell}(\omega)$	The backordered outgoing demand at a customer q in period t under scenario ω.
$\gamma_{qt}^{\ell}(\omega)$	The rejected incoming demand at a customer q in period t under scenario ω.
$\delta_{qt}^{\ell}(\omega)$	The rejected outgoing demand at a customer q in period t under scenario ω.
$\mathbf{C}(\omega)$	The vector of all continuous variables for each scenario

Below the second-stage model is presented in which $\mathbf{C}(\omega) = \{F_{pqtm}^{k}(\omega),\ I_{nt}^{e}(\omega),$ $L_{nt}^{e}(\omega), L_{nt}^{e+}(\omega), L_{nt}^{e-}(\omega),\ H_{nt}^{e+}(\omega), H_{nt}^{e-}(\omega), U_{qt}^{\ell}(\omega), B_{qt}^{\ell}(\omega), \gamma_{qt}^{\ell}(\omega), \delta_{qt}^{\ell}(\omega)\}$ denotes the vector of all continuous variables for each scenario ω. This model is applied after the strategic decisions from the first-stage model have been made and the uncertainty in demand is revealed. Therefore, the second-stage model is a collection of deterministic programming models over all possible scenarios. The objective function of the second-stage model for a single scenario $\omega \in \Omega$ is associated with all incurred operational costs, and can be formulated as (4.6.a) – (4.6.e):

$$\min_{\mathbf{C}(\omega)} \mathrm{q}(\mathbf{X}, \omega) = \sum_{(p,q)\in\mathcal{A}} \sum_{t\in T} \sum_{m\in M} \sum_{k\in K} c_{pqm}^{k} F_{pqtm}^{k}(\omega) \tag{4.6.a}$$

$$+ \sum_{t\in T} \sum_{n\in\mathbb{N}} h_n I_{nt}^{e}(\omega) \tag{4.6.b}$$

$$+ \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{O}\cup\mathbb{J}\}} \left(g_n^{+} L_{nt}^{e+}(\omega) + g_n^{-} L_{nt}^{e-}(\omega) + g_n L_{nt}^{e}(\omega)\right) \tag{4.6.c}$$

$$+ \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{I}\cup\mathbb{J}\}} \left(v_n^{+} H_{nt}^{e+}(\omega) + v_n^{-} H_{nt}^{e-}(\omega)\right) \tag{4.6.d}$$

$$+ \sum_{t\in T} \sum_{q\in\mathbb{J}} b_q \left(U_{qt}^{\ell}(\omega) + B_{qt}^{\ell}(\omega)\right) + \sum_{t\in T} \sum_{q\in\mathbb{J}} \nu_q \left(\gamma_{qt}^{\ell}(\omega) + \delta_{qt}^{\ell}(\omega)\right) \tag{4.6.e}$$

Expression (4.6.a) evaluates the total cost of intermodal transportation of laden and empty containers over all arcs involved in the considered network. This expression includes the transportation cost between dry ports and customers, between seaports and dry ports, and between seaports and customers, over all periods. Expression (4.6.b) represents the total holding cost of empty containers throughout all nodes in the network over all periods. Expression (4.6.c) evaluates the total cost associated with empty containers leasing operations, which include the cost of leases, returned containers, and net stock of leased empty containers over the planning periods within the network. Expression (4.6.d) refers to the total cost of importing/exporting empty containers at seaports over all periods. Finally, expression (4.6.e) indicates the total cost of backordered as well as rejected demand of incoming and outgoing containers at customers over the planning horizon. In the second stage of the model, the following constraints need to be followed.

- *Containers demand constraints*

Constraint (4.7) represents the flow of incoming laden containers from all possible dry ports and the set of seaports to a given customer during each period to fulfil its current uncertain demand. The possible backordered demand accumulated from previous periods as well as rejected demand are considered by this constraint. Note that $t - \tau_{\wp q m}$ represents the time when the containers are dispatched using transportation mode m to be delivered to a given customer in period t.

$$\sum_{\wp \in \mathbb{I}} \sum_{m \in M} F^{\ell}_{\wp q, t-\tau_{\wp q m}, m}(\omega) + \sum_{\wp \in \mathbb{O}} \sum_{m \in M} F^{\ell}_{\wp q, t-\tau_{\wp q m}, m}(\omega) + \gamma^{\ell}_{qt}(\omega) = D^{\ell}_{qt}(\omega) + U^{\ell}_{q, t-1}(\omega) - U^{\ell}_{qt}(\omega)$$

$$\forall q \in \mathbb{J}, t \in T, \omega \in \Omega \qquad (4.7)$$

Constraint (4.8) refers to the flow of outgoing laden containers, which should be dispatched from a given customer to all possible dry ports and seaports. This constraint aims to meet the current uncertain demand of outgoing (supply) containers from customer q and their accumulated backordered units from previous periods. The possible rejected demand of outgoing containers is also included here. It should be noted that the customer's outgoing demand in period t would be satisfied, as soon as laden containers are dispatched from the customer in period t. That is why the lead time is excluded from this constraint.

$$\sum_{\wp \in \mathbb{I}} \sum_{m \in M} F^{\ell}_{q \wp t m}(\omega) + \sum_{\wp \in \mathbb{O}} \sum_{m \in M} F^{\ell}_{q \wp t m}(\omega) + \delta^{\ell}_{qt}(\omega) = S^{\ell}_{qt}(\omega) + B^{\ell}_{q, t-1}(\omega) - B^{\ell}_{qt}(\omega)$$

$$\forall q \in \mathbb{J}, t \in T, \omega \in \Omega \qquad (4.8)$$

- *Container flow conservation constraints*

The flow conservation of laden containers at any possible dry port for raw materials and finished goods are ensured by constraints (4.9) and (4.10), respectively. More specifically, constraint (4.9) specifies that the inward containers from all seaports to a given dry port should be equal to the outflow of those containers from the given dry port to all customers. It should be noted that this constraint corresponds to the forward flow of containers. Constraint (4.10) relates to the outgoing flow from customers to a given dry port, which is equal to the flow of those containers from the given dry port to all seaports. This constraint reflects the backward flow of containers in the network considered here. Constraints (4.9) and (4.10) consider the time lag between the decisions to deploy the containers, i.e., $\tau_{\mathscr{p}\mathscr{q}m}$.

$$\sum_{\mathscr{q}\in\mathbb{O}}\sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p},t-\tau_{\mathscr{q}\mathscr{p}m},m}(\omega) = \sum_{\mathscr{q}\in\mathbb{J}}\sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q}tm}(\omega), \qquad \forall \mathscr{p}\in\mathbb{I}, t\in T, \omega\in\Omega \qquad (4.9)$$

$$\sum_{\mathscr{q}\in\mathbb{O}}\sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q}tm}(\omega) = \sum_{\mathscr{q}\in\mathbb{J}}\sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p},t-\tau_{\mathscr{p}\mathscr{q}m},m}(\omega), \qquad \forall \mathscr{p}\in\mathbb{I}, t\in T, \omega\in\Omega \qquad (4.10)$$

- *Container handling constraints*

Container handling constraints determine the inventory level of empty containers which are held in each period. This constraint is applied for each node of the network, including all seaports, dry ports, and customers, in (4.11), (4.12), and (4.13), respectively.

Constraint (4.11) quantifies the inventory level of available empty containers at a given customer at the end of each period. This is calculated by considering the accumulated inventory of empty containers from the previous periods as well as the inflow and outflow of laden and empty containers in the network. More specifically, the second (third) term of the right-hand side of this constraint includes: i) the inflow of laden and empty containers which are transported from all seaports (dry ports) to a given customer in periods $t-\tau_{\mathscr{p}\mathscr{q}m}-\theta$ and $t-\tau_{\mathscr{p}\mathscr{q}m}$, respectively; as well as ii) the outflow of laden and empty containers which are transported from a given customer to all seaports (dry ports) in periods $t+\theta$ and t, respectively. The arriving laden containers are emptied within a specific processing time θ at the given customer and then counted as empty containers for period t. Moreover, the inventory level of empty containers at a given customer is reduced when they are loaded with goods with a specific processing time of θ in order to be sent-out to the dry ports and seaports.

$$I_{qt}^{e}(\omega) = I_{q,t-1}^{e}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm}-\theta_q,m}^{\ell}(\omega) + F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qp,t+\theta_q,m}^{\ell}(\omega) - F_{qptm}^{e}(\omega)\right)$$
$$+ \sum_{p\in\mathbb{I}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm}-\theta_q,m}^{\ell}(\omega) + F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qp,t+\theta_q,m}^{\ell}(\omega) - F_{qptm}^{e}(\omega)\right)$$
$$\forall q \in \mathbb{J}, t \in T, \omega \in \Omega \qquad (4.11)$$

Constraint (4.12) represents the inventory level of available empty containers at each dry port in each period. This is equal to the accumulated inventories from previous periods as well as the inflow and outflow of empty containers in the network. The second (third) term of the right-hand side of this constraint considers the inflow of empty containers which are transported from the seaports (customers) to dry ports and the outflow of empty containers which are transported from the dry ports to the seaports. Note that, the number of leased and returned empty containers are taken into consideration at each dry port in each period.

$$I_{qt}^{e}(\omega) = I_{q,t-1}^{e}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qptm}^{e}(\omega)\right)$$
$$+ \sum_{p\in\mathbb{J}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qptm}^{e}(\omega)\right) + L_{qt}^{e+}(\omega) - L_{qt}^{e-}(\omega)$$
$$\forall q \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (4.12)$$

Constraint (4.13) specifies the inventory level of available empty containers at the seaport. It takes into account the accumulated inventories from the previous periods as well as the inflow and outflow of empty containers. The second (third) term of the right-hand side of this constraint is associated with the inflow of empty containers which are transported from the dry ports (customers) to the seaports and the outflow of empty containers which are transported from the seaports to the dry ports (customers). At each seaport, if the inventory level of empty containers is insufficient to meet the demand, empty containers can be imported from other seaports. The returned and exported empty containers are also considered in this constraint.

$$I_{qt}^{e}(\omega) = I_{q,t-1}^{e}(\omega) + \sum_{p\in\mathbb{I}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qptm}^{e}(\omega)\right)$$
$$+ \sum_{p\in\mathbb{J}}\sum_{m\in M}\left(F_{pq,t-\tau_{pqm},m}^{e}(\omega) - F_{qptm}^{e}(\omega)\right) + H_{qt}^{e+}(\omega) - H_{qt}^{e-}(\omega)$$
$$\forall q \in \mathbb{O}, t \in T, \omega \in \Omega \qquad (4.13)$$

- *Container inter-balancing constraints*

Container inter-balancing constraints imply that the total outflow of containers at each node of the network, including all seaports, dry ports, and customers, should not exceed the available inventory level of empty containers, which are represented by (4.14), (4.15), and (4.16), respectively. Constraint (4.14) ensures that the total outflow of containers from each customer cannot exceed its in-stock empty containers in period t. More specifically, the first term of the left-hand side refers to the total outflow of empty containers in period t and the number of empty containers which will be laden in $t+1$ from a given customer to all possible dry ports. Similarly, the total outflow of empty containers in period t and the number of empty containers which will be laden in $t+1$ from the customers to seaports is shown in the second term of the left-hand side of constraint (14). The inter-balancing constraints give a derived demand for empty containers. The flow of empty containers is driven by the flow of laden containers and determined internally by the shipping lines themselves rather than by external demand.

$$\sum_{q \in \mathbb{I}} \sum_{m \in M} \left(F^{e}_{pqtm}(\omega) + F^{\ell}_{pq,t+1,m}(\omega) \right) + \sum_{q \in \mathbb{O}} \sum_{m \in M} \left(F^{e}_{pqtm}(\omega) + F^{\ell}_{pq,t+1,m}(\omega) \right) \leq I^{e}_{pt}(\omega)$$

$$\forall p \in \mathbb{J}, t \in T, \omega \in \Omega \qquad (4.14)$$

Constraint (4.15) ensures that the net outflow of empty containers from a given dry port to all seaports and customers in each period cannot exceed the available inventory level of empty containers at the given dry port. Correspondingly, constraint (4.16) represents that the net outflow of empty containers from a given seaport to all dry ports and customers should be equal or less than the available inventory level of empty containers at the given seaport.

$$\sum_{q \in \mathbb{O}} \sum_{m \in M} F^{e}_{pqtm}(\omega) + \sum_{q \in \mathbb{J}} \sum_{m \in M} F^{e}_{pqtm}(\omega) \leq I^{e}_{pt}(\omega), \qquad \forall p \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (4.15)$$

$$\sum_{q \in \mathbb{I}} \sum_{m \in M} F^{e}_{pqtm}(\omega) + \sum_{q \in \mathbb{J}} \sum_{m \in M} F^{e}_{pqtm}(\omega) \leq I^{e}_{pt}(\omega), \qquad \forall p \in \mathbb{O}, t \in T, \omega \in \Omega \qquad (4.16)$$

- *Empty containers exchange constraints*

The empty containers exchange constraints guarantee equilibrium between the surplus and deficit of empty containers at the nodes of the network. More specifically, the net empty containers associated with the leased and returned decisions at each seaport and each dry port is given by (4.17). In addition, constraint (4.18) verifies that the total number of returned empty containers at each corresponding node (i.e. each seaport and each dry port) is equal or less than the total number of leased empty containers over the planning horizon. Constraint (4.19) ensures that at a given seaport, the total number of exported empty containers is equal or less than the total number of imported empty containers over the planning horizon.

$$L^{e}_{nt}(\omega) = L^{e}_{n,t-1}(\omega) + L^{e+}_{nt}(\omega) - L^{e-}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}, t \in T, \omega \in \Omega \qquad (4.17)$$

$$\sum_{t \in T} L^{e-}_{nt}(\omega) \leq \sum_{t \in T} L^{e+}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}, \omega \in \Omega \qquad (4.18)$$

$$\sum_{t \in T} H^{e-}_{nt}(\omega) \leq \sum_{t \in T} H^{e+}_{nt}(\omega) \qquad \forall n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}, \omega \in \Omega \qquad (4.19)$$

- *Capacity constraints*

Constraints (4.20) and (4.21) represent the limited capacity for storing empty containers at each node involved in the network in each period.

$$I^{e}_{qt}(\omega) \leq Cap_{q} X_{q} \qquad \forall q \in \mathbb{I}, t \in T, \omega \in \Omega \qquad (4.20)$$

$$I^{e}_{nt}(\omega) \leq Cap_{n} \qquad \forall n \in \mathbb{N} \setminus \mathbb{I}, t \in T, \omega \in \Omega \qquad (4.21)$$

- *Standard constraints*

The flow of containers throughout the whole network is planned based on the allocation decisions which are made by the first-stage model. This is shown by constraint (4.22), where $\mathcal{M}$ is a sufficiently big number.

$$F^{k}_{pqtm}(\omega) \leq \mathcal{M}.Y_{pq} \qquad (p,q) \in \mathcal{A},\ t \in T, m \in M, k \in K, \omega \in \Omega \qquad (4.22)$$

Constraint (4.23) indicates the standard nonnegative continuous variables of the model.

$$F^{k}_{pqtm}(\omega), I^{e}_{pt}(\omega), L^{e}_{pt}(\omega), L^{e+}_{pt}(\omega), L^{e-}_{pt}(\omega), H^{e+}_{pt}(\omega), H^{e-}_{pt}(\omega), U^{\ell}_{pt}(\omega), B^{\ell}_{pt}(\omega), \gamma^{\ell}_{qt}(\omega), \delta^{\ell}_{qt}(\omega) \geq 0$$

$$\forall p, q \in \mathbb{N}, t \in T, m \in M, k \in K, \omega \in \Omega \qquad (4.23)$$

In order to obtain the optimal solution for decision variables, the proposed two-stage stochastic programming model (4.1)−(4.23) should be solved. The uncertainty as well as the hierarchical decision making structure considered in the developed model leads to high complexity which makes the solution procedure quite challenging. To cope with this complexity, a sample average approximation together with a Benders decomposition are applied to solve the model. The complexity, the sample average approximation, and an accelerated Benders decomposition are discussed in the following chapter.

Chapter 5. Solution Procedure

5.1. Introduction

Solving the proposed two-stage stochastic programming model is difficult. The main reason for this difficulty is evaluating the expected value of the objective function for the 'true' model. For discrete distributions, the evaluation of expectation involves solving a large number of linear programming models for each scenario that corresponds to an uncertain parameter realisation. This thesis copes with this difficulty using the Sample Average Approximation approach (Kleywegt *et al.*, 2002; Santoso *et al.*, 2005). The solutions from the SAA approach converge to the optimal 'true' objective function value as the scenario sample size increases (Shapiro *et al.*, 2009). The set of sample scenarios are generated outside of the optimisation procedure using the Monte-Carlo sampling method (Shapiro, 2003). The quality of the solutions obtained by SAA was evaluated through a validation analysis with respect to the sample size and the number of samples. This was achieved by calculating the optimality gap between the optimal 'true' solution for the problem and the SAA model solution from a given sample of scenarios. Moreover, Benders decomposition (Benders, 1962) was applied to enhance the computational performance of the developed SAA model. Also, several acceleration methods were applied within the Benders decomposition method to improve the computational time.

5.2. Robust SAA model

The Monte-Carlo sampling method is employed to generate plausible future scenarios during the planning horizon. This approach uses statistical information on uncertain parameters to generate the sample. In this research, the uncertain containers demand over the planning horizon is characterised by a random variable. For dry port q in period t, the random variable of the demand size process follows Normal distribution function $Nr(.)$. The inverse of this distribution function, $Nr^{-1}(.)$, is used to obtain a realisation of the demand for incoming laden containers at customer q in period t under scenario ω. More specifically, for all $q \in \mathbb{I}$ and $t \in T$: 1) a uniformly pseudorandom number a on the interval $[0,1]$ is generated, 2) the inverse of the distribution $Nr^{-1}(a)$ is computed as the demand value. The values of parameters used in this procedure will be presented in Section 6.1.

In the SAA approach, we run the abovementioned Monte-Carlo sampling procedure for N time to generate a sample of independent container demand scenarios denoted by $\Omega^N = \{\omega^1, \omega^2, \dots, \omega^N\}$. The expected value of the 'true' objective function is estimated by the average over objective values $\mathrm{q}(\boldsymbol{X}, \omega)$, where $\omega \in \Omega^N$ and N is the number of scenarios randomly generated in an equiprobable manner. Therefore, the expected value of the 'true' objective function can be represented by $\sum_{\omega \in \Omega^N} \pi(\omega)\, \mathrm{q}(\mathbf{X}, \omega)$, with $\pi(\omega) = {}^1\!/_N$. Given the original two-stage stochastic problem (4.1) – (4.23), the equivalent deterministic linear programming model is constructed as follows:

$$
\begin{aligned}
\min \Bigg\{ & \sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} \\
& + {}^1\!/_N \sum_{\omega \in \Omega^N} \Bigg(\sum_{(p,q) \in \mathcal{A}} \sum_{t \in T} \sum_{m \in M} \sum_{k \in K} c^k_{pqm} F^k_{pqtm}(\omega) + \sum_{t \in T} \sum_{n \in \mathbb{N}} h_n I^e_{nt}(\omega) \\
& + \sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}} \left(g^+_n L^{e+}_{nt}(\omega) + g^-_n L^{e-}_{nt}(\omega) + g_n L^e_{nt}(\omega) \right) \\
& + \sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}} \left(v^+_n H^{e+}_{nt}(\omega) + v^-_n H^{e-}_{nt}(\omega) \right) + \sum_{t \in T} \sum_{q \in \mathbb{J}} b_q \left(U^\ell_{qt}(\omega) + B^\ell_{qt}(\omega) \right) \\
& + \sum_{t \in T} \sum_{q \in \mathbb{J}} v_q \left(\gamma^\ell_{qt}(\omega) + \delta^\ell_{qt}(\omega) \right) \Bigg) \Bigg\}, \qquad (5.1)
\end{aligned}
$$

subject to:

(4.2) – (4.5),

(4.7) – (4.23),

where the first two terms in (5.1) represent the first-stage objective function and the third term represents the expected objective function of the second-stage problem. Please note that the latter can be achieved by $\pi(\omega)\ \mathrm{q}(\boldsymbol{X}, \omega)$, where we set $\pi(\omega) = {}^1\!/_N$ as explained above and $\mathrm{q}(\boldsymbol{X}, \omega)$ is set to Equations (4.6.a) – (4.6.e).

The SAA method is applicable when a feasible solution exists and the problem has a finite objective value (Shapiro, 2003). However, in the context considered here, the customers' uncertain incoming and outgoing demand may not have an identical distribution function or known distribution parameters. This, in addition to the large number of scenarios, will most likely generate infeasible solutions to the stochastic programming (Neyshabouri and Berg, 2017). To tackle this challenge, a robust counterpart problem was identified for the mentioned SAA method to address solution robustness using an approach proposed by Mulvey *et al.* (1995).

In order to present the robust counterpart problem, the modelling approach proposed by Yu and Li (2000) was applied which is an extension to the work of Mulvey *et al.* (1995). This approach aims to achieve solution and model robustness. The former aims to find solutions which are optimal for all possible scenario realisations of the uncertain parameters; the latter aims to guarantee the feasibility of the obtained solutions over scenario realisations by considering a penalty function (see Mulvey et al. 1995). More specifically, an efficient robust programming model can generate a series of solutions that are progressively less sensitive to realisations of the data in a scenario set. The robust scenario-based stochastic formulation proposed by Mulvey *et al.* (1995) is briefly described as follows:

Consider a linear optimization model as follows:

$$\min f(x, y) = c^T x + d^T y \tag{5.2}$$

$$s.t. Ax = b, \tag{5.3}$$

$$Bx + Cy = e, \tag{5.4}$$

$$x, y \geq 0. \tag{5.5}$$

where $x \in R^{n_1}$ and $y \in R^{n_2}$ denote the vector of the design variables and the vector of the control variables, respectively. The design variables are the variables whose optimal value is not dependent on the realisation of uncertain parameters, and the control variables are those that are conditioned on the realization of uncertain parameters. Note that the coefficients of constraint (5.3) are fixed and “free of noise”, while those for constraint (5.4) are subject to noise. The model may lead to infeasibility under some scenarios due to the parameter uncertainty. Thus, the control variable $\mathfrak{U}(\omega)$, which illustrates the infeasibility of the model under scenario ω should be introduced. A robust optimization model can be defined as follows:

$$\min \sigma\big(x, y(\omega)\big) + \vartheta\rho(u_1, \mathfrak{U}(\omega)) \tag{5.6}$$

$$s.t.\ Ax = b, \tag{5.7}$$

$$B(\omega)x + C(\omega)y(\omega) + \mathfrak{U}(\omega) = e(\omega), \quad \forall \omega \in \Omega^N, \tag{5.8}$$

$$x, y(\omega) \geq 0, \quad \forall \omega \in \Omega^N. \tag{5.9}$$

The first part of the objective function (5.6) is the solution robustness, and the second part represents the model robustness weighted by ϖ.

Considering the robust formulation of (5.6)−(5.9) and recalling $\mathrm{q}(\mathbf{X}, \omega)$ as:

$$\begin{aligned}
\mathrm{q}(\mathbf{X}, \omega) = & \sum_{(p,q)\in\mathcal{A}} \sum_{t\in T} \sum_{m\in M} \sum_{k\in K} c^k_{pqm} F^k_{pqtm}(\omega) \\
& + \sum_{t\in T} \sum_{n\in\mathbb{N}} h_n I^e_{nt}(\omega) \\
& + \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{O}\cup\mathbb{J}\}} \left(g^+_n L^{e+}_{nt}(\omega) + g^-_n L^{e-}_{nt}(\omega) + g_n L^e_{nt}(\omega)\right) \\
& + \sum_{t\in T} \sum_{n\in\mathbb{N}\setminus\{\mathbb{I}\cup\mathbb{J}\}} \left(v^+_n H^{e+}_{nt}(\omega) + v^-_n H^{e-}_{nt}(\omega)\right) \\
& + \sum_{t\in T} \sum_{q\in\mathbb{J}} b_q \left(U^\ell_{qt}(\omega) + B^\ell_{qt}(\omega)\right) + \sum_{t\in T} \sum_{q\in\mathbb{J}} v_q \left(\gamma^\ell_{qt}(\omega) + \delta^\ell_{qt}(\omega)\right)
\end{aligned}$$

The model can be reformulated as the following robust optimisation problem:

$$\begin{aligned}
\min_{\mathbf{X},\mathbf{C}} & \sum_{p\in\mathbb{I}} f_p X_p + \sum_{(p,q)\in\mathcal{A}} d_{pq} Y_{pq} + \sum_{\omega\in\Omega^N} \pi(\omega)\, \mathrm{q}(\mathbf{X}, \omega) \\
& + \lambda \sum_{\omega\in\Omega} \pi(\omega) \left| \mathrm{q}(\mathbf{X}, \omega) - \sum_{\omega'\in\Omega^N\setminus\{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X}, \omega') \right| \\
+ & \sum_{\omega\in\Omega^N} \pi(\omega) \sum_{t\in T} \sum_{\forall q\in\mathbb{J}} \varpi \left(\left| \sum_{p\in\mathbb{I}} \sum_{m\in M} F^\ell_{pq,t-\tau_{pqm},m}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} F^\ell_{pq,t-\tau_{pqm},m}(\omega) - D^\ell_{qt}(\omega) \right| \right. \\
& \left. + \left| \sum_{p\in\mathbb{I}} \sum_{m\in M} F^\ell_{qptm}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} F^\ell_{qptm}(\omega) - S^\ell_{qt}(\omega) \right| \right)
\end{aligned} \tag{5.10}$$

subject to (4.7)−(4.23).

In the model (5.10), $|a|$ is an absolute term of a, λ denotes the determined weight of solution variance which measures the tradeoff between optimality and cost, and $\mathscr{v}$ is the weighting penalty for the lost sale or the overflow of containers at each customer $\mathscr{q} \in \mathbb{J}$ at period $t \in T$.

The term $\sum_{\omega\in\Omega^N} \pi(\omega)\, \mathrm{q}(\mathbf{X},\omega) + \lambda \sum_{\omega\in\Omega^N} \pi(\omega) \left|\mathrm{q}(\mathbf{X},\omega) - \sum_{\omega'\in\Omega^N\setminus\{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega')\right|$ in (5.10) corresponds to the solution robustness of $\sigma(o)$ where the first term is the cumulative expected cost and the second term is the variance of the cost. The two terms of $\sum_{\mathscr{p}\in\mathbb{I}} \sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}(\omega) + \sum_{\mathscr{p}\in\mathbb{O}} \sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}(\omega) - D^{\ell}_{\mathscr{q}t}(\omega)$ and $\sum_{\mathscr{p}\in\mathbb{I}} \sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p}tm}(\omega) + \sum_{\mathscr{p}\in\mathbb{O}} \sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p}tm}(\omega) - S^{\ell}_{\mathscr{q}t}(\omega)$ in (5.10) denote the model robustness of $\rho(o)$, which was used for penalizing the violations of the control constraints of (4.7) and (4.8), capable of adjusting the model in responding to changes in container demand realisations.

There are different approaches to derive the above models proposed by Mulvey *et al.* (1995) and Mulvey and Ruszczyński (1995). These approaches and their limitations are briefly described in the following subsection.

Two different techniques are discussed here to deal with the absolute value term in robust model (5.10): mean variance method and mean absolute deviation method.

- *Mean variance program*

This approach converts the model (5.10) to the following form:

$$\begin{aligned}\operatorname*{Min}_{\mathbf{X},\mathbf{C}} \sum_{\mathscr{p}\in\mathbb{I}} f_{\mathscr{p}} X_{\mathscr{p}} + \sum_{(\mathscr{p},\mathscr{q})\in\mathcal{A}} d_{\mathscr{p}\mathscr{q}} Y_{\mathscr{p}\mathscr{q}} + \sum_{\omega\in\Omega} \pi(\omega)\, \mathrm{q}(\mathbf{X},\omega) + \lambda \sum_{\omega\in\Omega} \pi(\omega) \left[\mathrm{q}(\mathbf{X},\omega) - \sum_{\omega'\in\Omega\setminus\{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega')\right]^2 \\ + \sum_{\omega\in\Omega} \pi(\omega) \sum_{t\in T} \sum_{\forall\mathscr{q}\in\mathbb{J}} \mathscr{v} \left(\left[\sum_{\mathscr{p}\in\mathbb{I}} \sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}(\omega) + \sum_{\mathscr{p}\in\mathbb{O}} \sum_{m\in M} F^{\ell}_{\mathscr{p}\mathscr{q},t-\tau_{\mathscr{p}\mathscr{q}m},m}(\omega) - D^{\ell}_{\mathscr{q}t}(\omega)\right]^2 \right. \\ \left. + \left[\sum_{\mathscr{p}\in\mathbb{I}} \sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p}tm}(\omega) + \sum_{\mathscr{p}\in\mathbb{O}} \sum_{m\in M} F^{\ell}_{\mathscr{q}\mathscr{p}tm}(\omega) - S^{\ell}_{\mathscr{q}t}(\omega)\right]^2 \right) \end{aligned} \tag{5.11}$$

subject to (4.7)−(4.23).

The model formulation derived from this approach has two limitations:

The quadratic terms in (5.11) involve many cross-products among variables and parameters within $\sigma(o)$ and $\rho(o)$. This leads to a computational burden for solving this model. Also, estimating the coefficients of these products is quite difficult.

The penalty cost of infeasibility , $\mathcal{v}$, is forced to be the same values for both positive and negative deviations for violations of the control constraints (4.7) and (4.8). The positive deviation for constraint (4.7) and (4.8) is $\mathfrak{U}_{qt}^{+}(\omega) = \sum_{p\in\mathbb{I}}\sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) - D_{qt}^{\ell}(\omega) \geq 0$ and $\mathcal{K}_{qt}^{+}(\omega) = \sum_{p\in\mathbb{I}}\sum_{m\in M} F_{qptm}^{\ell}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F_{qptm}^{\ell}(\omega) - S_{qt}^{\ell}(\omega) \geq 0$ respectively. Similarly the negative deviation for constraints (4.7) and (4.8) is $\mathfrak{U}_{qt}^{-}(\omega) = \sum_{p\in\mathbb{I}}\sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) - D_{qt}^{\ell}(\omega) \leq 0$ and $\mathcal{K}_{qt}^{-}(\omega) = \sum_{p\in\mathbb{I}}\sum_{m\in M} F_{qptm}^{\ell}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F_{qptm}^{\ell}(\omega) - S_{qt}^{\ell}(\omega) \leq 0$, respectively. This limitation implies that this approach cannot specify different $\mathcal{v}$ values for $\mathfrak{U}_{qt}^{+}(\omega)$ and $\mathfrak{U}_{qt}^{-}(\omega)$(or $\mathcal{K}_{qt}^{+}(\omega)$ and $\mathcal{K}_{qt}^{-}(\omega)$).

- *Mean absolute deviation program*

By using this approach, the robust optimization model is transformed as follows:

$$\min_{\mathbf{X},\mathbf{C}} \sum_{p\in\mathbb{I}} f_p X_p + \sum_{(p,q)\in\mathcal{A}} d_{pq} Y_{pq} + \sum_{\omega\in\Omega} \pi(\omega)\, \mathrm{q}(\mathbf{X},\omega) + \lambda \sum_{\omega\in\Omega} \pi(\omega) \left(\theta^{+}(\omega) + \theta^{-}(\omega)\right)$$
$$+ \sum_{\omega\in\Omega} \sum_{\forall q\in\mathbb{J}} \sum_{t\in T} \mathcal{v}^{+} \left(\mathfrak{U}_{qt}^{+}(\omega) + \mathcal{K}_{qt}^{+}(\omega)\right) + \mathcal{v}^{-} \left(\mathfrak{U}_{qt}^{-}(\omega) + \mathcal{K}_{qt}^{-}(\omega)\right) \tag{5.12}$$

subject to:

$$\mathrm{q}(\mathbf{X},\omega) - \sum_{\omega'\in\Omega} \pi(\omega')\, \mathrm{q}(\mathbf{X},\omega') = \theta^{+}(\omega) - \theta^{-}(\omega) \qquad \omega \in \Omega \tag{5.13}$$

$$\sum_{p\in\mathbb{I}} \sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} F_{pq,t-\tau_{pqm},m}^{\ell}(\omega) = D_{qt}^{\ell}(\omega) + \mathfrak{U}_{qt}^{+}(\omega) - \mathfrak{U}_{qt}^{-}(\omega) \qquad \forall q \in \mathbb{J}, t \in T, \omega \in \Omega \tag{5.14}$$

$$\sum_{p\in\mathbb{I}} \sum_{m\in M} F_{qptm}^{\ell}(\omega) + \sum_{p\in\mathbb{O}} \sum_{m\in M} F_{qptm}^{\ell}(\omega) = S_{qt}^{\ell}(\omega) + \mathcal{K}_{qt}^{+}(\omega) - \mathcal{K}_{qt}^{-}(\omega) \qquad \forall q \in \mathbb{J}, t \in T, \omega \in \Omega \tag{5.15}$$

(4.7)−(4.23).

In this model, $\theta^+(\omega)$ and $\theta^-(\omega)$ denote the deviations for violations of the mean and $\mathfrak{U}_{qt}(\omega)$ and $\mathcal{K}_{qt}(\omega)$ represent the deviations for violations of the control constraints.

Note that constraints (5.13)−(5.15) are equity expressions, where all $\theta^+(\omega)$, $\theta^-(\omega)$, $\mathfrak{U}_{qt}(\omega)$, and $\mathcal{K}_{qt}(\omega)$ appear in both the objective function and the constraint set. Equations (5.12)−(5.15) should be solved through introducing artificial variables into (5.13), (5.14) and (5.15) and using the "two phase" or "big M" method. The big M method change the model to the following program which is equivalent to (5.12)−(5.15):

$$\begin{aligned}\operatorname*{Min}_{\mathbf{X},\mathbf{C}} \sum_{p\in\mathbb{I}} f_p X_p &+ \sum_{(p,q)\in\mathcal{A}} d_{pq} Y_{pq} + \sum_{\omega\in\Omega} \pi(\omega)\, \mathrm{q}(\mathbf{X},\omega) + \lambda \sum_{\omega\in\Omega} \pi(\omega)\left(\theta^+(\omega)+\theta^-(\omega)\right) \\ &+ \sum_{\omega\in\Omega}\sum_{\forall q\in\mathbb{J}}\sum_{t\in T} v^+\left(\mathfrak{U}^+_{qt}(\omega)+\mathcal{K}^+_{qt}(\omega)\right) + v^-\left(\mathfrak{U}^-_{qt}(\omega)+\mathcal{K}^-_{qt}(\omega)\right) + M\sum_{\omega\in\Omega}(\alpha(\omega) \\ &\qquad + \sum_{q\in\mathbb{J}}\sum_{t\in T}\beta_{qt}(\omega))\end{aligned} \tag{5.16}$$

subject to:

$$\mathrm{q}(\mathbf{X},\omega) - \sum_{\omega'\in\Omega} \pi(\omega')\, \mathrm{q}(\mathbf{X},\omega') = \theta^+(\omega) - \theta^-(\omega) + \alpha(\omega) \qquad \omega\in\Omega \tag{5.17}$$

$$\sum_{p\in\mathbb{I}}\sum_{m\in M} F^{\ell}_{pq,t-\tau_{pqm},m}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F^{\ell}_{pq,t-\tau_{pqm},m}(\omega) - D^{\ell}_{qt}(\omega) - \mathfrak{U}^+_{qt}(\omega) + \mathfrak{U}^-_{qt}(\omega) + \beta_{qt}(\omega) = 0$$

$$\forall q\in\mathbb{J}, t\in T, \omega\in\Omega \tag{5.18}$$

$$\sum_{p\in\mathbb{I}}\sum_{m\in M} F^{\ell}_{qptm}(\omega) + \sum_{p\in\mathbb{O}}\sum_{m\in M} F^{\ell}_{qptm}(\omega) - S^{\ell}_{qt}(\omega) - \mathcal{K}^+_{qt}(\omega) + \mathcal{K}^-_{qt}(\omega) + \beta_{qt}(\omega) = 0$$

$$\forall q\in\mathbb{J}, t\in T, \omega\in\Omega \tag{5.19}$$

(4.7)−(4.23).

The M in the objective function (5.16) is a big positive number, and $\alpha(\omega)$ and $\beta_{qt}(\omega)$ are artificial variables. The restriction related to this approach is the large number of new non-negative variables added to the model. More specifically, the cumulative number of these new variables (i.e., $\theta^{+}(\omega)$, $\theta^{-}(\omega)$, $\mathfrak{U}_{qt}^{+}(\omega)$, $\mathfrak{U}_{qt}^{-}(\omega)$, $\mathcal{K}_{qt}^{+}(\omega)$, $\mathcal{K}_{qt}^{-}(\omega)$, $\alpha(\omega)$, $\beta_{qt}(\omega)$) would be:

$$N = 3\left(\Omega + \sum_{\omega=1}^{\Omega} J_{\omega}\right),$$

where Ω denotes the number of scenarios and J_{ω} represents the number of the control constraints under scenario ω.

Accordingly, the total number of constraints would be:

$$L = G + \Omega + \sum_{\omega=1}^{\Omega} J_{\omega},$$

where G denotes the number of constraints in (4.7)−(4.23).

These approaches, as discussed above, are highly restrictive. Hence, in order to deal with the non-linearity in (5.10), the following procedure is proposed.

The solution robustness of the SAA model, denoted by $\Phi(\mathbf{X},\omega)$, is built using the absolute deviation of the second-stage objective values over the number of scenarios, as follows:

$$\Phi(\mathbf{X},\omega) = \left| \mathrm{q}(\mathbf{X},\omega) - \sum_{\omega' \in \Omega \setminus \{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega') \right|. \qquad \forall \omega \in \Omega \tag{5.20}$$

Expression (5.20), which computes the solution robustness, should be included in the objective function of the SAA model. However, this makes the SAA model non-linear due to the presence of absolute function. Hence, a linearisation method is applied to ensure the solution space convexity.

Proposition 1. *As the minimisation objective function of the proposed robust SAA model contains the expression (5.20), this can be substituted by the following expressions:*

$$\Upsilon(\mathbf{X},\omega) = \mathrm{q}(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega')\mathrm{q}(\mathbf{X},\omega') + 2\rho(\omega), \qquad \forall \omega \in \Omega^{N} \tag{5.21}$$

where,

$$q(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') + \rho(\omega) \geq 0, \qquad \forall \omega \in \Omega^N \tag{5.22}$$

$$\rho(\omega) \geq 0. \qquad \forall \omega \in \Omega^N \tag{5.23}$$

Proof. Below, two possible cases of the proposition are verified.

Case 1 is where $q(\mathbf{X},\omega) - \sum_{\omega' \in \Omega \setminus \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') \geq 0$, then it is clear that $\rho(\omega) = 0$, when minimizing expression (5.21). In this case, $\Upsilon(\mathbf{X},\omega) = q(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') = \Phi(\mathbf{X},\omega)$.

Case 2 is where $q(\mathbf{X},\omega) - \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') < 0$. Considering the minimisation of $\Upsilon(\mathbf{X},\omega)$, we then have $\rho(\omega) = \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') - q(\mathbf{X},\omega)$ which results in $\Upsilon(\mathbf{X},\omega) = \sum_{\omega' \in \Omega - \{\omega\}} \pi(\omega') q(\mathbf{X},\omega') - q(\mathbf{X},\omega) = \Phi(\mathbf{X},\omega)$. For more information regarding this linearisation method, refer to Yu and Li (2000).

The stochastic programming model is very likely to return infeasible solutions due to high variability of scenario realisations (Birge and Louveaux, 2011). However, there is no constraint violation in the proposed model as it possesses complete recourse.

Lemma 1. The proposed two-stage stochastic programming model (4.6)−(4.23) is feasible for any possible scenario realisation.

Proof. The term (4.6.e) in objective function (4.6), i.e., $\sum_{t \in T} \sum_{q \in \mathbb{J}} b_q \left(U_{qt}^{\ell}(\omega) + B_{qt}^{\ell}(\omega) \right) + \sum_{t \in T} \sum_{q \in \mathbb{J}} v_q \left(\gamma_{qt}^{\ell}(\omega) + \delta_{qt}^{\ell}(\omega) \right)$, is a feasibility penalty function, which is deployed to penalise the violations of the control constraints of (4.7) and (4.8). In other words, any possible violation related to containers demand constraints is minimised using the relevant penalty costs, i.e., b_q, v_q, in the objective function. The other set of control constraints that can cause infeasibility in the proposed model is containers handling constraints (4.11)−(4.13). The violation of these constraints is also avoided by introducing leasing and importing variables $L_{nt}^{e+}(\omega)$, $L_{nt}^{e-}(\omega)$, $H_{nt}^{e+}(\omega)$, $H_{nt}^{e-}(\omega)$. Any possible violation of these constraints can be avoided by incorporating container leasing variables and including them into objective function using the relevant penalty costs in (4.6.c) and (4.6.d), i.e., $\sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}} \left(g_n^{+} L_{nt}^{e+}(\omega) + g_n^{-} L_{nt}^{e-}(\omega) + g_n L_{nt}^{e}(\omega) \right)$ and $\sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}} \left(v_n^{+} H_{nt}^{e+}(\omega) + v_n^{-} H_{nt}^{e-}(\omega) \right)$. Therefore, the proposed problem is feasible for any possible scenario realisation, where no feasibility cut is needed in MP when applying the Benders Decomposition algorithm to solve the model.

The problem formulation considering solution robustness is addressed by the following equations:

$$\begin{aligned}
\min \Bigg\{ & \sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} \\
& + {}^{1}\!/_{N} \sum_{\omega \in \Omega^N} \Bigg(\sum_{(p,q) \in \mathcal{A}} \sum_{t \in T} \sum_{m \in M} \sum_{k \in K} c^k_{pqm} F^k_{pqtm}(\omega) + \sum_{t \in T} \sum_{n \in \mathbb{N}} h_n I^e_{nt}(\omega) \\
& + \sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}} \left(g^+_n L^{e+}_{nt}(\omega) + g^-_n L^{e-}_{nt}(\omega) + g_n L^e_{nt}(\omega) \right) \\
& + \sum_{t \in T} \sum_{n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}} \left(v^+_n H^{e+}_{nt}(\omega) + v^-_n H^{e-}_{nt}(\omega) \right) + \sum_{t \in T} \sum_{q \in \mathbb{J}} b_q \left(U^\ell_{qt}(\omega) + B^\ell_{qt}(\omega) \right) \\
& + \sum_{t \in T} \sum_{q \in \mathbb{J}} v_q \left(\gamma^\ell_{qt}(\omega) + \delta^\ell_{qt}(\omega) \right) + \lambda \Upsilon(\omega) \Bigg) \Bigg\},
\end{aligned} \tag{5.24}$$

subject to constraints (4.2) – (4.5), (4.7) – (4.23), and (5.21) – (5.23), where λ is the cost of solution robustness. More precisely, λ refers to the weighting scale to measure the trade-off between the cost and dispersion minimization of second-stage objective values over all scenarios.

5.2. The SAA algorithm

In the previous subsection, the robust counterpart problem formulation for the SAA model was obtained. Hereafter we refer to this model as the "robust SAA problem". In this section, a validation analysis is presented that estimates the optimality gap between the objective value associated with the robust SAA problem, and the expected value of the 'true' problem (4.1) – (4.23).

Let $\mathcal{Z}^*_N(\mathbf{X}^*_N, \mathbf{C}^*_N)$ be the optimal objective value of the robust SAA model with scenario sample size N, where $\mathbf{X}^*_N$ and $\mathbf{C}^*_N$ denote the optimal solution vector of the first-stage and the second-stage model, respectively. When the sample size N increases towards infinity, it leads $\mathcal{Z}^*_N(\mathbf{X}^*_N, \mathbf{C}^*_N)$ to converge to the optimal value for the 'true' solution. However, obtaining the optimal value of the true problem, $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$, involves solving the problem for an enormously large number of scenarios. Therefore, statistical confidence intervals are provided that estimate lower and upper bounds on the quality of the approximate solutions based on the work of Shapiro *et al.* (2009). The statistical lower bound and the statistical upper bound for the true optimal objective are estimated by *averaging* and *sampling* procedures, respectively. Finally, the optimality gap of the objective values is calculated. These procedures are presented below.

- *Averaging procedure*

In this procedure, a valid lower bound for the optimal value $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ for the 'true' problem is estimated. In order to do that, R independent samples with the size of N scenarios are generated. Let $\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)$ be the optimal objective value of sample $r = 1, \ldots, R$, of the robust SAA model with N scenarios, and $(\mathbf{X}_N^r, \mathbf{C}_N^r)$ be the corresponding solution vectors. The average of the R replications of the robust SAA model is computed as follows:

$$\bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R) = \frac{1}{R}\sum_{r=1}^{R} \mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r) \quad (5.25)$$

which is an unbiased estimator of the lower bound for the expectation of optimal value $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$. Since the generated samples are independent and have identical distribution, the standard deviation of this estimator can be computed as follows:

$$\hat{\sigma}_N^R = \sqrt{\frac{1}{(R-1)R}\sum_{r=1}^{R}\left(\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r) - \bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R)\right)^2} \quad (5.26)$$

An approximate 100(1-α) % confidence lower bound for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ can be applied using the average and standard deviation of R SAA programs as follows:

$$\mathcal{L}_{N,1-\alpha}^R = \bar{\mathcal{Z}}_N^R(\mathbf{X}_N^R, \mathbf{C}_N^R) - \mathbf{t}_{\alpha,R-1}\hat{\sigma}_N^R \quad (5.27)$$

Here $\mathbf{t}_{\alpha,R-1}$ is the α-critical value of the t-distribution with $R - 1$ degrees of freedom.

- *Sampling Procedure*

The valid estimate of an upper bound on the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ can be computed by sampling. This can be obtained by solving the second-stage of the robust SAA model for a large enough sample of N' scenarios ($\omega \in \Omega^{N'} \subset \Omega$) generated independently, where $N' \gg N$. Let $\bar{\mathbf{X}}_N$ denote the first-stage solution of the initial robust SAA model with sample size N. In this case, we use the best computed $\overline{\mathbf{X}}_N$ among R replications as an input. Let $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ denote the optimal objective value of the robust SAA model with sample size N'. As a result, we have $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}) > \mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ since $\overline{\mathbf{X}}_N$ is a feasible solution of the true problem. Therefore, $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ is an estimation of an upper bound of $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$. It should be clear that $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ involves solving N' second-stage models which produces an optimal objective value per scenario denoted by $\hat{\mathcal{Z}}_\omega\left(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_\omega\right)$, where $\hat{\mathbf{C}}_\omega$ is the solution of the robust SAA second-stage model of scenario $\omega \in \Omega^{N'} \subset \Omega$. Obtaining $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ involves solving the second-stage model per scenario at a time, $\omega \in \Omega^{N'} \subset \Omega$. This is much easier to solve these problems for one scenario at a time. Additionally, since an independent and identical distribution is used to generate the sample N', the variance of this upper bound can be estimated as given in (5.28):

$$\sigma^2_{N'}(\overline{\mathbf{X}}) = \frac{1}{(N'-1)N'} \sum_{\omega=1}^{N'} (\hat{\mathcal{Z}}_\omega(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_\omega) - \hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}))^2 \tag{5.28}$$

Using the above mean and variance, an approximate 100 (1-α) % confidence upper bound for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ is given as:

$$\mathcal{U}_{N',1-\alpha} = \hat{\mathcal{Z}}_{N'}\left(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'}\right) + \mathbf{z}_\alpha\, \sigma_{N'}(\overline{\mathbf{X}}) \tag{5.29}$$

where $\mathbf{z}_\alpha$ is the standard normal critical value with a 100(1-α)% confidence level. Consequently, an approximate 100(1-α)% confidence interval for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ is obtained by $(\mathcal{L}^R_{N,1-\alpha}, \mathcal{U}_{N',1-\alpha})$. Then, the statistical optimality gap and the statistical optimality gap percentage can be calculated by (5.30) and (5.31), respectively:

$$gap_{N,R,N'} = \mathcal{U}_{N',1-\alpha} - \mathcal{L}^R_{N,1-\alpha} \tag{5.30}$$

$$gap_{N,R,N'}\% = \frac{gap_{N,R,N'}}{\mathcal{U}_{N',1-\alpha}} \times 100\% \tag{5.31}$$

The calculated optimality gap is then used to set up a stopping criterion for the algorithm.

The SAA algorithm corresponding to the objective function (5.24) subject to constraints (4.2) – (4.5), (4.7) – (4.23), and (5.21) – (5.23) is summarized in Procedure 5.1.

Inputs: $N, N', R \in \mathcal{N}, \alpha \in [0,1]$, where $\mathcal{N}$ represents the set of Natural numbers.

Pre-processing

For sample $r = 1 \dots R$

Run the Monte-Carlo sampling method to generate random numbers for demand of raw materials' containers with the given distribution function.

`Averaging procedure` (N, R, α)

For sample $r = 1 \dots R$

Solve the robust SAA model (5.24) s.t. (4.2) – (4.5), (4.7) – (4.23), and (5.21) – (5.23), and obtain the objective value $\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)$ and solution $(\mathbf{X}_N^r, \mathbf{C}_N^r)$.

Calculate the average and variance of the objective functions of R robust SAA models using (5.25) and (5.26), respectively.

Compute an approximate 100(1-α) % confidence lower bound using (5.27).

Calculate $\mathcal{Z}_N^R = \min_r\{\mathcal{Z}_N^r(\mathbf{X}_N^r, \mathbf{C}_N^r)\}$ and $\overline{\mathbf{X}}_N = \operatorname{argmin}\{\mathcal{Z}_N^R\}$, which the latter corresponds to the best first-stage design solution found among R samples of the robust SAA model with size N.

`Sampling procedure` $(\overline{\mathbf{X}}_N, N', \alpha)$

For scenario $\omega = 1$ to N'

Solve the robust SAA model (5.24) s.t. (4.2) – (4.5), (4.7) – (4.23), and (5.21) – (5.23) for ω with the obtained first-stage design solution $\overline{\mathbf{X}}_N$ and get the optimal objective value $\hat{\mathcal{Z}}_\omega(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_\omega)$.

Compute the average of optimal objective values as $\hat{\mathcal{Z}}_{N'}(\overline{\mathbf{X}}_N, \hat{\mathbf{C}}_{N'})$ and its variance using (5.28).

Compute an approximate 100(1-α) % confidence upper bound using (5.29).

`Optimality gap`

Calculate the statistical optimality gap percentage using (5.31). If this gap is acceptable, stop. Otherwise, increase N and R by 10 and 1, respectively, and return to step 1.

`Output`: An approximate 100(1-α) % confidence interval for the expectation of optimal $\mathcal{Z}^*(\mathbf{X}^*, \mathbf{C}^*)$ and the associated robust solution $(\mathbf{X}^*, \mathbf{C}^*)$.

Procedure 5.1. The SAA algorithm corresponding to the robust model.

5.3. Benders Decomposition Algorithm

In this section, the abovementioned SAA algorithm is improved by employing the Benders decomposition method as well as several acceleration schemes. Step 2a. of the outlined SAA algorithm in procedure 5.1 requests to solve the robust SAA model repetitively. Although this model contains much fewer scenarios than the true problem (4.1)–(4.23), this two-stage stochastic model is an NP-hard problem which has the same NP-hardness property as a classical capacitated facility location problem (Balinski, 1965). Therefore, a solution approach is proposed inspired by a method introduced by Van Slyke and Wets (1969), which is the stochastic version of classical Benders decomposition applied to two-stage stochastic programming problems. The effectiveness of the method for solving various large optimisation problems including SAA two-stage stochastic programming has been demonstrated in the previous studies, e.g., Santoso et al. (2005). In the following, the Benders decomposition method is described to enhance the SAA algorithm corresponding to the robust model.

In order to implement the Benders decomposition method, one need to separate the robust SAA problem into a master problem (MP) that involves first-stage decision variables, and Benders sub problems (BSP) to optimize the second stage decision variables. The BSP of the proposed model can be formulated by fixing the first-stage variables to the given values at iteration it. The objective function of the BSP is:

$$\min\Big\{{}^{1}\!/_{N}\sum_{\omega\in\Omega^N}\Big(\sum_{(\wp,q)\in\mathcal{A}}\sum_{t\in T}\sum_{m\in M}\sum_{k\in K}c^k_{\wp q m}F^k_{\wp q t m}(\omega)+\sum_{t\in T}\sum_{n\in\mathbb{N}}h_n I^e_{nt}(\omega)+\sum_{t\in T}\sum_{n\in\mathbb{N}\setminus\{\mathbb{O}\cup\mathbb{J}\}}\big(g^+_n L^{e+}_{nt}(\omega)+g^-_n L^{e-}_{nt}(\omega)+g_n L^e_{nt}(\omega)\big)+\sum_{t\in T}\sum_{n\in\mathbb{N}\setminus\{\mathbb{I}\cup\mathbb{J}\}}\big(v^+_n H^{e+}_{nt}(\omega)+v^-_n H^{e-}_{nt}(\omega)\big)+\sum_{t\in T}\sum_{q\in\mathbb{J}}b_q\left(U^\ell_{qt}(\omega)+B^\ell_{qt}(\omega)\right)+\sum_{t\in T}\sum_{q\in\mathbb{J}}\nu_q\left(\gamma^\ell_{qt}(\omega)+\delta^\ell_{qt}(\omega)\right)+\lambda\Upsilon(\omega)\Big)\Big\} \tag{5.32}$$

The objective function (5.32) is constructed from objective function (5.24) by excluding the binary terms, i.e. $\sum_{\wp\in\mathbb{I}}f_\wp X_\wp+\sum_{(\wp,q)\in\mathcal{A}}d_{\wp q}Y_{\wp q}$. The constraints for the BSP are equations (4.7) – (4.23), and (5.21) – (5.23) in which the first-stage variables have been fixed to given values. The given values are updated at each iteration from solving the following MP. Before presenting the MP, it should be pointed out since the modelling approach used in this research possesses complete recourse, the BSP is feasible for the given values of first-stage variables, and an optimality cut can be deducted from an optimal solution to the dual of the sub-problem (DSP).

Let $\boldsymbol{\chi}$ be the vector of the DSP's variables corresponding to constraints (4.7) – (4.23), and (5.21) – (5.23). $\hat{\chi}$ denotes an element in $\boldsymbol{\chi}$, and also the extreme points of the dual polyhedron obtained from solving the dual sub-problem (DSP). The superscripts associated with $\hat{\chi}$ indicate the corresponding constraint. For example, $\hat{\chi}^{4,20}_{qt}(\omega)$ denotes the dual variable relating to constraint 4.20 for $q\in\mathbb{I}, t\in T$. The MP, which produces a lower bound (LB) for the objective function of original robust SAA model at each iteration it, can be formulated based on the defined DSP variables as follows:

$$\text{MP: } \min \left\{ \sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} + \gamma \right\} \tag{5.33}$$

s.t. (4.2) – (4.5),

$$\begin{aligned} \gamma \geq \sum_{\omega \in \Omega^N} {}^{1}\!/_{N} \Bigg(\sum_{t \in T} \Bigg[& \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} D^{\ell}_{qt}(\omega)\, \hat{\chi}^{4.7}_{kpqtm}(\omega) + \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} S^{\ell}_{qt}(\omega)\, \hat{\chi}^{4.8}_{kpqtm}(\omega) \\ & - \sum_{q \in \mathbb{I}} Cap_q X_{q,it}\, \hat{\chi}^{4.20}_{qt}(\omega) - \sum_{q \in \forall n \in \mathbb{N} \setminus \mathbb{I}} Cap_q\, \hat{\chi}^{4.21}_{qt}(\omega) \\ & - \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} \mathcal{M} . Y_{pq,it}\, \hat{\chi}^{4.22}_{kpqtm}(\omega) \Bigg] \Bigg) \end{aligned} \tag{5.34}$$

$$\gamma \geq 0 \tag{5.35}$$

Constraint (5.34) determines the optimality cut. This optimality cut is derived based on constraint (3.41) and the procedure outlined in Section 3.6. More specifically, the first two-terms of the right-hand side of (5.34) are related to the dual formulation associated with constraint (4.7) and constraint (4.8), where $\hat{\chi}^{4.7}_{kpqtm}(\omega)$ and $\hat{\chi}^{4.8}_{kpqtm}(\omega)$ denote dual variables relating to these constraints, respectively. Likewise, the third, fourth and fifth terms of the right-hand side of optimality cut (5.34) are dual formulations related to constraints (4.20), (4.21), and (4.22), respectively, where $\hat{\chi}^{4.20}_{qt}(\omega)$, $\hat{\chi}^{4.21}_{qt}(\omega)$, and $\hat{\chi}^{4.22}_{kpqtm}(\omega)$ denote dual variables of constraints (4.20), (4.21), and (4.22), respectively. It should be noted that for deriving the optimality cut (5.34), the dual variables related to each of constraints in the primal subproblem have been taken into account. However, the right-hand side of most of constraints (i.e., all constraints except (4.7), (4.8), (4.20), (4.21) and (4.22)) in the primal sub-problem is equal to 0. Therefore, they would be excluded from the optimality cut (5.34) as they would not appear in the dual problem's value function. Furthermore, the signs of terms in (5.34) are selected based on the type of constraints (e.g., equality, less than equal, and greater than equal) in the primal sub-problem.

At each iteration of BD, one first solves the MP to obtain the values of first-stage decisions. Then, these values are used to solve DSP to obtain an extreme point and a new optimality cut (5.34) is included in the MP. This procedure is conducted iteratively until the stopping criterion is met. The stopping criterion is established as a small percentage gap between the best upper and lower bounds. Although the described Benders decomposition method is a finite scheme, this algorithm may require a large number of iterations to converge for large optimization models (Santoso et al. 2005). In order to improve the slow convergence of outlined Benders decomposition method, a number of accelerating methods are used in the following subsections.

5.3.1. Multi-cut framework

The number of iterations within Benders decomposition algorithm can be reduced significantly using the multi-cut framework since more dual information is provided for the MP as shown by (Birge and Louveaux, 1988). In the abovementioned BD method, only one cut is added at each iteration, which approximates the sample average of the second-stage objective value function at the current solution. Instead, we add N cuts to the MP by decomposing the BSP into several independent BSPs in each iteration. These cuts approximate the independent second-stage objective functions corresponding to each of individual N scenarios. Thus, we determine the cuts for each scenario and define the new MP as follows:

$$\min \left\{ \sum_{\wp \in \mathbb{I}} f_{\wp} X_{\wp} + \sum_{(\wp,q) \in \mathcal{A}} d_{\wp q} Y_{\wp q} + \sum_{\omega \in \Omega^N} \gamma(\omega) \right\} \tag{5.36}$$

s.t. (4.2) – (4.5),

$$\begin{aligned} \gamma(\omega) \geq \sum_{t \in T} \Bigg[& \sum_{k \in K} \sum_{\wp, q \in \mathbb{N}} \sum_{m \in M} D_{qt}^{\ell}(\omega)\, \hat{\chi}_{k\wp qtm}^{4.7}(\omega) + \sum_{k \in K} \sum_{\wp, q \in \mathbb{N}} \sum_{m \in M} S_{qt}^{\ell}(\omega)\, \hat{\chi}_{k\wp qtm}^{4.8}(\omega) \\ & - \sum_{q \in \mathbb{I}} Cap_q X_{q,it}\, \hat{\chi}_{qt}^{4.20}(\omega) - \sum_{q \in \forall n \in \mathbb{N} \setminus \mathbb{I}} Cap_q\, \hat{\chi}_{qt}^{4.21}(\omega) \\ & - \sum_{k \in K} \sum_{\wp, q \in \mathbb{N}} \sum_{m \in M} \mathcal{M}.\, Y_{\wp q,it}\, \hat{\chi}_{k\wp qtm}^{4.22}(\omega) \Bigg] \end{aligned} \tag{5.37}$$

$$\gamma(\omega) \geq 0, \quad \omega \in \Omega^N \tag{5.38}$$

Note that the cut (5.37) should be added to the MP only if the lower bound, given by the master problem for a specific scenario, is smaller than the objective function of the subproblem for that scenario. This can help avoiding redundant cuts, and thus improves the algorithm's computational time. Applying this multi-cut framework can provide a better approximation of the sample average of the second-stage objective value functions due to the disaggregation of the optimality cut. This accelerated method results in fewer number of BD iterations at the expense of a larger MP.

5.3.2. Knapsack Inequalities

Santoso et al. (2005) showed that including knapsack inequalities together with optimality cut leads to an improved solution from the MP. They indicated that state-of-the-art solvers such as CPLEX can derive a variety of valid inequalities from the knapsack inequality, which accelerates the convergence of the Benders decomposition method. Let UB^* be the current best known upper bound. Since $UB^* > \sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} + \sum_{\omega \in \Omega^N} \gamma_\omega$, one can add the following valid knapsack inequality to the master problem in iteration $it + 1$:

$$\begin{aligned} &\sum_{p \in \mathbb{I}} f_p X_p + \sum_{(p,q) \in \mathcal{A}} d_{pq} Y_{pq} - \sum_{q \in \mathbb{I}} Cap_q X_{q,it}\, \hat{\chi}_{qt}^{4.20}(\omega) - \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} \mathcal{M}.\, Y_{pq,it}\, \hat{\chi}_{kpqtm}^{4.22}(\omega) \\ &\leq UB^* - \sum_{t \in T} \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} D_{qt}^{\ell}(\omega)\, \hat{\chi}_{kpqtm}^{4.7}(\omega) - \sum_{t \in T} \sum_{k \in K} \sum_{p,q \in \mathbb{N}} \sum_{m \in M} S_{qt}^{\ell}(\omega)\, \hat{\chi}_{kpqtm}^{4.8}(\omega) \\ &\qquad + \sum_{t \in T} \sum_{q \in \forall n \in \mathbb{N} \setminus \mathbb{I}} Cap_q\, \hat{\chi}_{qt}^{4.21}(\omega) \end{aligned} \tag{5.39}$$

The adoption of above Knapsack Inequalities leads to a restricted master problem at each iteration which can accelerate the convergence. More specifically, such a valid inequality could eliminate infeasible space of the master problem, thus improve the quality of generated lower bounds. Accordingly, the gap between the upper bound and the lower bound will be tightened and the algorithm will converge to the optimal solution faster (Saharidis *et al.*, 2011).

5.3.3. Pareto-optimal cuts generation scheme

In this section, an acceleration procedure proposed by Magnanti and Wong (1981) is used to strengthen the optimality cuts of Benders decomposition method by generating Pareto-optimal cuts. An optimality cut is called pareto-optimal if there is no other cut to make it redundant. Likewise, the optimal dual solution corresponding to the Pareto-optimal cut is referred to as Pareto-optimal. In BSPs with network structure as in this thesis, the DSP normally has multiple optimal solutions, among which the Pareto-optimal generates the strongest cut. To generate a Pareto-optimal cut, consider the robust two-stage stochastic model as the general problem $Min\ c_1^T\boldsymbol{C} + c_2^T\mathbf{X}$, s.t. $A\boldsymbol{C} + B\mathbf{X} \geq b, \boldsymbol{C} \geq 0, \mathbf{X} \in \{0,1\}$. Fixing integer variables $\mathbf{X} = \widetilde{\mathbf{X}}$, one can write the general form of SP as $Min\ c_1^T\boldsymbol{C},$ s.t.$A\boldsymbol{C} \geq b - B\ \widetilde{\mathbf{X}}, \boldsymbol{C} \geq 0$ and then its DSP is $Max\ \left(b - B\ \widetilde{\mathbf{X}}\right)^T\boldsymbol{\chi}$, s.t. $A^T\boldsymbol{\chi} \leq c_1^T$, $\boldsymbol{\chi} \geq 0$. Let $\mathbf{X}^c$ be a core point of the solution space of MP, and $\boldsymbol{\chi}^*$ be the optimal solution of the DSP. A Pareto-optimal cut can be obtained by solving the following problem, which is referred to as Magnati-Wong problem:

$$Max\ (b - B\ \mathbf{X}^c\ )^T\boldsymbol{\chi}, \text{ s.t. } A^T\boldsymbol{\chi} \leq c_1^T, \left(b - B\ \widetilde{\mathbf{X}}\right)^T\boldsymbol{\chi} \leq \left(b - B\ \widetilde{\mathbf{X}}\right)^T\boldsymbol{\chi}^*, \boldsymbol{\chi} \geq 0 \tag{5.40}$$

At each iteration, the challenge is to identify and update a core point which should lie inside the relative interior of the convex hull of the sub-region defined by the MP variables. To combat this problem, Papadakos (2008) proved that instead of a core point $\mathbf{X}^c$, one can use a convex combination of the current MP solution and the previously used core point to obtain a new core point at each iteration as $\mathbf{X}_{it}^c \leftarrow \varphi\mathbf{X}_{it-1}^c + (1-\varphi)\mathbf{X}_{it}^{MP}, 0 < \varphi < 1$. It should be noted that for the first iteration, $\mathbf{X}_0^c$ is set to the solution of MP.

In the following the proposed accelerated Benders decomposition algorithm is summarised. In the first step, we generate initial feasible solutions for DSP and MP. In step 2, the MP with multi-cut framework and knapsack inequalities is generated and then solved to obtain a lower bound and the first-stage solutions. In the next step, we solve the DSP, generate the Pareto-optimal cut from the corresponding Magnati-Wong problem's solutions, and update the upper bound. Finally, we add the Pareto-optimal cut to the MP and update the core point. The pseudo-code of the proposed accelerated Benders decomposition algorithm is illustrated in Procedure 5.2.

Inputs: $UB_0 = +\infty$, $LB_0 = -\infty$, $\widetilde{\mathbf{X}}_0$, φ, ϵ

Initialisation

Solve the DSP corresponding to arbitrary feasible solutions of $\widetilde{\mathbf{X}}_0$ to obtain an initial dual feasible solution $\widetilde{\boldsymbol{\chi}}_0$

Solve MP and set the core point equal to its solution

$it = 0$

While $(UB_{it} - LB_{it}) > \epsilon$

Master problem ($\widetilde{\boldsymbol{\chi}}_{it}$)

Solve MP with multi-cut and knapsack inequalities, i.e. (5.36)– (5.39), using $\widetilde{\boldsymbol{\chi}}_{it}$

Update LB_{it}

Update $\widetilde{\mathbf{X}}_{it}$

Dual sub-problem ($\widetilde{\mathbf{X}}_{it}, \omega \in \Omega^N$)

For each scenario $\omega \in \Omega^N$ solve DSPs using $\widetilde{\mathbf{X}}_{it}$

If solved to optimality

Generate a Pareto-optimal cut

Update $\widetilde{\boldsymbol{\chi}}_{it}$

Update UB_{it}

End if

Update

Add generated cuts to the MP

$it = it + 1$

Update the core point ($\mathbf{X}^c_{it} \leftarrow \varphi \mathbf{X}^c_{it-1} + (1-\varphi)\mathbf{X}^{MP}_{it}, 0 < \varphi < 1$)

End while

Output: Return $\widetilde{\mathbf{X}}_{it}$ as the optimal solution, and UB_{it} as the optimal objective value.

Procedure 5.2. The accelerated Benders decomposition algorithm

The presented solution strategy in this chapter is applied to solve the proposed model in Chapter 4. A comprehensive numerical experiment is employed to demonstrate the applicability of network design model and the efficiency of the solution strategy techniques. These are discussed in the following chapter.

Chapter 6. Computational Study

6.1. Context for study

In this chapter, a hypothetical case of designing a hinterland container shipping network is described and the results obtained by the proposed solution procedure is presented. The performance indicators and a sensitivity analysis are discussed to provide further managerial insights of interest to shipping line companies. The provided managerial insights are based on the choices of parameters in this study. Finally, the performance experiments of proposed accelerated Benders decomposition algorithm are provided. The solution procedure was coded in CPLEX 12.8.0 and tested on a personal computer with a 3.2 gigahertz Intel Core 5 processor and 16 gigabytes of RAM.

The hypothetical case study was based in North Carolina State which includes a seaport at Wilmington and fifty manufacturers. A set of eight predesignated locations were selected as the candidate points for establishing dry ports in the state. These points were chosen from sites in cities with more than 70,000 inhabitants. A one-year planning horizon ($T = 12$) was considered as the temporal scope of this study. Below the input data used in this study is presented.

An average fixed cost of \$120 per TEU was considered for opening a dry port at each candidate location (Ambrosino and Sciomachen, 2014). The available storage capacity at dry port candidate location $\wp \in \mathbb{I}$ was randomly generated within $[2.0, 5.0]\times 10^4$ TEUs. A storage capacity of 2000 TEUs was taken for each manufacturer and 10,000 TEUs for the seaport. According to Ballou (2004), the unit cost for transporting container type k on arc $(\wp, q) \in \mathcal{A}$ follows a flat rate for each transportation mode, which is calculated as $c^k_{\wp q m} = \mathrm{t}_{\wp q m}\beta_m$, where $\mathrm{t}_{\wp q m}$ denotes the transportation time on arc $(\wp, q) \in \mathcal{A}$ using transportation mode m, and β_m denotes the transportation cost parameter of transportation mode m. The transportation time for mode m on arc $(\wp, q) \in \mathcal{A}$ is estimated by $\mathrm{t}_{\wp q m} = \frac{\Delta_{\wp q}}{V_m}$, where $\Delta_{\wp q}$ denotes the travel distance from node $\wp$ to node q, and V_m represents the average speed of transportation mode m. The set of transportation modes include road and rail, $M = \{1,2\}$. Let set $\beta_1 = \$3.88$ and $V_1 = 60$ mph for the road mode, and $\beta_2 = \$0.05$ and $V_2 = 24$ mph for the rail mode according to FAF3 dataset used in Ballou (2004). The unit cost of holding an empty container at the seaport, a dry port, and a manufacturer are set to \$2, \$4, and \$8, respectively which are adopted from (Dang *et al.*, 2012).

The Normal distribution is good for modelling economic variables such as demand (Kamath and Pakkala, 2002). Therefore, incoming demand follows $D_{qt}^{\ell}(\omega) \sim Nr\left(\mu_{D_{qt}^{\ell}}, \sigma_{D_{qt}^{\ell}}\right)$, where $\mu_{D_{qt}^{\ell}}$ and $\sigma_{D_{qt}^{\ell}}$ refer to the mean and standard deviation of Normal probability distribution function, respectively. The supply of outgoing goods $S_{qt}^{\ell}(\omega)$ could be calculated accordingly since a linear proportional relationship between incoming and outgoing goods is considered. With regards to the Normal distribution, the random value of incoming goods demand for a given customer $q \in \mathbb{J}$ over period $t \in T$ are given as $\mu_{D_{qt}^{\ell}} \in [6000{,}7000]$, where the mean-standard deviation ratio is set to $\mu_{D_{qt}^{\ell}}/\sigma_{D_{qt}^{\ell}} = 10$. To avoid generating negative values for containers demand, Lognormal distribution is used to generate random demands. Hence, Normal distribution parameters (i.e., $\mu_{D_{qt}^{\ell}}$ and $\sigma_{D_{qt}^{\ell}}$) should be scaled in order to generate the data with Lognormal distribution. The mean and standard deviation of Lognormal distribution are denoted by $\mu'_{D_{qt}^{\ell}}$ and $\sigma'_{D_{qt}^{\ell}}$, respectively. The computation of Lognormal scaled parameters from normal parameters are given as:

$$\mu'_{D_{qt}^{\ell}} = e^{(\mu_{D_{qt}^{\ell}} + \frac{\sigma_{D_{qt}^{\ell}}}{2})},$$

$$\sigma'_{D_{qt}^{\ell}} = \sqrt{e^{\left(2\,\mu_{D_{qt}^{\ell}} + \sigma_{D_{qt}^{\ell}}{}^{2}\right)}(e^{\sigma_{D_{qt}^{\ell}}{}^{2}} - 1)}$$

Following the work of Ambrosino and Sciomachen (2014), different cost structures were taken into account, with two levels of fixed cost for opening a dry port and two levels of empty container holding costs, as shown in Table 6.1.

Table 6.1. Cost structure.

Cost structures	fixed cost for opening a dry port ($)	cost of holding an empty container ($)*
(a)	$[1.8,4.0]\times10^{6}$	$[2, 4, 8]\times10^{-1}$
(b)	$[1.8,4.0]\times10^{6}$	$[2, 4, 8]\times10$
(c)	$[4.0;7.5]\times10^{6}$	$[2, 4, 8]\times10^{-1}$
(d)	$[4.0;7.5]\times10^{6}$	$[2, 4, 8]\times10$

* $[h_{o}, h_{i}, h_{j}]$, $\forall o \in \mathbb{O}, i \in \mathbb{I}, j \in \mathbb{J}$.

As mentioned before, the quality of solutions in the SAA method is related to the sample size. We calibrated, tested and validated the SAA model. For this validation, three different samples of size

$N = 20$, $N = 40$, and $N = 50$ and the four cost structures as shown in Table 6.1 were considered. This design gave rise to 12 instances which were used to estimate the statistical optimality gap. Table 6.2 shows the results obtained from the SAA model, where $R = 4$ and $N' = 150$ were used to calculate the statistical optimality gap percentage with a 95% confidence interval using equation (5.31). The number of samples and their size are adopted from (Amiri-Aref *et al.*, 2018).

Table 6.3. Statistical optimality gap.

	Sample size											
	$N = 20$				$N = 40$				$N = 50$			
Cost structure	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Solution time (seconds)	242.4	156.9	261.4	217.7	93.2	614.5	603.6	2621.0	973.8	799.8	1056.4	3506.3
$gap_{N,R,N'}$%	1.08	1.07	1.08	1.08	1.03	1.02	1.03	1.02	**1.01**	**1.00**	**1.01**	**1.00**

When N scenarios were used in the proposed SAA model with a planning horizon of 12 periods; a combination of 12×N sample scenarios were included for solving each instance. The results show that the quality of the SAA model solutions increased; the percentage optimality gap decreased as the sample size N increased. This validation analysis demonstrates that with the maximum considered sample size, the SAA method generated satisfactory optimality gaps of approximately 1%. Accordingly, the sample size $N = 50$ is used for numerical experiments.

In order to attain comprehensive results, the problem was tested across different values of the solution robustness and rejected demand coefficients. Accordingly, a wide range of values of λ and ν were used for each problem instance which were $\lambda = \{10^{-3}, 10^{-2}, 10^{-1}, 1\}$ and $\nu = \{1000, 2000, 3000, 4000, 5000\}$. The range of these values are adopted from various studies (Mulvey *et al.*, 1995; Zarrinpoor *et al.*, 2018). The combination of different values of these coefficients as well as the four cost structure levels shown in Table 6.1 yielded 80 problem instances.

6.2. Key performance indicators

In the robust SAA model presented in this thesis, a cost-based objective function was used for designing dry port container networks. In this section three further key performance indicators (KPIs) are defined: *service level*, *fill rate*, and *inventory turnover*, to provide further insights into the performance of network designs. The three KPIs are supplements to the cost-based objective function and can be calculated based on the optimal solution of the robust SAA model.

Let denote $\widehat{U}^{\ell}_{qt}(\omega)$ and $\widehat{B}^{\ell}_{qt}(\omega)$ as the solution value of variables $U^{\ell}_{qt}(\omega)$ and $B^{\ell}_{qt}(\omega)$, respectively. Also, the service level, which corresponds to the demand of raw materials and the supply of finished goods for each scenario is denoted by $\eta_r(\omega)$ and $\eta_s(\omega)$, respectively. Let $\xi_r(\omega)$ and $\xi_s(\omega)$ be the fill rate corresponding to the demand of raw materials and the supply of finished goods for each scenario, respectively. The service level is defined as the fraction of customer demand that is met through available stock. The service level for the demand of raw materials (in TEUs) for each scenario over all periods and all customers (manufacturers) is calculated as $\eta_r(\omega) = 1 - \frac{\sum_{q\in\mathbb{J}}\sum_{t\in T}\widehat{U}^{\ell}_{qt}(\omega)}{\sum_{q\in\mathbb{J}}\sum_{t\in T}D^{\ell}_{qt}(\omega)} \times 100\%$. Then the expected value of $\eta_r(\omega)$ over all scenarios is estimated by:

$$\bar{\eta}_r = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\eta_r(\omega) \tag{6.1}$$

Likewise, the service level corresponding to the supply of finished goods for each scenario over all periods and all customers (manufacturers) is calculated by $\eta_s(\omega) = 1 - \frac{\sum_{q\in\mathbb{J}}\sum_{t\in T}\widehat{B}^{\ell}_{qt}(\omega)}{\sum_{q\in\mathbb{J}}\sum_{t\in T}D^{\ell}_{qt}(\omega)} \times 100\%$, and its expected value over all scenarios by:

$$\bar{\eta}_s = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\eta_s(\omega) \tag{6.2}$$

The fill rate is defined as the percentage of demand orders which are satisfied on time and in full. The fill rate corresponding to the demand orders of raw materials for each scenario over all periods and all customers (manufacturers) is calculated by:

$$\xi_r(\omega) = \frac{\sum_{q\in\mathbb{J}}\sum_{t\in T}\widetilde{U}^{\ell}_{qt}(\omega)}{|\mathbb{J}||T|} \times 100\% \tag{6.3}$$

where $\widetilde{U}^{\ell}_{qt}(\omega) = 1$ if $\widehat{U}^{\ell}_{qt}(\omega) = 0$ and zero, otherwise. Then the expected value of $\xi_r(\omega)$ over all scenarios is computed by $\bar{\xi}_r = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\xi_r(\omega)$. Likewise, the fill rate corresponding to the supply orders of finished goods is calculated by $\xi_s(\omega) = \frac{\sum_{q\in\mathbb{J}}\sum_{t\in T}\tilde{B}^{\ell}_{qt}(\omega)}{|\mathbb{J}||T|} \times 100\%$, where $\tilde{B}^{\ell}_{qt}(\omega) = 1$ if $\widehat{B}^{\ell}_{qt}(\omega) = 0$ and zero, otherwise, and its expected value by:

$$\bar{\xi}_s = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\xi_s(\omega) \tag{6.4}$$

The inventory turnover of empty containers reflects the number of times that empty containers were replenished at all dry ports over the planning horizon. The total throughput of empty containers from all dry ports for each scenario $\omega \in \Omega^N$ is denoted by $\psi(\omega) =$

$\sum_{\wp\in\mathbb{I}}\sum_{t\in T}\sum_{m\in M}\left(\sum_{q\in\mathbb{O}}\hat{F}^{e}_{\wp q t m}(\omega)+\sum_{q\in\mathbb{J}}\hat{F}^{e}_{\wp q t m}(\omega)\right)$, where $\hat{F}^{e}_{\wp q t m}(\omega)$ represents the solution of the flow of empty containers from dry port $\wp \in \mathbb{I}$ to the seaport $q \in \mathbb{O}$ and manufacturer $q \in \mathbb{J}$. Let also denote the average inventory level of empty containers in each period by $\iota(\omega) = \frac{1}{|T|}\left(\sum_{\wp\in\mathbb{I}}\sum_{t\in T}\hat{I}^{e}_{\wp t}(\omega)\right)$, where $\hat{I}^{e}_{\wp t}(\omega)$ represents the solution corresponding to the decision variable $I^{e}_{\wp t}(\omega)$. Therefore, the inventory turnover of empty containers for each scenario can be indicated by $\varepsilon(\omega) = \frac{\psi(\omega)}{\iota(\omega)}$ and its expected value by:

$$\bar{\varepsilon} = \frac{1}{|\Omega^N|}\sum_{\omega\in\Omega^N}\varepsilon(\omega) \tag{6.5}$$

6.3. Numerical results and discussion

In this section the solutions generated by the proposed two-stage stochastic model is discussed. In Section 6.3.1, the solutions obtained from the 80 explained instances are analysed regarding to the network configuration, container flow decisions, and key performance indicators. In Section 6.3.2, the performance of the robustness approach is discussed. Finally, 6.3.3, is dedicated to the computational efficiency of applied solution procedure employing the proposed accelerated Benders decomposition algorithm.

6.3.1. Solution analysis

- *Network configuration*

Dry ports location decisions are presented for all the instances based on various cost structures and different values of λ shown in Figure 6.1. This illustrates the significant impact of operational costs on shipping network design. The horizontal axis represents the cost structures, and the vertical axis shows the average number of dry ports to be opened over all instances. With low holding costs for keeping empty containers, i.e. cost structures (a) and (c), the shipping network tends to open more facilities (approximately 7 dry ports) in order to achieve more robust solutions. This outcome indicates that under these cost structures it is more cost-effective to decentralise the storage of empty containers by opening more dry ports across the region to cope with the existing uncertainty. On the other hand, centralisation of empty containers storage is recommended under cost structures (b) and (d) which aims to obtain more robust solutions with minimal cost.

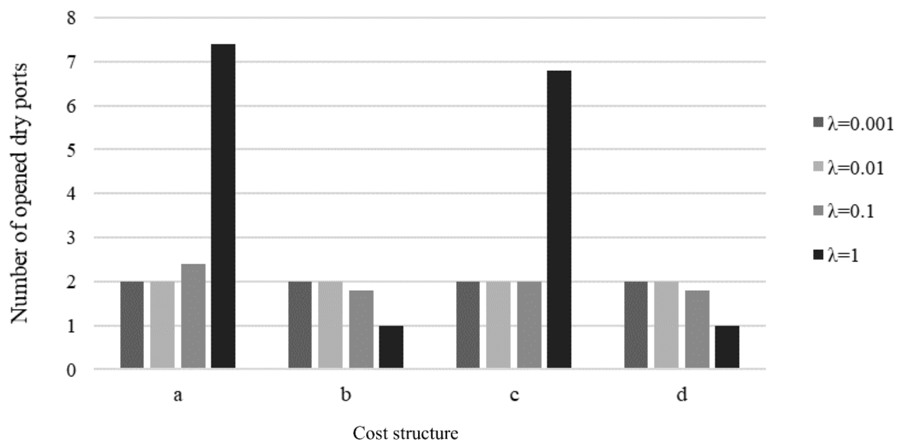


Figure 6.1. Location decisions on the number of opened dry ports.

To have a better view of the network configuration, one of the instances described above is chosen and the corresponding solution is plotted. Figure 6.2 shows the location of dry ports and their allocation to manufacturers. The location of the seaport, dry ports, and manufacturers are shown as orange, red, and blue icons, respectively. For this instance, the developed model proposes two dry ports which are located fairly close to the seaport with a sufficient distance from/to all manufacturers. The distant manufacturers, located far away from the seaport, are mainly multiple-sourced by dry ports.

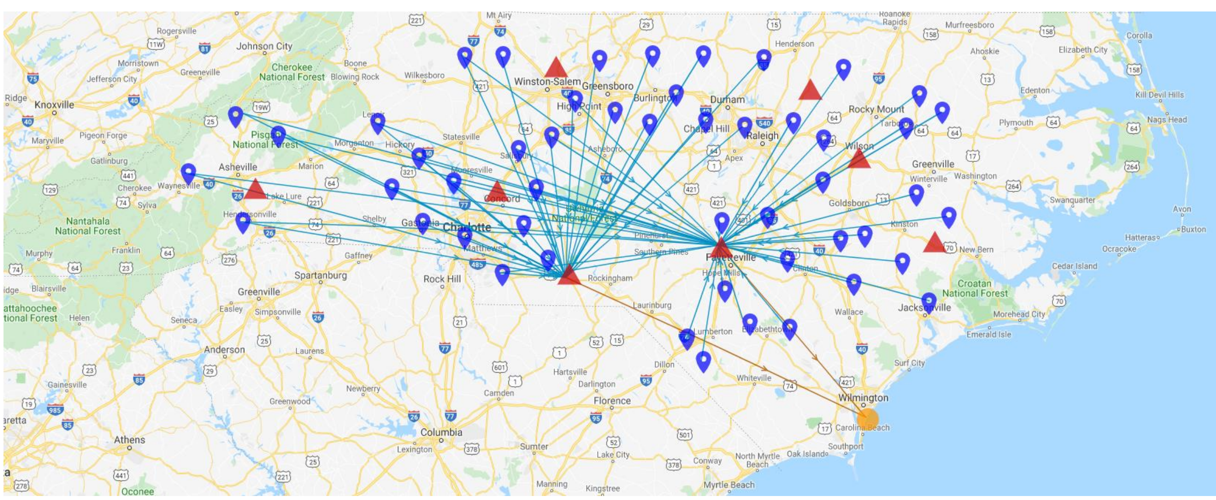


Figure 6.2. Geographical presentation of the network in North Carolina.

Figure 6.3 provides a comparison to illustrate the effectiveness of the obtained solutions from the models developed in this study. In this figure, different cost components related to the above optimal network and a given feasible, but not optimal, solution are provided. Also, the total cost which involves containers transportation cost, holding cost, leasing cost, and dry port locating cost is shown in the figure.

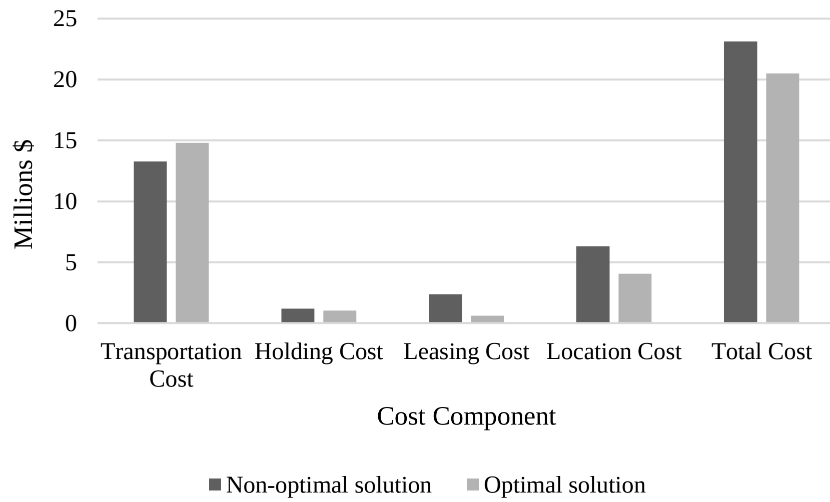


Figure 6.3. Cost components results.

It can be seen that the transportation cost of achieved optimal solution is higher than the given solution. However, the given non-optimal solution suggests utilising three DPs. Accordingly, the locating cost of non-optimal design is higher than the optimal container network solution. Most importantly, one can observe that the total cost related to the optimal dry port container network configuration is lower than the given solution for the network. This indicates that the proposed models can minimise the total costs related to the network design by making a trade-off among different cost components. The detailed network configuration results related to the location-allocation are provided in Tables 6.3-6.6.

Table 6.4. Network configuration decisions results of cost structure (a)				
λ	ν	Nb Dry Ports	Allocation decision	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	67%	33%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	68%	32%
	2000	2	70%	30%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	3	100%	0%
	2000	3	100%	0%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2.4	81%	19%
1	1000	5	88%	12%
	2000	8	0%	100%
	3000	8	0%	100%
	4000	8	0%	100%
	5000	8	0%	100%
Average		7.4	18%	82%

Table 6.5. Network configuration decisions results of cost structure (b).

λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	1	100%	0%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		1.8	74%	26%
1	1000	1	100%	0%
	2000	1	100%	0%
	3000	1	100%	0%
	4000	1	100%	0%
	5000	1	100%	0%
Average		1	100%	0%

Table 6.6. Network configuration decisions results of cost structure (c).

λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	34%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	67%	33%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	2	88%	12%
	2000	2	88%	12%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	76%	24%
1	1000	2	100%	0%
	2000	8	0%	100%
	3000	8	0%	100%
	4000	8	0%	100%
	5000	8	0%	100%
Average		6.8	20%	80%

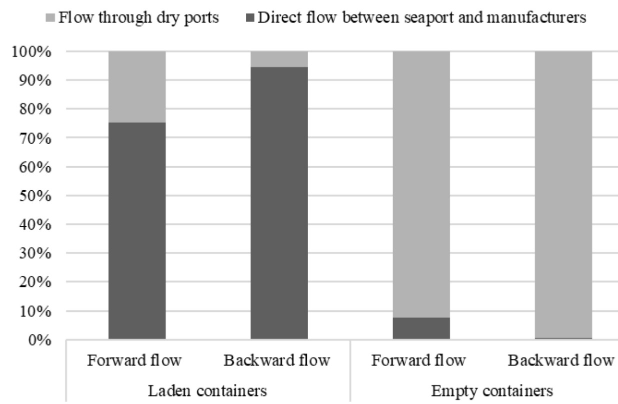
Table 6.7. Network configuration decisions results of cost structure (d)

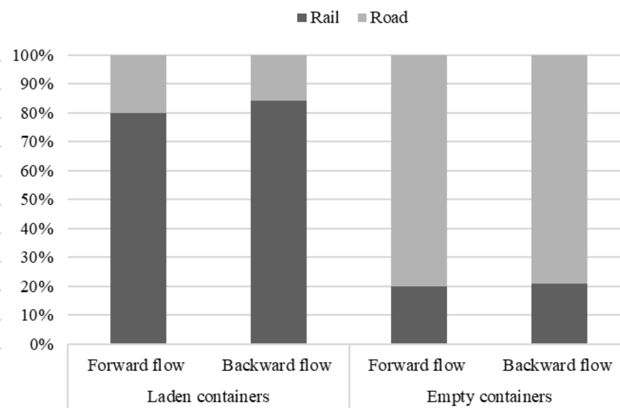
λ	ν	Nb Dry Ports	Allocation/Sourcing rule	
			Single-sourcing	Multiple-sourcing
0.001	1000	2	68%	32%
	2000	2	70%	30%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.01	1000	2	68%	32%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		2	68%	32%
0.1	1000	1	100%	0%
	2000	2	68%	32%
	3000	2	68%	32%
	4000	2	68%	32%
	5000	2	68%	32%
Average		1.8	74%	26%
1	1000	1	100%	0%
	2000	1	100%	0%
	3000	1	100%	0%
	4000	1	100%	0%
	5000	1	100%	0%
Average		1	100%	0%

The results regarding to the number of opened dry ports and allocation decisions obtained from all described problem instances are presented in Tables 6.3-6.6. It should be noted that Tables 6.3, 6.4, 6.5, and 6.6 are related to the cost structures (a), (b), (c), and (d) respectively. The allocation decisions are shown according to the sourcing strategies of single-sourcing and multiple-sourcing, where the former shows the percentage of manufacturers allocated to only one opened dry port, while the latter denotes the percentage of manufacturers allocated to more than one established dry ports. Furthermore, the average of mentioned results over different values of ν for a given λ are provided for further transparency.

- *Containers flow decisions*

Figure 6.4 summarises the flow of empty and laden containers in the network and the mode of transport applied. Figure 6.4(a) shows that the laden containers were mostly transported directly between the seaport and manufacturers, whilst the empty containers were mainly transported between these nodes through dry ports. Moreover, the results suggest that the backward flow of laden containers from manufacturers to the seaport contributed to a higher percentage of direct flow than the forward flow from the seaport to manufacturers. Figure 6.4(a) also shows that the usage of dry ports is more practical for empty container repositioning especially in the backward flow, i.e., from manufacturers to the seaport. This confirms the importance of deploying dry ports in the container shipping networks for the repositioning of empty containers. Figure 6.4(b) shows that a higher percentage of laden containers were transported by rail, whilst road was the main transportation mode for the flow of empty containers. Each transportation mode was used almost equally in the forward and backward laden containers flow. The results were similar for empty containers. The percentage flow using rail for laden containers was higher for empty containers, since the transportation costs were reduced by taking the advantage of the economies of scale for direct flows which involved longer distances than the indirect flows through dry ports.





(a) Containers flow.

(b) Transportation modality.

Figure 6.4. Transportation in the container shipping network.

Overall, rail was used more for the direct movement of laden containers, whilst road was employed more for the indirect flow of empty containers. The detailed results obtained for the flow of empty and laden containers using considered transportation modes for different cost structures are presented in Tables 6.7-6.10.

The details of numerical results associated with the solution of the intermodal transportation problem for both laden and empty containers are provided in Tables 6.7-6.10 corresponding to cost structures (a)−(d). More specifically, the percentage of laden containers flow (LCF) and empty containers flow (ECF) throughout the network using both available transportation modes of road and rail are presented. Also, the flow solutions are categorised according to the dry-ports deployment, where “Direct” flows refer to the flows of containers which are transported between nodes without using dry ports, while “Indirect” flows denote the flows of containers which are moved via established dry ports. Moreover, the average of flow percentages over different values of ν for a given λ, as well as the total average of flow percentages over all different values of ν and λ, are provided for each cost structure.

Table 6.8. Flow decisions results for cost structure (a)									
λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.67%	84.36%	4.59%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.68%	84.35%	4.59%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.68%	84.34%	4.61%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.68%	84.35%	4.59%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.62%	84.36%	4.65%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.67%	84.35%	4.61%	1.38%
0.01	1000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.66%	84.37%	4.59%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.59%	84.36%	4.67%	1.38%
	4000	6.24%	56.13%	18.82%	18.82%	9.60%	84.35%	4.67%	1.38%
	5000	6.21%	55.93%	18.93%	18.93%	9.58%	84.37%	4.67%	1.38%
Average		6.27%	56.46%	18.63%	18.63%	9.61%	84.36%	4.65%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	9.59%	84.34%	6.07%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	9.68%	84.32%	6.00%	0.00%
	3000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
Average		7.81%	70.31%	10.94%	10.94%	9.63%	84.33%	5.22%	0.83%
1	1000	10.00%	90.00%	0.00%	0.00%	10.16%	83.60%	6.24%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	9.72%	83.86%	6.41%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
Average		7.81%	70.31%	10.94%	10.94%	9.63%	84.33%	5.22%	0.83%
Total Average		**7.60%**	**68.39%**	**12.00%**	**12.00%**	**9.64%**	**84.29%**	**5.18%**	**0.90%**

Table 6.7. Flow decisions results for cost structure (a) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	10.07%	0.00%	62.95%	26.98%	3.31%	0.00%	74.66%	22.03%
	2000	0.00%	0.00%	70.00%	30.00%	1.65%	0.00%	74.26%	24.08%
	3000	0.00%	0.00%	70.00%	30.00%	1.95%	0.00%	74.03%	24.01%
	4000	28.21%	0.00%	50.25%	21.54%	3.53%	0.00%	74.46%	22.02%
	5000	0.00%	0.00%	70.00%	30.00%	0.20%	0.00%	77.80%	22.00%
Average		7.66%	0.00%	64.64%	27.70%	2.13%	0.00%	75.04%	22.83%
0.01	1000	0.00%	0.00%	70.00%	30.00%	0.93%	0.00%	77.04%	22.02%
	2000	0.00%	0.00%	70.00%	30.00%	0.20%	0.00%	75.84%	23.96%
	3000	0.00%	0.00%	70.00%	30.00%	0.10%	0.00%	77.91%	21.99%
	4000	0.00%	0.00%	70.00%	30.00%	0.07%	0.00%	77.94%	21.99%
	5000	0.00%	0.00%	70.00%	30.00%	0.09%	0.00%	77.92%	21.99%
Average		0.00%	0.00%	70.00%	30.00%	0.28%	0.00%	77.33%	22.39%
0.1	1000	0.00%	0.00%	100.0%	0.00%	0.00%	0.00%	100.0%	0.00%
	2000	0.00%	0.00%	100.0%	0.00%	0.00%	0.00%	96.67%	3.33%
	3000	0.00%	0.00%	70.00%	30.00%	0.90%	0.00%	77.07%	22.02%
	4000	0.00%	0.00%	70.00%	30.00%	1.25%	0.00%	76.72%	22.03%
	5000	0.00%	0.00%	70.00%	30.00%	0.71%	0.00%	77.26%	22.03%
Average		0.00%	0.00%	82.00%	18.00%	0.57%	0.00%	85.54%	13.88%
1	1000	0.00%	0.00%	97.59%	2.41%	0.00%	0.00%	90.09%	9.91%
	2000	0.00%	0.00%	99.57%	0.43%	0.00%	0.00%	100.0%	0.00%
	3000	0.00%	0.00%	98.42%	1.58%	0.00%	0.00%	100.0%	0.00%
	4000	0.00%	0.00%	98.37%	1.63%	0.00%	0.00%	100.0%	0.00%
	5000	0.00%	0.00%	98.45%	1.55%	0.00%	0.00%	100.0%	0.00%
Average		0.00%	0.00%	82.00%	18.00%	0.57%	0.00%	85.54%	13.88%
Total Average		**1.91%**	**0.00%**	**78.78%**	**19.31%**	**0.74%**	**0.00%**	**83.98%**	**15.27%**

Table 6.9. Flow decisions results for cost structure (b)

λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.56%	84.41%	4.65%	1.38%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		6.28%	56.53%	18.59%	18.59%	9.61%	84.39%	4.63%	1.38%
0.01	1000	6.30%	56.70%	18.50%	18.50%	9.72%	84.36%	4.54%	1.38%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		6.28%	56.53%	18.59%	18.59%	9.64%	84.38%	4.61%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	11.92%	83.64%	3.29%	1.15%
	2000	6.24%	56.13%	18.82%	18.82%	9.62%	84.39%	4.62%	1.38%
	3000	6.20%	55.76%	19.02%	19.02%	9.63%	84.38%	4.62%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.67%	84.37%	4.59%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.58%	84.38%	4.66%	1.38%
Average		7.02%	63.19%	14.89%	14.89%	10.08%	84.23%	4.36%	1.33%
1	1000	10.71%	89.29%	0.00%	0.00%	24.59%	75.41%	0.00%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	25.81%	74.19%	0.00%	0.00%
	3000	10.0%	90.0%	0.00%	0.00%	27.0%	73.0%	0.00%	0.00%
	4000	10.0%	90.0%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	5000	10.0%	90.0%	0.00%	0.00%	12.24%	83.32%	3.29%	1.15%
Average		10.14%	89.86%	0.00%	0.00%	23.32%	75.79%	0.66%	0.23%
Total Average		**7.43%**	**66.53%**	**13.02%**	**13.02%**	**13.17%**	**82.20%**	**3.56%**	**1.08%**

Table 6.8. Flow decisions results for cost structure (b) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	5.88%	0.00%	65.88%	28.24%	0.24%	0.00%	77.81%	21.95%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		5.88%	0.00%	65.88%	28.24%	0.74%	0.00%	76.87%	22.39%
0.01	1000	12.09%	0.00%	61.54%	26.37%	3.27%	0.00%	72.77%	23.96%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		7.12%	0.00%	65.01%	27.87%	1.35%	0.00%	75.86%	22.79%
0.1	1000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
	2000	5.88%	0.00%	65.88%	28.24%	1.23%	0.00%	76.78%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.59%	0.00%	77.43%	21.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.81%	0.00%	75.19%	24.01%
	5000	5.88%	0.00%	65.88%	28.24%	0.85%	0.00%	77.14%	22.01%
Average		11.37%	0.00%	66.04%	22.59%	0.70%	0.00%	74.21%	25.09%
1	1000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	69.99%	30.00%
	2000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	65.62%	34.37%
	3000	33.33%	0.0%	66.7%	0.0%	0.0%	0.0%	65.6%	34.4%
	4000	33.34%	0.00%	66.66%	0.00%	0.0%	0.0%	64.52%	35.48%
	5000	33.34%	0.00%	66.66%	0.00%	0.0%	0.0%	64.52%	35.48%
Average		33.34%	0.00%	66.67%	0.00%	0.00%	0.00%	66.05%	33.95%
Total Average		**14.43%**	**0.00%**	**65.90%**	**19.67%**	**0.70%**	**0.00%**	**73.25%**	**26.05%**

Table 6.10. Flow decisions results for cost structure (c)

λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.59%	84.36%	4.67%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.61%	84.36%	4.66%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.61%	84.36%	4.65%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.60%	84.35%	4.68%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.63%	84.35%	4.65%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.61%	84.36%	4.66%	1.38%
0.01	1000	6.30%	56.70%	18.50%	18.50%	9.70%	84.36%	4.56%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.59%	84.36%	4.68%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.59%	84.35%	4.68%	1.38%
	4000	6.36%	57.26%	18.19%	18.19%	9.59%	84.39%	4.64%	1.38%
	5000	6.31%	56.81%	18.44%	18.44%	9.63%	84.35%	4.64%	1.38%
Average		6.31%	56.77%	18.46%	18.46%	9.62%	84.36%	4.64%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	9.59%	84.46%	4.70%	1.25%
	2000	10.00%	90.00%	0.00%	0.00%	9.61%	84.33%	4.68%	1.38%
	3000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.62%	84.33%	4.67%	1.38%
Average		7.81%	70.31%	10.94%	10.94%	9.61%	84.36%	4.68%	1.35%
1	1000	10.00%	90.00%	0.00%	0.00%	10.52%	83.00%	5.32%	1.17%
	2000	10.00%	90.00%	0.00%	0.00%	9.72%	83.86%	6.41%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	9.49%	84.35%	6.16%	0.00%
Average		10.00%	90.00%	0.00%	0.00%	9.74%	83.98%	6.04%	0.23%
Total Average		**7.61%**	**68.47%**	**11.96%**	**11.96%**	**9.65%**	**84.26%**	**5.01%**	**1.09%**

Table 6.9. Flow decisions results for cost structure (c) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	0.00%	0.0%	70.00%	30.00%	1.97%	0.00%	76.03%	22.01%
	2000	0.00%	0.0%	70.00%	30.00%	0.21%	0.00%	77.79%	22.00%
	3000	0.00%	0.0%	70.00%	30.00%	1.97%	0.00%	76.02%	22.01%
	4000	0.00%	0.0%	70.00%	30.00%	0.01%	0.00%	77.99%	22.00%
	5000	0.00%	0.0%	70.00%	30.00%	1.82%	0.00%	76.14%	22.04%
Average		0.00%	0.00%	70.00%	30.00%	1.20%	0.00%	76.79%	22.01%
0.01	1000	0.00%	0.0%	70.00%	30.00%	0.18%	0.00%	75.87%	23.96%
	2000	0.00%	0.0%	70.00%	30.00%	0.07%	0.00%	77.94%	21.98%
	3000	0.00%	0.0%	70.00%	30.00%	0.12%	0.00%	77.89%	21.99%
	4000	0.00%	0.0%	70.00%	30.00%	0.20%	0.00%	77.81%	21.99%
	5000	0.00%	0.0%	70.00%	30.00%	0.23%	0.00%	77.77%	22.00%
Average		0.00%	0.00%	70.00%	30.00%	0.16%	0.00%	77.46%	22.38%
0.1	1000	0.00%	0.0%	70.00%	30.00%	0.00%	0.00%	76.67%	23.33%
	2000	0.00%	0.0%	70.00%	30.00%	0.00%	0.00%	76.93%	23.07%
	3000	0.00%	0.00%	70.00%	30.00%	0.03%	0.00%	77.97%	22.00%
	4000	0.00%	0.00%	70.00%	30.00%	0.85%	0.00%	77.12%	22.02%
	5000	0.00%	0.00%	70.00%	30.00%	0.98%	0.00%	76.99%	22.03%
Average		0.00%	0.00%	70.00%	30.00%	0.37%	0.00%	77.14%	22.49%
1	1000	0.00%	0.00%	100%	0.00%	0.00%	0.00%	81.63%	18.37%
	2000	0.00%	0.00%	99.57%	0.43%	0.00%	0.00%	100%	0.00%
	3000	0.00%	0.00%	98.42%	1.58%	0.00%	0.00%	100%	0.00%
	4000	0.00%	0.00%	98.37%	1.63%	0.00%	0.00%	100%	0.00%
	5000	0.00%	0.00%	98.45%	1.55%	0.00%	0.00%	100%	0.00%
Average		0.00%	0.00%	98.96%	1.04%	0.00%	0.00%	96.33%	3.67%
Total Average		**0.00%**	**0.00%**	**77.24%**	**22.76%**	**0.43%**	**0.00%**	**81.93%**	**17.64%**

Table 6.11. Flow decisions results for cost structure (d)

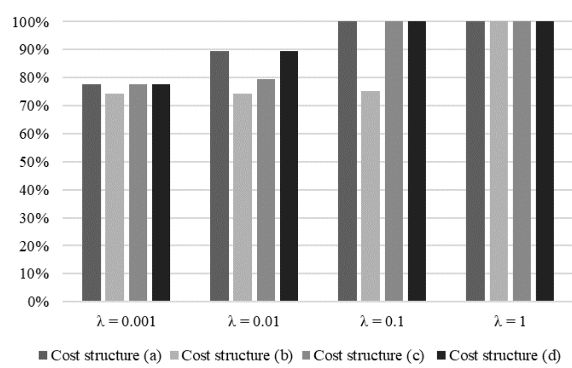
λ	ν	LCF (forward)				LCF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	6.30%	56.70%	18.50%	18.50%	9.54%	84.41%	4.68%	1.38%
	2000	6.53%	58.75%	17.36%	17.36%	9.56%	84.40%	4.67%	1.38%
	3000	6.21%	55.93%	18.93%	18.93%	9.73%	84.36%	4.54%	1.38%
	4000	6.18%	55.62%	19.10%	19.10%	9.54%	84.40%	4.68%	1.38%
	5000	6.34%	57.03%	18.32%	18.32%	9.55%	84.40%	4.68%	1.38%
Average		6.31%	56.81%	18.44%	18.44%	9.58%	84.39%	4.65%	1.38%
0.01	1000	6.35%	57.18%	18.23%	18.23%	9.57%	84.34%	4.72%	1.38%
	2000	6.30%	56.70%	18.50%	18.50%	9.42%	84.41%	4.80%	1.38%
	3000	6.26%	56.38%	18.68%	18.68%	9.55%	84.39%	4.68%	1.38%
	4000	6.24%	56.13%	18.82%	18.82%	9.55%	84.39%	4.69%	1.38%
	5000	6.21%	55.93%	18.93%	18.93%	9.64%	84.36%	4.63%	1.38%
Average		6.27%	56.46%	18.63%	18.63%	9.55%	84.38%	4.70%	1.38%
0.1	1000	10.00%	90.00%	0.00%	0.00%	11.92%	83.64%	3.29%	1.15%
	2000	7.04%	63.33%	14.82%	14.82%	9.60%	84.33%	4.69%	1.38%
	3000	6.35%	57.18%	18.23%	18.23%	9.60%	84.33%	4.69%	1.38%
	4000	6.35%	57.18%	18.23%	18.23%	9.60%	84.33%	4.69%	1.38%
	5000	6.35%	57.18%	18.23%	18.23%	9.57%	84.34%	6.09%	0.00%
Average		7.22%	64.97%	13.90%	13.90%	10.06%	84.19%	4.69%	1.06%
1	1000	10.71%	89.29%	0.00%	0.00%	24.59%	75.41%	0.00%	0.00%
	2000	10.00%	90.00%	0.00%	0.00%	25.81%	74.19%	0.00%	0.00%
	3000	10.00%	90.00%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	4000	10.00%	90.00%	0.00%	0.00%	26.98%	73.02%	0.00%	0.00%
	5000	10.00%	90.00%	0.00%	0.00%	12.24%	83.32%	3.29%	1.15%
Average		10.14%	89.86%	0.00%	0.00%	23.32%	75.79%	0.66%	0.23%
Total Average		**7.49%**	**67.03%**	**12.74%**	**12.74%**	**13.13%**	**82.19%**	**3.68%**	**1.01%**

Table 6.10. Flow decisions results for cost structure (d) (cont.).									
λ	ν	ECF (forward)				ECF (backward)			
		Direct		Indirect		Direct		Indirect	
		Road	Rail	Road	Rail	Road	Rail	Road	Rail
0.001	1000	5.88%	0.00%	65.88%	28.24%	2.18%	0.00%	75.88%	21.94%
	2000	5.88%	0.00%	65.88%	28.24%	0.14%	0.00%	77.88%	21.98%
	3000	5.88%	0.00%	65.88%	28.24%	0.32%	0.00%	73.70%	25.98%
	4000	5.88%	0.00%	65.88%	28.24%	0.43%	0.00%	77.59%	21.98%
	5000	5.88%	0.00%	65.88%	28.24%	0.09%	0.00%	77.92%	21.99%
Average		5.88%	0.00%	65.88%	28.24%	0.63%	0.00%	76.59%	22.77%
0.01	1000	5.88%	0.00%	65.88%	28.24%	2.06%	0.00%	75.97%	21.98%
	2000	5.88%	0.00%	65.88%	28.24%	1.59%	0.00%	76.46%	21.96%
	3000	5.88%	0.00%	65.88%	28.24%	0.16%	0.00%	77.87%	21.97%
	4000	5.88%	0.00%	65.88%	28.24%	0.15%	0.00%	77.86%	21.98%
	5000	5.88%	0.00%	65.88%	28.24%	1.97%	0.00%	74.04%	23.99%
Average		5.88%	0.00%	65.88%	28.24%	1.19%	0.00%	76.44%	22.38%
0.1	1000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	63.89%	36.11%
	2000	5.88%	0.00%	65.88%	28.24%	0.00%	0.00%	76.67%	23.33%
	3000	5.88%	0.00%	65.88%	28.24%	0.87%	0.00%	77.14%	21.99%
	4000	5.88%	0.00%	65.88%	28.24%	1.16%	0.00%	76.85%	21.99%
	5000	5.88%	0.00%	94.12%	0.00%	0.12%	0.00%	99.88%	0.00%
Average		11.37%	0.00%	71.68%	16.94%	0.43%	0.00%	78.89%	20.68%
1	1000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	69.99%	30.00%
	2000	33.34%	0.00%	66.66%	0.00%	0.01%	0.00%	65.62%	34.37%
	3000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	65.62%	34.37%
	4000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
	5000	33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	64.52%	35.48%
Average		33.34%	0.00%	66.66%	0.00%	0.00%	0.00%	66.05%	33.94%
Total Average		**14.12%**	**0.00%**	**67.53%**	**18.36%**	**0.56%**	**0.00%**	**74.49%**	**24.94%**

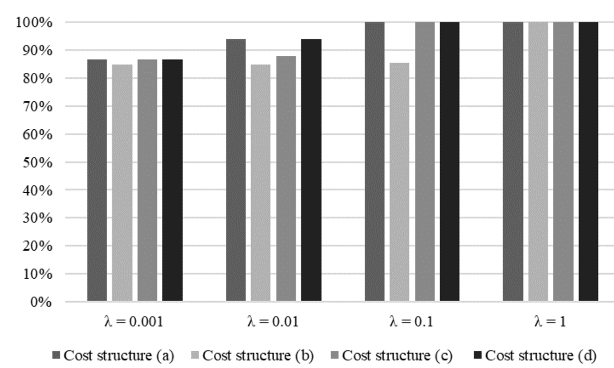
- *Key performance Indicators*

The results obtained from computing the KPIs are summarised in Figure 6.5, in order to underline the impact of different cost structures and solution robustness coefficient. In the first glance, one can observe that in Figures 6.5(a) and 6.5(b) both $\bar{\eta}_r$ and $\bar{\eta}_s$, obtained from (6.1) and (6.2) respectively, were improved significantly as the solution robustness coefficient increased. Overall, the results show that increasing robustness led to higher service levels for the supply of finished goods compared to the one for the demand of raw materials. One also may argue that both $\bar{\xi}_r$ and $\bar{\xi}_s$, obtained from (6.3) and (6.4) respectively, had almost equal growth with respect to increase in the value of λ, as shown in Figures 6.5(c) and 6.5(d). This similarity in the value of fill rate is due to the linear proportional relationship between the demand and supply of containers at the manufacturers' locations. In general, this result confirms the importance of the incorporation of robustness considerations into container shipping network design in order to achieve the maximal desired service level and fill rate.

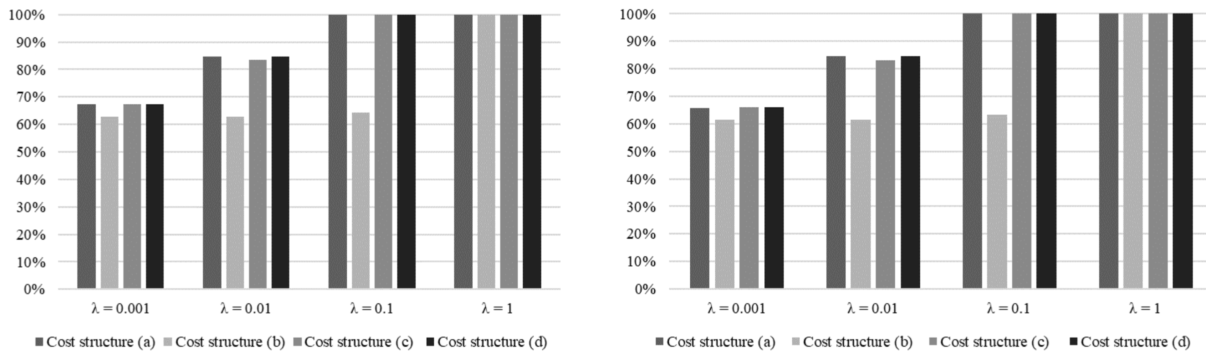
Moreover, Figures 6.5(a) and 6.5(b) show that when the fixed opening cost is relatively low, by increasing the holding cost of empty containers, both service levels $\bar{\eta}_r$ and $\bar{\eta}_s$ may decrease. This is because satisfying the demand and supply of laden containers is closely interrelated to the inventory level of empty containers across the shipping network. Figures 6.5(c) and 6.5(d) demonstrate a similar behaviour with regards to the cost structure.



(a) Service level for the demand of raw materials.



(b) Service level for the supply of finished goods.



(c) Fill rate for the demand of raw materials. (d) Fill rate for the supply of finished goods.

Figure 6.5. Service level and fill rate in considered shipping network.

The values of $\bar{\varepsilon}$ obtained from (6.5) with regards to different cost structures and solution robustness coefficients is summarised in Figure 6.6.

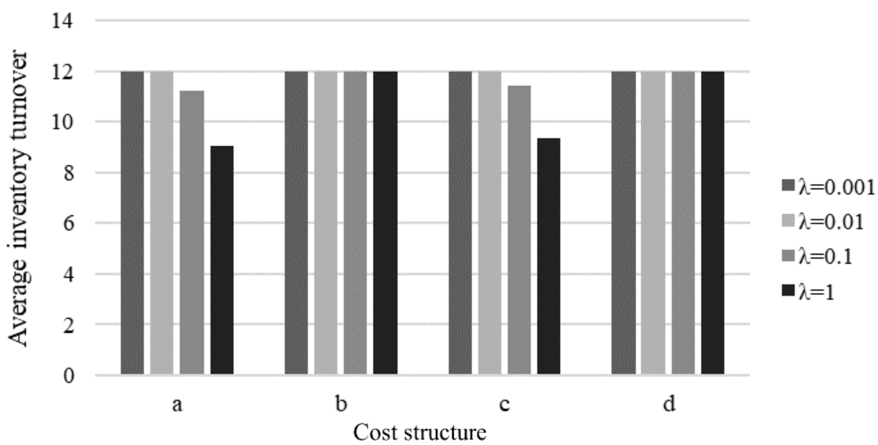


Figure 6.6. Average inventory turnover of empty containers.

Overall, it can be seen that the expected value of inventory turnover $\bar{\varepsilon}$ increased as the inventory holding cost of empty containers increased. This is due to the fact that when the inventory holding cost was high, a lower inventory level of empty containers was preferred at dry ports, increasing the number of replenishments. Moreover, Figure 6.6 shows that lower inventory turnover occurred when seeking to achieve higher solution robustness, which implies that higher inventory levels were needed in order to avoid the effect of uncertainty which lowers the inventory turnover indicator. It is worthwhile to mention that the inventory turnover for instances with high holding costs suggests a monthly replenishment policy with the case study data, since a twelve-month planning horizon was considered. Detailed outputs regarding KPIs over all instances are provided in Tables 6.11-6.14.

The obtained numerical results regarding to described KPIs of service level, fill rate, and inventory turnover are reported for different values of rejected demand and solution robustness coefficients, and different cost structures in Tables 6.11-6.14. Also, the average of these KPIs over different values of ν is presented for each given λ.

Table 6.12. KPIs results for cost structure (a)

λ	ν	Service Level: Raw materials ($\bar{\eta}_r$)	Service Level: Finished goods ($\bar{\eta}_s$)	Fill Rate: Raw materials ($\bar{\xi}_r$)	Fill Rate: Finished Goods ($\bar{\xi}_s$)	Inventory Turnover ($\bar{\varepsilon}$)
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	100%	100%	100%	100%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	77%	77%	84%	91%	12
	5000	69%	69%	79%	88%	12
Average		85%	85%	89%	94%	12
0.1	1000	100%	100%	100%	100%	10.08
	2000	100%	100%	100%	100%	10.08
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	11.232
1	1000	100%	100%	100%	100%	5.76
	2000	100%	100%	100%	100%	8.64
	3000	100%	100%	100%	100%	10.32
	4000	100%	100%	100%	100%	10.32
	5000	100%	100%	100%	100%	10.32
Average		100%	100%	100%	100%	9.072

Table 6.13. KPIs results for cost structure (b)

λ	ν	Service Level		Fill Rate		Inventory Turnover ($\bar{\varepsilon}$)
		Raw materials ($\bar{\eta}_r$)	Finished goods ($\bar{\eta}_s$)	Raw materials ($\bar{\xi}_r$)	Finished Goods ($\bar{\xi}_s$)	
0.001	1000	92%	92%	95%	97%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		63%	62%	74%	85%	12
0.01	1000	92%	92%	95%	97%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		63%	62%	74%	85%	12
0.1	1000	100%	100%	100%	100%	12
	2000	77%	77%	84%	91%	12
	3000	62%	62%	73%	85%	12
	4000	49%	46%	65%	79%	12
	5000	34%	31%	54%	73%	12
Average		64%	63%	75%	86%	12
1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12

Table 6.14. KPIs results for cost structure (c)						
λ	ν	Service Level		Fill Rate		Inventory Turnover ($\bar{\varepsilon}$)
		Raw materials ($\bar{\eta}_r$)	Finished goods ($\bar{\eta}_s$)	Raw materials ($\bar{\xi}_r$)	Finished Goods ($\bar{\xi}_s$)	
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	92%	92%	95%	97%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	80%	77%	65%	79%	12
	5000	69%	69%	54%	73%	12
Average		70%	69%	80%	88%	12
0.1	1000	100%	100%	100%	100%	10.08
	2000	100%	100%	100%	100%	11.04
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	11.42
1	1000	100%	100%	100%	100%	7.2
	2000	100%	100%	100%	100%	8.64
	3000	100%	100%	100%	100%	10.32
	4000	100%	100%	100%	100%	10.32
	5000	100%	100%	100%	100%	10.32
Average		100%	100%	100%	100%	9.36

Table 6.15. KPIs results for cost structure (d)						
λ	ν	Service Level		Fill Rate		Inventory Turnover ($\bar{\varepsilon}$)
		Raw materials ($\bar{\eta}_r$)	Finished goods ($\bar{\eta}_s$)	Raw materials ($\bar{\xi}_r$)	Finished Goods ($\bar{\xi}_s$)	
0.001	1000	92%	92%	95%	97%	12
	2000	80%	77%	86%	91%	12
	3000	69%	69%	79%	88%	12
	4000	54%	54%	68%	82%	12
	5000	42%	38%	60%	76%	12
Average		67%	66%	78%	87%	12
0.01	1000	100%	100%	100%	100%	12
	2000	92%	92%	95%	97%	12
	3000	85%	85%	89%	94%	12
	4000	77%	77%	84%	91%	12
	5000	69%	69%	79%	88%	12
Average		85%	85%	89%	94%	12
0.1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12
1	1000	100%	100%	100%	100%	12
	2000	100%	100%	100%	100%	12
	3000	100%	100%	100%	100%	12
	4000	100%	100%	100%	100%	12
	5000	100%	100%	100%	100%	12
Average		100%	100%	100%	100%	12

6.3.2. Performance of robustness

The proposed modelling approach provides decision-makers with reliable and robust solutions related to target inventory levels, empty container repositioning and intermodal transportation of laden containers. In relation to robust optimisation, the performance and reliability of solutions is measured by computing the standard deviation-to-mean ratio, which is referred to as the *coefficient of variation* (Birge, 1982). Table 6.15 shows the minimum, mean, and maximum values over all instances. The results demonstrate that the minimum coefficient of variation (CV) for all of the operational decisions was less than 1%. However, the mean values of CV relating to the decision of empty containers inventory level at dry ports and direct flow of empty containers from seaports are 4.29% and 3.54% respectively. The former relatively high CV was due to the risk pooling effect relating to uncertain demand for empty containers at the dry ports. It implies that the uncertainty in laden container demand created high variability in decisions related to empty container repositioning, which emphasises the fact that the demand of empty and laden containers in the shipping network design were highly interrelated. The latter can be explained by the impact of network configuration on the variability of operational decisions. The repositioning of empty containers at the seaport which was coupled with importing and leasing operational decisions was another reason for this issue.

Table 6.16. Coefficient of variation values (%).

	$I^{e}_{qt}(\omega)$	$F^{\ell}_{pqtm}(\omega)$				$F^{e}_{pqtm}(\omega)$			
	$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$
Min	0.77	0.15	0.10	0.00	0.00	0.00	0.00	0.06	0.10
Mean	4.29	0.24	0.14	0.08	0.15	1.42	3.54	0.17	0.15
Max	5.58	0.52	0.38	0.16	0.98	2.80	19.61	0.38	0.19

In addition, the average value of CVs over ν associated with each cost structure and different values of λ is analysed. The corresponding value for the solution of variable $F^{\ell}_{pqtm}(\omega)$, $p \in \mathbb{I}, q \in \mathbb{J}$ is illustrated in Figure 6.7, where the horizontal axis represents the four cost structures considered and the vertical axis shows the average of CVs as a percentage. This shows that the value of CVs had a decreasing trend with regards to the incremental weight assigned to the solution robustness for all cost structures. This reduction of CVs was more significant when the cost of holding empty containers was relatively low, i.e., cost structures (a) and (c).

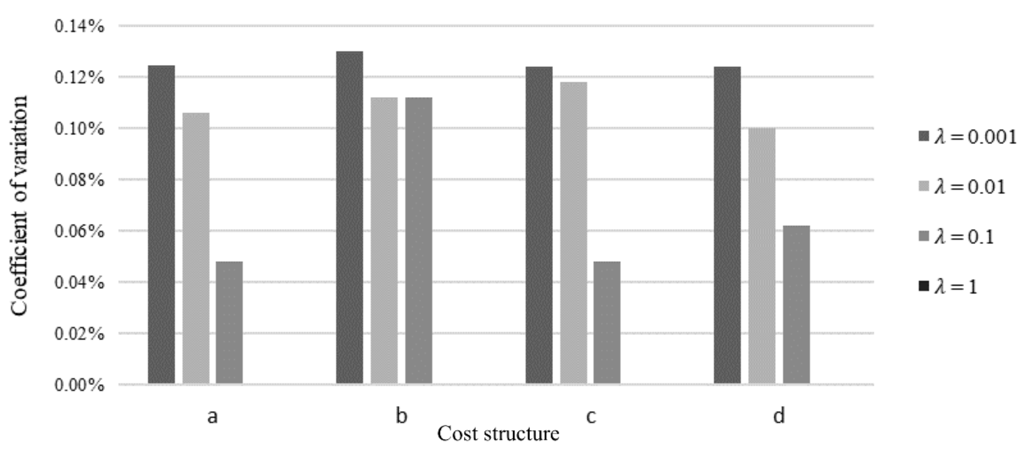


Figure 6.7. Coefficient of variations of flow for different cost structures.

The standard deviation-to-mean ratio for all decision variables for all instances is analysed (see Tables 6.16–6.19). Overall, it can be concluded that the proposed robust modelling approach returned solutions with low variability.

The numerical results related to the CV of the solution of second-stage decision variables, represented by $\hat{I}^{e}_{qt}(\omega), \hat{F}^{\ell}_{pqtm}(\omega), \hat{U}^{\ell}_{qt}(\omega)$, and $\hat{B}^{\ell}_{qt}(\omega)$ are provided in Tables 6.16−6.19 for all 80 described instances.

Table 6.17. Coefficient of variation for cost structure (a)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}, q \in \mathbb{J}$	$p \in \mathbb{J}, q \in \mathbb{O}$	$p \in \mathbb{I}, q \in \mathbb{J}$	$p \in \mathbb{J}, q \in \mathbb{I}$	$p \in \mathbb{O}, q \in \mathbb{J}$	$p \in \mathbb{J}, q \in \mathbb{O}$	$p \in \mathbb{I}, q \in \mathbb{J}$	$p \in \mathbb{J}, q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.808%	0.182%	0.112%	0.094%	0.142%	1.596%	2.035%	0.155%	0.142%	0.540%	0.558%
	2000	4.808%	0.243%	0.112%	0.109%	0.142%	0.000%	2.693%	0.155%	0.141%	0.341%	0.316%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	2.750%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	1.370%	2.050%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	0.000%	7.680%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.900%	0.150%	0.140%	0.000%	0.000%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	4.630%	0.150%	0.140%	0.540%	0.560%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	0.000%	7.000%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.240%	0.110%	0.120%	0.140%	0.000%	10.360%	0.150%	0.140%	0.310%	0.320%
	5000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	8.150%	0.150%	0.140%	0.260%	0.270%
0.1	1000	1.370%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	2000	1.330%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.980%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.370%	0.150%	0.140%	0.000%	0.000%
	5000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	4.380%	0.150%	0.140%	0.000%	0.000%
1	1000	1.720%	0.150%	0.140%	0.000%	0.160%	0.000%	0.000%	0.120%	0.150%	0.000%	0.000%
	2000	0.770%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.160%	0.000%	0.000%
	3000	0.850%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	4000	0.830%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	5000	0.840%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%

Table 6.18. Coefficient of variation for cost structure (b)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	3.410%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
0.01	1000	4.810%	0.180%	0.110%	0.000%	0.140%	2.520%	2.130%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
0.1	1000	5.580%	0.150%	0.150%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	2000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	3.350%	0.150%	0.140%	0.310%	0.320%
	3000	4.810%	0.290%	0.110%	0.140%	0.140%	2.770%	4.880%	0.150%	0.140%	0.230%	0.230%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	2.770%	4.050%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	2.770%	4.010%	0.150%	0.140%	0.150%	0.140%
1	1000	5.580%	0.520%	0.380%	0.000%	0.000%	2.770%	1.610%	0.380%	0.190%	0.000%	0.000%
	2000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	1.960%	0.380%	0.180%	0.000%	0.000%
	3000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	2.770%	0.380%	0.180%	0.000%	0.000%
	4000	5.580%	0.500%	0.380%	0.000%	0.980%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	5000	5.580%	0.150%	0.180%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%

Table 6.19. Coefficient of variation for cost structure (c)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	2.660%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.110%	0.140%	0.000%	6.820%	0.150%	0.140%	0.340%	0.320%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	0.000%	2.720%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	0.000%	8.010%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	0.000%	2.750%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	4.180%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	0.000%	6.540%	0.150%	0.140%	0.540%	0.550%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	0.000%	9.180%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.320%	0.110%	0.140%	0.140%	0.000%	6.890%	0.150%	0.140%	0.190%	0.180%
	5000	4.810%	0.340%	0.110%	0.160%	0.140%	0.000%	7.350%	0.150%	0.140%	0.150%	0.140%
0.1	1000	1.670%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.180%	0.000%	0.000%
	2000	1.700%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.190%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	19.610%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	4.070%	0.150%	0.140%	0.000%	0.000%
	5000	4.810%	0.150%	0.110%	0.080%	0.140%	0.000%	3.750%	0.150%	0.140%	0.000%	0.000%
1	1000	2.520%	0.150%	0.140%	0.000%	0.160%	0.000%	0.000%	0.150%	0.160%	0.000%	0.000%
	2000	0.770%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.070%	0.160%	0.000%	0.000%
	3000	0.850%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	4000	0.830%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%
	5000	0.840%	0.150%	0.110%	0.000%	0.140%	0.000%	0.000%	0.060%	0.180%	0.000%	0.000%

Table 6.20. Coefficient of variation for cost structure (d)

λ	ν	$\hat{I}^{e}_{qt}(\omega)$	$\hat{F}^{\ell}_{pqtm}(\omega)$				$\hat{F}^{e}_{pqtm}(\omega)$				$\hat{U}^{\ell}_{qt}(\omega)$	$\hat{B}^{\ell}_{qt}(\omega)$
		$q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$p \in \mathbb{O}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{O}$	$p \in \mathbb{I}$, $q \in \mathbb{J}$	$p \in \mathbb{J}$, $q \in \mathbb{I}$	$q \in \mathbb{J}$	$q \in \mathbb{J}$
0.001	1000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	2.580%	0.150%	0.140%	0.540%	0.560%
	2000	4.810%	0.240%	0.110%	0.110%	0.140%	2.770%	5.010%	0.150%	0.140%	0.340%	0.320%
	3000	4.810%	0.270%	0.110%	0.130%	0.140%	2.770%	6.300%	0.150%	0.140%	0.260%	0.270%
	4000	4.810%	0.300%	0.110%	0.140%	0.140%	2.770%	5.630%	0.150%	0.140%	0.200%	0.200%
	5000	4.810%	0.330%	0.110%	0.150%	0.140%	2.770%	8.500%	0.150%	0.140%	0.170%	0.160%
0.01	1000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	2.820%	0.150%	0.140%	0.000%	0.000%
	2000	4.810%	0.180%	0.110%	0.090%	0.140%	2.770%	3.050%	0.150%	0.140%	0.540%	0.560%
	3000	4.810%	0.210%	0.110%	0.110%	0.140%	2.770%	5.260%	0.150%	0.140%	0.390%	0.390%
	4000	4.810%	0.240%	0.110%	0.120%	0.140%	2.770%	7.000%	0.150%	0.140%	0.310%	0.320%
	5000	4.800%	0.300%	0.100%	0.100%	0.100%	2.800%	2.800%	0.200%	0.100%	0.300%	0.300%
0.1	1000	5.580%	0.150%	0.150%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	2000	4.810%	0.150%	0.110%	0.070%	0.140%	2.770%	0.000%	0.150%	0.180%	0.000%	0.000%
	3000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	4.320%	0.150%	0.140%	0.000%	0.000%
	4000	4.810%	0.150%	0.110%	0.080%	0.140%	2.770%	3.730%	0.150%	0.140%	0.000%	0.000%
	5000	4.160%	0.150%	0.110%	0.080%	0.140%	2.770%	11.420%	0.150%	0.140%	0.000%	0.000%
1	1000	5.580%	0.520%	0.380%	0.000%	0.000%	2.770%	1.610%	0.380%	0.190%	0.000%	0.000%
	2000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	1.960%	0.380%	0.180%	0.000%	0.000%
	3000	5.580%	0.500%	0.380%	0.000%	0.000%	2.770%	2.770%	0.380%	0.180%	0.000%	0.000%
	4000	5.580%	0.500%	0.380%	0.000%	0.980%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%
	5000	5.580%	0.150%	0.180%	0.000%	0.190%	2.770%	0.000%	0.380%	0.180%	0.000%	0.000%

6.3.3. Computational efficiency of the accelerated Benders decomposition algorithm

The computational efficiencies of the enhanced SAA procedure which was achieved by adding knapsack inequalities and utilizing Pareto-optimal cuts were measured in terms of computational time. The accelerated BD algorithm was coded in CPLEX 12.8.0 and tested on a personal computer with a 3.2 gigahertz Intel Core 5 processor and 16 gigabytes of RAM. We set a maximum computational time of 2 hours for these tests. Two sorts of experiment were conducted in order to assess the efficiency of proposed accelerated BD method. The first set of experiments focused on the impact of network size on the computational efficiency. Table 6.20 provides the characteristics of test sets for this experiment, and the effect of acceleration methods on the computational times of Cplex (i.e., extensive formulation), standard BD, and accelerated BD algorithm. These experiments were conducted for $N = 50$ sampled scenarios. The results indicate that the model for smaller networks such as tests 1 can be solved directly by Cplex with smaller computational time compared to the BD method and its accelerated version. However, as the network size grew by increasing the number of nodes, both standard BD and the proposed accelerated BD strategy outperformed Cplex in terms of computational time. The computational times in Table 6.20 show that the proposed BD algorithm together with multi-cut scheme, knapsack inequalities, and Pareto-cut strategy outperformed Cplex and standard BD for larger networks.

It worth mentioning that in the computational experiments, a core point approximation $\bar{\mathbf{X}}_{it}^{c}$ was initialised with a feasible solution to the MP and then the approximation was updated at each iteration by $\mathbf{X}_{it}^{c} \leftarrow \varphi \mathbf{X}_{it-1}^{c} + (1-\varphi)\mathbf{X}_{it}^{MP}$. According to emprical observations of Papadakos (2008) and Oliveira et al. (2014), φ was set to 0.5. Also, the stopping criterion was set to when the objective value gap was below a threshold value 0.01.

Table 6.21. Computational results of the first experiment set.

Test set	$\lvert\mathbb{I}\rvert$	$\lvert\mathbb{J}\rvert$	Number of variables	Number of constraints	Computational time (seconds)		
					Cplex	Bender decomposition	Accelerated Bender decomposition
1	2	10	82742	58550	110.43	243.54	245.76
2	3	20	202463	156020	410.4	289.08	326.17
3	5	30	429315	359160	2089	398.09	377.96
4	6	40	653086	560680	5349	518	492.03
5	8	50	1025608	909490	7200	7200	7200

The second experiment was designed to investigate the effect of the number of scenarios on the performance of the algorithm. Table 6.21 shows the characteristics of second group of test sets and compares the effect of acceleration methods on the computational times for different number of scenarios. A network instance with the size of $|\mathbb{I}| = 5$ and $|\mathbb{J}| = 20$ was selected for this experiment and implement both standard and accelerated BD algorithms. Results suggest that both standard and the proposed accelerated Bender decomposition method outperformed Cplex, specially for instances with a higher number of scenarios. Yet, it was observed that the computational time for the accelerated BD algorithm in comparison with the standard BD was slightly higher within instances with a larger number of scenarios. The reason lies in the fact that despite employing Pareto-optimal cuts which helped to reduce the number of iterations, the computational time needed to solve the Magnati-Wong problem increased, when a larger number of scenarios were involved. Overall, the results confirm the advantage of using several accelerating frameworks in the proposed solution strategy, since the accelerated BD algorithm was more time-efficient compared to Cplex and the standard BD.

Table 6.22. Computational results of the second experiment set.

Test set	Number of scenarios	Number of variables	Number of constraints	**Computational time** (seconds)		
				Cplex	**Bender decomposition**	**Accelerated Benders decomposition**
6	10	145985	121310	10.86	24.72	22.90
7	20	291865	242500	31.63	30.01	28.38
8	30	437745	363690	70.77	54.23	55.12
9	40	583625	484880	233.12	88.30	96.85
10	50	729505	606070	412.49	285.6	298.3

6.4. Summary

Previous research on inland container shipping network design has dealt with decision-making at different levels in an isolated manner. This research has designed, developed and implemented a novel integrated dry port network design model that integrates strategic decision making (dry port location, allocation and the provision of arcs between nodes in the network) with operational decision making (selection of transportation modes, empty container repositioning and inventory planning) that takes into account the stochastic nature of demand. The incoming and outgoing demand of laden containers were considered as uncertain parameters in this problem and were generated using Monte-Carlo sampling. This procedure resulted in a number of demand and supply realisations with high variability, which alongside with the hierarchical decision-making structure involved in this problem, led to us proposing a robust two-stage stochastic programming model that adopted the SAA method.

The experimental results revealed that the network configuration obtained from the proposed modelling approach varied according to the cost structure and the solution robustness coefficient. More specifically, in order to achieve more robust solutions, the number of opened dry ports declined when the holding cost of empty containers grew (and vice versa). The results, also, show that the direct transportation of laden containers between the seaport and manufacturers was mostly performed by rail, whereas road was mainly employed for the movement of empty containers between the seaport and manufacturers through dry ports, confirming the pivotal significance of dry ports relevant to ECR in the container shipping networks. Furthermore, service level, fill rate, and inventory turnover were evaluated. The results imply that both service level and fill rate improve when aiming for higher solution robustness by increasing the value of λ, whilst these KPIs may decrease with high holding costs. It was also observed that the inventory turnover of empty containers declined with high empty container holding costs, and also with high values of λ.

A validation procedure was adopted to investigate the performance of the enhanced SAA approach by approximating an optimality gap of 95%. The results from 80 instances based on a hypothetical case study, disclosed some practical and managerial insights confirming the significance of hinterland container shipping network design and its operational decisions under uncertainty. The findings showed that the applied robust modelling approach produced solutions with low variability with the average CV of less than 1% under an uncertain environment. This was confirmed by the fact that this metric decreased for all operational decisions when the value of solution robustness coefficient rose.

Finally, two sets of experiments were used for testing. They were analysed to validate the effectiveness and efficiency of the proposed solution algorithm. The computational times of Cplex, the standard Benders decomposition method, and the accelerated Benders decomposition algorithm were compared. The results confirmed that applying the acceleration methods to the Benders decomposition algorithm can improve the computational performance for various problems with different network sizes and different number of scenarios.

Overall, it is believed the proposed model and solution procedure can be applied to address practical cases efficiently to achieve more reliable hinterland container shipping networks in terms of cost and operational performance. As future research directions, incorporating the inventory policies within the modelling process would be appealing. It also would be interesting to investigate the uncertainty in other parameters such as capacities, transportation and operational costs.

Chapter 7. Conclusions and Future Research Directions

This chapter outlines the outcomes and highlights the contributions of this research and provides recommendations for possible further studies in this area.

This research studied the main issues and problems associated with container shipping network design with a focus on the inland part of the networks. It was explained in Chapter 1 that these challenges have arisen due to the rapid growth of containerisation use. Further development of the seaports' hinterland infrastructure and operations is required to address problems associated with limited physical capacity and resultant congestion at seaports. Furthermore, the container shipping industry confronts additional difficulties due to the uncertainties. More precisely, the decisions related to container shipments including container flow and container inventory management should take the uncertain environment into consideration. The empty container repositioning problem aspect of inland container network design was reviewed as well. The role and development of dry ports in the seaports' hinterland container networks were presented as a combined seaport-hinterland solution to address the previously mentioned challenges. In light of these challenges, the aim of this thesis was to study decision-making problems in the design of hinterland container networks including dry port establishment taking into account uncertainties. More specifically, the research purpose was to develop mathematical models to integrate strategic and operational level decisions relating to dry port container network design and operations taking into account uncertainties.

In Chapter 2, a review of relevant studies was presented to further elaborate the research objectives and to identify research gaps. The review included studies of container network design, empty container repositioning, dry port development, and facility location problems. The detailed review of these topics revealed a number of research gaps that requires more study: inland container network design optimisation with dry port employment; taking practical container uncertainties and periodic fluctuations into consideration in the optimisation process; and the integration of different decision-making levels in the considered network design problem. Then, Chapter 3 presented the research methodology that was employed to contribute to the research gaps. The research was conducted through developing mathematical models. The models were built using the two-stage stochastic programming approach. The developed models were then solved efficiently by combining various solution methods including sample average approximation, robust optimisation, and an accelerated Benders Decomposition algorithm.

A two-stage stochastic programming model, as presented in Chapter 4, was proposed to formulate the stated decision-making problems. The first-stage problem was dealt with the strategic location-allocation decisions of dry ports in the network. The second-stage problem was concentrated on optimising operational decisions including container intermodal transportation, empty container repositioning, leasing, and inventory planning. The model involved a detailed set of operational constraints including containers demand, container flow conservation, container handling, container inter-balancing, empty containers exchange, and storage capacity. Then, Chapter 5 presented a robust SAA procedure to provide a scenario-based stochastic model and optimise both model robustness and solution robustness. Although the quality of solutions was supported through a statistical validation procedure, a Benders decomposition algorithm was adopted to enhance the computational efficiency and reliability of solution methods. The solution strategy was further improved by proposing three different acceleration schemes: multi-cut framework, Knapsack inequalities, and pareto-optimal cuts generation scheme.

Finally, the proposed mathematical models and solution methods were applied to a hypothetical case study in Chapter 6. The solution results were tested and calibrated by conducting thorough numerical experiments. For the analysis of these experiments, a large number of problem instances were used and examined. The obtained solutions were verified according to the network configuration, container flow decisions, and robustness performance. Furthermore, three different key performance indicators (i.e., service level, fill rate, and inventory turnover) were analysed to demonstrate the applicability of developed network design models and provide possible relevant insights. Finally, the efficiency and computational performance of enhanced SAA method using accelerated Benders decomposition were confirmed by two different sets of experiment.

7.1. Research contributions highlights

This thesis involves various contributions to theory, practice, and solution method.

The literature review identified several research gaps that required further studies. In the context of container network design, almost all previous studies disregarded the strategic decisions impact on the network design (see Table 2.1). Furthermore, the container network design research was mainly studied the seaborne part without taking the inland network into consideration. Moreover, the uncertainty involved in designing container networks were overlooked by previous research. The developed research in this thesis, took all these features into consideration simultaneously. The strategic decisions of location-allocation and their impact on the operations of the designed container network were taken into consideration by developing a two-stage mathematical model. Additionally, the model focused on the inland network design to assist the seaports hinterland to overcome capacity, accessibility, and congestion problems. The uncertainty nature of the container shipping industry was also considered in the modelling process.

The literature review relating to the empty container repositioning problem demonstrated the need for more studies. The main gap highlighted in this context was the absence of research that could examine the effect of strategic-level decision making on ECR problem (Tables 2.3−2.4). Although the ECR problem is considered as an operational level decision, it can be influenced by the strategic level decisions of facility location significantly. The dry ports number and location can determine the quantity and timing of empty containers delivery and inventory. This research attempted to incorporate this into the modelling procedure. In addition, this thesis contributed to the literature by optimising the ECR problem together with the laden containers flow through designing an intermodal network under uncertain and periodic environment.

This work also reviewed the literature associated with the dry port concept and its development as a solution for improving the seaports' hinterland operations. The review from decision-making perspective (see Section 2.4), indicated that previous research in designing dry port networks were solely focused on the strategic level. However, the dry port strategic level decisions should be taken jointly with the operational level decisions including container transportation, ECR, and empty container inventory planning in order to achieve a global optimum solution. This study formulated all these optimisation problems simultaneously. Furthermore, the main solution method in dry port network design literature was the direct use of standard solvers. This research developed several advanced solution methods to improve the computational efficiency and solve large scale instances.

7.2. Research results

The research proposed a novel mathematical model to formulate the dry port container network design. It is believed that this is the first research effort which integrates strategic and operational planning of such a network in an uncertain environment. In other words, the strategic dry port locational decisions were optimised jointly with operational decisions relating to container intermodal transportation, empty container repositioning, leasing, and inventory planning.

Based on the identified research gaps in the literature and the motivation of the research in the introduction chapter, this research attempted to achieve the following research aims:

1. *To comprehensively review and obtain knowledge about container network design, empty container repositioning, dry port development and location decision-making.*

This study provided a comprehensive literature review including container network design, ECR network flow models, ECR inventory control models, inland container network design using dry port, decision-making in dry port development, and facility location problem. The review of studies associated with facility location problem focused on discrete models, facility location under uncertainty, location-inventory problem modelling, dynamic location-inventory models, and location-inventory transportation models.

2. *To formulate mathematical models to identify the optimal number and location of dry ports in the seaport hinterland container network and the allocation of customers to established dry ports under periodic and uncertain container demand.*

A two-stage stochastic programming model was formulated to discover the optimal number and location of dry ports within the hinterland container network. Concurrently, the allocation of customers to opened dry ports was identified at the strategic level. Figure 6.2 illustrates the optimal number and location of DPs and their optimal allocation to customers for a specific problem instance. Furthermore, findings obtained from the formulated mathematical model revealed that the number of DPs should be increased when the inventory holding cost is low. The results also suggested that in order to achieve optimal allocation decisions at the strategic level, customers located far from the seaport should be allocated to more than one dry port. These findings confirm the optimality location-allocation solutions derived from the mathematical model.

3. *To determine the optimal decisions related to laden container flow, empty container flow, empty container leasing, and empty container inventory throughout the dry port container network.*

The second-stage of the developed stochastic programming model formulated the operational level decisions to find the optimal values for both laden and empty containers flow, empty container leasing, and empty container inventory level. The dynamic and stochastic nature of customers' container demand was incorporated by introducing a robust SAA procedure which ensured the reliability of solutions in the stochastic environment. The research findings verified that the employment of a robust optimisation approach could provide high quality solutions for container network design, which are less sensitive to the uncertainty of customers' container demand. Additionally, the robustness incorporation led to container networks with high performance from practical implementation. The optimal values of these operational decisions were validated through 80 different problem instances applied to a hypothetical container network design case study.

4. *To develop efficient solution methods in order to handle the complexity of the proposed models and to obtain high quality and robust solutions for the integrated dry port container network design problem.*

The research proposed an accelerated Benders Decomposition algorithm for solving the developed model. The combination of three different acceleration techniques (i.e., multi-cut framework, Knapsack inequalities, and pareto-optimal cuts generation scheme) performed better than Cplex solvers and standard BD, particularly for large scale networks. Furthermore, it was shown that the proposed acceleration scheme outperformed Cplex solvers and standard BD for large number of scenarios by generating optimal solutions with lower computational time.

5. *To provide key performance indicators for shipping lines to improve their service level, fill rate, and inventory turnover in dry port container networks.*

From the practical contribution point of view, this thesis presented a comprehensive analysis on important performance indicators of container networks. It was shown that the container network designer could enhance the service level of the network by assigning a higher weight to the solution robustness. The dependency of networks' service level to critical parameters including empty container's holding cost was shown by this research. Furthermore, the sensitivity of empty containers' inventory turnover to the weight of solution robustness was discovered by this research. More specifically, this research found that aiming for higher solution robustness could improve both fill rate and service level. Additionally, findings of the research suggests that high inventory holding costs and high levels of solution robustness could lower the inventory turnover of empty containers in the network.

7.3. Future research directions

There are a number of ways that this work can be further extended in the future. In this study the uncertainty of the industry was focused on container demand. One possible future study could be the consideration of uncertainty in other parameters including travel times, transportation capacities, and operational costs. Furthermore, the inventory control policy for the storage of empty containers at dry ports can be taken into consideration within the optimisation model. Dry ports can also be considered as the inland hub locations for consolidation, storage and intermodal transportation of containers. Accordingly, decisions related to the allocation of dry ports to each other as well as the optimal containers flow between dry ports (i.e., lateral transhipment decisions) should be studied further to examine possible lower costs through economies of scale. Additionally, the capacity of dry ports can be considered as a first-stage decision variable since the fixed costs of opening dry ports highly depend on their capacity. Finally, the described problem can be extended by developing models for global scale container network design. This requires the integration of decisions in both maritime and inland networks which may lead to extremely complex models.

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Appendix A.

Table 23. Notation of sets, parameters, and decision variables.

Sets

$\mathcal{I}$	Set of retailers indexed by i.
$\mathcal{J}$	Set of distribution centres indexed by j.
$\mathbb{N}$	Set of nodes indexed by n.
$\mathbb{O}$	Set of seaports indexed by p, q.
$\mathbb{I}$	Set of candidate dry port locations indexed by p, q.
$\mathbb{J}$	Set of customers indexed by p, q.
$\mathcal{A}$	Set of arcs.
Ω	Set of scenarios indexed by ω.
T	Set of periods indexed by t.
K	Set of containers indexed by k, ℓ, e.
M	Set of available transportation modes indexed by m.
R	Set of independent samples indexed by r.

Parameters

A_j	The ordering cost at distribution centre $j \in \mathcal{J}$.
H_j	The unit holding cost at distribution centre $j \in \mathcal{J}$.
T_j	Total available product at distribution centre $j \in \mathcal{J}$.
Q_j	The order size at distribution centre $j \in \mathcal{J}$.
D_j	The annual demand at distribution centre $j \in \mathcal{J}$.
F_j	The fixed cost of locating distribution centre $j \in \mathcal{J}$.
l_j	The mean of delivery lead time from the supplier to distribution centre $j \in \mathcal{J}$.
δ_j^2	The variance of delivery lead time from the supplier to distribution centre $j \in \mathcal{J}$.
μ_i	The mean of demand at retailer $i \in \mathcal{I}$.
σ_i^2	The variance of demand at retailer $i \in \mathcal{I}$.
t_j	The unit transportation cost from the supplier to distribution centre $j \in \mathcal{J}$.
t_{ij}	The unit transportation cost from retailer $i \in \mathcal{I}$ to distribution centre $j \in \mathcal{J}$.
z_α	The standard normal deviate such that $P(Z \leq z_\alpha) = \alpha$.
α	The service level at distribution centre $j \in \mathcal{J}$.
ψ	Number of working days per year.
w_i	Quantity delivered to retailer $i \in \mathcal{I}$.
$q_i(0)$	Inventory cost function at retailer $i \in \mathcal{I}$.
$C_i(0)$	Cumulative demand distribution function at retailer $i \in \mathcal{I}$.
Cap_n	The storage capacity of node $n \in \mathbb{N}$

f_{p}	The fixed cost for opening a dry port at node $p \in \mathbb{I}$.
d_{pq}	The fixed cost for allocating node p to node q.
c^k_{pqm}	The unit cost of transporting container type k on arc $(p, q) \in \mathcal{A}$ using mode m .
h_n	The unit cost of holding an empty container at node $n \in \mathbb{N}$.
g^+_n / g^-_n	The unit cost of leasing/returning an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
g_n	The unit cost of leased empty containers' net stock per container per period at node $n \in \mathbb{N} \setminus \{\mathbb{O} \cup \mathbb{J}\}$.
v^+_n / v^-_n	The unit importing/exporting cost of an empty container at node $n \in \mathbb{N} \setminus \{\mathbb{I} \cup \mathbb{J}\}$.
b_q	The unit backorder cost per container at customer $q \in \mathbb{J}$.
ν_q	The unit cost of rejected demand per container at customer $q \in \mathbb{J}$.
τ_{pqm}	The transportation lead-time on arc $(p, q) \in \mathcal{A}$ using mode $m \in M$.
θ_q	The containers processing time associate with the loading and unloading of finished goods/raw materials at customers $q \in \mathbb{J}$.
$\pi(\omega)$	The occurrence probability of scenario ω.
$D^{\ell}_{qt}(\omega)$	The demand for incoming laden containers at customer q in period t under scenario ω.
$S^{\ell}_{qt}(\omega)$	The demand for outgoing laden containers at customer q in period t under scenario ω.
ϑ	The cost of model robustness.
λ	The cost of solution robustness.
t_{pqm}	The transportation time on arc $(p, q) \in \mathcal{A}$ using transportation mode m.
β_m	The transportation cost parameter of transportation mode m.
Δ_{pq}	The travel distance from node p to node q.
V_m	The average speed of transportation mode m.
$\mu_{D^{\ell}_{qt}}$	The mean of lognormal distribution function related to demand at customer q in period t.
$\sigma_{D^{\ell}_{qt}}$	The standard deviation of lognormal distribution function related to demand at customer q in period t.
${\mu'}_{D^{\ell}_{qt}}$	The mean of Normal distribution function related to demand at customer q in period t.
${\sigma'}_{D^{\ell}_{qt}}$	The standard deviation of Normal distribution function related to demand at customer q in period t.

Decision Variables

X_j	Binary variable associated with location of retailer.
Y_{ij}	Binary variable associated with the demand allocation of an arc from node i to node j.
X_p	Binary variable associated with location of dry port p.
Y_{pq}	Binary variable associated with the demand allocation of an arc from node p to q.
$F^k_{pqtm}(\omega)$	The flow of container type k on arc $(p, q) \in \mathcal{A}$ in period t using mode m under scenario ω.
$I^e_{nt}(\omega)$	The inventory level of empty containers at node $n \in \mathbb{N}$ in period t under scenario ω.
$L^e_{nt}(\omega)$	The net stock of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$L^{e+}_{nt}(\omega)$	The number of leased empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.

$L_{nt}^{e-}(\omega)$	The number of returned empty containers at node $n \in \mathbb{I}$ in period t under scenario ω.
$H_{nt}^{e+}(\omega)$	The number of imported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$H_{nt}^{e-}(\omega)$	The number of exported empty containers at node $n \in \mathbb{O}$ in period t under scenario ω.
$U_{qt}^{\ell}(\omega)$	The backordered incoming demand at a customer q in period t under scenario ω.
$B_{qt}^{\ell}(\omega)$	The backordered outgoing demand at a customer q in period t under scenario ω.
$\gamma_{qt}^{\ell}(\omega)$	The rejected incoming demand at a customer q in period t under scenario ω.
$\delta_{qt}^{\ell}(\omega)$	The rejected outgoing demand at a customer q in period t under scenario ω.
$\mathbf{C}(\omega)$	The vector of all continuous variables for each scenario
$\mathfrak{U}(\omega)/\mathcal{K}(\omega)$	The infeasibility control variables under scenario ω.
$\Phi(\mathbf{X}, \omega)$	The solution robustness associate with first stage decisions **X** under scenario ω.
$\boldsymbol{\chi}$	The vector of the dual sub-problem's variables.
$\hat{\chi}$	The extreme points of the dual polyhedron obtained from solving the dual sub-problem.
γ	The optimality cut variable in Benders decomposition algorithm.
$\eta_r(\omega)$	The service level related to the demand of raw materials under scenario ω.
$\eta_s(\omega)$	The service level related to the supply of finished goods under scenario ω.
$\xi_r(\omega)$	The fill rate related to the demand of raw materials under scenario ω.
$\xi_s(\omega)$	The fill rate related to the supply of finished goods under scenario ω.
$\psi(\omega)$	The total throughput of empty containers from all dry ports under scenario ω.
$\iota(\omega)$	The average inventory level of empty containers in each period under scenario ω.
$\varepsilon(\omega)$	The inventory turnover of empty containers under scenario ω.

Appendix B.

Table 24. Parameters' value.

Parameter		Value/Distribution
$Cap_{\wp}$	$\wp \in \mathbb{O}$	10,000 TEUs
$Cap_{\wp}$	$\wp \in \mathbb{I}$	[2.0, 5.0]×10^4 TEUs
$Cap_{\wp}$	$\wp \in \mathbb{J}$	2000 TEUs
β_1		\$3.88
β_2		\$0.05
V_1		60 mph
V_2		24 mph
$\mu_{D^{\ell}_{qt}}$	$q \in \mathbb{J}, t \in T$	[6000,7000] TEUs
$\sigma_{D^{\ell}_{qt}}$	$q \in \mathbb{J}, t \in T$	$0.1\mu_{D^{\ell}_{qt}}$
$f_{\wp}$	$\wp \in \mathbb{I}$	[1.8,4.0]×10^6 for *low* opening costs
$f_{\wp}$	$\wp \in \mathbb{I}$	[4.0;7.5]×10^6 for *high* opening costs
$[h_{o}, h_{i}, h_{j}]$	$\forall o \in \mathbb{O}, i \in \mathbb{I}, j \in \mathbb{J}$	[2, 4, 8]×10^{-1} for *low* holding costs
$[h_{o}, h_{i}, h_{j}]$	$\forall o \in \mathbb{O}, i \in \mathbb{I}, j \in \mathbb{J}$	[2, 4, 8]×10 for *high* holding costs
λ		$\{10^{-3}, 10^{-2}, 10^{-1}, 1\}$
ν		$\{1000, 2000, 3000, 4000, 5000\}$

Appendix C.

The Cplex OPL code relating to the robust two-stage stochastic programming model

```
 int nbport = 1;
 int nbdepots = ...;
 int nbcustomers = ...;
 int nbperiods = 13;
 int nbmodes = 2;
 int nbscenarios = ...;
 float Lambda = 1;
 float Nu = 5000;


 range port = 1..nbport;
 range depots = 1..nbdepots;
 range customers = 1..nbcustomers;
 range periods = 1..nbperiods;
 range modes = 1..nbmodes;
 range scenarios = 1..nbscenarios;

 float Prob = 1/nbscenarios;



 //Parameters for laden containers

 // Travelling time
 int Tij[1..nbdepots, 1..nbcustomers*nbmodes]= ... ;
 int Tijm[i in 1..nbdepots, j in 1..nbcustomers, m in
1..nbmodes]= Tij[i,    m+nbmodes*(j-1)];

 int Toj[1..nbport, 1..nbcustomers*nbmodes]= ...;
 int Tojm[o in port, j in customers, m in modes]=Toj[o,
m+nbmodes*(j-1)];

 int Toi[1..nbport, 1..nbdepots*nbmodes]= ...;
 int Toim[o in port, i in depots, m in modes]=Toi[o,
m+nbmodes*(i-1)];

 int Wjm[customers][modes]=...; // processing time
(loading/unloading) at manufacturer j
 // Transportation costs
```

```
 float TCij[1..nbdepots, 1..nbcustomers*nbmodes]= ... ;
 float TCijm[i in depots, j in customers, m in modes]=TCij[i,
m+nbmodes*(j-1)];

 float TCoj[1..nbport, 1..nbcustomers*nbmodes]= ...;
 float TCojm[o in port, j in customers, m in modes]=TCoj[o,
m+nbmodes*(j-1)];

 float TCoi[1..nbport, 1..nbdepots*nbmodes]= ...;
 float TCoim[o in port, i in depots, m in modes]=TCoi[o,
m+nbmodes*(i-1)];

 float CONSij[1..nbdepots, 1..nbcustomers*nbmodes]= ... ;
 float CONSijm[i in depots, j in customers, m in
modes]=CONSij[i, m+nbmodes*(j-1)];

 float CONSoj[1..nbport, 1..nbcustomers*nbmodes]= ...;
 float CONSojm[o in port, j in customers, m in modes]=CONSoj[o,
m+nbmodes*(j-1)];

 float CONSoi[1..nbport, 1..nbdepots*nbmodes]= ...;
 float CONSoim[o in port, i in depots, m in modes]=CONSoi[o,
m+nbmodes*(i-1)];

 // Operatioal costs

 float OCij [1..nbdepots, 1..nbcustomers*nbmodes] = ...;
 float OCijm [i in depots, j in customers, m in modes]= OCij[i,
m+nbmodes*(j-1)];

 float OCoi [1..nbport, 1..nbdepots*nbmodes] = ...;
 float OCoim [o in port, i in depots, m in modes]= OCoi[o,
m+nbmodes*(i-1)];

 float OCoj [1..nbport, 1..nbcustomers*nbmodes] = ...;
 float OCojm [o in port, j in customers, m in modes]= OCoj[o,
m+nbmodes*(j-1)];
 //Demand and supply

float Djts[1..nbcustomers, 1..nbperiods*nbscenarios] = ...;
float Djtss[j in customers, t in periods, s in scenarios] =
Djts[j,s+nbscenarios*(t-1)];

float Sjts[1..nbcustomers, 1..nbperiods*nbscenarios] = ...;
float Sjtss[j in customers, t in periods, s in scenarios] =
Sjts[j,s+nbscenarios*(t-1)];
```

```
 //Penalty cost
 float CUj [customers]=...;
 float CKj [customers]=...;
 //Parameters for empty containers

 // Transportation costs
 float TCEij[1..nbdepots, 1..nbcustomers*nbmodes]= ... ;
 float TCEijm[i in depots, j in customers, m in modes]=TCEij[i,
m+nbmodes*(j-1)];

 float TCEoi[1..nbport, 1..nbdepots*nbmodes]= ...;
 float TCEoim[o in port, i in depots, m in modes]=TCEoi[o,
m+nbmodes*(i-1)];

 float TCEoj[1..nbport, 1..nbcustomers*nbmodes]= ...;
 float TCEojm[o in port, j in customers, m in modes]=TCEoj[o,
m+nbmodes*(j-1)];

 // Operatioal costs
 float OCEij [1..nbdepots, 1..nbcustomers*nbmodes] = ...;
 float OCEijm [i in depots, j in customers, m in modes]=
OCEij[i, m+nbmodes*(j-1)];

 float OCEoi [1..nbport, 1..nbdepots*nbmodes] = ...;
 float OCEoim [o in port, i in depots, m in modes]= OCEoi[o,
m+nbmodes*(i-1)];

 float OCEoj [1..nbport, 1..nbcustomers*nbmodes] = ...;
 float OCEojm [o in port, j in customers, m in modes]= OCEoj[o,
m+nbmodes*(j-1)];

 // Holding costs
 float Ho[port]=...;
 float Hi[depots]=...;
 float Hj[customers]=...;

 // Borrowing/returning Importing/exporting costs
 float CBo [port]=...;
 float CRo [port]=...;
 float CBi [depots]=...;
 float CRi [depots]=...;
 float CIMo [port]=...;
 float CEXo [port]=...;

 float CBNo [port]=...;
 float CBNi [depots]=...;
 //float CIMNo [port]=...;
```

```
//Capacities
float CAPi [depots]=...;
float CAPo [port]=...;
float CAPj [customers]=...;

//Fixed opening cost
float Fi[depots]=...;


// Decision Variables

//First stage decision variables
dvar boolean X[depots];
dvar boolean Y[depots][customers];

//Second stage decision variables

//Direct shipment between port and customers
dvar float+ Wojtsm
[port][customers][periods][scenarios][modes];
dvar float+ Wjotsm
[customers][port][periods][scenarios][modes];

dvar float+ WEojtsm
[port][customers][periods][scenarios][modes];
dvar float+ WEjotsm
[customers][port][periods][scenarios][modes];

//shipment between depots and customers
dvar float+ Fijtsm
[depots][customers][periods][scenarios][modes];
dvar float+ Fjitsm
[customers][depots][periods][scenarios][modes];

dvar float+ FEijtsm
[depots][customers][periods][scenarios][modes];
dvar float+ FEjitsm
[customers][depots][periods][scenarios][modes];

//Indirect shipment between port and customers where depots are
intermediate nodes
dvar float+ Goitsm [port][depots][periods][scenarios][modes];
dvar float+ Giotsm [depots][port][periods][scenarios][modes];

dvar float+ GEoitsm [port][depots][periods][scenarios][modes];
dvar float+ GEiotsm [depots][port][periods][scenarios][modes];
```

```
 //total
 dexpr float Cshipment = sum(o in port, i in depots, j in
customers, t in periods, s in scenarios, m in modes)
 (Wojtsm[o][j][t][s][m]+ Wjotsm[j][o][t][s][m]+
Fijtsm[i][j][t][s][m]+ Fjitsm[j][i][t][s][m]+
Goitsm[o][i][t][s][m]+ Giotsm[i][o][t][s][m]);

 dexpr float CEshipment = sum(o in port, i in depots, j in
customers, t in periods, s in scenarios, m in modes)
 (WEojtsm[o][j][t][s][m]+ WEjotsm[j][o][t][s][m]+
FEijtsm[i][j][t][s][m]+ FEjitsm[j][i][t][s][m]+
GEoitsm[o][i][t][s][m]+ GEiotsm[i][o][t][s][m]);

 // Stock volume of empty containers
 dvar float+ SEots [port][periods][scenarios];
 dvar float+ SEits [depots][periods][scenarios];
 dvar float+ SEjts [customers][periods][scenarios];

 dvar float+ INITSEis[depots][scenarios];
 dvar float+ INITSEjs[customers][scenarios];
 dvar float+ INITSEos[port][scenarios];


 //total
 dexpr float Cinventory = sum(o in port, i in depots, j in
customers, t in periods, s in scenarios)
(SEots[o][t][s]+SEits[i][t][s]+SEjts[j][t][s]) ;

 //Borrowing/returning Importing/exporting Volumes
 dvar float+ Bots [port][periods][scenarios];
 dvar float+ Rots [port][periods][scenarios];
 dvar float+ Bits [depots][periods][scenarios];
 dvar float+ Rits [depots][periods][scenarios];
 dvar float+ IMots [port][periods][scenarios];
 dvar float+ EXots [port][periods][scenarios];

 //total
 dexpr float Cborrowing = sum(o in port, i in depots, t in
periods, s in scenarios) (Bots[o][t][s]+ Bits[i][t][s]+
IMots[o][t][s]);
 dexpr float Creturning = sum(o in port, i in depots, t in
periods, s in scenarios) (Rots[o][t][s]+ Rits[i][t][s]+
EXots[o][t][s]);

 //Net Borrowing/returning volumes
 dvar float+ BNots [port][periods][scenarios];
```

```
 dvar float+ BNits [depots][periods][scenarios];
 //dvar float+ IMNots [port][periods][scenarios];

 //backorder volume
 dvar float+ Ujts [customers][periods][scenarios];
 dvar float+ Kjts [customers][periods][scenarios];

 //total
 dexpr float Cbackorder = sum(j in customers, t in periods, s in
scenarios) (Ujts[j][t][s] + Kjts[j][t][s]);

  dexpr float RawBackorderCost = (sum(s in scenarios, t in
periods, j in customers) CUj[j]*Ujts[j][t][s])/nbscenarios;
  dexpr float FinishBackorderCost = (sum(s in scenarios, t in
periods, j in customers) CKj[j]*Kjts[j][t][s])/nbscenarios;

 // robust making terms

// dvar float+ ZS[scenarios];
 dvar float+ Tet[scenarios]; // tetta
 dvar float+ Infeas1[customers][periods][scenarios];
 dvar float+ Infeas2[customers][periods][scenarios];


constraint Location; //constraint allocation;

//Second Stage Objective Function Components

// Transport Cost
dexpr float Transport[s in scenarios]=
sum(i in depots, j in customers, t in periods, m in modes)
((CONSijm[i][j][m]+TCijm[i][j][m]+OCijm[i][j][m])*(Fijtsm[i][j][
t][s][m]+Fjitsm[j][i][t][s][m])
+(CONSijm[i][j][m]+TCEijm[i][j][m]+OCEijm[i][j][m])*(FEijtsm[i][
j][t][s][m]+FEjitsm[j][i][t][s][m]))
+sum(o in port,i in depots, j in customers, t in periods, m in
modes)((CONSoim[o][i][m]+TCoim[o][i][m]+OCoim[o][i][m])*(Goitsm[
o][i][t][s][m]+Giotsm[i][o][t][s][m])
+(CONSoim[o][i][m]+TCEoim[o][i][m]+OCEoim[o][i][m])*(GEoitsm[o][
i][t][s][m]+GEiotsm[i][o][t][s][m]))
+sum (o in port, j in customers, t in periods, m in modes)
((CONSojm[o][j][m]+TCojm[o][j][m]+OCojm[o][j][m])*(Wojtsm[o][j][
t][s][m]+Wjotsm[j][o][t][s][m])
+(CONSojm[o][j][m]+TCEojm[o][j][m]+OCEojm[o][j][m])*(WEojtsm[o][
j][t][s][m]+WEjotsm[j][o][t][s][m]));

// Holding Cost
```

```
dexpr float Holding[s in scenarios]=
sum (t in periods)(sum (o in port)Ho[o]*SEots[o][t][s]+sum (i in
depots)Hi[i]*SEits[i][t][s]+ sum(j in
customers)Hj[j]*SEjts[j][t][s]);

//Borrowing Cost
dexpr float Borrowing[s in scenarios]=
sum (t in periods)(sum (o in
port)(CBo[o]*Bots[o][t][s]+CRo[o]*Rots[o][t][s]+CBNo[o]*BNots[o]
[t][s])
+sum(i in
depots)(CBi[i]*Bits[i][t][s]+CRi[i]*Rits[i][t][s]+CBNi[i]*BNits[
i][t][s])
+sum (o in port)
(CIMo[o]*IMots[o][t][s]+CEXo[o]*EXots[o][t][s]));

//Backorder Cost
dexpr float RawBackorder [s in scenarios]=
sum(t in periods, j in customers) (CUj[j]*Ujts[j][t][s]);

dexpr float FinishedBackorder [s in scenarios]=
sum(t in periods, j in customers) (CKj[j]*Kjts[j][t][s]);


dexpr float ZS[s in scenarios] = Transport[s] + Holding[s] +
Borrowing[s] + RawBackorder[s] + FinishedBackorder[s];

//First Stage Objective Function

dexpr float Locating =sum (i in depots)Fi[i]*X[i] + sum (i in
depots, j in customers) Y[i][j] ;

//RO Objective Function

dexpr float Mean= sum (s in scenarios) Prob*ZS[s];
dexpr float SD= sum(s in scenarios) Prob*(ZS[s] - (sum(c in
scenarios:c!=s)Prob*ZS[c]) + 2*Tet[s]);
dexpr float SolutionRobust= Mean + (Lambda*SD);
dexpr float ModelRobust= Prob* sum(s in scenarios, t in periods,
j in customers) (Infeas1[j][t][s] +Infeas2[j][t][s]);

dexpr float SD1 = sum(s in scenarios) Prob*(ZS[s]-(sum(c in
scenarios:c!=s)Prob*ZS[c]));
dexpr float TotalZS1 = sum(s in scenarios) Prob*ZS[s];
dexpr float TotalZS2 = (sum(s in scenarios) ZS[s])/nbscenarios;
dexpr float TotalCost2 = Locating + Mean;
```

```
//Model Formulation
dexpr float obj = Locating + Mean + (Nu*ModelRobust);

float temp;
execute{
var before = new Date();
temp = before.getTime();
}

minimize obj;

subject to{

Location =
forall (i in depots, j in customers)
 Y[i][j]<=X[i];

forall (j in customers)
 sum(i in depots)Y[i][j] >= 1 ;

SupplyDemandFLow:

forall (j in customers, t in periods: t-1 in periods, s in
scenarios)
cons7:
sum (i in depots, m in modes: t-Tijm[i][j][m] in periods)
Fijtsm[i][j][t- Tijm[i][j][m]][s][m]+sum(o in port,m in modes:
t- Tojm[o][j][m] in periods) Wojtsm[o][j][t-
Tojm[o][j][m]][s][m] == Djtss[j][t][s] + Ujts[j][t-1][s]-
Ujts[j][t][s] -Infeas1[j][t][s];

forall (j in customers, t in periods: t-1 in periods, s in
scenarios)
cons8:
sum (i in depots, m in modes) Fjitsm[j][i][t][s][m]+sum(o in
port, m in modes) Wjotsm[j][o][t][s][m] == Sjtss[j][t][s] +
Kjts[j][t-1][s]-Kjts[j][t][s] -Infeas2[j][t][s];

//************************************************************
*****************************************************

EmptyContainersAvailability:

cons9:
```

```
forall (j in customers, t in periods: t-1 in periods, s in
scenarios)
sum(i in depots, m in modes) Fjitsm[j][i][t][s][m] + sum (o in
port, m in modes) Wjotsm[j][o][t][s][m] +
sum(i in depots, m in modes) FEjitsm[j][i][t-1][s][m] + sum (o
in port, m in modes) WEjotsm[j][o][t-1][s][m]<= SEjts[j][t-
1][s];

//**************************************************************
****************************************************
cons10:
forall (j in customers, t in periods: t==1, s in scenarios)
sum(i in depots, m in modes) FEjitsm[j][i][t][s][m] + sum (o in
port, m in modes) WEjotsm[j][o][t][s][m] <= SEjts[j][t][s];

//**************************************************************
****************************************************
cons11:
forall (i in depots, t in periods, s in scenarios)
sum(j in customers, m in modes) FEijtsm[i][j][t][s][m] + sum (o
in port, m in modes) GEiotsm[i][o][t][s][m] <= SEits[i][t][s];

cons12:
forall (o in port, t in periods, s in scenarios)
sum(i in depots, m in modes) GEoitsm[o][i][t][s][m] + sum (j in
customers, m in modes) WEojtsm[o][j][t][s][m] <= SEots[o][t][s];

InventoryLevel:

cons13:
forall (j in customers, t in periods: t-1 in periods, s in
scenarios)
SEjts[j][t][s] == SEjts[j][t-1][s]+ sum(o in port, m in modes:
t-Tojm[o][j][m]-Wjm[j][m] in periods) Wojtsm[o][j][t-
Tojm[o][j][m]-Wjm[j][m]][s][m]
+sum (i in depots, m in modes :t-Tijm[i][j][m]-Wjm[j][m] in
periods) Fijtsm[i][j][t-Tijm[i][j][m]-Wjm[j][m]][s][m]
+sum(o in port, m in modes :t-Tojm[o][j][m] in
periods)WEojtsm[o][j][t-Tojm[o][j][m]][s][m]
+sum(i in depots, m in modes :t-Tijm[i][j][m] in
periods)FEijtsm[i][j][t-Tijm[i][j][m]][s][m]
-sum(o in port, m in modes :t+Wjm[j][m]in
periods)Wjotsm[j][o][t+Wjm[j][m]][s][m]
-sum(i in depots, m in modes :t+Wjm[j][m]in
periods)Fjitsm[j][i][t+Wjm[j][m]][s][m]
-sum(o in port, m in modes)WEjotsm[j][o][t][s][m]
-sum(i in depots, m in modes)FEjitsm[j][i][t][s][m];
```

```
cons14:
forall (j in customers, t in periods: t==1, s in scenarios)
SEjts[j][t][s] == 0
-sum(o in port, m in modes :t+Wjm[j][m]in
periods)Wjotsm[j][o][t+Wjm[j][m]][s][m]
-sum(i in depots, m in modes :t+Wjm[j][m]in
periods)Fjitsm[j][i][t+Wjm[j][m]][s][m]
-sum(o in port, m in modes)WEjotsm[j][o][t][s][m]
-sum(i in depots, m in modes)FEjitsm[j][i][t][s][m];

//*************************************************************
*******************************************************

cons15:
forall (i in depots, t in periods: t-1 in periods, s in
scenarios)
SEits[i][t][s] == SEits[i][t-1][s]
+ sum(j in customers, m in modes: t-Tijm[i][j][m] in
periods)FEjitsm[j][i][t-Tijm[i][j][m]][s][m]
+ sum(o in port, m in modes:t-Toim[o][i][m] in periods)
GEoitsm[o][i][t-Toim[o][i][m]][s][m]
- sum(j in customers, m in modes)FEijtsm[i][j][t][s][m]
- sum (o in port, m in modes) GEiotsm[i][o][t][s][m]
+ Bits[i][t][s] - Rits[i][t][s];

cons16:
forall (i in depots, s in scenarios, t in periods:t==1)
SEits[i][t][s] == 0
- sum(j in customers, m in modes)FEijtsm[i][j][t][s][m]
- sum (o in port, m in modes) GEiotsm[i][o][t][s][m]
+ Bits[i][t][s] - Rits[i][t][s];

//*************************************************************
********************************************************
cons17:
forall (o in port, t in periods: t-1 in periods, s in scenarios)
SEots [o][t][s] == SEots[o][t-1][s]
- sum(i in depots, o in port, m in modes) GEoitsm[o][i][t][s][m]
+ sum(i in depots, m in modes: t-Toim[o][i][m] in
periods)GEiotsm[i][o][t-Toim[o][i][m]][s][m]
+ sum (j in customers, m in modes: t-Tojm[o][j][m] in periods)
WEjotsm[j][o][t-Tojm[o][j][m]][s][m]
- sum(j in customers, m in modes) WEojtsm[o][j][t][s][m]
+ Bots[o][t][s] - Rots[o][t][s] + IMots[o][t][s] -
EXots[o][t][s];
```

```
cons18:
forall (o in port, t in periods: t==1, s in scenarios)
SEots [o][t][s] == 0
- sum(i in depots, o in port, m in modes) GEoitsm[o][i][t][s][m]
- sum(j in customers, m in modes) WEojtsm[o][j][t][s][m]
+ Bots[o][t][s] - Rots[o][t][s] + IMots[o][t][s] -
EXots[o][t][s];

//*************************************************************
********************************************************
BorrowReturn:

cons19:
forall (i in depots, s in scenarios)
sum(t in periods)Bits[i][t][s] >= sum(t in
periods)Rits[i][t][s];

cons20:
forall (o in port, s in scenarios)
sum(t in periods) Bots[o][t][s] >= sum(t in periods)
Rots[o][t][s];

cons21:
forall (o in port, s in scenarios)
sum(t in periods) IMots[o][t][s] >= sum(t in periods)
EXots[o][t][s];

CapacityNodes:

forall(i in depots, t in periods, s in scenarios)
cons22:
SEits[i][t][s] <= CAPi[i]*X[i];

forall (o in port, t in periods, s in scenarios)
cons23:
SEots[o][t][s] <= CAPo[o];

forall(j in customers, t in periods, s in scenarios)
cons24:
SEjts[j][t][s] <= CAPj[j];

IntermediateFlowConservation:

cons25:
forall (i in depots, t in periods, s in scenarios)
```

```
sum(m in modes, j in customers) Fijtsm[i][j][t][s][m] == sum (o
in port, m in modes:t-Toim[o][i][m] in periods) Goitsm[o][i][t-
Toim[o][i][m]][s][m];

cons26:
forall (i in depots, t in periods, s in scenarios)
sum(j in customers, m in modes: t-Tijm[i][j][m] in periods)
Fjitsm[j][i][t-Tijm[i][j][m]][s][m] == sum(o in port, m in
modes) Giotsm[i][o][t][s][m];

//**************************************************************
*******************
cons27:
forall (i in depots, t in periods: t<2, s in scenarios)
sum (o in port, m in modes) Goitsm[o][i][t][s][m] == sum (m in
modes, j in customers) Fijtsm[i][j][t][s][m];

cons28:
forall (i in depots, t in periods: t<2, s in scenarios)
sum (j in customers,m in modes) Fjitsm[j][i][t][s][m] == sum(o
in port, m in modes) Giotsm[i][o][t][s][m];

//**************************************************************
********************
NetBorrowing:

cons29:
forall (o in port, t in periods: t-1 in periods, s in scenarios)
 BNots[o][t][s] == BNots[o][t-1][s] + Bots[o][t][s] - Rots
[o][t][s];

cons30:
forall (i in depots, t in periods: t-1 in periods, s in
scenarios)
 BNits[i][t][s] == BNits[i][t-1][s] + Bits[i][t][s] - Rits
[i][t][s];

cons31:
forall (o in port, t in periods: t==1, s in scenarios)
 BNots[o][t][s] == Bots[o][t][s] - Rots [o][t][s];

cons32:
forall (i in depots, t in periods: t==1, s in scenarios)
 BNits[i][t][s] == Bits[i][t][s] - Rits [i][t][s];

//**************************************************************
******************
```

```
forall ( i in depots, j in customers, t in periods, s in
scenarios, m in modes)
cons37:
Fijtsm[i][j][t][s][m] <= 100000000*Y[i][j];

forall ( j in customers, i in depots, t in periods, s in
scenarios, m in modes)
cons38:
Fjitsm[j][i][t][s][m] <= 100000000*Y[i][j];

forall ( i in depots, j in customers, t in periods, s in
scenarios, m in modes)
cons39:
FEijtsm[i][j][t][s][m] <= 100000000*Y[i][j];

forall ( j in customers, i in depots, t in periods, s in
scenarios, m in modes)
cons40:
FEjitsm[j][i][t][s][m] <= 100000000*Y[i][j];

}

   execute
     {
     writeln("x=",X, " Y=",Y);
     writeln("  Obj=",obj);
     var after = new Date();
      writeln("solving time ~= ",(after.getTime()-temp)/1000, "
seconds")

     }
```

Appendix D.

The Cplex script code relating to standard Benders Decomposition Algorithm

```
int ar[i in 1..1]=520000;

main {

 thisOplModel.settings.mainEndEnabled = true;

var xb = new Array();
var yb = new Array(10);
    for(var i=1;i<=8;i++){
    yb[i]=new Array(10);
    for(var j=1;j<=50;j++){
    yb[i][j]=new Array(10);}}
var zb =0;
var gammb = 0;

     var du7 = new Array(10);
     for(var k=1;k<=10;k++){
      du7[k]= new Array(10);
     for(var j=1;j<=50;j++){
      du7[k][j]= new Array(10);
     for(var t=2;t<=13;t++){
      du7[k][j][t]= new Array(10);
     for(var s=1;s<=5;s++){
      du7[k][j][t][s] = 1;}}}}

     var du8 = new Array(10);
     for(var k=1;k<=10;k++){
      du8[k]= new Array(10);
     for(var j=1;j<=50;j++){
      du8[k][j]= new Array(10);
     for(var t=2;t<=13;t++){
      du8[k][j][t]= new Array(10);
     for(var s=1;s<=5;s++){
      du8[k][j][t][s] = 1;}}}}

     var du22 = new Array(5200);
     for(var k=1;k<=10;k++){
      du22[k]= new Array(10);
     for(var i=1;i<=8;i++){
      du22[k][i]= new Array(80);
     for(var t=1;t<=13;t++){
```

```
 du22[k][i][t]= new Array(1040);
for(var s=1;s<=5;s++){
 du22[k][i][t][s] = 1;}}}}

var du23 = new Array(650);
for(var k=1;k<=10;k++){
 du23[k]= new Array(10);
for(var o=1;o<=1;o++){
 du23[k][o]= new Array(10);
for(var t=1;t<=13;t++){
 du23[k][o][t]= new Array(130);
for(var s=1;s<=5;s++){
 du23[k][o][t][s] = 1;}}}}

var du24 = new Array(2500);
for(var k=1;k<=10;k++){
 du24[k]= new Array(10);
for(var j=1;j<=50;j++){
 du24[k][j]= new Array(500);
for(var t=1;t<=13;t++){
 du24[k][j][t]= new Array(2500);
for(var s=1;s<=5;s++){
 du24[k][j][t][s] = 1;}}}}

var du37 = new Array(520000);
for(var k=1;k<=10;k++){
 du37[k]= new Array(10);
for(var i=1;i<=8;i++){
 du37[k][i]= new Array(80);
for(var j=1;j<=50;j++){
 du37[k][i][j]= new Array(4000);
for(var t=1;t<=13;t++){
 du37[k][i][j][t] = new Array(52000);
for(var s=1;s<=5;s++){
 du37[k][i][j][t][s] = new Array(260000);
for(var m=1;m<=2;m++){
 du37[k][i][j][t][s][m] = 1; }}}}}}

var du38 = new Array(520000);
for(var k=1;k<=10;k++){
 du38[k]= new Array(10);
for(var j=1;j<=50;j++){
 du38[k][j]= new Array(500);
for(var i=1;i<=8;i++){
 du38[k][j][i]= new Array(4000);
for(var t=1;t<=13;t++){
 du38[k][j][i][t] = new Array(52000);
```

```
      for(var s=1;s<=5;s++){
       du38[k][j][i][t][s] = new Array(260000);
      for(var m=1;m<=2;m++){
       du38[k][j][i][t][s][m] = 1; }}}}}}

      var du39 = new Array(520000);
      for(var k=1;k<=10;k++){
       du39[k]= new Array(10);
      for(var i=1;i<=8;i++){
       du39[k][i]= new Array(80);
      for(var j=1;j<=50;j++){
       du39[k][i][j]= new Array(4000);
      for(var t=1;t<=13;t++){
       du39[k][i][j][t] = new Array(52000);
      for(var s=1;s<=5;s++){
       du39[k][i][j][t][s] = new Array(260000);
      for(var m=1;m<=2;m++){
       du39[k][i][j][t][s][m] = 1; }}}}}}

     var du40 = new Array(520000);
      for(var k=1;k<=10;k++){
       du40[k]= new Array(10);
      for(var j=1;j<=50;j++){
       du40[k][j]= new Array(500);
      for(var i=1;i<=8;i++){
       du40[k][j][i]= new Array(4000);
      for(var t=1;t<=13;t++){
       du40[k][j][i][t] = new Array(52000);
      for(var s=1;s<=5;s++){
       du40[k][j][i][t][s] = new Array(260000);
      for(var m=1;m<=2;m++){
       du40[k][j][i][t][s][m] = 1; }}}}}}

var LB = -Infinity;
var UB = +Infinity;
var counter=1;
//uk[counter]=new Array();
var ofile = new IloOplOutputFile("modelRun.txt");
var modelSource = new IloOplModelSource("SubProblem0.mod");
var modelDef = new IloOplModelDefinition(modelSource);
var model = new IloOplModel(modelDef, cplex);
var Data = new IloOplDataSource("SubProblem0.dat");
var f=new IloOplOutputFile("export.csv");

    model.addDataSource(Data);
    model.settings.mainEndEnabled = true;
    model.generate();
```

```
  if (cplex.solve()) {  for(var s=1;s<=5;s++)
                                   for(var t=2;t<=13;t++)
                                        for(var j=1;j<=50;j++)

     {du7[counter][j][t][s]= model.cons7[j][t][s].dual;}
                           for(var s=1;s<=5;s++)
                                   for(var t=2;t<=13;t++)
                                        for(var j=1;j<=50;j++)

     {du8[counter][j][t][s]= model.cons8[j][t][s].dual;}
                              for(var s=1;s<=5;s++)
                                   for(var t=2;t<=13;t++)
                                        for(var i=1;i<=8;i++)

     {du22[counter][i][t][s]= model.cons22[i][t][s].dual;}
                              for(var s=1;s<=5;s++)
                                   for(var t=1;t<=13;t++)
                                        for(var o=1;o<=1;o++)

     {du23[counter][o][t][s]= model.cons23[o][t][s].dual;}
                              for(var s=1;s<=5;s++)
                                   for(var t=1;t<=13;t++)
                                        for(var j=1;j<=50;j++)

     {du24[counter][j][t][s]= model.cons24[j][t][s].dual;}
                              for(var m=1;m<=2;m++)
                                  for(var s=1;s<=5;s++)
                                       for(var t=1;t<=13;t++)
                                            for(var
j=1;j<=50;j++)
                                                 for(var
i=1;i<=8;i++)

     {du37[counter][i][j][t][s][m]=
model.cons37[i][j][t][s][m].dual;}
                              for(var m=1;m<=2;m++)
                                  for(var s=1;s<=5;s++)
                                       for(var t=1;t<=13;t++)
                                            for(var
i=1;i<=8;i++)
                                                for(var
j=1;j<=50;j++)

{du38[counter][j][i][t][s][m]=
model.cons38[j][i][t][s][m].dual;}
                              for(var m=1;m<=2;m++)
                                  for(var s=1;s<=5;s++)
```

```
                                        for(var t=1;t<=13;t++)
                                             for(var
j=1;j<=50;j++)
                                                  for(var
i=1;i<=8;i++)

     {du39[counter][i][j][t][s][m]=
model.cons39[i][j][t][s][m].dual;}
                               for(var m=1;m<=2;m++)
                                   for(var s=1;s<=5;s++)
                                        for(var t=1;t<=13;t++)
                                             for(var
i=1;i<=8;i++)
                                                 for(var
j=1;j<=50;j++)

{du40[counter][j][i][t][s][m]=
model.cons40[j][i][t][s][m].dual;}

                                             }



          else {writeln("No solution Found")}
               model.end();
     while (UB-LB >= 10){writeln(counter);
     var masterSource = new
IloOplModelSource("MasterProblem.mod");
     var masterDef = new IloOplModelDefinition(masterSource);
     var master = new IloOplModel(masterDef, cplex);
    var masterData = new IloOplDataSource("MasterProblem.dat");

    master.settings.mainEndEnabled = true;
    var data2= new IloOplDataElements();
     data2.counters=thisOplModel.ar;
    data2.counters[1]=counter;
    master.addDataSource(data2);
    master.addDataSource(masterData);
    masterData = master.dataElements;

    for(var s=1;s<=5;s++) for(var t=2;t<=13;t++) for(var
j=1;j<=50;j++) for(var k=1;k<=counter;k++)
master.dualcons7[k][j][t][s]=du7[k][j][t][s];
    for(var s=1;s<=5;s++) for(var t=2;t<=13;t++) for(var
j=1;j<=50;j++) for(var k=1;k<=counter;k++)
master.dualcons8[k][j][t][s]=du8[k][j][t][s];
```

```
    for(var s=1;s<=5;s++) for(var t=1;t<=13;t++) for(var
i=1;i<=8;i++) for(var k=1;k<=counter;k++)
master.dualcons22[k][i][t][s]=du22[k][i][t][s];
    for(var s=1;s<=5;s++) for(var t=1;t<=13;t++) for(var
o=1;o<=1;o++)  for(var k=1;k<=counter;k++)
master.dualcons23[k][o][t][s]=du23[k][o][t][s];
    for(var s=1;s<=5;s++) for(var t=1;t<=13;t++) for(var
j=1;j<=50;j++) for(var k=1;k<=counter;k++)
master.dualcons24[k][j][t][s]=du24[k][j][t][s];
    for(var m=1;m<=2;m++) for(var s=1;s<=5;s++)  for(var
t=1;t<=13;t++) for(var j=1;j<=50;j++) for(var i=1;i<=8;i++)
for(var k=1;k<=counter;k++)
master.dualcons37[k][i][j][t][s][m]=du37[k][i][j][t][s][m];
    for(var m=1;m<=2;m++) for(var s=1;s<=5;s++)  for(var
t=1;t<=13;t++) for(var i=1;i<=8;i++)  for(var j=1;j<=50;j++)
for(var k=1;k<=counter;k++)
master.dualcons38[k][j][i][t][s][m]=du38[k][j][i][t][s][m];
    for(var m=1;m<=2;m++) for(var s=1;s<=5;s++)  for(var
t=1;t<=13;t++) for(var j=1;j<=50;j++) for(var i=1;i<=8;i++)
for(var k=1;k<=counter;k++)
master.dualcons39[k][i][j][t][s][m]=du39[k][i][j][t][s][m];
    for(var m=1;m<=2;m++) for(var s=1;s<=5;s++)  for(var
t=1;t<=13;t++) for(var i=1;i<=8;i++)  for(var j=1;j<=50;j++)
for(var k=1;k<=counter;k++)
master.dualcons40[k][j][i][t][s][m]=du40[k][j][i][t][s][m];

    master.generate();
    if (cplex.solve()) {master.postProcess();
    zb=master.Z.solutionValue;
    LB=zb;
    for(var i=1; i<=8; i++)xb[i] = master.X[i].solutionValue;
    for(var j=1; j<=50; j++) for(var i=1; i<=8; i++)yb[i][j] =
master.Y[i][j].solutionValue;
    var Zfirst=master.FirstStage.solutionValue;

}
 else {writeln("No solution Found")}
      master.end();
     counter++;

 var DualSource = new IloOplModelSource("SubProblem.mod");
 var DualDef = new IloOplModelDefinition(DualSource);
 var Dualmodel = new IloOplModel(DualDef, cplex);
 var DualData = new IloOplDataSource("SubProblem.dat");
    Dualmodel.addDataSource(DualData);
     Dualmodel.settings.mainEndEnabled = true;
    DualData = Dualmodel.dataElements;
```

```
    for(var i=1; i<=8; i++) {xb[i]=DualData.X[i];}
    for(var j=1; j<=50; j++) for(var i=1; i<=8; i++)
{yb[i][j]=DualData.Y[i][j];}
    Dualmodel.generate();
    if (cplex.solve()) {
          Dualmodel.postProcess();
          for(var s=1;s<=5;s++)
                    for(var t=2;t<=13;t++)
                         for(var j=1;j<=50;j++)

     {du7[counter][j][t][s]=Dualmodel.cons7[j][t][s].dual;}
              for(var s=1;s<=5;s++)
                    for(var t=2;t<=13;t++)
                         for(var j=1;j<=50;j++)

     {du8[counter][j][t][s]=Dualmodel.cons8[j][t][s].dual;}
               for(var s=1;s<=5;s++)
                   for(var t=1;t<=13;t++)
                         for(var i=1;i<=8;i++)

     {du22[counter][i][t][s]=Dualmodel.cons22[i][t][s].dual;}
               for(var s=1;s<=5;s++)
                   for(var t=1;t<=13;t++)
                         for(var o=1;o<=1;o++)

     {du23[counter][o][t][s]=Dualmodel.cons23[o][t][s].dual;}

               for(var s=1;s<=5;s++)
                   for(var t=1;t<=13;t++)
                         for(var j=1;j<=50;j++)

     {du24[counter][j][t][s]=Dualmodel.cons24[j][t][s].dual;}
               for(var m=1;m<=2;m++)
                   for(var s=1;s<=5;s++)
                        for(var t=1;t<=13;t++)
                             for(var j=1;j<=50;j++)
                                  for(var i=1;i<=8;i++)

     {du37[counter][i][j][t][s][m]=Dualmodel.cons37[i][j][t][s][
m].dual;}
               for(var m=1;m<=2;m++)
                   for(var s=1;s<=5;s++)
                        for(var t=1;t<=13;t++)
                             for(var i=1;i<=8;i++)
                                 for(var j=1;j<=50;j++)
```

```
{du38[counter][j][i][t][s][m]=Dualmodel.cons38[j][i][t][s][m].du
al;}
               for(var m=1;m<=2;m++)
                   for(var s=1;s<=5;s++)
                        for(var t=1;t<=13;t++)
                             for(var j=1;j<=50;j++)
                                  for(var i=1;i<=8;i++)

{du39[counter][i][j][t][s][m]=Dualmodel.cons39[i][j][t][s][m].du
al;}
                              for(var m=1;m<=2;m++)
                   for(var s=1;s<=5;s++)
                        for(var t=1;t<=13;t++)
                             for(var i=1;i<=8;i++)
                                 for(var j=1;j<=50;j++)

{du40[counter][j][i][t][s][m]=Dualmodel.cons40[j][i][t][s][m].du
al;}

     gammb=cplex.getObjValue();
     UB=Zfirst+ (0.2*gammb);
   }
     else {writeln("No solution found");}
     Dualmodel.end();

     writeln ("UB: ", UB," ,LB: ", LB);
}
     writeln ("objective value: ", UB);
     f.writeln("X: ", xb);
     f.writeln("Y: ", yb);

}
```