Variable Rate Swap Model

Variable rate swap is an interest rate swap that has two legs: one fixed rate leg and a variable rate leg. The variable leg involves fixed rate payments for an initial period of time and a floating rate for the rest. The floating rate on that portion is defined as a minimum of two index rates.

The fixed rate leg is similar to a fixed rate bond. The bond price is computed by discounting each coupon descripted at https://finpricing.com/lib/FiBond.html

We treat the two index rates as assets whose values are lognormally distributed random variable. Their pricing procedure uses discount factors retrieved from the EURIBOR curve.

The present value of the minimum of two assets, s_1 and s_2, is given by the formula

$$p = D(T)[\bar{s}_1 N(\frac{\ln(\bar{s}_2 / \bar{s}_1) - \sigma^2 / 2}{\sigma}) + \bar{s}_2 N(\frac{\ln(\bar{s}_1 / \bar{s}_2) - \sigma^2 / 2}{\sigma})] \qquad (1)$$

where

- $\sigma^2 = (\sigma_1{}^2 + \sigma_2{}^2 - 2\rho\sigma_1\sigma_2)T$,
- σ_1 and σ_2 are the volatilities of s_1 and s_2 respectively,
- ρ is the correlation of the two risk factors,
- T is the maturity time,
- $\bar{s}_1$ and $\bar{s}_2$ are the expected values of s_1 and s_2 at maturity (future values),
- $D(T)$ is the discount factor implied by the yield curve.

It is forward two index rates as retrieved from the yield curves that should be used for $\bar{s}_1$ and $\bar{s}_2$.

The minimum of two values, s_1 and s_2, can be expressed as

$$p = min(s_1,s_2) = s_1 - max(s_1\text{-}s_2, 0) \qquad (2)$$

If s_1 and s_2 are two risky assets, the present value of p at some future time T is

$$PV(p) = P(0,T)E_T(p) \qquad (3)$$

where $P(0,T)$ is the discount factor implied by the yield curve. Assuming the asset values lognormally distributed, the expectation of $max(s_1\text{-}s_2, 0)$ at maturity can be found as the value of the so called "Exchange-One-Asset-for-Another" European option [2,3] and is given by the following equation:

$$v = \bar{s}_1 N(\frac{\ln(\bar{s}_1 / \bar{s}_2) + \sigma^2 / 2}{\sigma}) - \bar{s}_2 N(\frac{\ln(\bar{s}_1 / \bar{s}_2) - \sigma^2 / 2}{\sigma}) \qquad (4)$$

where

- $\sigma^2 = (\sigma_1{}^2 + \sigma_2{}^2 - 2\rho\sigma_1\sigma_2)T$
- σ_1 and σ_2 are the volatilities of s_1 and s_2 respectively,
- ρ is the correlation of the two risk factors
- T is the maturity time
- $\bar{s}_1$ and $\bar{s}_2$ are the expected values of s_1 and s_2 at maturity (future values).

Combined, Eqs. (2), (3) and (4) yield

$$p = P(0,T)[\bar{s}_1 N(\frac{\ln(\bar{s}_2 / \bar{s}_1) - \sigma^2 / 2}{\sigma}) + \bar{s}_2 N(\frac{\ln(\bar{s}_1 / \bar{s}_2) - \sigma^2 / 2}{\sigma})] \qquad (5)$$