GIC Pricing Model

Guaranteed investment certificate (GIC) is a type of guaranteed investment certificate, which offers a return based on a corresponding GIC rate and the performance of a basket of certain stock and bond market indices.

The payoff at maturity from a GIC can be shown equal to the invested principal plus principal times the sum of the minimum guaranteed interest rate and the payoff from a European call option on the arithmetic average of the basket price, where the basket price is given by a weighted sum of the index levels.

We assume that each of the basket's underlying stock and bond market indices follows geometric Brownian motion with drift under its respective risk neutral probability measure. Each index process is then expressed under the Canadian risk neutral probability measure by means of a corresponding quanto adjustment.

We approximate the basket price arithmetic average by a shifted lognormal random variable. Here the defining parameters for the approximating random variable are uniquely determined by matching the true, first three moments of the basket price arithmetic average. The European call option price is then given analytically by a Black Scholes type of formula.

To determine the rate of return, the percentage change in each index level from the initial level is calculated. Here the initial level is set two days after purchase, while the final level is set to the arithmetic average of the last 11 month-ending index levels and the level one day prior to maturity.

The rate of return is then given by a weighted sum of the GIC rate and the percentage change in each index level described above, but bounded below by a minimum guaranteed interest rate. At maturity, the GIC holder receives the invested principal plus the principal times this rate of return.

The payoff at maturity from a GIC can be shown equal to the invested principal plus these principal times the sum of the minimum guaranteed interest rate and the payoff from a European call option on the arithmetic average of a basket price at the 12 points above, where the basket price is given by a weighted sum of the index levels above.

We consider the pricing of this call option. We assume that each of the underlying stock and bond market indices in the basket follows geometric Brownian motion with drift under their respective risk neutral probability measures. Each index process is then expressed under the Canadian risk neutral probability measure by means of a corresponding quanto adjustment.

Observe that the basket price arithmetic average is not lognormally distributed. Corporate Treasury's approach towards pricing the call option, then, is to approximate the basket price arithmetic average using a shifted lognormal random variable. Here the defining parameters for the approximating random variable are uniquely determined by matching the first three moments of the basket price arithmetic average. The call option price is then given analytically.

The GIC specification includes

- the maturity (either 3 or 5 years),
- initial levels for the five indices above, set to the respective index closing level on the second business day after the date of purchase,

- twelve averaging times, respectively set to the last business day in each of the eleven months that precede the month in which the GIC matures and the business day that immediately precedes the maturity date,
- a GIC interest rate with corresponding compounding frequency, and
- a participation rate, which is the percentage of the net return on the index components.

For each index, let

* I denote the index closing level,
* ω denote the corresponding weight,
* IIL denote the initial index level,
* the final index level (FIL) denote the arithmetic average of the index closing level at each of the twelve points defined above, that is, $FIL = \frac{1}{12}\sum I$,
* the total return (TR) equal the percentage change in final index price from the initial price, that is, $TR = \frac{FIL - IIL}{IIL}$, and
* the weighted return (WR) equal the weight times the total return, $\omega \frac{FIL - IIL}{IIL}$.

Then

* the weighted basket return (WBR) equals the sum of each index weighted return, that is, $WBR = \sum WR$, and
* the basket final return (B_FR) equals the weighted basket return multiplied by the participation rate, that is, $B\_FR = \beta \times WBR$ (where β denotes the participation rate).

Similarly for the GIC component, let

* ω denote the corresponding weight,
* r and f respectively denote the annualized GIC rate of interest (expressed as a percentage) and corresponding compounding frequency, and

* weighted return (WR) equal the compounded interest over the maturity, T, that is,

$$WR = \left(1 + \frac{r}{f}\right)^{Tf} - 1.$$

Then the GIC final return (GIC_FR) equals the GIC weight times the final return, that is,

$$GIC\_FR = \omega \times WR.$$

The total return on the GIC equals the sum of the GIC and basket respective final returns, but bounded below by the GIC final return, that is,

$$\max\left(GIC\_FR + B\_FR,\ GIC\_FR \right),$$

where GIC_FR and B_FR are formally defined in Section 2. Furthermore the payoff at maturity equals the invested principal plus the principal times the total return rate above, that is,

$$P \times \left(1 + \max\left(GIC\_FR + B\_FR,\ GIC\_FR \right)\right),$$

where P denotes the invested principal.

Let

* P denote the GIC invested principal,
* T denote the GIC maturity,
* $\{t\}$ denote the set of 12 averaging times,
* I denote a respective index level,
* ω denote the weight corresponding to the index level,
* IIL denote the initial level for the index,
* β denote the participation rate, and
* GIC_FR denote the GIC final return.

The GIC payoff at maturity then equals (see Appendix A.1)

$$P\left(1 + GIC\_FR + \beta \max\left[\frac{1}{12}\sum_{i=1}^{12} Z_{t_i} - \sum_{i=1}^{5} \omega_i ,\ 0\right]\right)$$

where $Z = \sum_{j=1}^{5} \alpha_j I^j$, with $\alpha = \dfrac{\omega}{IIL}$, denotes the basket level.

From the above we see that the GIC payoff at maturity includes a term,

$$\beta \max\left[\frac{1}{12}\sum_{i=1}^{12} Z_{t_i} - \sum_{i=1}^{5} \omega_i , 0\right],$$

which is the payoff from a European call option on the arithmetic average of the basket price. In this vetting report we consider the pricing of the option with payoff of the form above at maturity.

We assume that the index price, I, follows geometric Brownian motion with drift under the respective risk neutral probability measure, that is,

$$dI = I([r - q]dt + \sigma d\hat{W}),$$

where

* r, q and σ respectively denote constant risk free rate, dividend yield and volatility parameters, and
* $\hat{W}$ denotes standard Brownian motion.

Let X denote the value of one unit of foreign currency in terms of Canadian currency. Furthermore assume that X satisfies under the Canadian risk neutral probability measure a stochastic differential equation of the form

$$dX = X\left(\left[r_c - r\right]dt + \sigma_X dB\right)$$

where

* r_c denotes the Canadian risk free rate,

* σ_X denotes a constant volatility parameter, and

* B denotes standard Brownian motion.

Then, under the Canadian risk neutral probability measure,

$$dI = I\left(\mu dt + \sigma dW\right),$$

where

* $\mu = r - q - \rho_X \sigma \sigma_X$,

* W denotes standard Brownian motion, and

* ρ_X denotes the constant instantaneous correlation coefficient between the Brownian motions W and B.

We further assume that the Brownian motions W^i and W^j, respectively corresponding the i^{th} and j^{th} index levels, have instantaneous correlation coefficient ρ_{ij}.

You can find more details at

https://finpricing.com/lib/EqConvertible.html