

Quanto Option Valuation

A quanto option is an option whose payout is made in a currency other than that of the underlying security, based on a fixed exchange rate. The term “quanto” is abbreviation for “quantity adjusted” that refers to the feature where the payoff of an option is determined by the financial price of index in one currency but the actual payout is realized in another currency.

It is a common trend that domestic investors allocate their assets in foreign stock markets. The major reason behind this investment trend is that the regulation liberalization spreads throughout in the global security markets. Domestic investors who put their money into foreign stock markets will encounter both of the foreign stock price risk and foreign exchange rate risk.

Therefore, it is natural for the financial markets to design some kinds of financial tools, such as quanto options that can help those domestic investors who buy foreign stocks to hedge their foreign stock price risk and exchange rate risk as well.

Quantos are attractive because they shield the purchaser from exchange rate fluctuations. If a US investor were to invest directly in the Japanese stocks that comprise the Nikkei, he would be exposed to both fluctuations in the Nikkei index and fluctuations in the USD/JPY exchange rate. Essentially, a quanto has an embedded currency forward with a variable notional amount. It is that variable notional amount that give quantos their name—“quanto” is short for “quantity adjusting.”

The quanto feature of such options allows an investor to participate in the return of an industrial or economic sector with an international exposure without the foreign exchange risk exposure. It means that the underlying instrument price is in one currency, while payments are denominated in another currency. Such a feature allows counterparties to take advantage of global underlying instrument returns without incurring currency exposures.

The payoff of a quanto option depends on the joint process of the stock price and exchange rate.

Denote F_1 as the forward quanto price specified in a contract at time 0 to be paid at time t_1 for the underlying asset.

The forward quanto price under dividend yield is given by

$$F_j = S_0 \cdot e^{(r_j - d_y - \rho\sigma_j\sigma_c)t_j}$$

The forward quanto price under discrete dividend is given by

$$F_j = \left(S_0 - \sum_{l=1}^{n_j} d_l \cdot e^{-r_l t_l} \right) \cdot e^{(r_j - \rho\sigma_j\sigma_c)t_j}$$

Under the risk-neutral measure, $F_j = E[S_j]$; in particular, the forward quanto prices reflect the risk-free interest rate and any dividend yield (positive or negative) on the underlying asset.

Reference:

<https://finpricing.com/lib/EqBarrier.html>