# Identifying Textual Clusters with Non-negative Matrix Factorization

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#### **How Do We Compare Manuscripts?**

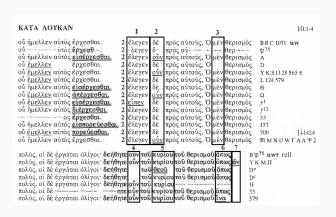
· Start with collation—aligning texts at variation units

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KATA AOTKAN
                                                                                    10.1-4
οῦ ήμελλεν αύτὸς ἔρχεσθαι.
                               2 έλεγεν δὲ πρὸς αύτούς. Ὁ μὲν θερισμὸς
                                                                        B R C 1071 uw
ού ..... υτὸς ἔρχεσθ.....
                              2 -λεγεν δ. πρός αύ...... θεο......
                                                                         2375
ού ήμελλεν αύτὸς εισέρχεσθαι.
                              2 έλεγεν ούν πρός αὐτούς, Ό μὲν θερισμός
ດນີ້ ຂັບຂຸ້ນຂຸ້ນ
                ἔργεσθαι.
                               2 ἔλεγεν δὲ πρὸς αὐτούς, Ὁ
                                                              θερισμός
οὖ ἔμελλεν αύτὸς ἔρχεσθαι.
                               2 έλεγεν ούν ποὸς αὐτούς. Ό μὲν θεοισμὸς
                                                                         YKSIT28565 T
ού εμελλεν αυτός εργεσθαι.
                               2 έλεγεν δὲ πρὸς αὐτούς, Ὁ μὲν θερισμὸς
                                                                        L 124 579
ού ήμελλεν αύτὸς εισέργεσθαι.
                               2 έλεγεν δὲ πρὸς αὐτούς, Ὁ μὲν θερισμὸς
ού ἔμελλεν αύτὸς ἀπέργεσθαι.
                              2 έλεγεν ούν πρὸς αὐτούς, Ὁ μὲν θερισμὸς
                                                                         Ω
ού εμελλεν αύτὸς εἰσέργεσθαι.
                              2 είπεν δὲ πρὸς αὐτούς, Ὁ μὲν θερισμὸς
                                                                        f1
ού ήμελλεν αύτὸς διέρχεσθαι.
                               2 έλεγεν δὲ πρὸς αὐτούς. Ὁ μὲν θερισμὸς
ος ἔρχεσθαι.
                               2 έλεγεν δὲ πρὸς αὐτούς, Ὁ μὲν θερισμὸς
ού ημελλεν αύτὸς εισπορεύεσθαι, 2 έλεγεν δὲ πρὸς αύτούς, Ο μὲν θερισμὸς
                                                                        157
                               2 έλεγεν δὲ πρὸς αὐτούς. Ὁ μὲν θερισμὸς
ού εμελλεν αύτὸς πορεύεσθαι.
                                                                        700
                                                                                    Г↓1424
                              2 έλεγεν ούν πρός αύτούς, Ο μέν θερισμός
ού ήμελλεν αύτὸς ἔργεσθαι.
                                                                        TRMNUWFAA 42
πολύς, οι δὲ ἐργάται όλίγοι δεήθητε ούν τοῦ κυρίου τοῦ θερισμοῦ ὅπως
                                                                     B 9\75 HW7 rell
πολύς, οι δὲ ἐργάται όλίγοι δεήθητε ούν τοῦ κυρίου τοῦ θερισμοῦ ὅπως ἀν
                                                                     YKMII
πολύς, οἱ δέ έργάται όλίγοι δεήθητε
                                    τοῦ θεοῦ τοῦ θερισμοῦ ὅπως
                                                                     n*
πολύς, οί δε εργάται όλίνοι δεήθητε τοῦ κυρίου τοῦ θερισμοῦ όπως
                                                                     DC
πολύς, οί δε εργάται όλίνοι: δεήθητε ούν τού κυρίου .....
                                                                      Н
πολύς, οι δὲ ...... ητε ούν τοῦ κυρίου τοῦ θερισμοῦ ὅπως
                                                                     33
πολύς, οι δὲ ἐργάται όλίνοι: δεήθητε ούν τοῦ κυρίου τοῦ θερισμοῦ ἴνα
                                                                     579
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(Source: Swanson, New Testament Greek Manuscripts, Luke, 183)

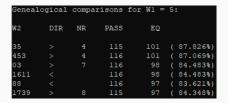
#### **How Do We Compare Manuscripts?**

· Start with collation—aligning texts at variation units



#### How Do We Compare Manuscripts?

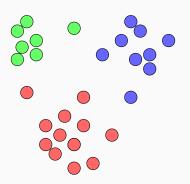
- This provides a simple basis of comparison between pairs of manuscripts
  - · Number of units where both agree
  - For a proportion, divide by number of units where the readings of both are known
- "Pre-genealogical coherence" in the Coherence-Based Genealogical Method (CBGM)



 Can we use mutual agreement to classify manuscripts into groups?

#### The Quantitative Method

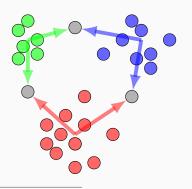
- Colwell and Tune: if manuscripts agree significantly more with one another than they do with other manuscripts, then they form a family, or text-type<sup>1</sup>
  - $\cdot \geq 70\%$  with one another, and  $\geq 10\%$  more than with others



<sup>1.</sup> Colwell and Tune, "Quantitative Relationships."

#### The Quantitative Method

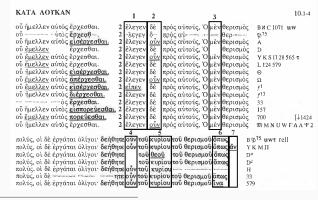
- · Problems:
  - All units (including those involving singular readings and common scribal errors) have equal weight
  - Mixture in the transmission process is a problem<sup>2</sup>



2. Epp, "Textual Clusters."

#### The Quantitative Method

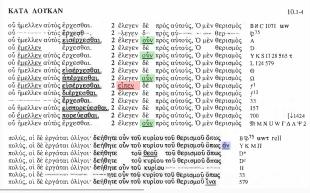
- For efficiency and accuracy, comparisons should be done on the basis of informative points of variation, or even readings<sup>3</sup>
- · But how do we know which ones are the most informative?



<sup>3.</sup> Colwell, "Method in Locating."

#### The Claremont Profile Method

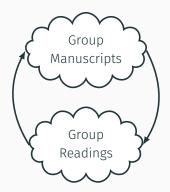
- Start with an established set of manuscript groups<sup>4</sup>
- Filter out variation units involving common types of variation and singular / subsingular readings to get a set of test passages
- Readings supported by group manuscripts = the group's profile



<sup>4.</sup> Wisse, Profile Method.

#### The Claremont Profile Method

- This allows us to isolate informative readings for group classification
- Also robust to mixture
- But it needs manuscript groups to be established first!
- "Good manuscripts have good readings, and good readings are found in good manuscripts"



- Non-negative matrix factorization (NMF), a machine learning technique, uses this circular relationship to solve both problems
- Represent our collation as a matrix A with a row for each variant reading and a column for each manuscript
- m rows by n columns

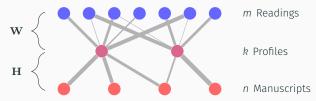
|   |        |        | $\mathfrak{P}^{75}$ | Α | В | D | K | $f^1$ | 579 |
|---|--------|--------|---------------------|---|---|---|---|-------|-----|
| - | Unit 1 | ἔλεγεν | 1                   | 1 | 1 | 1 | 1 | 0     | 1   |
|   |        | εἶπεν  | 0                   | 0 | 0 | 0 | 0 | 1     | 0   |
|   | Unit 2 | 36     | 1                   | 0 | 1 | 1 | 0 | 1     | 1   |
|   |        | οὖν    | 0                   | 1 | 0 | 0 | 1 | 0     | 0   |
|   | Unit 3 | μὲν    | 0                   | 1 | 1 | 0 | 1 | 1     | 1   |
|   |        | omit   | 0                   | 0 | 0 | 1 | 0 | 0     | 0   |
| - | Unit 4 | οὖν    | 1                   | 1 | 1 | 0 | 1 | 1     | 1   |
|   |        | omit   | 0                   | 0 | 0 | 1 | 0 | 0     | 0   |
|   | Unit 5 | κυρίου | 1                   | 1 | 1 | 0 | 1 | 1     | 1   |
|   |        | θεοῦ   | 0                   | 0 | 0 | 1 | 0 | 0     | 0   |
|   | Unit 6 | őπως   | 1                   | 1 | 1 | 1 | 1 | 1     | 0   |
|   |        | ἵνα    | 0                   | 0 | 0 | 0 | 0 | 0     | 1   |
|   | Unit 7 | omit   | 1                   | 1 | 1 | 1 | 0 | 1     | 1   |
|   |        | ầν     | 0                   | 0 | 0 | 0 | 1 | 0     | 0   |
|   |        |        |                     |   |   |   |   |       |     |

 The goal is to approximate this original matrix as the product of two smaller matrices with non-negative entries:

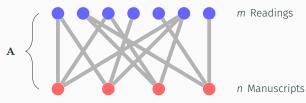
#### $\mathbf{A} \approx \mathbf{WH}$

- Specify a number k of underlying textual profiles (there are metrics for finding good choices)
- $\mathbf{W}$ : m rows and k columns; defines group readings
- **H**: *k* rows and *n* columns; defines makeup of manuscripts in terms of profiles

• Use two sets of relationships with a few components...



...to reconstruct the large set of original relationships



- The process:
  - 1. Start with guesses for  ${f W}$  and  ${f H}$
  - 2. Fix  $\mathbf{W}$ , optimize the weights in  $\mathbf{H}$  (Quantitative Method)
  - 3. Fix  ${f H}$ , optimize the weights in  ${f W}$  (Claremont Profile Method)
  - 4. Repeat steps 2 and 3 until the difference between  ${\bf A}$  and  ${\bf WH}$  no longer decreases



• Guaranteed to terminate with *locally optimal* groupings in  ${f W}$  and  ${f H}^5$ 

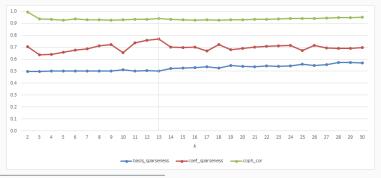
<sup>5.</sup> Grippo and Sciandrone, "On the Convergence."

- Tommy Wasserman's collation of Jude contains 1346 variant readings and 560 manuscripts (including lectionaries)<sup>6</sup>
- Filtering out 42 fragmentary manuscripts (< 300 known readings) yields a matrix  $\bf A$  with m=1346 rows and n=518 columns
- The fragmentary manuscripts can be classified after groups are established<sup>7</sup>

<sup>6.</sup> Wasserman, The Epistle of Jude.

<sup>7.</sup> For details, see the appendix of McCollum, "Biclustering Readings and Manuscripts."

- We select the number of profiles *k* based on several factors:
  - · Overlap of readings in profiles
  - · Mixture of profiles in manuscripts
  - Consistency of manuscript groupings when random starting points are used (the cophenetic correlation coefficient)<sup>8</sup>



8. Brunet et al., "Metagenes and Molecular Pattern Discovery."

• The k = 13 groups identified by NMF correspond to groups in the Catholic Epistles identified in the literature

| Members (by Gregory-Aland number)                | Group                            |  |  |
|--|----------------------------------|--|--|
| 920, 1277, 1859, 1719, 452, 1857, 1871, 941,     | K (von Soden)                    |  |  |
| 1103, 1352, etc.                                 |                                  |  |  |
| 141, 204, 394, 444, 1101, 1723, 1737, 1752,      | K <sup>r</sup> (von Soden)       |  |  |
| 1865, 2221, etc.                                 |                                  |  |  |
| 390, 1863, 912, 234, 1861, 2085, 1753, 2279, 42, | K <sup>c</sup> (von Soden)       |  |  |
| 996, etc.  |                                  |  |  |
| L606, L938, L145, L840, L740, L2106, L2394,      | Lectionary (Colwell)             |  |  |
| L809, L1279, L62, etc.                           |                                  |  |  |
| 606, 454, 641, 103, 221, 2125, 314, 250, 1888,   | O, Θδ Commentaries               |  |  |
| 393, etc.  | (von Soden)                      |  |  |
| 619, 1780, 1175, 330, 1769, 2516, 917, 451,      | f <sup>1780</sup> (unidentified) |  |  |
| 1162, 601, etc.                                  |                                  |  |  |
| 1563, 1718, 1425, 1359, 1066, 0142, 056          | f <sup>0142</sup> (unidentified) |  |  |

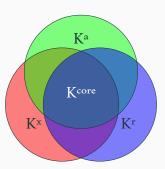
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| Members (by Gregory-Aland number)                                  | Group                           |  |
|--|---------------------------------|--|
| 03, 623, $\mathfrak{P}^{72}$ , 81, 5, 326, 33, 1837, 93, 665, etc. | H (von Soden)                   |  |
| 321, 918, 307, 453, 2197, 2818, 1678, 94, 2186,                    | f453 (Spencer, Wachtel,         |  |
| 1840, etc.   | Howe)                           |  |
| 323, 1241, 322, 1739, 1881, 2298, 6                                | f <sup>1739</sup> (Zuntz, Geer) |  |
| 1505, 2495, 1611, 1292, 630, 2200, 1765, 1832,                     | f <sup>2138</sup> /Harklean     |  |
| 2494, 876, etc.  | (Amphoux)                       |  |
| 1843, 1869, 506, 1903, 489, 927, 203, 1868,                        | l (von Soden)                   |  |
| 1729, 1873, etc.   |                                 |  |
| 915, 88, 459, 104, 1846, 1838, 1842, 1845                          | f <sup>915</sup> (unidentified) |  |

- Applying NMF to Morrill's collation of all continuous-text manuscripts of John 18 illustrates some of the idiosyncrasies of the method and how to deal with them<sup>9</sup>
- Significantly larger and more "square" collation: m=1545 variant readings and n=1610 manuscripts after filtering out fragmentary manuscripts (< 350 known readings)
- $\cdot$  (Recall that the collation matrix for Jude was 1346 imes 518)

<sup>9.</sup> Morrill, "Complete Collation and Analysis."

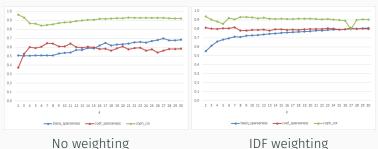
- Applying NMF to the matrix as-is separates readings common to multiple groups into their own "core" profiles
- No manuscripts belong to these profiles, but many appear "mixed" with it
- Symptom of volume and similarity of manuscripts, especially Byzantine ones



• To remedy this, weigh readings in the original matrix by their inverse document frequency (IDF)10

$$\log \frac{n}{\#\{\text{MSS with reading}\}}$$

- Removing singular readings is helpful in this setting
- Encourages NMF to isolate unique group readings in profiles



10. Jones, "Statistical Interpretation."

IDF weighting

 $\cdot$  With k=12, NMF identifies known groups from the literature

| Members (by Gregory-Aland number)                  | Group                      |  |
|--|----------------------------|--|
| 2605, 492, 1215, 2897, 1090, 1567, 1210, 851,      | Kx (von Soden)             |  |
| 494, 2406, etc.                                    |                            |  |
| 47, 1126, 61, 1138, 58, 56, 189, 1236, 825, 1614,  | K <sup>r</sup> (von Soden) |  |
| etc.   |                            |  |
| 2902, 1219, 1079, 489, 114, 2404, 389, 2193,       | Ka (von Soden)             |  |
| 699, 1627, etc.                                    |                            |  |
| 1534, 741, 857, 744, 2735, 1160, 817, 1261,        | Θε Commen-                 |  |
| 2470, 833, etc.                                    | taries (von                |  |
|  | Soden)                     |  |
| 892, 977, 555, 16, 152, 513, 1243, 829, 348, 1579, | $f^{16}+f^{1216}$          |  |
| etc.   | (Wisse)                    |  |
| 1663, 1413, 2291, 86, 569, 71, 1170, 1014, 1531,   | M27+Cl1531                 |  |
| 2705, etc.   | (Wisse)                    |  |

• With k = 12, NMF identifies known groups from the literature

| Members (by Gregory-Aland number)                 | Group                |
|---|----------------------|
| 01, 032, 05, 579, 1654, 2561, 1242                | Egyptian             |
| 1820, 2129, 865, 033, 019, 1819, 213, 03, 33,     | Alexandrian          |
| 1321, etc.  |                      |
| 1, 1582, 357, 138, 565, 209, 994, 2713, 2575,     | f1 (Lake)            |
| 1784, etc.  |                      |
| 13, 788, 826, 828, 543, 69, 346, 1689, 124, 2786, | f13 (Lake and Lake,  |
| etc.  | Geerlings)           |
| 2524, 1001, 1268, 2397, 352, 2728, 132, 175,      | Cl1001+Cl352 (Wisse) |
| 1701, 2252, etc.                                  |                      |
| 1446, 1050, 706, 1457, 827, 2620, 1128, 0211,     | Cl827 (Wisse)        |
| 2707, 1402, etc.                                  |                      |

# **Concluding Observations**

- In John 18, Gregory-Aland 03 (Codex Vaticanus, B) stands out as an instructive example
- Appears to be mixed between the "Egyptian" and "Alexandrian" profiles, but could preserve a text earlier than both

|              | "03"   |
|--------------|--------|
| Kx           | 0.0290 |
| Cl1001+Cl352 | 0.0000 |
| Theophylact  | 0.0000 |
| f13          | 0.0000 |
| f1           | 0.0000 |
| Alexandrian  | 1.1248 |
| Egyptian     | 1.0556 |
| Kr           | 0.0000 |
| Ka           | 0.0000 |
| M27+Cl1531   | 0.0000 |
| f16+f1216    | 0.0000 |
| CI827        | 0.0000 |

- · NMF identifies relationships, but not their directions
- Pre-genealogical, but not genealogical



# **Concluding Observations**

- The advantage: few assumptions and editorial decisions are required
- Intended for use in "pre-processing" (manuscript and test reading selection)
- · Useful for other applications (new manuscript classification)
- Work in progress: applying NMF to ~2000 manuscripts in the pericope adulterae (with Maurice A. Robinson)

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