Systematic study of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge symmetry

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We review in a systematic way how anomaly free $SU(3)_c \otimes SU(3)_L \otimes U(1)_x$ models without exotic electric charges can be constructed, using as basis closed sets of fermions which includes each one the particles and antiparticles of all the electrically charged fields. Our analysis reproduces not only the known models in the literature, but also shows the existence of several more independent models for one and three families not considered so far. A phenomenological analysis of the new models is done, where the lowest limits at a 95 % CL on the gauge boson masses are presented.

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I. INTRODUCTION

The impressive success of the Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with the color sector $SU(3)_c$ confined and the flavor sector $SU(2)_L \otimes U(1)_Y$ hidden and broken spontaneously by the minimal Higgs mechanism [1], has not been able enough to provide explanation for several fundamental issues, among them: the hierarchical masses and the mixing angles for both, the quark and the lepton sectors [2–6], charge quantization [7–12], the strong CP violation [13–16], the small neutrino masses and their oscillations [17, 18], and last but not least, the presence of dark matter and dark energy in the universe [19–23]. Because of this, many physicists believe that the SM does not stand for the final theory, representing only an effective model originated from a more fundamental one.

Minimal extensions of the SM arise either by adding new fields, or by enlarging the local gauge group (adding a right handed neutrino field constitute its simples extension, something that ameliorate, but not solve some of the problems mentioned above).

Simple extensions of the local gauge group consider an electroweak sector with an extra abelian symmetry $SU(2)_L \otimes U(1)_x \otimes U(1)_z$ [24, 25], or either the so called left right symmetric model $SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)}$ [26–29], and also $SU(3)_L \otimes U(1)_X$, being the last one we are going to consider in this study [30–35].

One way to obtain new models without changing the symmetry groups is studying alternative embeddings [36–44], the best-known examples in the literature correspond to flipped $SU(5)$ [36] and the alternative left-right-model [39, 40]. A complete list of possible embedings in E_6 is given in the references [42, 44]. Part of our analysis is to obtain the alternative embeddings for some of the well-known 3-3-1 models in the literature.

This document is organized as follows: in Section II we review some of the best-known 3-3-1 models without exotic electric charges in the literature. In section III we introduce the irreducible anomaly-free sets, which are the basis of our classification. In section IV We study the different possibilities for the embeddings of the SM particles in the anomaly-free sets. As we will see, for a given embedding, the collider constraints could be quite different. In section V, we present the procedure to obtain the collider constraints.

II. 3-3-1 MODELS

In what follows we assume that the electroweak gauge group is $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (3-3-1 for short) in which the electroweak sector of the standard model $SU(2)_L \otimes U(1)_Y$ is extended to $SU(3)_L \otimes U(1)_X$. We also assume that, as in the SM, the color group $SU(3)_c$ is vector-like (free of anomalies) and that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform only under the two fundamental representations of $SU(3)_L$ (the 3 and 3^*).

Two classes of models will show up: universal one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the several families.

For the 3-3-1 models, the most general electric charge operator in the extended electroweak sector is

$$Q = a\lambda_3 + \frac{1}{\sqrt{3}}b\lambda_8 + XI_3, \qquad (2.1)$$

where λ_α, $\alpha = 1, 2, \ldots, 8$ are the Gell-Mann matrices for $SU(3)_L$ normalized as $\mathrm{Tr}(\lambda_\alpha\lambda_\beta) = 2\delta_{\alpha\beta}$ and $I_3 = Dg(1, 1, 1)$ is the diagonal 3×3 unit matrix. If one assumes $a = 1/2$, the isospin $SU(2)_L$ of the SM is entirely embedded in $SU(3)_L$; b is a free parameter which fixes the model and the X values are obtained by anomaly cancellation. The 8 gauge fields A^α_μ of $SU(3)_L$ may be written as [34, 35]

$$\sum_\alpha \lambda_\alpha A^\alpha_\mu = \sqrt{2}\begin{pmatrix} D^0_{1\mu} & W^+_\mu & K^{(b+1/2)}_\mu \\ W^-_\mu & D^0_{2\mu} & K^{(b-1/2)}_\mu \\ K^{-(b+1/2)}_\mu & K^{-(b-1/2)}_\mu & D^0_{3\mu} \end{pmatrix}, \tag{2.2}$$

where $D^0_{1\mu} = A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6}$, $D^0_{2\mu} = -A^3_\mu/\sqrt{2} + A^8_\mu/\sqrt{6}$, and $D^0_{3\mu} = -2A^8_\mu/\sqrt{6}$. The upper indices on the gauge bosons in Eq. (2.2) stand for the electric charge of the particles, some of them being functions of the b parameter [46].

The breaking of the 3-3-1 gauge symmetry down to $SU(3)_c \otimes U(1)_Q$, as shown somewhere [35], is properly achieved by using the Higgs scalar fields

$$\phi_1(1,3,1/3) \oplus \phi_2(1,3,-2/3),$$

with the following Vacuum Expectation Values (VEV's):

$$\langle\phi_1(1,3,1/3)\rangle = (0, v', V),$$

and

$$\langle\phi_2(1,3,-2/3)\rangle = (v, 0, 0),$$

with $V \gg v \sim v' \equiv 174$ GeV. To provide with masses to the charged fermion fields in all the models, the following Higgs scalar sector must be used:

$$\phi(1,3,-2/3) \oplus \phi'(1,3,1/3) \oplus \phi''(1,3,1/3),$$

with the following VEV's:

$$\begin{aligned} \langle\phi(1,3,-2/3)\rangle &= (v,0,0), \\ \langle\phi'(1,3,1/3)\rangle &= (0,v',0), \\ \langle\phi''(1,3,1/3)\rangle &= (0,0,V), \end{aligned}$$

with $V \gg v \sim v' \equiv 174$ GeV. To provide with masses to the neutral fermion fields is a model dependent matter and should be analyzed case by case.

A. The Minimal Model

In Ref. [31, 33] it has been shown that, for $b = 3/2$, the following fermion structure is free of all the gauge anomalies: $\psi^T_{lL} = (\nu^0_l, l^-, l^+)_L \sim (1,3,0)$, $Q^T_{iL} = (d_i, u_i, X_i)_L \sim (3,3^*,-1/3)$, $Q^T_{3L} = (u_3, d_3, Y) \sim (3,3,2/3)$, where $l = e, \mu, \tau$ is a family lepton index, $i = 1,2$ for the first two quark families, and the numbers after the similarity sign means 3-3-1 representations. The right handed fields are $u^c_{aL} \sim (3^*,1,-2/3)$, $d^c_{aL} \sim (3^*,1,1/3)$, $X^c_{iL} \sim (3^*,1,4/3)$ and $Y^c_L \sim (3^*,1,-5/3)$, where $a = 1,2,3$ is the quark family index, and there are two exotic quarks with electric charge $-4/3$ (X_i) and other with electric charge $5/3$ (Y). This version is called minimal in the literature, because it does not make use of exotic leptons, including possible right-handed neutrinos.

B. 3-3-1 Models Without Exotic Electric Charges

If one wishes to avoid exotic electric charges in the fermion and boson sectors as the ones present in the minimal (3-3-1) model, one must choose $b = 1/2$ in Eq. (2.1) as shown in Ref [34, 35].

To begin with our systematic analysis, let us start with closed fermion structures consisting of only one left handed $SU(3)_L$ triplet and right handed singlets, where for "closed" we mean structures containing the antiparticles of all the electric charged particles. Following the notation in Ref. [34, 35], there are only six of such structures containing at least one of the fermion fields in one family of the SM, or in its minimal extension with right-handed neutrino fields:

- $S_1 = [(\nu^0_e, e^-, E^-_1) \oplus e^+ \oplus E^+_1]_L$ with quantum numbers $(1,3,-2/3)$; $(1,1,1)$ and $(1,1,1)$ respectively.
- $S_2 = [(e^-, \nu^0_e, N^0_1) \oplus e^+]_L$ with quantum numbers $(1,3^*,-1/3)$ and $(1,1,1)$ respectively.
- $S_3 = [(d,u,U) \oplus u^c \oplus d^c \oplus U^c]_L$ with quantum numbers $(3,3^*,1/3)$; $(3^*,1,-2/3)$; $(3^*,1,1/3)$ and $(3^*,1,-2/3)$ respectively.
- $S_4 = [(u,d,D) \oplus u^c \oplus d^c \oplus D^c]_L$ with quantum numbers $(3,3,0)$; $(3^*,1,-2/3)$; $(3^*,1,1/3)$ and $(3^*,1,1/3)$ respectively.
- $S_5 = [(N^0_2, E^+_2, e^+) \oplus E^-_2 \oplus e^-]_L$ with quantum numbers $(1,3^*,2/3)$ $(1,1,-1)$, and $(1,1,-1)$ respectively.
- $S_6 = [(E^+_3, N^0_3, N^0_4) \oplus E^-_3]_L$ with quantum numbers $(1,3,1/3)$ and $(1,1,-1)$ respectively,

where for phenomenological reasons we allow for the precense of several exotic leptons (charged and neutral), but only one exotic quark of each type (down-type or up-type). In the former sets, N^0_1 and N^0_4 can play the role of the right handed neutrino field ν^{0c}_e in an SO(10) basis.

Notice that the value $b = 1/2$ in Eq. (2.1) implies that the electric charge of the last two components of a 3 or a 3^* of $SU(3)_L$ are of the same value. At this point, our approach is different to the one presented in Ref. [34, 35], the difference being that only one fundamental representation of $SU(3)_L$ (triplet or anti-triplet) is used in each set, instead of the composite ones present in the original reference [31–33] (such composite lepton structures will appear anon in our systematic analysis).
The several gauge anomalies calculated for these six sets are shown in Table I; where notice that the anomaly values for S_1, S_2, S_3 and S_4 coincide with the ones presented in Ref [34, 35], being the values for S_5 and S_6 new results.

Now, if we want to consider only one family of quarks, either the sets S_3 or S_4 are enough, but for 3 quark families, one of the following combinations must be used: $3S_3$, $3S_4$, $(2S_3 + S_4)$ and $(S_3 + 2S_4)$, where the first two

Anomalies	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_C]^2U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2U(1)_X$	$-2/3$	$-1/3$	1	0	2/3	1/3
$[Grav]^2U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	$-4/3$	$-2/3$	-10/9	-8/9
$[SU(3)_L]^3$	1	-1	-3	3	-1	1

TABLE I: Anomalies for some 3-3-1 fermion fields structures

Anomalies	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}
$[SU(3)_C]^2U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2U(1)_X$	1/3	-1/3	0	-1	1	0
$[Grav]^2U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	$-2/9$	2/9	2/3	4/3	-4/3	-2/3
$[SU(3)_L]^3$	-2	2	-3	3	-3	3

TABLE II: Anomalies for the 3-3-1 non Vector-Like lepton fields structures

ones are associated with universal models in the quark sector.

Right from Table 1 it is simple to read the following sets free of anomalies:

- **Model I:** $2S_2 + S_4 + S_5$,
- **Model J:** $2S_1 + S_3 + S_6$,
- **Model A:** $3S_2 + S_3 + 2S_4$,
- **Model B:** $3S_1 + 2S_3 + S_4$,

where the structures **I** and **J** [45] contain only one family of quarks, and **A** and **B** are three family quark models (here we are following the notation in Ref. [45]). But, can we view **I** and **J** as one family ("universal") anomaly free models? the answer is yes if we allow models with exotic electrons and new electric neutral particles. As a matter of fact, we can writte the particle content for Model **I** as [45]:

$$[(e^-,\nu_e^0,N_1^0)\oplus e^+\oplus(E_1^-,N_2^0,N_3^0)\oplus E_1^+\oplus (N_4^0,E_2^+,E_3^+)\oplus E_2^-\oplus E_3^-\oplus(u,d,D)\oplus u^c\oplus d^c\oplus D^c]_L.$$

Ugly as it may be, due to the precense of several exotic leptons, some with the same quantun numbers of the ordinary ones, we may said that this model is not yet excluded from present phenomenology.

In a similar way, the particle content of the structure **J** can be written as [45]:

$$[(\nu_e^0,e^-,E_1^-)\oplus e^+\oplus E_1^+\oplus(N_1,E_2^-,E_3^-)\oplus E_2^+\oplus E_3^+\oplus (E_4^+,N_2^0,N_3^0)\oplus E_4^-\oplus(d,u,U)\oplus u^c\oplus d^c\oplus U^c]_L.$$

The other two structures **A** and **B** correspond to two well known non universal models already present in the literature; **A** being named as a "3-3-1 model with right-handed neutrinos" [47–49] and **B** named as a "3-3-1 model with exotic charged leptons" [50–52].

An unrealistic two family anomaly free structure is for example $S_1 + S_2 + S_3 + S_4$ (unrealistic because there is strong evidence for at least three families in nature [1]).

The next strategy is to use the lepton sets S_1, S_2, S_5 and S_6 to build non vector-like new sets of leptons (vector-like sets are free of anomalies by definition [1], and quark sector anomalies must cancel out with those of the lepton sector). Notice that vector-like sets as for example S_1+S_5 and S_2+S_6 are free of anomalies and not suitable for constructing realistic models due to the non zero anomalies in the quark sector. For the same reason we exclude from our analysis vector like structures as $(\nu_e^0,e^-,E^-)\oplus(N^0,e^+,E^+)\sim(1,3,-2/3)\oplus(1,3^*,2/3)$ and $(e^-,\nu_e^0,N_1^0)\oplus(E^+,N_2^0,N_3^0)\sim(1,3^*,-1/3)\oplus(1,3,1/3)$.

To be systematic, let us start first with sets of leptons containing only two $SU(3)_L$ triplets or anti-triplets:

- $S_7 = [(e^-,\nu_e^0,N_1^0)\oplus(N_2^0,E^+,e^+)\oplus E^-]_L$ with quantum numbers $(1,3^*,-1/3)$; $(1,3^*,2/3)$ and $(1,1,-1)$ respectively.
- $S_8 = [(\nu_e^0,e^-,E^-)\oplus(E^+,N_1^0,N_2^0)\oplus e^+]_L$ with quantum numbers $(1,3,-2/3)$, $(1,3,1/3)$ and $(1,1,1)$ respectively.

Notice again $S_7 + S_8$ is vector-like and in consequence is free of anomalies and unsuitable for our building process.

The next step is to include lepton sets with three $SU(3)_L$ triplets or anti-triplets:

- $S_9 = [(e^-,\nu_e,N_1^0)\oplus(E^-,N_2^0,N_3^0)\oplus(N_4^0,E^+,e^+)]_L$ with quantum numbers $(1,3^*,-1/3)$;$(1,3^*,-1/3)$ and $(1,3^*,2/3)$ respectively.
- $S_{10} = [(\nu_e,e^-,E_1^-)\oplus(E_2^+,N_1^0,N_2^0)\oplus e^+\oplus (N_3^0,E_2^-,E_3^-)\oplus E_1^+\oplus E_3^+]_L$ with quantum numbers $(1,3,-2/3)$; $(1,3,1/3)$; $(1,1,1)$; $(1,3,-2/3)$; $(1,1,1)$, and $(1,1,1)$ respectively.
- $S_{11} = [(e^-,\nu_e,N_1^0)\oplus(N_2^0,E_1^+,e^+)\oplus(N_3^0,E_2^+,E_3^+)\oplus E_1^-\oplus E_2^-\oplus E_3^-]_L$ with quantum numbers $(1,3^*,-1/3)$; $(1,3^*,2/3)$; $(1,3^*,2/3)$; $(1,1,-1)$; $(1,1,-1)$, and $(1,1,-1)$ respectively.
- $S_{12} = [(\nu_e^0,e^-,E_1^-)\oplus(E_1^+,N_1^0,N_2^0)\oplus (E_2^+,N_3^0,N_4^0)\oplus e^+\oplus E_2^-]_L$ with quantum numbers $(1,3,-2/3)$; $(1,3,1/3)$; $(1,3,1/3)$; $(1,1,1)$, and $(1,1,-1)$; respectively.

The anomalies for these new lepton sets are given in Table II.

At this step we can stop combining new sets, and analyze the rich structure we have gotten so far with the anomaly values presented in the former two Tables:

A simple computer program allow us to construct Table III, which by the way is our main result of the first part of this paper. Let us see:

III. IRREDUCIBLE ANOMALY FREE SETS

Table III lists all the basic and irreducible sets of multiplets of quarks and leptons which are free of anomalies (hereafter, Irreducible Anomaly-Free-Set (IAFS)), classified according to their quark content. These sets can be combined in several different ways in order to construct anomaly free three family models. In the first column in Table III the index i lists the IAFSs, the second column shows various possible irreducible lepton sets free of anomalies. A closer look to the sets of the second column shows that all of them are vector-like structures, not suitable to build simple models, but useful to build complex anomaly free models, as we will see ahead. Column three contains one of the most important results of our analysis. As a matter of fact, it lists 20 universal models (one family models), where only four of them are reported in the literature so far. Let us see:

$Q_1^I = S_4 + S_9$ called *carbon copy one* or **"Model G"** in Ref. [34, 35]. This structure can be embedded in the unification group E(6) according to Ref. [53], with some phenomenology of this structure already presented in the same reference.

$Q_2^I = S_3 + S_{10}$ called *carbon copy two* or **"Model H"** in Ref. [34, 35]. It can be embedded in the unification group $SU(6) \otimes U(1)$ according to Ref. [41], with some phenomenology of this structure already presented in the same reference.

$Q_5^I = 2S_1 + S_3 + S_6$ and $Q_6^I = 2S_2 + S_4 + S_5$ were introduced in Re. [45] where they were named as **"Model J"** and **"Model I"**, respectively.

The other 16 structures correspond to totally new, one family (universal) models, which will be analyzed in the second part of this paper. But there is much more in the third column of Table III: we can construct three family models using sets in column three not only by iterating three times every structure, but also combining them by taking two equal and one diferent (producing in this way 380 three family models), or by combining three different sets (producing in this way 1140 three family models). Let us see some of them:
$2Q_1^I + Q_2^I = 2S_4 + 2S_9 + S_3 + S_{10}$ which is denoted as **Model E** in Ref. [34, 35], called *hybrid model one* in that paper.
$Q_1^I + 2Q_2^I = S_4 + S_9 + 2S_3 + 2S_{10}$ which is denoted as **Model F** in Ref. [34, 35], called *hybrid model two* in that paper.

Column four in Table III shows an amusing result; aparently it is related to unphysical structures (only two families of quarks). But the point is that each one of the seven entries in this column can be combined with each one of the 20 entries in the third column, in order to produce a three family quark structure, for a total of 140 anomaly free three family models, most of them new ones. Let us see some examples:
$Q_1^I + Q_1^{II} = S_1 + S_2 + S_3 + 2S_4 + S_9$ which is denoted as **Model C** in Ref. [34, 35], called *a model with unique lepton generation one* in such paper.
$Q_2^I + Q_1^{II} = S_1 + S_2 + 2S_3 + S_4 + S_{10}$ which is denoted as **Model D** in Ref. [34, 35], called *a model with unique lepton generation two* in the same paper.

The last column in Table III shows the simplest and most economical three family structures available for 3-3-1 models without exotic electric charges. As a matter of fact, Q_1^{III} is the well known 3-3-1 model with right handed neutrinos [47–49], and Q_2^{III} is the 3-3-1 model with exotic electrons [50–52]. All of these are well known models and they are summarized in Table IV.

At this point, any AFS containing at least three families (as explained above) allows the construction of new models by including one or more IAFSs of vector-like leptons from the second column of Table III. In this way we end up with an immeasurable number of anomaly free fermion structures, constructed out of the 12 fermion structures (10 lepton and two quark) introduced in Sec. II B.

Model count summary: leaving aside the IAFSs of the second column in Table III (L), we can make a very illuminating analysis to exemplify the model building process. Each one of the IAFSs in the third column (Q^I) in Table III contains, at least, one family of SM leptons and quarks, so that, if we threefold this structure, one IAFS for each family, it is possible to have a universal model. Following this procedure, we can obtain 20 models. Another possibility is to choose the same IAFSs for two families and a different one for the third family, in this case we have $19 \times 20 = 380$ possibilities. Taking a different IAFS for each family we will have $\binom{20}{3} = 1140$ different models. Adding these results with the 2 models coming from the fifth column in Table III, and the $7 \times 20 = 140$ combinations between the IAFSs in the columns Q^I and Q^{II}, we obtain a total of 1682 models with a minimum content of exotic quarks. It is clear that from the arbitrary union of IAFSs it is possible to obtain an infinite number of models, but in general, these models will have too many exotic particles.

	Irreducible anomaly free sets			
i	Vector-like lepton sets (L_i)	One quark set (Q_i^I)	Two quark sets (Q_i^{II})	Three quark sets (Q_i^{III})
1	$S_1 + S_5$	$S_4 + S_9$	$S_1 + S_2 + S_3 + S_4$	$3S_2 + S_3 + 2S_4$
2	$S_2 + S_6$	$S_3 + S_{10}$	$2S_1 + S_3 + S_4 + S_7$	$3S_1 + 2S_3 + S_4$
3	$S_7 + S_8$	$S_2 + S_4 + S_7$	$2S_2 + S_3 + S_4 + S_8$	
4	$S_{10} + S_{11}$	$S_1 + S_3 + S_8$	$3S_2 + S_3 + S_4 + S_{12}$	
5	$S_9 + S_{12}$	$2S_1 + S_3 + S_6$	$3S_1 + 2S_3 + S_{12}$	
6	$S_1 + S_6 + S_7$	$2S_2 + S_4 + S_5$	$3S_2 + 2S_4 + S_{11}$	
7	$S_6 + S_8 + S_9$	$S_1 + S_4 + 2S_7$	$3S_1 + S_3 + S_4 + S_{11}$	
8	$S_2 + S_5 + S_8$	$S_2 + S_3 + 2S_8$		
9	$S_5 + S_7 + S_{10}$	$S_1 + S_2 + S_3 + S_{12}$		
10	$S_2 + S_7 + S_{12}$	$S_1 + S_2 + S_4 + S_{11}$		
11	$S_1 + S_8 + S_{11}$	$S_4 + 3S_7 + S_{10}$		
12	$S_1 + 2S_6 + S_9$	$S_3 + 3S_8 + S_9$		
13	$S_6 + 2S_7 + S_{10}$	$2S_1 + S_3 + S_7 + S_{12}$		
14	$S_5 + 2S_8 + S_9$	$2S_1 + S_4 + S_7 + S_{11}$		
15	$S_5 + S_6 + S_9 + S_{10}$	$2S_2 + S_3 + S_8 + S_{12}$		
16	$S_2 + 2S_5 + S_{10}$	$2S_2 + S_4 + S_8 + S_{11}$		
17	$S_1 + 2S_7 + S_{12}$	$3S_2 + S_3 + 2S_{12}$		
18	$S_1 + S_2 + S_{11} + S_{12}$	$3S_2 + S_4 + S_{11} + S_{12}$		
19	$S_2 + 2S_8 + S_{11}$	$3S_1 + S_3 + S_{11} + S_{12}$		
20	$2S_1 + S_6 + S_{11}$	$3S_1 + S_4 + 2S_{11}$		
21	$2S_2 + S_5 + S_{12}$			

TABLE III: IAFSs. Any general Anomaly Free-Set (AFS) containing quarks, must be a combination of IAFSs (i.e., L_i, Q^I, Q^{II} and Q^{III}) even for more than three families. For leptons, the second column (L) is not exhaustive and it was not possible to account for all the possibilities.

Name	Model	AFS
Model **A**	Q_1^{III}	$3S_2 + S_3 + 2S_4$
Model **B**	Q_2^{III}	$3S_1 + 2S_3 + S_4$
Model **C**	$Q_1^I + Q_1^{II}$	$S_1 + S_2 + S_3 + 2S_4 + S_9$
Model **D**	$Q_2^I + Q_1^{II}$	$S_1 + S_2 + 2S_3 + S_4 + S_{10}$
Model **E**	$Q_2^I + 2\,Q_1^I$	$2S_4 + 2S_9 + S_3 + S_{10}$
Model **F**	$2\,Q_2^I + Q_1^I$	$S_4 + S_9 + 2S_3 + 2S_{10}$
Model **G**	$3\,Q_1^I$	$3(S_4 + S_9)$
Model **H**	$3\,Q_2^I$	$3(S_3 + S_{10})$
Model **I**	$3\,Q_6^I$	$3(2S_2 + S_4 + S_5)$
Model **J**	$3\,Q_5^I$	$3(2S_1 + S_3 + S_6)$

TABLE IV: The 3-3-1 models already reported in the literature. Particular embeddings are assumed for the AFSs, which are well known in the literature [34, 35].

IV. ALTERNATIVE EMBEDDINGS FOR THE SM FERMIONS

From the previous section, we have several anomaly-free representations for the 3-3-1 gauge group. From now on we will make a distinction between an Anomaly Free-Set (AFS) and a particular embedding corresponding to a distinguishable phenomenological model. In these representations, there are several ways to assign the SM particles within the available multiplets. For the quark sector, this identification is easy since there are only two sets of multiplets of quarks, S_3 and S_4, depending on whether the SM quark doublet is within a $SU(3)_L$ triplet or an anti-triplet. For the lepton sector, there are several sets of multiplets S_i by allowing the right-handed charged lepton to be the third component of a $SU(3)_L$ triplet or a singlet. Once the S_i are chosen for quarks and leptons, there are still several choices for the SM particles within the multiplets.

In order to illustrate the possible embeddings, we will consider the AFSs associated with the models **A**, **E**, **G**, Q_3^I, **I** and Q_7^I . For model **A**, its particle content is indicated in Table IV, and there is only one possible identification for the SM particles in the lepton sector ($3S_2^{\ell+e^+}$), in this case we have three identical copies of the set of multiplets S_2. In this case the SM left-handed lepton doublet (ℓ) is embedded in the $SU(3)_L$ triplet of S_2 and the right-handed charged lepton (e^+) in the singlet. Additionally, the model contains three exotic neutral particles, N^0. In Tables V and VI the SM particle content is shown in the superscript of the fermion sets S_i, avoiding any mention of exotic particles.

Another interesting example is the model **E** which has a rich content of particles and therefore its corresponding AFS has many possible embeddings, as shown in Table IV and V. For example, we can choose the following embedding for the SM particles:

$$
\begin{aligned}
2S_9 + S_{10} = 2&\left[\underbrace{(e^-,\nu_e^0,N_1^0)}_{\text{SM}}\oplus(N_4^0,E^+,\underbrace{e^+}_{\text{SM}})\oplus(E^-,N_2^0,N_3^0)\right]_L \\
&+\left[\underbrace{(\nu_e,e^-,E_1^-)\oplus e^+}_{\text{SM}}\oplus(E_2^+,N_1^0,N_2^0)\oplus(N_3^0,E_2^-,E_3^-)\oplus E_1^+\oplus E_3^+\right]_L .
\end{aligned}
$$

In this embedding, two SM lepton families are put in $2S_9$ and one SM family in S_{10}. In S_9 the right-handed lepton is in the third component of an anti-triplet. The third family is embedded into S_{10} where the right-handed charged lepton is a singlet. The sets, S_9 and S_{10}, have different particle content and quantum numbers. This particular embedding is usually known as the model **E**, which is not universal in the lepton sector and it corresponds to the model E^1 in Table V.

In the two sets of particles, $2S_9$, there are 4 anti-triplets of $SU(3)_L$ with identical quantum numbers, by embedding the three left-handed lepton doublets of the SM in these anti-triplets and the three SM right-handed leptons into the S_{10} singlets, we obtain a new embedding which corresponds to the model E^5 in Table V, as follows:

$$
\begin{aligned}
&\left[\underbrace{(e^-,\nu_e^0,N_1^0)\oplus(E^-,N_2^0,N_3^0)\oplus(e^-,\nu_e^0,N_1^0)}_{\text{SM}}\oplus(E^-,N_2^0,N_3^0)\oplus(N_4^0,E^+,e^+)\oplus(N_4^0,E^+,e^+)\right]_L \\
&+\left[\underbrace{e^+\oplus E_1^+\oplus E_3^+}_{\text{SM}}\oplus(\nu_e,e^-,E_1^-)\oplus(E_2^+,N_1^0,N_2^0)\oplus(N_3^0,E_2^-,E_3^-)\right]_L .
\end{aligned}
$$

At a phenomenological level, the last embedding, which is equivalent to the model **A**, exceeds it by the exotic vector-likes: $(E^-,N_2^0,N_3^0)\oplus(E_2^+,N_1^0,N_2^0)$, $(N_4^0,E^+,e^+)\oplus(\nu_e,e^-,E_1^-)$ and $(N_4^0,E^+,e^+)\oplus(N_3^0,E_2^-,E_3^-)$. This result implies that adding vector-like lepton content to an anomaly-free set of fermions could result in a different non-trivial model by taking a different embedding. The embeddings for the models I^j and Q_7^{Ij} are given in the respective Tables V and VI. These models are always universal in the quark sector, which is very convenient to avoid FCNC. There are four I^j embeddings, two universal and two non-universal as shown in Table V. The I^1 model has the same SM particle content as model **A**, with additional exotic leptons. There are 20 possible Q_7^{Ij} embeddings, with only 4 of them universal, as listed in Table VI. Interesting examples of AFSs that do not contain the vector-like structures listed in column 2 in Table III are: model **G** and the models Q_3^{Ij}. Model **G** is universal and it has only one embedding, while the Q_3^{Ij} models are universal in the quark sector and for some of its embeddings in the lepton sector.

V. COLLIDER CONSTRAINTS

In general, each one of the possible embeddings has a different phenomenology. For collider constraints only the Z' couplings to the SM particles matter [54–56]. In the lepton sector, these couplings depend on whether left-handed lepton doublet ℓ is in a $SU(3)_L$ triplet or anti-triplet. In the same way, the Z' coupling for the right-handed charged lepton e^+ depends on whether is a $SU(3)_L$ singlet or it is the third component of a $SU(3)_L$ anti-triplet. Similar caveats hold for the SM quark doublets q and the right-handed quarks as shown in the Table VII.

We obtain the lower limit on the Z' mass in Table VII, from the intersection of the 95% CL upper limit on the cross-section from searches of high-mass dilepton resonances at the ATLAS experiment [57] with the theoretical cross-section reported in [58]. In the reference [57] the upper limits on the cross-section go up to 6 TeV, however, we extrapolate these results up to 7 TeV in order to obtain the restrictions for the simplified models $C_4+\bar{q}$ and C_4+q in Table VII. The cross-section depends on the Z' charges as they are given the Appendix A. The ATLAS data was obtained from proton-proton collisions at a center-of-mass energy of $\sqrt{s}=13$ TeV during Run 2 of the Large Hadron Collider and correspond to an integrated luminosity of 139 fb^{-1}. Further details are shown in references [44, 58, 59]. We obtain the constraints in Tables V and VI from those shown in Table VII. We only report lower limits for embeddings for which it is possible to choose the same Z' charges for the first two families. Under these assumptions, for the models **A**, C^j, **E** , **G**, **I**, $Q_3^{I_j}$ and $Q_7^{I_j}$, in Tables V and VI, the left-handed quark doublet q is part of a $SU(3)_L$ triplet. For the remaining models, in the mentioned tables, q is part of a $SU(3)_L$ anti-triplet.

Model	j	SM Lepton Embeddings	Universal —	$2+1$ —	Lepton Configuration —	LHC-Lower limit (TeV)
A	-	$3S_2^{\bar{\ell}+e^+}$	✓	×	$3C_2$	4.87
B	-	$3S_1^{\ell+e^+}$	✓	×	$3C_1$	5.53
C^j	1	$S_1^{\ell+e^+}+S_2^{\bar{\ell}+e^+}+S_9^{\bar{\ell}+e'^+}$	×	×	$C_1+C_2+C_3$	
	2	$(S_1^{\ell}+S_9^{e'^+})+S_2^{\bar{\ell}+e^+}+(S_9^{\bar{\ell}}+S_1^{e^+})$	×	✓	$2C_2+C_4$	4.87
D^j	1	$S_1^{\ell+e^+}+S_2^{\bar{\ell}+e^+}+S_{10}^{\ell+e^+}$	×	✓	$2C_1+C_2$	5.53
	2	$S_1^{\ell+e^+}+S_{10}^{2\ell+2e^+}$	✓	×	$3C_1$	5.53
E^j	1	$2S_9^{\bar{\ell}+e'^+}+S_{10}^{\ell+e^+}$	×	✓	C_1+2C_3	5.75
	2	$S_9^{2\bar{\ell}}+S_{10}^{\ell+3e^+}$	×	✓	C_1+2C_2	4.98
	3	$S_9^{2\bar{\ell}+e'^+}+S_9^{\bar{\ell}+e'^+}+S_{10}^{e^+}$	×	✓	C_2+2C_3	5.75
	4	$S_9^{2\bar{\ell}+e'^+}+S_9^{\bar{\ell}}+S_{10}^{2e^+}$	×	✓	$2C_2+C_3$	4.98
	5	$S_9^{2\bar{\ell}}+S_9^{\bar{\ell}}+S_{10}^{3e^+}$	✓	×	$3C_2$	4.98
	6	$S_9^{2\bar{\ell}+e'^+}+S_{10}^{\ell+2e^+}$	×	×	$C_1+C_2+C_3$	
	7	$(S_9^{2\ell+e'^+}+S_{10}^{e^+})+(S_{10}^{\ell}+S_9^{e'^+})$	×	×	$C_2+C_3+C_4$	
	8	$(S_9^{2\ell}+S_{10}^{2e^+})+(S_{10}^{\ell}+S_9^{e'^+})$	×	✓	$2C_2+C_4$	4.98
	9	$S_9^{\bar{\ell}+e'^+}+S_{10}^{2\ell+2e^+}$	×	✓	$2C_1+C_3$	5.53
	10	$S_9^{\ell+e'^+}+(S_{10}^{2\ell+e^+}+S_9^{e'^+})$	×	×	$C_1+C_3+C_4$	
	11	$S_9^{\bar{\ell}}+S_{10}^{2\ell+3e^+}$	×	✓	$2C_1+C_2$	5.53
	12	$(S_9^{\bar{\ell}}+S_{10}^{e^+})+(S_{10}^{2\ell+e^+}+S_9^{e'^+})$	×	×	$C_1+C_2+C_4$	
	13	$(S_9^{\bar{\ell}}+S_{10}^{e^+})+(S_{10}^{2\ell}+2S_9^{e'^+})$	×	✓	C_2+2C_4	7.0
F^j	1	$S_9^{\bar{\ell}+e'^+}+S_{10}^{2\ell+2e^+}$	×	✓	$2C_1+C_3$	5.53
	2	$(S_9^{\bar{\ell}}+S_{10}^{e^+})+S_{10}^{2\ell+2e^+}$	×	✓	$2C_1+C_2$	5.53
	3	$(S_9^{\bar{\ell}}+S_{10}^{e^+})+(S_{10}^{2\ell+e^+}+S_9^{e'^+})$	×	×	$C_1+C_2+C_4$	
	4	$(S_9^{2\bar{\ell}+e'^+}+S_{10}^{e^+})+S_{10}^{\ell+e^+}$	×	×	$C_1+C_2+C_3$	
	5	$(S_9^{2\bar{\ell}}+S_{10}^{2e^+})+S_{10}^{\ell+e^+}$	×	✓	C_1+2C_2	4.98
	6	$(S_9^{2\bar{\ell}}+S_{10}^{2e^+})+(S_{10}^{\ell}+S_9^{e'^+})$	×	✓	$2C_2+C_4$	4.98
	7	$S_{10}^{2\ell+2e^+}+S_{10}^{\ell+e^+}$	✓	×	$3C_1$	5.53
	8	$S_{10}^{2\ell+2e^+}+S_{10}^{\ell}+S_9^{e'^+}$	×	✓	$2C_1+C_4$	5.53
G	-	$3S_9^{\bar{\ell}+e'^+}$	✓	×	$3C_3$	5.53
H	-	$3S_{10}^{\ell+e^+}$	✓	×	$3C_1$	5.53
I^j	1	$3S_2^{\bar{\ell}+e^+}$	✓	×	$3C_2$	4.87
	2	$2S_2^{\bar{\ell}+e^+}+S_2^{\bar{\ell}}+S_5^{e'^+}$	×	✓	$2C_2+C_3$	4.87
	3	$S_2^{\bar{\ell}+e^+}+2S_2^{\bar{\ell}}+2S_5^{e'^+}$	×	✓	C_2+2C_3	5.53
	4	$3S_2^{\bar{\ell}}+3S_5^{e'^+}$	✓	×	$3C_3$	5.53
J	-	$3S_1^{\ell+e^+}$	✓	×	$3C_1$	5.53

TABLE V: Alternative embeddings for the classical AFSs. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and e'^{+} (e^{+}) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The lepton content of the model C_i was defined in Table VII. The check mark ✓ means that at least two families (2+1) or three families (universal) have the same charges under the gauge symmetry, the cross × stands for the opposite. LHC constraints are obtained from Table VII for embeddings for which we can choose the same Z' charges for the first two families, otherwise we leave the space blank.

Model	j	SM Lepton Embeddings	Universal	$2+1$	Lepton Configuration	LHC-Lower limit (TeV)
Q_3^{Ij}	1	$3S_2^{\bar{\ell}+e^+}$	✓	×	$3C_2$	4.87
	2	$2S_2^{\bar{\ell}+e^+} + (S_2^{\bar{\ell}} + S_7^{e'^+})$	×	✓	$2C_2 + C_3$	4.87
	3	$S_2^{\bar{\ell}+e^+} + (2S_2^{\bar{\ell}} + 2S_7^{e'^+})$	×	✓	$C_2 + 2C_3$	5.53
	4	$3S_2^{\bar{\ell}} + 3S_7^{e'^+}$	✓	×	$3C_3$	5.53
Q_7^{Ij}	1	$3S_1^{\ell+e^+}$	✓	×	$3C_1$	5.33
	2	$2S_1^{\ell+e^+} + (S_1^{\ell} + S_7^{e'^+})$	×	✓	$2C_1 + C_4$	5.33
	3	$S_1^{\ell+e^+} + (2S_1^{\ell} + 2S_7^{e'^+})$	×	✓	$C_1 + 2C_4$	6.52
	4	$3S_1^{\ell} + 3S_7^{e'^+}$	✓	×	$3C_4$	6.52
	5	$2S_1^{\ell+e^+} + S_7^{\bar{\ell}+e'^+}$	×	✓	$2C_1 + C_3$	5.33
	6	$2S_1^{\ell+e^+} + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	✓	$2C_1 + C_2$	5.33
	7	$S_1^{\ell+e^+} + (S_1^{\ell} + S_7^{e'^+}) + S_7^{\bar{\ell}+e'^+}$	×	×	$C_1 + C_3 + C_4$	
	8	$S_1^{\ell+e^+} + (S_1^{\ell} + S_7^{e'^+}) + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	×	$C_1 + C_2 + C_4$	
	9	$(2S_1^{\ell} + 2S_7^{e'^+}) + S_7^{\bar{\ell}+e'^+}$	×	✓	$C_3 + 2C_4$	6.52
	10	$(2S_1^{\ell} + 2S_7^{e'^+}) + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	✓	$C_2 + 2C_4$	6.52
	11	$S_1^{\ell+e^+} + 2S_7^{\bar{\ell}+e'^+}$	×	✓	$C_1 + 2C_3$	5.53
	12	$S_1^{\ell+e^+} + S_7^{\bar{\ell}+e'^+} + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	×	$C_1 + C_2 + C_3$	
	13	$S_1^{\ell+e^+} + (2S_7^{\bar{\ell}} + 2S_1^{e^+})$	×	✓	$C_1 + 2C_2$	4.87
	14	$(S_1^{\ell} + S_7^{e'^+}) + 2S_7^{\bar{\ell}+e'^+}$	×	✓	$2C_3 + C_4$	5.53
	15	$(S_1^{\ell} + S_7^{e'^+}) + S_7^{\bar{\ell}+e'^+} + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	×	$C_2 + C_3 + C_4$	
	16	$(S_1^{\ell} + S_7^{e'^+}) + (2S_7^{\bar{\ell}} + 2S_1^{e^+})$	×	✓	$2C_2 + C_4$	4.87
	17	$3S_7^{\bar{\ell}+e'^+}$	✓	×	$3C_3$	5.53
	18	$2S_7^{\bar{\ell}+e'^+} + (S_7^{\bar{\ell}} + S_1^{e^+})$	×	✓	$C_2 + 2C_3$	5.53
	19	$S_7^{\bar{\ell}+e'^+} + (2S_7^{\bar{\ell}} + 2S_1^{e^+})$	×	✓	$2C_2 + C_3$	4.87
	20	$3S_7^{\bar{\ell}} + 3S_1^{e^+}$	✓	×	$3C_2$	4.87

TABLE VI: Alternative embeddings for new anomaly-free sets. The superscripts correspond to the particle content of the SM, where ℓ ($\bar{\ell}$) stands for a left-handed lepton doublet embedded in a $SU(3)_L$ triplet (anti-triplet), and e'^{+} (e^{+}) is the right-handed charged lepton embedded in a $SU(3)_L$ triplet (singlet). The lepton content of the model C_i was defined in Table VII. The check mark ✓ means that at least two families (2+1) or three families (universal) have the same charges under the gauge symmetry, the cross × stands for the opposite. LHC constraints are obtained from Table VII for embeddings for which we can choose the same Z' charges for the first two families, otherwise we leave the space blank.

Model	SM Particle embedding	Short Notation	LHC-Lower limit in TeV
$C_1+\bar{q}$	$\ell\subset 3,\ \ e^+\subset 1,\ \ q\subset 3^*$	ℓ+e^++$\bar{q}$	5.53
C_1+q	$\ell\subset 3,\ \ e^+\subset 1,\ \ q\subset 3$	ℓ+e^++q	5.33
$C_2+\bar{q}$	$\ell\subset 3^*,\ e^+\subset 1,\ \ q\subset 3^*$	$\bar{\ell}$+e^++$\bar{q}$	4.98
C_2+q	$\ell\subset 3^*,\ e^+\subset 1,\ \ q\subset 3$	$\bar{\ell}$+ e^++q	4.87
$C_3+\bar{q}$	$\ell\subset 3^*,\ e^+\subset 3^*,\ q\subset 3^*$	$\bar{\ell}$+ e'^++$\bar{q}$	5.75
C_3+q	$\ell\subset 3^*,\ e^+\subset 3^*,\ q\subset 3$	$\bar{\ell}$+ e'^++q	5.53
$C_4+\bar{q}$	$\ell\subset 3,\ \ e^+\subset 3^*,\ q\subset 3^*$	ℓ+ e'^++$\bar{q}$	7.00
C_4+q	$\ell\subset 3,\ \ e^+\subset 3^*,\ q\subset 3$	ℓ+ e'^++q	6.52

TABLE VII: Simplified 3-3-1 models. The collider constraints depend on whether the SM left-handed doublets ℓ or q are contained in a $SU(3)_L$ triplet or anti-triplet. In short notation q, $\bar{q}$ or ℓ, $\bar{\ell}$. The notation $e^{+\prime}$ is used if the right-handed lepton is the third component of an anti-triplet, otherwise e^+. In the first column, C_1 corresponds to the embedding $\ell\subset 3$, $e^+\subset 1$, in a similar way are defined the remaining C_i. The Z' charges for the C_i, q and $\bar{q}$ are given in Tables VIII to XIII, respectively. In the last column the 95% CL lower limits on the Z' mass, coming from the Drell-Yan process $\bar{p}p\to Z'_\mu$. To obtain the constraints, we assumed that the first two families have the same Z' charges.

VI. CONCLUSIONS

Restricting ourselves to models without exotic electric charges, we have built 12 sets of particles S_i from triplets, antitriplets and singlets of $SU(3)_L\otimes U(1)_X$. These sets are constructed in such a way that they contain the charged particles and their respective antiparticles, following a similar procedure to that in references [34, 35]. With these sets, we built the IAFSs L_i, Q_i^I, Q_i^{II} and Q_i^{III} depending on their quark content, as it is shown in Table III. From the IAFSs it is possible to systematically build 3-3-1 models. It is important to realize that if we restrict the AFSs to a minimum content of vector-like structures (i.e, L_i), having a lepton and quark sector consistent with the SM, our analysis is reduced to the AFSs that contain the classical 3-3-1 models reported in [34, 35]. However, if we allow alternative embeddings for SM particles within S_i, we get new phenomenological distinguishable models. Table V lists all the possible embeddings for the sets of fermions that originate the models reported in references [34, 35]. In Tables V and VI, C^1, D^1, E^1, F^1 and I^1 correspond to the models **C**, **D**, **E**, **F** and **I** in references [34, 35].

By combining the IAFSs from Table III, it is possible to find a large number of models. By restricting to models with a minimal content of exotic fermions, we found 1682 models which could be of phenomenological interest. To exemplify the new realistic AFSs that can be formed, we reported some of them, with their corresponding embeddings, in Table VI. We also report LHC constraints for models with the first two families having SM fermions with identical charges, including some of the classical 3-3-1 models, as reported in Table VII. From this Table we can see that, independent of the model, the mass value of the new neutral gauge boson for all the 3-3-1 models without exotic electric charges is above 4.87 TeV. It is important to note that for models without exotic electric charges, our phenomenological analysis in table VII considers all possible structures under the $SU(3)_L$ gauge symmetry, and therefore it goes beyond the models we have considered in the manuscript.

VII. ACKNOWLEDGMENTS

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Appendix A: Z' Charges

$C_1, \quad \ell \subset 3,\, e \subset 1$ (as in S_1)		
Fields	Vectorial	Axial
ν_e	$-\frac{1}{2}\left(\frac{\cos\theta}{\delta}+\sin\theta\right)$	$-\frac{1}{2}\left(\frac{\cos\theta}{\delta}+\sin\theta\right)$
e	$\frac{1}{2}\left[\sin\theta(1-4\sin^2\theta_W)-\frac{\cos\theta}{\delta}(1+2\sin^2\theta_W)\right]$	$\frac{1}{2}\left[\sin\theta-\frac{\cos\theta}{\delta}(1-2\sin^2\theta_W)\right]$

TABLE VIII: $\bar{f}f \to Z'_\mu$

$C_2, \quad \ell \subset 3^*,\, e \subset 1$ (as in S_2)		
Fields	Vectorial	Axial
ν_e	$\frac{\cos\theta}{\delta}\left(\frac{1}{2}-\sin^2\theta_W\right)-\frac{\sin\theta}{2}$	$\frac{\cos\theta}{\delta}\left(\frac{1}{2}-\sin^2\theta_W\right)-\frac{\sin\theta}{2}$
e	$\sin\theta\left(\frac{1}{2}-2\sin\theta_W^2\right)+\frac{\cos\theta}{\delta}\left(\frac{1}{2}-2\sin\theta_W^2\right)$	$\frac{1}{2}\left(\frac{\cos\theta}{\delta}+\sin\theta\right)$

TABLE IX: $\bar{f}f \to Z'_\mu$

$C_3, \quad \ell \subset 3^*,\, e \subset 3^*$ (as in S_7)		
Fields	Vectorial	Axial
ν_e	$\frac{1}{2}\left[\frac{\cos\theta}{\delta}(1-2\sin^2\theta_W)-\sin\theta\right]$	$\frac{1}{2}\left[\frac{\cos\theta}{\delta}(1-2\sin^2\theta_W)-\sin\theta\right]$
e	$\sin\theta\left(\frac{1}{2}-2\sin^2\theta_W\right)+\frac{3\cos\theta}{2\delta}(1-2\sin^2\theta_W)$	$\frac{1}{2}\left[\sin\theta-\frac{\cos\theta}{\delta}(1-2\sin^2\theta_W)\right]$

TABLE X: $\bar{f}f \to Z'_\mu$

$C_4, \quad \ell \subset 3,\, e \subset 3^*$		
Fields	Vectorial	Axial
ν_e	$\frac{1}{2}\left(\sin\theta+\frac{\cos\theta}{\delta}\right)$	$\frac{1}{2}\left(\sin\theta+\frac{\cos\theta}{\delta}\right)$
e	$\frac{1}{2}\left[\sin\theta(1-4\sin^2\theta_W)+\frac{\cos\theta}{\delta}(1-2\sin^2\theta_W)\right]$	$\frac{1}{2}\left[\sin\theta(1-2\sin^2\theta_W)-\frac{3\cos\theta}{\delta}\right]$

TABLE XI: $\bar{f}f \to Z'_\mu$

$\bar{q}, \quad q \subset 3^*$ (as in S_3)		
Fields	Vectorial	Axial
u	$\frac{1}{6}\left[\sin\theta\left(-3+8\sin^2\theta_W\right)+\frac{\cos\theta}{\delta}\left(3+2\sin^2\theta_W\right)\right]$	$\frac{1}{2}\left[\frac{\cos\theta}{\delta}\left(1-2\sin^2\theta_W\right)-\sin\theta\right]$
d	$\left[\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right]\left[\sin\theta+\frac{\cos\theta}{\delta}\right]$	$\frac{1}{2}\left[\sin\theta+\frac{\cos\theta}{\delta}\right]$

TABLE XII: $\bar{f}f \to Z'_\mu$

$q, \quad q \subset 3$ (as in S_4)		
Fields	Vectorial	Axial
u	$\frac{1}{6}\left[\sin\theta\left(-3+8\sin^2\theta_W\right)+\frac{\cos\theta}{\delta}\left(5-8\cos^2\theta_W\right)\right]$	$-\frac{1}{2}\left[\frac{\cos\theta}{\delta}+\sin\theta\right]$
d	$\frac{1}{6}\left[\sin\theta\left(3-4\sin^2\theta_W\right)-\frac{\cos\theta}{\delta}\left(3-2\sin^2\theta_W\right)\right]$	$\frac{1}{2}\left[\sin\theta-\frac{\cos\theta}{\delta}\left(1-2\sin^2\theta_W\right)\right]$

TABLE XIII: $\bar{f}f \to Z'_\mu$

where θ is an angle mixing between Z and Z' bosons, θ_W is the Weinberg angle and $\delta = \sqrt{4\cos^2\theta_W - 1}$.

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