Challenging Predictions of Inflationary Models with CMB Data

Richik Bhattacharya
Indian Institute of Science Education and Research

Atanu Debnath
Acharya Prafulla Chandra Roy Government College, University of North Bengal,

Esha Sajjanhar
Ashoka University, Sonipat

Shravani Sardeshpande
National Institute of Technology, Rourkela

Pablo Tenorio Hernández
Universidad de las Américas Puebla

See next page for additional authors

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Richik Bhattacharya[a1], **Atanu Debnath**[b2], **Esha Sajjanhar**[c3], **Shravani Sardeshpande**[d4], **Pablo Tenorio Hernández**[e5], and **José Ricardo Torres Heredia**[e6]

Mentor: Saúl Ramos-Sánchez[f7]

[a] *Indian Institute of Science Education and Research, Kolkata, India*

[b] *Acharya Prafulla Chandra Roy Government College, University of North Bengal, West Bengal, India*

[c] *Ashoka University, Sonipat, Haryana, India*

[d] *National Institute of Technology, Rourkela, Odisha, India*

[e] *Universidad de las Américas Puebla, UDLAP, Ex-Hacienda Sta. Catarina Mártir, Cholula, Puebla, México*

[f] *Instituto de Física, Universidad Nacional Autónoma de México, POB 20-364, Cd.Mx. 01000, México*

Abstract

Cosmic inflation offers the best known explanations for many of the observed features of the Universe, such as its flatness. An imprint of the qualities of this mechanism is left in the cosmic microwave background (CMB), which can be instrumental to confirm inflation. Unfortunately, there is a plethora of inflationary models, which are *a priori* in the same footing. It is conceivable that contrasting the predictions of the various models with the measured values of the parameters of CMB data and other cosmological observables shall allow one to single out the successful theory of inflation. In this work we provide a first contribution to this endeavor, by computing the degree of agreement between Planck data and the values of the scalar spectral tilt and the tensor-to-scalar ratio, predicted by different inflationary models. We observe that inflationary models based on power-law potentials and axion monodromy are disfavored.

[1] rb20ms168@iiserkol.ac.in

[2] atanudebnath5001@gmail.com

[3] esha.sajjanhar_ug24@ashoka.edu.in

[4] 422ph2080@nitrkl.ac.in

[5] pablo.tenoriohz@udlap.mx

[6] jose.torresha@udlap.mx

[7] ramos@fisica.unam.mx

1 Introduction

The theory of inflation resolves previously open questions about our universe, mainly accounting for its flatness, homogeneity and isotropy, as well as the solutions to the horizon and monopole problems. Inflation consists of a period of rapid exponential expansion of the universe when it was about 10^{-34} s of age [1, 2]. The *cosmological principle*, which postulates the universe to be homogeneous and isotropic, is the basis for our current understanding of cosmology. Its most important prediction is the Cosmic Microwave Background radiation (CMB), which at first approximation has the homogeneous and isotropic spectrum of blackbody radiation. However, plenty of small perturbations in the CMB lead to slight deviations from its homogeneity. The inflationary paradigm offers one important primordial source of these perturbations. To study them, we use the CMB primordial *power spectrum*

$$P_{\mathcal{R}}(k) = \mathcal{A}_s \left(\frac{k}{k_\star}\right)^{n_s-1}, \tag{1}$$

where we include the amplitude $\mathcal{A}_s$ and the scalar spectral index or tilt n_s, which are both observable, and $k_\star$ denotes the pivot value of the wavenumber k associated with the experiment [1, 3].

Inflation is described at first order by a homogeneous scalar field $\phi(t)$ known as *inflaton*. This field has the property that if its potential energy is large compared to its kinetic energy, then the pressure it induces in the universe is negative. As a consequence of the space-time dynamics, this means that the inflaton produces an exponential expansion of the universe if it is moving slowly along its potential. This defines the meaning of *slow-roll* in this framework. If this slow-rolling happens and the potential is almost flat, inflation can occur [2].

The Planck Collaboration data offers precise constraints for the parameters of inflation [4], in particular for the index n_s and for the tensor-to-scalar ratio r, which is the quotient of the amplitude of the tensor power spectrum and $\mathcal{A}_s$. In figure 2, we display the 1σ and 2σ CL admissible values for these parameters.

This work is organized as follows. In section 2, we discuss the general features of cosmic inflation. Section 3 is devoted to an inspection of some important features of various relevant inflationary models, including the analytical and numerical predictions for the relevant parameters of this work. Finally, in section 4 we discuss the theoretical predictions of the models and how they are compared to the CMB data.

2 Cosmic Inflation

Cosmic inflation is a period of accelerated expansion of the universe preceding the Hot Big Bang, during which the scale factor $a(t)$ grew exponentially with time. This means that the horizon grew by a factor of e^N as compared to the pre-inflation horizon, where N is known as the number of e-folds of inflation. This allows the entire last scattering surface to be in causal contact pre-inflation, thereby providing a resolution to the horizon problem of the standard model [2].

A homogeneous inflaton scalar field $\phi(t)$ is introduced in order to account for the dynamics of cosmic inflation. The quantum fluctuations of the inflaton and gravitational fields make inflationary models perturbative in nature. Such fluctuations are the seeds for cosmic structures such as galaxies and clusters [2].

From the assumption that the universe is a perfect fluid with density $\rho(t)$, we can write the stress-energy tensor subject to an equation of state $p = w\rho$, where p is the isotropic pressure and w is constant [5]. Using the definition of the stress-energy tensor, one directly arrives at

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + V(\phi) \qquad \text{and} \qquad p = \tfrac{1}{2}\dot{\phi}^2 - V(\phi)\,, \tag{2}$$

where in both equations the left term is the kinetic energy density and the right term is the potential energy density [5]. Hence, the equation of state is described in this case by

$$\omega = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \tag{3}$$

Therefore, if the potential energy density dominates, i.e. $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$, then we have $\omega \approx -1$, which implies that the universe grows exponentially fast [5]. The dynamics of the universe during the inflationary period are completely determined by the evolution of the inflaton, which is governed by the Klein-Gordon-like equation

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0\,, \tag{4}$$

where H is the Hubble parameter that sets the rate of expansion of the Universe. To quantify the speed of the inflaton dynamics, we define the inflationary parameter $\epsilon = -\dot{H}/H^2$. The condition for inflation to occur thus becomes $\epsilon < 1$, and $\epsilon \approx 1$ pinpoints the ending of inflation. Imposing in addition $|\ddot{\phi}| \ll 3H\dot{\phi}$ in eq. (4) during inflation to ensure that it lasts long enough, we have

$$3H\dot{\phi} \approx -V'(\phi). \tag{5}$$

This is the *slow-roll* equation of motion which describes the case of an inflaton field rolling slowly down its potential in field space. With these approximations, including the second slow-roll parameter $\eta = -\ddot{\phi}/H\dot{\phi}$, we obtain[8]

$$\epsilon \approx \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \text{and} \qquad \eta \approx \frac{V''}{V}. \tag{6}$$

With the help of the condition $\epsilon(\phi_{\text{end}}) \approx 1$, the value of ϕ_{end}, i.e. the value of the inflaton field at the end of inflation, can be obtained. Then, using the limit on the number of e-folds

$$N \approx \int_{\phi_{\text{end}}}^{\phi_{\text{ini}}} \frac{V}{V'}\,\mathrm{d}\phi\,, \tag{7}$$

[8] In this paper, we assume natural units, such that the reduced Planck mass is the unity.

we get ϕ_{ini}, i.e. the value of the inflaton at the beginning of inflation. Now, at $\phi = \phi_{\text{ini}}$ we can calculate the *scalar spectral index* n_s and the *tensor-to-scalar ratio* r given as

$$n_s \approx 1 + 2\eta - 6\epsilon \qquad \text{and} \qquad r \approx 16\,\epsilon. \tag{8}$$

Recent observations indicate that these parameters are constrained to be $n_s = 0.9649 \pm 0.0042$ and $r < 0.123$ (at 2σ C.L.) [4,6]. The precise bounds are depicted in figure 2, accessed indirectly from [3] using `WebPlotDigitizer` tool [7].

3 Inflationary models

Many theoretical frameworks have been proposed to describe cosmic inflation. Such models are characterized by the expressions for their scalar field potential. Their validity can be confirmed experimentally by the observation of the parameters discussed in the previous section, such as the tensor-to-scalar ratio r, the scalar spectral index n_s and the amplitude of the scalar power spectrum $\mathcal{A}_s$ from the observation of anisotropies in the CMB [8].

We focus here on the family of inflationary models whose potentials are plotted in figure 1. In the following, we present a brief overview of these models, along with the resulting analytic expressions of n_s and r, whenever possible, and their respective predictions for the observable parameters, see table 1. In figure 2, we compare the resulting predictions with the bounds observed by the Planck Collaboration.

Natural inflation. In 1990, Katherine Freese and others [9] presented the case in which spontaneous symmetry breaking giving rise to pseudo Nambu-Goldstone bosons, otherwise called *axions*, could have a potential of the form $V(\phi) = \Lambda^4[1 + \cos(\phi/\mu)]$. They showed that such axion field could give rise to successful inflation in the scale $\mu \approx m_P$ and $\Lambda \approx m_{GUT} \approx 10^{15}\,\text{GeV}$ [9]. This model is now known as natural inflation. In this case, the slow-roll parameters are given by

$$\epsilon = \frac{1}{2\mu^2}\tan^2\frac{\phi}{2\mu} \qquad \text{and} \qquad \eta = \epsilon - \frac{1}{2\mu^2}. \tag{9}$$

Considering the ending of inflation when $\epsilon \approx 1$ and using eq. (7), we obtain

$$\phi_{\text{ini}} = 2\mu \arctan\sqrt{\frac{2\mu^2\exp(\frac{-N}{\mu^2})}{1 + 2\mu^2 - 2\mu^2\exp(\frac{-N}{\mu^2})}}\,, \qquad \phi_{\text{end}} = 2\mu \arcsin\sqrt{\frac{2\mu^2}{2\mu^2+1}}. \tag{10}$$

Further, from eq. (8), at $\phi = \phi_{\text{ini}}$, the observable parameters are given by

$$n_s(N) = 1 - \frac{1}{\mu^2} - \frac{4}{(2\mu^2+1)\exp\left(\frac{N}{\mu^2}\right) - 2\mu^2}, \tag{11}$$

$$r(N) = \frac{16}{(2\mu^2+1)\exp\left(\frac{N}{\mu^2}\right) - 2\mu^2}. \tag{12}$$

The numerical value for these parameters are listed in table 1.

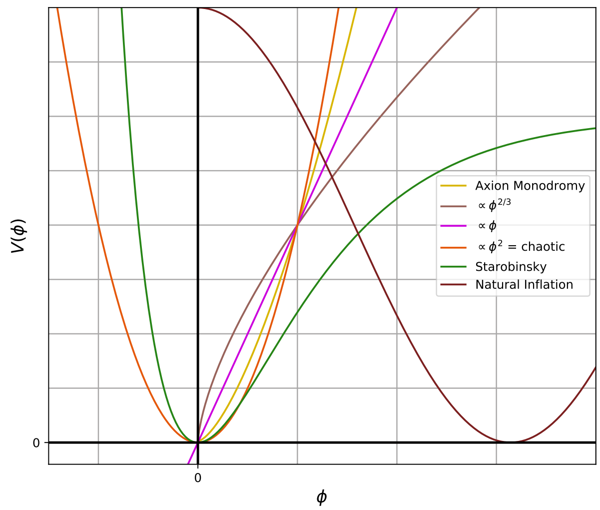

Figure 1: Scalar potentials $V(\phi)$ of various inflationary models discussed here. Here, in order to simplify the comparison between models, all the parameters are set to 1.

Chaotic inflation. In 1983, Andrei Linde attempted to formulate a theory of inflation that could naturally arise from the chaotic initial conditions of the universe [10]. These conditions generally entail the fields described by either a polynomial or an exponential potential. The simplest of these without any loss of generality is given by the potential $V(\phi) = \Lambda^4(\phi/\mu)^p$. A common specific case of this is that of a free massive field with a quadratic potential $V(\phi) = m^2\phi^2/2$, with $m = \sqrt{2}\Lambda^2/\mu$ being the mass of the inflaton. For $p = 1$, this is a linear potential, leading to $V''(\phi) = \eta = 0$ and ϵ independent of ϕ. This means that upon starting, inflation does not stop unless an additional mechanism is considered [11]. Following the same steps as in the previous case, for different values of p and N, we find

$$\phi^2_{\rm end} = \frac{p^2}{2}, \qquad \phi^2_{\rm ini} = 2p\left[N + \frac{p}{4}\right], \qquad n_s(N) = 1 - \frac{p+2}{2N + \frac{p}{2}}, \qquad r(N) = \frac{4p}{N + \frac{p}{4}}\,. \tag{13}$$

Starobinsky R^2 Inflation. We can express the Starobinsky potential as $V(\phi) = \Lambda^4[1-e^{-\phi/\mu}]^2$. The presence of a free parameter μ adds complexity to this inflationary potential by making μ-dependent the observables n_s and r. We found that the value $\mu = 3/2$ provides an optimal fit with the experimental data and, hence, we use it as our standard. The observable parameters, with the aid of analytical approximations, are found to be

$$\phi_{\rm end} = \mu \ln\left[\frac{\sqrt{2}}{\mu}\right], \qquad \phi_{\rm ini} = \mu \ln\frac{2N}{\mu^2}, \qquad n_s(N) \approx 1 - \frac{2}{N} - \frac{3\mu^2}{2N^2}, \qquad r(N) \approx \frac{16\mu^2}{2N^2}\,. \tag{14}$$

Axion monodromy. The general axion monodromy potential can be written as

$$V(\phi) = \Lambda^4\left[\left(\frac{\phi}{\mu}\right)^p + b\cos\left(\frac{\phi}{f} + \gamma\right)\right], \tag{15}$$

		Inflationary models										
N		$p=\frac{1}{4}$	$p=\frac{1}{3}$	$p=\frac{1}{2}$	$p=\frac{2}{3}$	$p=\frac{3}{4}$	$p=1$	$p=\frac{4}{3}$	$p=2$	Natural	Starobinsky	Axion monodromy
50	r	0.019	0.027	0.039	0.053	0.059	0.079	0.106	0.158	0.072	0.0072	0.126
	n_s	0.978	0.977	0.975	0.973	0.973	0.970	0.967	0.960	0.954	0.959	0.966
70	r	0.014	0.019	0.029	0.038	0.043	0.057	0.076	0.113	0.037	0.0037	0.081
	n_s	0.984	0.983	0.982	0.981	0.980	0.979	0.976	0.972	0.963	0.971	0.972

Table 1: Predicted values of the observables n_s and r calculated analytically for $N = 50$ and $N = 70$. For different models, different values of the parameters were so chosen, such that the observables are in the best agreement with the Planck data [4].

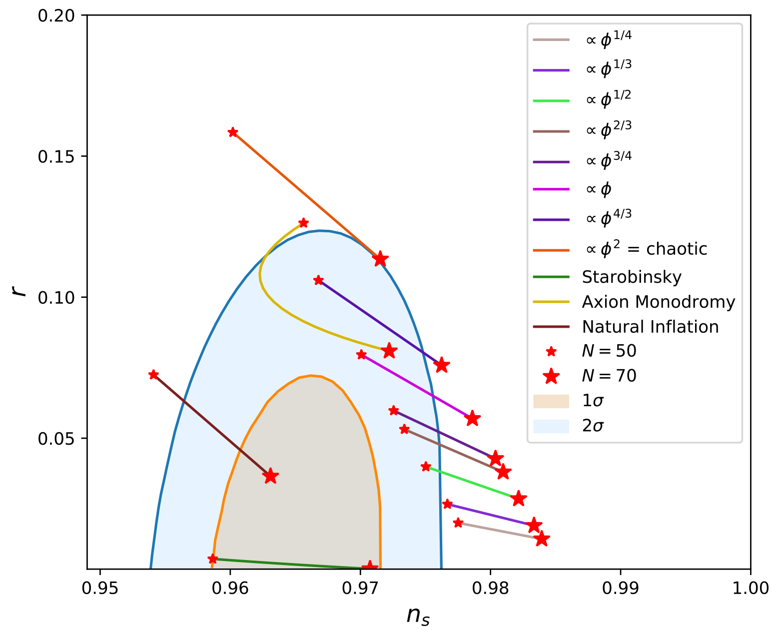

Figure 2: Planck constraints on n_s and r for a family of inflationary models. The outer blue and inner gray regions respectively denote the 2σ and 1σ bounds on the values of n_s and r [4]. Each curve corresponds to the values of the parameters for an inflationary model, with different number of e-folds, $50 \leq N \leq 70$. The models based on $V(\phi) \propto \phi^p$ are disfavored. For the values considered in this work, axion monodromy appears to be mostly disfavored as it lies naturally in the 2σ region.

where there are six free parameters, $\{\Lambda, \mu, f, p, b, \gamma\}$. Note that for $b \ll 1$, the oscillatory modulation acts as a perturbation of chaotic inflation. The large number of free parameters in inflationary models based on axion monodromy makes it significantly more complex to study them analytically. Thus, we use the Newton-Raphson method to numerically evaluate the values of r and n_s, arbitrarily choosing $b = 0.1$, $f = 0.8$, $\mu = 1.2$, $\gamma = \frac{\pi}{2}$ and $p = 3/2$. Table 1 displays the numerical values of the relevant observables. Interestingly, the oscillatory modulation with non-trivial phase γ contributes to improve the results of chaotic inflation, as can also be seen in figure 2.

4 Discussion

Our computations facilitate an analysis of the feasibility of various inflationary models on the basis of the degree of their agreement with Planck data. From figure 2 one can discern the range of N values for which different inflationary models give reasonable (within 2σ CL) values of n_s and r. The 1σ region imposes tighter constraints on the values of observable parameters than the 2σ region. As more recent results indicate [12], the models which predict the values of n_s and r to fall inside the 1σ region might be the only admissible inflationary scenarios.

We observe that many power-law potentials $V(\phi) \propto \phi^p$ (with $p = 2, 1/3, 1/4$) are ruled out since their predictions of n_s or r, or both, for all the values of N lie outside the 2σ CL region. Furthermore, the power-law potentials with $p = 1/2, 2/3, 3/4$ seem also unlikely as their validity is limited to a very small range of e-fold values, which lies in the 2σ CL region. However, natural inflation scenarios give more compelling results compared to other models, especially in the range of higher N.

In the case of axion monodromy, although the value of b is very small, it grants a significant contribution to the values of n_s and r. A slight oscillatory behavior is introduced by b to the potential. Furthermore, the presence of a large number of free parameters assuming different possible values gives rise to the uncertainty in predicting the viability of the general axion monodromy model. For the value parameters considered in this paper, axion monodromy seems to be disfavored but an alternate choice of parameters might improve its feasibility and even make the values of n_s and r fall in 1σ region.

The best fit appears to come from the Starobinsky potential as it lies in the 1σ region for all $50 \leq N \leq 70$. But with recent attempts to identify a lower bound on the values of these parameters, its validity might come into question because of its prediction of very small r value [12].

The constraints on these observable parameters are being tightened with the combination of newer and more precise data from various probes like BICEP/KECK 18 and baryonic acoustic oscillation data (BAO). With the improved data we expect many of the currently acceptable inflationary models to be conclusively ruled out. This must be investigated elsewhere, taking into account the latest and future (forecast) results.

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