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Systems Statistical Engineering – Systems Hierarchical Constraint Propagation

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SYSTEMS STATISTICAL ENGINEERING – SYSTEMS HIERARCHICAL CONSTRAINT PROPAGATION

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Abstract
Cotter (ASEM-IAC 2012, 2015, 2016, 2017): (1) identified the gaps in knowledge that statistical engineering needed to address and set forth a working definition of and body of knowledge for statistical engineering; (2) proposed a systemic causal Bayesian hierarchical model that addressed the knowledge gap needed to integrate deterministic mathematical engineering causal models within a stochastic framework; (3) specified the modeling methodology through which statistical engineering models could be developed, diagnosed, and applied to predict systemic mission performance; and (4) proposed revisions to and integration of IDEF0 as the framework for developing hierarchical qualitative systems models. In the last work, Cotter (2017) noted that a necessary dimension of the systems statistical engineering body of knowledge is hierarchical constraint propagation to assure that imposed environmental economic, legal, political, social, and technical constraints are consistently decomposed to subsystems , modules, and components and that modules, and subsystems socio-technical constraints are mapped to systemic mission performance.. This paper presents systems theory, constraint propagation theory, and Bayesian constrained regression theory relevant to the problem of systemic hierarchical constraint propagation and sets forth the theoretical basis for their integration into the systems statistical engineering body of knowledge.

Keywords
Causal Bayesian Hierarchical Models, Hierarchical Systems, Systems Statistical Engineering

Introduction
Cotter (2015, 2016, 2017) developed the quantitative causal Bayesian hierarchal model as,

$$\begin{aligned} &\text{Min } \mathbf{Y}_{\text{Total}} = f(\mathbf{w}'(\mathbf{Y}_{\text{pred}} - \mathbf{T})) \\ &\text{s.t.} \\ &\mathbf{Y} = \mathbf{F}(pa_{\text{i}}, u_{\text{xi}})\boldsymbol{\beta} + \mathbf{F}(pa_{\text{j}}, u_{\text{zj}})\boldsymbol{\gamma} + \varepsilon \\ &\mathbf{LB}_{\text{X}} \le \mathbf{F}(pa_{\text{i}}, u_{\text{xi}}) \le \mathbf{UB}_{\text{X}} \\ \text{possibly} \quad &\mathbf{LB}_{\text{Z}} \le \mathbf{F}(pa_{\text{j}}, u_{\text{zj}}) \le \mathbf{UB}_{\text{Z}} \end{aligned} \tag{1}$$

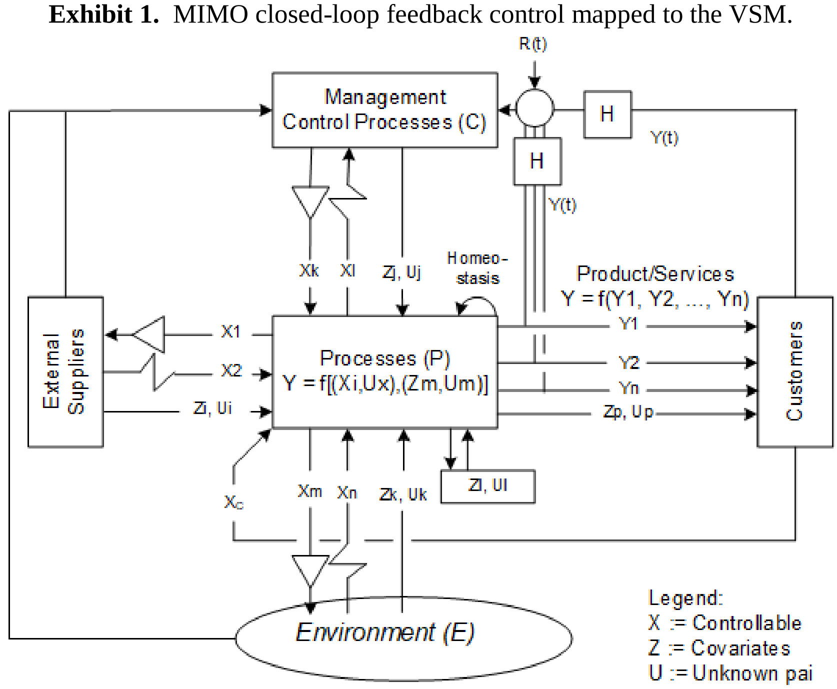
and demonstrated the applicability of this model to hierarchical propagation of mission outcomes versus requirements topologies to sub-system, module, and component functional requirements. SSE is designed to model dynamic systems stochastic-causal functions-to-mission topologies through the integration of general and complex systems governance, the cybernetics of Stafford Beer's Viable System Model, control theory, constraint propagation, and causal Bayesian hierarchical regression.

The core of Systems Statistical Engineering (SSE) is to be a true systems product and process design and performance improvement methodology. As such, SSE extends the current Six Sigma CT-drill-down improvement methodology to a broader systems framework in which the problem and its solution effects on systemic performance and mission outcomes are modeled holistically. General systems theory (Ashby 1956, Bertalanffy 1968, Boulding , Rapoport 1986) provides the general systemic modeling principles. Complex systems governance (Keating 2014, Keating and Katina 2015, 2016) specifies a reference model of the performance functions necessary to maintain complex system viability. SSE is designed to operate within the Metasystems Two (M2) *Information and*

Communications, Three (M3) *System Operations and Control*, and Four (M4) *System Development* functions. The Metasystem Two (M2) function assures consistent and accurate information flows. The Metasystem Three (M3) function monitors, controls, and corrects performance. Metasystem (M4) generates and maintains system models of current and future performance, mission outcomes and system viability. Within the Metasystem Four Star (M4*) *Learning and Transformation* subfunction, SSE facilitates learning through the correction of system design errors or omissions and transformation of the system to new performance levels. Within the Metasystem Four Prime (M4') *Environmental Scanning* subfunction, SSE identifies and maps environmental variation and variety to the variation and variety induced in the subsystem, process, or component under study with the objective of redesign toward improvement in robust systemic mission outcomes. Stafford Beer's Viable System Model (1972, 1979, 1985) provides the recursive cybernetic structure necessary and sufficient to guide the redesign of systemic constraints and connections of low variety components, processes, and subsystems into high variety absorbing systems.

Whereas Beer's VSM provides a systemic description of the System 3 control function, control theory (Bubnicki 2002, Özbay, 2000) provides the means to model dynamic uncertainty and bounded stability in multi-input-multi-output (MIMO) infinite dimension feedback control systems such as the VSM. As illustrated in Exhibit 1, MIMO closed-loop feedback control can be mapped to Beer's simplified VSM (adapted from Beer, 1985, Figure 7) to provide an augmented theory of control not provided by either alone. Product or process performance $\mathbf{Y}(t)$ is compared to the reference input $\mathbf{R}(t)$, and the error $\boldsymbol{e}(t) = \mathbf{Y}(t) - \mathbf{R}(t)$ is fed back to the control processes (C). Here "control processes" are a generalization of a physical controller to indicate joint managerial and machine control. This generalization is necessary to map systemic hierarchical constraint propagation. The **Z** inputs are known covariate disturbances that are either uncontrollable or controllable but not considered sufficiently important to control. The **Z•** covariates and **U•** unknown structural and random disturbances map to the ε error term in the Equation (1) transformation function but may also bias $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ coefficient estimates. This separate mapping allows joint causal Bayesian hierarchical and state-space control modeling. The (**Z•, U•**) disturbance mappings are important error sources not considered by Beer in his VSM specification. Beer's First Principle of Organization (1985, p. 30) translated Ashby's Law of Requisite Variety into a necessary condition of variety matching for control: "Managerial, operational, and environmental varieties, diffusing through an institution system, tend to equate; they should be designed to do so with minimal damage to people and to cost." As Exhibit 1 illustrates, variety matching is a theoretical construct that cannot be attained in application due to losses induced by the **Z•**, **U•** disturbances mapped into the output **Z**p, **U**p product or service process disturbances. The practical control objective is to Min $\mathbf{Y}_{Total} = f(\mathbf{w}'(\mathbf{Y}_{pred} - \mathbf{T}))$ and re-engineer the product or process to reduce **Z**p, **U**p disturbance losses as in the Six Sigma methodology.

Exhibit 1. MIMO closed-loop feedback control mapped to the VSM.



The VSM focus on managing variety, as opposed to variation, provides a useful way to map and measure hierarchical environmental, supply, customer, and systemic constraint propagation. Ashby's Law requires that control processes absorb all variety generated by the system. This requires that control processes must attenuate environmental variety and amplify transformation process (P) variety. Exhibit 1 illustrates both. Variety attenuation is illustrated by the arrows leading from the environment and external suppliers through the transformation process to control processes and variety amplification from control processes through the transformation process to external suppliers and the environment. However, it is insufficient to know only which sources of variety must be attenuated and amplified. Control attenuation and amplification boundaries must be aligned with and at least as large as environmental and supplier variety boundaries. Otherwise, excess **Z•**, **U•** disturbances will be amplified through the transformation process as **Z**p, **U**p finished product or service process variation or variety. Finally, note that Beer's original formulation of the VSM did not specify the dynamics of control. Rather, variety amplification and attenuation were concepts. Causal Bayesian hierarchical modeling admits identification of variation and variety dynamics. Positive or negative shifts or trends in $\mathbf{Y}_{pred} - \mathbf{T}$ indicate that particular sources of variation or variety have switched from stable random variation or variety to needing attenuation to amplification and vice versa. Statistically significant changes in the $\boldsymbol{\beta}$ or $\boldsymbol{\gamma}$ model coefficients suggest investigation for boundary misalignment and possible systemic state changes.

System Constraint Boundaries Identification and Variety/Variation

Integration of process control theory and statistical process control (Box and Luceno, 2009) provides the means of partitioning variation and variety. Control theory bounded-input bounded-output (BIBO) stability analysis can be applied to identify system viability constraint boundaries and provide a useful means of maximizing constraint boundary alignment. Beer (1979, 1981) provided little guidance on the measurement of variety. Following Shannon's definition of self-information, in *The Heart of Enterprise* (p. 520) Beer specified selection entropy as $H = -\boldsymbol{\Sigma}\, p_i \log_2 p_i$ which reduces to $H = -\boldsymbol{\Sigma} \log_2 V$ for unbiased probabilities as the measure of system variety. Other information measurements include the joint entropy of two discrete random variables, conditional entropy, Kullback-Leibler divergence, mutual transformation, and differential entropy. Espejo and Harden (1989, p. 82) considered variety as "… roughly equivalent …" to information but referred to variety as the number of possible systemic states (1989, p. 82). Information measurements are based on proper probability mass functions or probability density functions, which measure variation in information content or entropy. From statistical control theory, as long as the system remains in a state of control, dynamic homeostasis, it cannot be considered as having changed states. Thus, information measurements of variation are not sufficient measures of variety as changes in systemic states. These observations admit the following definition of systemic variety:

> **Definition – Systemic BIBO Stability**: For any technical, social, and information system (or system-of-systems) configuration, systemic variety is the number of possible changes in dynamic homeostasis states within the system's (or system-of-systems') supremum bounded-input bound-output (BIBO) stability constraints of viability.

This definition is supported by theorical and empirical BIBO stability research. Sup(BIBO) is necessary to account for fuzziness of any given system's viability boundary. Since system **Y** outputs must be bounded in order to provide outputs usable to the system for self-maintenance (the Homeostasis feedback loop in Exhibit 1) and usable by its customer systems as inputs, multivariate BI[($\mathbf{LB}_X$, $\mathbf{UB}_X$), ($\mathbf{LB}_Z$, $\mathbf{UB}_Z$)]-BO(**Y**) are estimates of systemic viability bounds. Any state change outside the multivariate BIBO() bounds must result in loss of systemic viability. Accordingly, systemic dynamic homeostasis bounds must be less than BIBO() bounds. From Exhibit 1, variation and viability state changes can be measured only in the $\mathbf{Y} = \mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} \pm \varepsilon$ output.

> **System Viability Theorem**: Let PERF(**F**[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) be the lower bound and PERF(**F**[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) be the upper bound of system design limits controllable by a systems designer, where PERF() limits are product and service limits set by (M3) *System Operations and Control* to provide BO(**Y**) outputs usable by the system for self-maintenance or by (M4) *System Development* to provide BO(**Y**)` outputs usable by its customer systems as inputs for transformation into their respective output **Y**(product, service). Then, either BO(**F**[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) **<** PERF(**F**[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) < BO(**Y**) < PERF(**F**[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) **=** BO(**F**[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) or BO(**F**[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) **=** PERF(**F**[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) < BO(**Y**) < PERF(**F**[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) **<** BO(**F**[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) is required for minimum system viability.

Proof: (Necessary) If $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, then induced entropy variety $>_{\text{everywhere}}$ system absorption variety rendering the system not viable in any state. (Sufficient) If $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) = \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) = \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, then induced entropy variety = system absorption variety rendering the system viable in exactly one state of dynamic homeostasis stability.

Corollary (**One-degree of design freedom**): Exclusively, either $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)])$ or $\text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ is required for exactly one and only one state change from dynamic homeostasis stability.

State Change Theorem: A dynamic homeostasis state change occurs if and only if $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)])$ or $\text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$.

Variation Theorem: System variation occurs if $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y} \leq \text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) \leq \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, where LNTL() and UNTL() are the lower and upper natural statistical control limits respectively.

Proof: (Necessary) If $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \text{BO}(\mathbf{Y}) \leq \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, the system remains in its dynamic homeostasis stable state, and a change in systemic state cannot have occurred. Hence, only random variation is present. (Sufficient) If $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)])$ or $\text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, the system has changed from its $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y} \leq \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ dynamic homeostasis stable state, and some Yi of BO($\mathbf{Y}$) are not useable by the system for self-maintenance or not useable by the system's customer systems. Hence, induced entropy variety > system absorption variety rendering the system in a state of reduced viability. If $\text{BO}(\mathbf{Y}) < \text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)])$ or $\text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) > \text{BO}(\mathbf{Y})$, then BO($\mathbf{Y}$) is not usable by the system for self maintenance and not useable by the system's customer systems. Hence, entropy variety > system variety rendering the system not viable.

Corollary (**Design degrees of freedom**): The combinations of ways that BO($\mathbf{Y}$) can change from dynamic homeostasis stability to $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)])$ or $\text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \text{BO}(\mathbf{Y}) < \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ is the design degrees of freedom.

Corollary (**Design variation degrees of freedom**): The combinations of ways that $\mathbf{Y}$ can change from its in-control state to either $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \mathbf{Y} < \text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon)$ or $\text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) < \mathbf{Y} < \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ is the design variation degrees of freedom.

In the above theory development, two-sided bounds were considered. One-sided bounds can be considered with no change in theoretical support by setting $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) = \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) = \text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) = 0$ for one-sided upper bounds or $\text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) = \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) = \text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) = \infty$ for one-sided lower bounds.

Thus, viable systems design can be viewed through the lens of feedback control filter design. Any combination of socio-technical-information components, modules, and subsystems will yield a multivariate $\text{BI}[(\mathbf{LB}_X, \mathbf{UB}_X), (\mathbf{LB}_Z, \mathbf{UB}_Z)]\text{-BO}[\mathbf{Y}]$ output. The problem is to identify the combination of socio-technical-information components, modules, and subsystems will yield the multivariate BO[$\mathbf{Y}$] outputs that maximize usability for system self-maintenance and usability inputs for the system's customers' transformation into their respective output $\mathbf{Y}$(product, service). Once max(BO[$\mathbf{Y}$]) has been identified, the design problem then becomes one of aligning the constraint boundaries such that $\text{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y} \leq \text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) < \text{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) = \text{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ to maximize system viability robustness. Then, statistical system identification methods can be applied to specify the form of the causal Bayesian hierarchical model that Min $\mathbf{Y}_{\text{Total}} = f(\mathbf{w}'(\mathbf{Y}_{\text{pred}} - \mathbf{T}))$.

Bayesian Hierarchical/Supply-Chain Systems Constraint Propagation and Satisfaction

As a body of knowledge, constraint satisfaction of combinatorial optimization problems emerged in operations research during the 1960s. In this paradigm, constraint satisfaction problems were formulated as a set of variables whose state of optimality must satisfy a set of limitations defining a feasible solution region. Foundational concepts such as arc and path consistency were developed in the 1970s. In the 1980s, constraint satisfaction was extended to logic programming in artificial intelligence to address goal provability under constraints. In the last four decades, constraint satisfaction problems (CSPs) in bioinformatics, decision preference, networking, planning, and scheduling have been studied extensively in both operations research and artificial intelligence (Barták, 2010). Ryu (1998, 1999) extended CSPs to the construction of hierarchical constraint satisfaction of product taxonomies for individual electronic shopping over the Internet. Ryu's constraint satisfaction hierarchy construction assumes the each individual shopper has the capacity to resolve pairwise product attribute preference differences. In the case of socio-technical systems with competing customer, user, and stakeholder preferences, Ryu's assumption of pairwise preference resolution may not hold strictly. Barták (1997, 1998) proposed a cell methodology that combines the advantages of refining and local propagation methods to solve over-constrained preference hierarchies such as those with competing customer, user, and stakeholder preferences. Barták's method assumes that conflicting preferences exist *a priori* and does not address setting product or service attribute or characteristic design specification and process control constraints. Integration and extension of Ryu's and Barták's methods can address both the setting of design specifications and solving over-constrained preference hierarchies. Finally, CSPs have not been extended to the current problem of maximizing BO[$\mathbf{Y}$] under managerial design control constraints $C(\mathbf{X}_k, \ldots, \mathbf{X}_l, \mathbf{Z}_j, \mathbf{U}_j)$ subject to environmental constraints $E(\mathbf{X}_m, \ldots, \mathbf{X}_n, \mathbf{Z}_k, \mathbf{U}_k)$ shown in Figure 1. This work addresses hierarchical CSPs that must integrate conflicting product and service design specifications and managerial design process control constraints subject to environmental constraints.

Under sup(BIBO) stability-viability, constraint propagation and satisfaction is a problem of finding an integral solution of both hierarchical environmental-systemic variety absorption through amplification and attenuation as specified by Beer and system identification of throughput $\mathrm{BO}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \mathrm{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) < \mathrm{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y} \leq \mathrm{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) < \mathrm{PERF}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]) < \mathrm{BO}(\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ constraint boundaries to maximize system viability robustness. Dimensions of the constraint propagation and satisfaction problem involve integration of:

- Horizontal hierarchical supply-chain versus hierarchical environmental and systemic constraints.
- Static versus dynamic constraints.
- Deterministic fixed versus probabilistic/fuzzy versus possibilistic constraints.
- Technical-economic versus human preference constraints.

This work addresses only the first bullet.

Arguably, the most important constraints are the $\mathbf{Y} = f(\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_n)$ products and services performance outputs demanded by customers, users, and stakeholders and by the system itself for self-maintenance. These constrained targeted performance outputs are what make the system "purposeful" (Beer, 1959). Consider a product or service component $\mathbf{Y}_i$ performance output measureable by an r tuple $(y_1, y_2, \ldots, y_r)$ set of attribute and characteristic values such that the product or service is fully specified by its $\mathbf{Y}$ performance values. Since characteristics are measured on either a continuous interval or ratio scale, any y_t characteristic that falls within or outside of its $\mathrm{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq y_i \leq \mathrm{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ specification limits can be treated as an attribute. Thus, the balance of this discussion will refer to any composite of attribute or characteristic performance output simply as yi. Extending the arguments of Ryu and Barták, let $\mathbf{Y}_i$ be the component domain of attributes and characteristics $f(y_1 \cup y_2 \cup \ldots \cup y_r)$ that yield $\mathrm{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y}_i \leq \mathrm{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ values. The domain y_t is termed an *ordered domain* if a strict total ordering relation can be defined on it. The domain y_t is termed a *clustered domain* if a directed tree structure can be defined on it. Express a branch of a clustered domain y_t as $B(y, y')$. The set of transformations $f(y_1, y_r, r-1)$ that yield $\mathrm{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq y_i \leq \mathrm{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ is defined to hold if $B(y_1, y_2), B(y_2, y_3), \ldots, B(y_{k-1}, y_k)$ hold for $k \geq 2$. The Cartesian product of component domains $\mathbf{Y}_i = f(y_1 \cup y_2 \cup \ldots \cup y_r)$ is the *attribute or characteristic design structure space*, and the Cartesian product of product or service domains $\mathbf{Y} = \mathbf{F}(\mathbf{Y}_1 \cup \mathbf{Y}_2 \cup \ldots \mathbf{Y}_n)$ is the *product or service design structure space*.

Define a constraint c_i on a y_i as an expression that establishes a preference relations $\rangle_c$ or an indifference relation $\sim_c$ on y_i. For a categorical product or service variable, the fundamental relation is a Boolean constraint that divides y_i into two sets y_1 and y_2 such that for all $c_i \in y_1$, $E(S \mid c, y) = 0$, and for all $c_i \in y_2$, $E(S \mid c, y) = 1$, where the function $E(S \mid c, y)$ measures the degree of constraint satisfaction by that component. Similarly, define a constraint set C_i on a component $\mathbf{Y}_i$. or constraint set $\mathbf{C}$ on product $\mathbf{Y}$. The fundamental Boolean constraint that divides $\mathbf{Y}_i$ or $\mathbf{Y}$

into two sets $\mathbf{Y}_1$ and $\mathbf{Y}_2$ such that for all C_i = PERF(f[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) $\leq \mathbf{Y}_i \leq$ PERF(f[($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) or C_i = PERF($\mathbf{F}$[($\mathbf{LB}_X$, $\mathbf{LB}_Z$)]) $\leq \mathbf{Y} \leq$ PERF($\mathbf{F}$ [($\mathbf{UB}_X$, $\mathbf{UB}_Z$)]) $\in \mathbf{Y}_1$, E(S | C, Y) = 0 and E(S | $\mathbf{C}$, $\mathbf{Y}$) = 0, and for all $C_i \in \mathbf{Y}_2$, E(S | C, Y) = 1 and E(S | $\mathbf{C}$, $\mathbf{Y}$) = 1, where the function E(S | C, Y) measures the degree of constraint satisfaction by that component and E(S | $\mathbf{C}$, $\mathbf{Y}$) measures the degree of constraint satisfaction by the product. Let E(S | c, y) = 0, E(S | C, Y) = 0, and E(S | $\mathbf{C}$, $\mathbf{Y}$) = 0 mean that y_i, $\mathbf{Y}_i$, or $\mathbf{Y}$ do not satisfy the constraint or constraint set and E(S | c, y) = 1, E(S | C, Y) = 1, and E(S | $\mathbf{C}$, $\mathbf{Y}$) = 1 mean that y_i, $\mathbf{Y}_i$, or $\mathbf{Y}$ satisfies the constraint or constraint set.

A continuous variable constraint is expressed as $r(c, y)$ where c is now an arithmetic constant and r is one of the relations =, ≤, <, or ≠. Similarly, define a set of variable product or service constraints as $R(C, \text{Y})$, where R is a set of relations (=, ≤, <, or ≠). For an arithmetic constraint c, the degree of characteristic constraint satisfaction is measured by:

$$E(S \mid c, y) = (ppm_{AQL} - ppm_{ci}) / ppm_{AQL}$$

$$\delta(c, y) = \begin{cases} E(S \mid f(y_i) = \max\{|\mu(y_i) + k\sigma = c_i|\}) & \text{for } c_i = \text{desired target value} \\ E(S \mid f(y_i) = \mu(y_i) + k\sigma \leq \max c_i) & \\ E(S \mid \min c_i \leq f(y_i) = \mu(y_i) \pm k\sigma \leq \max c_i) & \\ E(S \mid \min c_i \leq f(y_i) = \mu(y_i) - k\sigma) & \\ E(S \mid f(y_i) = \min\{|\mu(y_i) + k\sigma = c_i|\}) & \text{for } c_i \neq \text{undesired target value} \end{cases} \quad (2)$$

where ppm_{AQL} = parts per million defects at the sampling plan acceptable quality level, ppm_{ci} = parts per million defects at the constraint c_i given the current characteristic design, $\mu(y_i)$ is a measure of central location, σ is a measure of spread from a proper identically, independently distributed (IID) statistical distribution, and k is a constant. By extension, the degree of component and product satisfaction is defined for vectors or matrices $\mathbf{Y}_i = f(y_1 \cup y_2 \cup \ldots \cup y_r)$, $\boldsymbol{\mu}(\mathbf{Y}_i)$, $\boldsymbol{\Sigma}(\mathbf{Y}_i)$, and $\mathbf{C}_i$ as E(S | C, Y) and for $\mathbf{Y} = \mathbf{F}(\mathbf{Y}_1 \cup \mathbf{Y}_2 \cup \ldots \mathbf{Y}_n)$, $\boldsymbol{\mu}(\mathbf{Y})$, $\boldsymbol{\Sigma}(\mathbf{Y})$, and $\mathbf{C}$ as E(S | $\mathbf{C}$, $\mathbf{Y}$).

The above specifications of constraint satisfaction assume that the constraints are independent and not conflicting. For non-independent conflicting constraints, Barták's (1997, 1998) method of defining constraint cells provides a useful means of resolving satisfaction conflicts in product and service design structure spaces.

Definition – constraint cell: (1) Let C or $\mathbf{C}$ be a finite non-empty set of constraints with the same satisfaction level for attribute and characteristic sets $\mathbf{Y}_i$ or $\mathbf{Y}$. Let (In, Out $\subseteq \mathbf{Y}_i$ or $\mathbf{Y}$) be a design set of attributes and characteristics such that In ∪ Out = $\mathbf{Y}_i$ or $\mathbf{Y}$ and In ∩ Out = ∅. Define a *constraint cell* as a triple (C, In, Out) or ($\mathbf{C}$, In, Out), and enable a constraint cell in the form ({}, {}, C) or ({}, {}, $\mathbf{C}$) containing only the output variable $\mathbf{Y}_i$ or $\mathbf{Y}$. (2) Call the sets In and Out as input and output sets of attributes and characteristics. (3) Constraint cell (C, In, Out) or ($\mathbf{C}$, In, Out) determines each attribute and characteristic from the set Out.

Defining constraint sets in a cell admits processing conflicts between constraint sets with the same satisfaction level within design hierarchies and cycles of constraints.

Definition – constraint cells classification: Classify constraint cells into the following groups.

- *free attributes and characteristics cell* ({}, {}, {$\mathbf{Y}_i$} or {$\mathbf{Y}$})
- *functional cell* ({C @ 1} or {$\mathbf{C}$ @ 1}, In, Out) such that Out ≠ ∅ and for evaluation function θ of attributes and characteristics from In there exists a unique valuation V from Out such that CθV or $\mathbf{C}$θV holds.
- generative cell (C, In, Out) or ($\mathbf{C}$, In, Out) such that C ≠ ∅ or $\mathbf{C}$ ≠ ∅ and (C, In, Out) or ($\mathbf{C}$, In, Out) is not a functional cell.
- *test cell* (C, In, ∅) or ($\mathbf{C}$, In, ∅)
- *potentially unsatisfied cell* is a generative or test cell.

Definition – internal satisfaction strength: The *internal satisfaction* of a constraint cell (C, In, Out) or ($\mathbf{C}$, In, Out) is the satisfaction strength in C or $\mathbf{C}$. The internal satisfaction strength of the constraint cell ({}, {}, {$\mathbf{Y}_i$} or {$\mathbf{Y}$}) is free, which is the satisfaction strength that is weaker than any other of the constraints. Define the internal satisfaction constraint of Cell as E(S | C, Y) or E(S | $\mathbf{C}$,

Y). We denote that Cell is stronger than Cell′ if and only if $E(S \mid C, Y) > E(S \mid C', Y)$ or $E(S \mid \mathbf{C}, \mathbf{Y}) > E(S \mid \mathbf{C}', \mathbf{Y})$.

Definition – conflict-free hierarchy decomposition: Let H be a constraint hierarchy and sets $\mathbf{Y}_i$ or **Y** be a set of all attributes and characteristics from H. Define the finite set CC of constraint cells a conflict-free decomposition of the hierarchy H if and only if the following conditions hold: (1) $\forall$ Cell $\in$ CC Cell = (*C*, In, Out) or (**C**, In, Out) and C or C $\subseteq$ H and In $\subseteq \mathbf{Y}_i$ or **Y** and Out $\subseteq \mathbf{Y}_i$ or **Y**. (2) $\forall\, c \in$ H $\exists!$ Cell $\in$ CC Cell = (*C*, In, Out) or (**C**, In, Out) and $c \in C$. (3) $\forall\, y \in \mathbf{Y}_i\ \exists!$ Cell $\in$ CC Cell = (*C*, In, Out) or (**C**, In, Out) and $y \in$ Out.

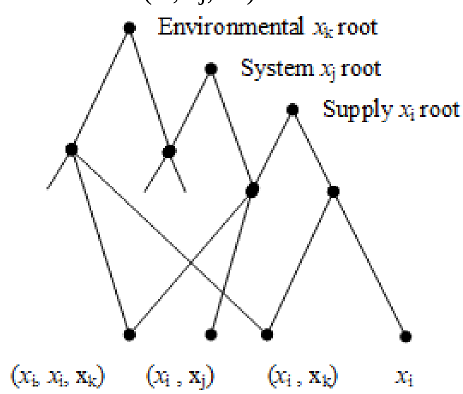
Definition – constraint hierarchy network: Let CC be a conflict-free decomposition of H. Define the directed acyclic graph (CC, E) with nodes CC and edges E a *constraint hierarchy network* of H if and only if the following conditions hold: (1) $\forall$ Cell, Cell′ $\in$ CC, Cell = (*C*, In, Out) or (**C**, In, Out) and (*C*′, In′, Out′) or (**C**′, In′, Out′) and In $\cap$ Out $\neq \varnothing \Rightarrow$ (Cell, Cell′) $\in$ E. (2) $\forall$ Cell, Cell′ $\in$ CC, (Cell is G/T and Cell′ < Cell) $\Rightarrow E(S \mid C', Y')$ or $E(S \mid \mathbf{C}', \mathbf{Y}')$ < as $E(S \mid C, Y)$ or $E(S \mid \mathbf{C}, \mathbf{Y})$. (3) $\forall$ Cell, Cell′, Cell″ $\in$ CC, Cell, Cell′, Cell″ are G/T and Cell < Cell′, Cell < Cell″ $\Rightarrow$ (Cell′ = Cell″ $\vee$ Cell′ < Cell″ $\vee$ Cell′ > Cell″).

With the above definitions adapted from Barták's (1997, 1998) method, SSE can now apply any hierarchy network construction algorithm to build and resolve product and service design satisfaction conflicts within representative hierarchical $\mathbf{Y} = f(\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_n)$ products and services performance networks.

With customer, user, stakeholder product and service constraint hierarchies and systemic maintenance constraints defined, the design problem is to specify transformation processes that yield $\text{LNTL}(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y}_i \leq \text{UNTL}(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon)$ aligned with $\text{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y}_i \leq \text{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ that maximizes $E(S \mid C, Y) \to 1$ and hierarchically $\text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y} \leq \text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon)$ aligned with $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y} \leq \text{PERF}(\mathbf{F}\,[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ that maximizes $E(S \mid \mathbf{C}, \mathbf{Y}) \to 1$. In the hierarchical environmental/system network of Exhibit 1, constraints on product or service y_i, $\mathbf{Y}_i$, and **Y** performance are propagated through the β and γ coefficient transfer functions for the functionality specified in the ***X*** input variables conditional on the **Z**, and **U** inputs in equation (1). Returning to Ryu (1998, 1999), define a constraint set $c = (c_i, c_j, c_k)$ on x_i supplied, x_j systemic, and x_k environmental inputs as an expression that establishes a preference relation $\rangle_c$ or an indifference relation $\sim_c$ on (x_i, x_j, x_k). Any constraint set $(\{\}, c_j, c_k)$ on the $\mathbf{Z}\bullet$ covariates and $\mathbf{U}\bullet$ unknown structural and random disturbances map to the ε error term in the Equation (1) transformation function but may also place unknown constraints on $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ coefficient estimates. Categorical Boolean and relational and continuous variable constraints $r(c, x)$ and $R(C, X)$ constraints are specified as in the above discussion. For supplied, environmental, and systemic input x's, the constraint satisfaction problem is one of maintaining hierarchical constraint consistency rather than resolving customer, user, and stakeholder constraint satisfaction conflicts.

Dechter and Pearl's (1989) and Dechter's (2006) Adaptive-Consistency algorithm of hierarchical tree decomposition can be extended to the construction of consistent hierarchical constraint satisfaction of product and service taxonomies as illustrated in Exhibit 2.

Exhibit 2. (xi, xj, xk) root-directed tree.



Only four input variable sets are possible from the generation of a root-directed tree: (1) set $((x_i, c_i), (x_j, c_j), (x_k, c_k))$, (2) set $((x_i, c_i), (x_j, c_j))$, (3) set $((x_i, c_i), (x_k, c_k))$, or (4) set (x_i, c_i). Excluding the root nodes, each (x, c) end node may have multiple parent nodes. However, by the superposition principle, the environmental, system, and supply (x, c) root-directed trees may be generated separately and overlaid to form the composite root-directed tree. Then, in each (x, c) root-directed tree, each node (excluding the root) will have one parent node directed toward it and possibly several child nodes directed away from it. For the SSE problem of $((x_i, c_i), (x_j, c_j), (x_k, c_k))$ decomposition, the following tree-solving algorithm may be applied for the input variables.

Tree-solving
Input: A hierarchical tree network N• = (D, X, C)

1. Generate a width-1 ordering d = $(x_1, c_1), (x_2, c_2) \ldots (x_n, c_n)$.
2. Let $X_{p(i)}$ denote the parent of (x_i, c_i) in the rooted ordered tree.
3. For i = n to 1 do
   Revise $((X_{p(i)}, X_i)$;
   If the domain D of $X_{p(i)}$ is empty, exit (no solution exists).
4. End For.
5. Repeat steps 1 – 4 for the (x_j, c_j) system tree substituting j for i and the (x_k, c_k) environmental tree substituting k for i.
6. Form the $((x_i, c_i), (x_j, c_j), (x_k, c_k))$ composite root-directed tree by overlaying the (x_j, c_j) system and (x_k, c_k) environmental trees on the (x_i, c_i) supply tree.

By the superposition principle, the definitions and theorems for acyclic trees, tree-width and induced-width, algorithm adaptive consistency (AC) tree decomposition, and hypertree decomposition hold for each individual tree and the composite tree. Therefore, constraint consistency is maintained within the $((x_i, c_i), (x_j, c_j), (x_k, c_k))$ composite root-directed tree spread across the process-of-interest/scaffolding interaction structure.

Composite Tree Theorem: The composite tree from the tree-solving algorithm specifies the process-of-interest/scaffolding constraint interaction structure of the system of interest.

Proof: Dechter (2006) proofs of theorems for Algorithm Adaptive-Tree Consistency.

A generic process-of-interest/scaffolding decomposition is illustrated in Exhibit 3.

The final remaining question to address is identification of the $\max(c_i, c_j, c_k)$ least restrictive constraint on each $\mathbf{Y_i} = f[(x_i, c_i) \cup (x_j, c_j) \cup (x_k, c_k))$ and hierarchically $\mathbf{Y} = \mathbf{F}(\mathbf{Y}_1 \cup \mathbf{Y}_2 \cup \ldots \mathbf{Y}_n)$, and the effect of $\max(c_i, c_j, c_k)$ on alignment of $\text{LNTL}(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y}_i \leq \text{UNTL}(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon)$ with $\text{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y}_i \leq \text{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$ and reduction of component $E(S \mid C, Y) \rightarrow 1$ and hierarchically on alignment of $\text{LNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \leq \mathbf{Y} \leq \text{UNTL}(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon)$ with $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y} \leq \text{PERF}(\mathbf{F}\ [(\mathbf{UB}_X, \mathbf{UB}_Z)])$ and reduction of product $E(S \mid \mathbf{C}, \mathbf{Y}) \rightarrow 1$.

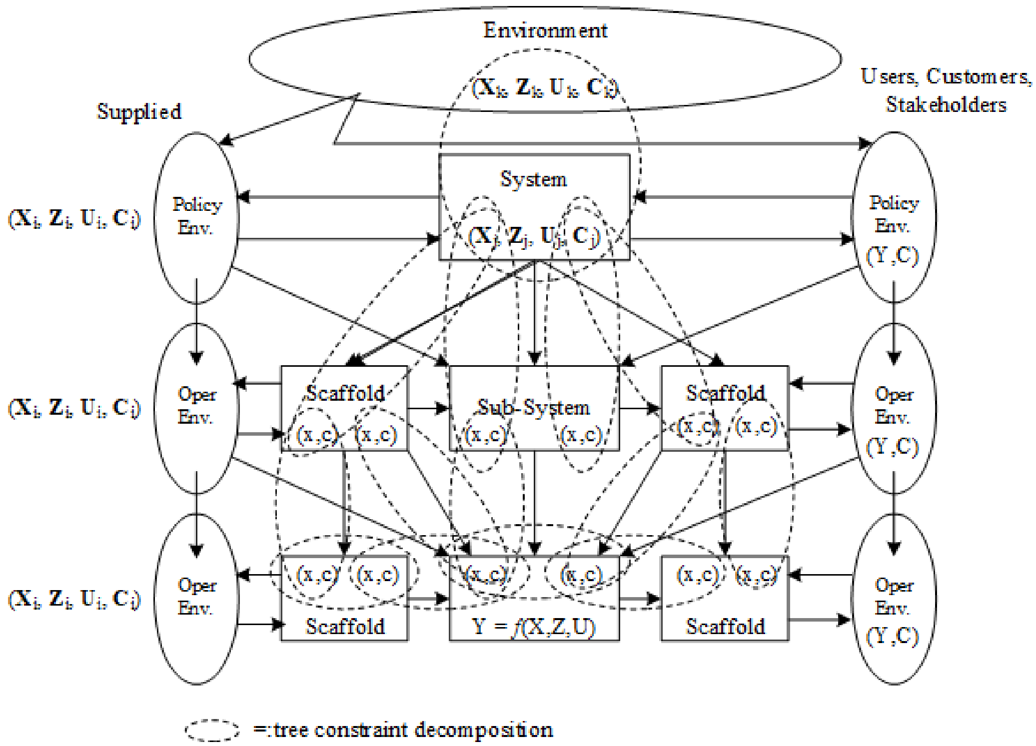
Most Restrictive Input Constraint Theorem: The $\text{LNTL}[(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \mid \max(c_i, c_j, c_k)] \leq \mathbf{Y}_i \leq \text{UNTL}[(f(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) \mid \max(c_i, c_j, c_k)]$ that minimizes $E(S \mid C, Y)$ and $E(S \mid \mathbf{C}, \mathbf{Y})$ is the most restrictive (c_i, c_j, c_k) constraint set.

Proof: Part 1. Let $(c_i, c_j, c_k) > (c_i, c_j, c_k)' > \ldots > (c_i, c_j, c_k)'' \in$ CC be all possible environmental, system, and input constraint sets. Then, $\max((c_i, c_j, c_k) > (c_i, c_j, c_k)' > \ldots > (c_i, c_j, c_k)'') = (c_i, c_j, c_k)$. Counter argument: Let $(c_i, c_j, c_k) < (c_i, c_j, c_k)' > \ldots > (c_i, c_j, c_k)'' \in$ CC be all possible environmental, system, and input constraint sets. Then, $\max((c_i, c_j, c_k) > (c_i, c_j, c_k)' > \ldots > (c_i, c_j, c_k)'') = (c_i, c_j, c_k)'$. Thus, $\max((c_i, c_j, c_k) \cup (c_i, c_j, c_k)' \cup \ldots \cup (c_i, c_j, c_k)'') \in$ CC induces the maximum variation into an attribute or characteristic, module, and the system.

Part 2. The operation $\max(c_i, c_j, c_k)$ induces variance into The $\text{LNTL}[(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon) \mid \max(c_i, c_j, c_k)] \leq \mathbf{Y}_i \leq \text{UNTL}[(\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon) \mid \max(c_i, c_j, c_k)]$ either as $\mu(y_i)$ - Target(y_i) bias, and likewise into $\mu(\mathbf{Y}_i)$ - Target($\mathbf{Y}_i$) and $\mu(\mathbf{Y})$ - Target($\mathbf{Y}$) bias; as an increase in variance in $\text{LNTL}[] \leq \mathbf{Y}_i \leq \text{UNTL}[]$ width relative to $\text{PERF}(f[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y}_i \leq \text{PERF}(f[(\mathbf{UB}_X, \mathbf{UB}_Z)])$, and likewise $\text{LNTL}[] \leq \mathbf{Y} \leq \text{UNTL}[]$ width relative to $\text{PERF}(\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]) \leq \mathbf{Y} \leq$

PERF($\mathbf{f}[(\mathbf{UB}_X, \mathbf{UB}_Z)]$); or as both. Either bias misalignment or variance increase must increase ppm_{ci} and decrease E(S | *c*, *y*), E(S | *C*, *Y*) , and E(S | **C**, **Y**) for all proper IID statistical error distributions.

Exhibit 3. Generic process-of-interest/scaffolding decomposition.



Conclusions

This research has advanced development of the Systems Statistical Engineering body of knowledge by the:

1. Integration of control theory into general systems theory and complex systems governance theory.
2. Partitioning of systemic variation from viability and providing a measure for system viability.
3. Integration of hierarchical constraint propagation into the SSE hierarchical causal Bayesian modeling framework.

This integration was the first step toward systemic modeling of throughput BO($\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]$) < PERF($\mathbf{F}[(\mathbf{LB}_X, \mathbf{LB}_Z)]$) < LNTL($\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} - \varepsilon$) $\leq \mathbf{Y} \leq$ UNTL($\mathbf{F}(pa_i, u_{xi})\boldsymbol{\beta} + \mathbf{F}(pa_j, u_{zj})\boldsymbol{\gamma} + \varepsilon$) < PERF($\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]$) < BO($\mathbf{F}[(\mathbf{UB}_X, \mathbf{UB}_Z)]$) constraint boundaries to maximize E(S | *C*, *Y*) component and E(S | C, Y) product constraint satisfaction and ultimately system viability robustness.

Continuing Research

Continuing research into the development of a Systems Statistical Engineering body of knowledge is directed toward:

- Static versus dynamic constraints.
- Deterministic fixed versus probabilistic/fuzzy versus possibilistic constraints.
- Technical-economic versus human preference constraints.
- Systems boundary interface decomposition and synthesis.
- Systems performance functional activation decomposition and synthesis.
- Systems with nonrecursive directed acyclic graph feedback loops.
- Model synthesis and verification.

References

Ashby, W. (1956) *An Introduction to Cybernetics*. London: Chapman & Hall.

Barták, R. (2010) History of Constraint Programming. In Cochran, J., Cox Jr., L, Keskinocak, P., Kharoufeh, P., and Smith, J. (Eds.), *Wiley Encyclopedia of Operations Research and Management Science, 1*, New York: Wiley.

Bertalanffy, L. (1968) *General System Theory: Foundations*, *Development*, *Applications*. New York: George Braziller.

Beer, S. (1959) *Cybernetics and Management*. New York: Wiley.

Beer, S. (1979) *The Heart of Enterprise*. London: Wiley.

Beer, S. (1981) *Brain of the Firm*. London: Wiley

Beer, S. (1985) *Diagnosiing the System for Organizations*. London: Wiley.

Boulding, K. (1956) General Systems Theory - The Skeleton of Science. *Management Science*, 2(3), 197-208.

Box, G. and Luceno, A. (2009) *Statistical Control by Monitoring and Feedback Adjustment*. New York: Wiley.

Dechter, R. and Pearl, J. (1989) Tree Clustering for Constraint Networks. *Artificial Intelligence*, 38(3), 353-366.

Dechter, R. (2006) Tractable Structures for Constraint Statisfaction Problems. In Rossi, F., van Beek P, and Walsh, T. (Eds.), *Handbook of Constraint Programming*, Oxford: Elsevier, 133-168.

Cotter, T. (2015) Statistical Engineering: A Causal-Stochastic Modeling Research Update. *Proceedings of the ASEM 2015 International Conference*. Indianapolis, Indiana.

Cotter, T. (2016) A Hierarchical Statistical Engineering Modeling Methodology. *Proceedings of the ASEM 2016 International Conference*. Charlotte, North Carolina.

Cotter, T. (2017) Integrating IDEF0 into a Systems Framework for Statistical Engineering *Proceedings of the ASEM 2017 International Conference*. Huntsville, Alabama.

Espejo, R. and Harnden, R. (1989) *The Viable System Model: Interpretations and Applications of Stafford Beer's VSM*. New York: Wiley.

Keating, C. (2014) Governance Implications for Meeting Challenges in the System of Systems Engineering Field. *Proceedings of the 9th International Conference on Systems of Systems Engineering*. Adelaide, Australia, June 9-13, 2014, pp. 154-159.

Keating, C. and Katina, P. (2015) Editorial: Foundational Perspectives for the Emerging Complex System Governance Field. *International Journal of System of Systems Engineering* 6(1/2), 1-14.

Keating, C. and Katina, P. (2016) Complex System Governance Development: First Generation Methodology. *International Journal of System of Systems Engineering* 7(1/2/3), 43-74.

Rapoport, A. (1986) *General System Theory: Essential Concepts and Applications*. Boca Raton: CRC Press.

Ryu, Y. (1998) Dynamic Construction of Product Taxonomy Hierarchies for Assisted Shopping in the Electronic Marketplace. *Proceedings of the Thirty-First Hawaii International Conference on System Sciences*, January 6-9, 1998 5, Kohala Coast, Hawaii, pp. 196-204.

Ryu, Y. (1999) A Hierarchical Constraint Satisfaction Approach to Product Selection for Electronic Shopping Support. *IEEE Transactions on Systems, Man, and Cybernetics – Part A* 29(6), pp. 525-532.

Shannon, C. E. (1948) A Mathematical Theory of Communication. *Bell System Technical Journal*, 27.

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