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Finite Volume Matrix Elements of Two-Body States With One Current Insertion

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Finite volume matrix elements of two-body states with one current insertion

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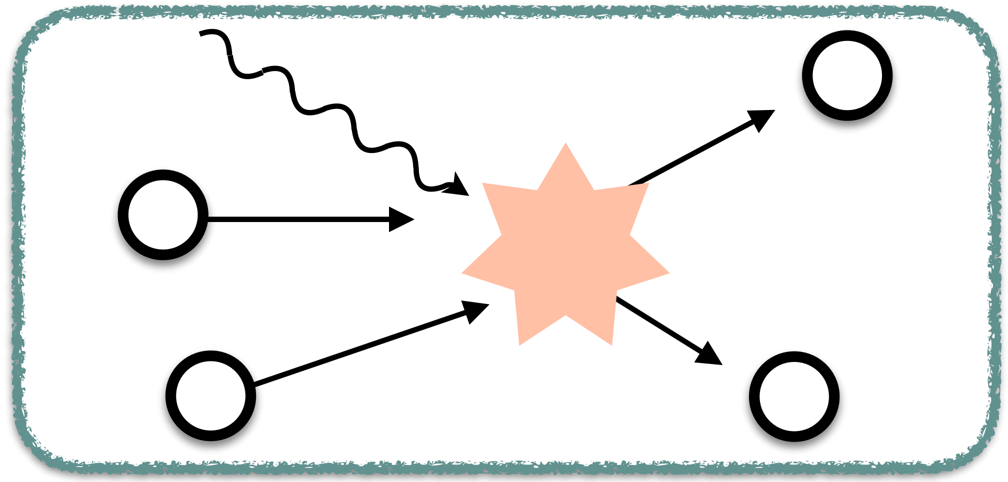
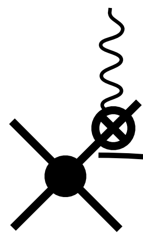
William and Mary

Santa Fe 2019

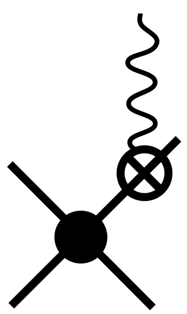
Two-to-two scattering **with current**

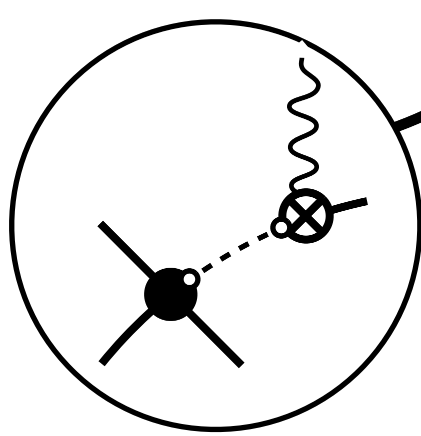


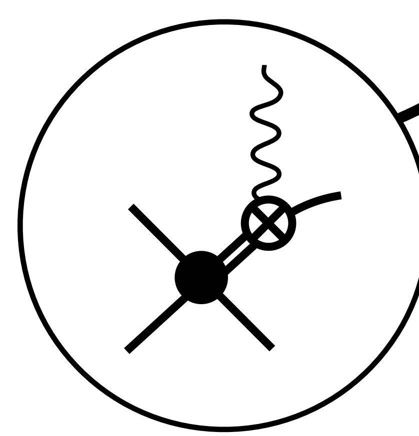
Kinematic divergences

$$i\mathcal{W}_{\mu_1\cdots\mu_n} =$$  $$+ \cdots$$

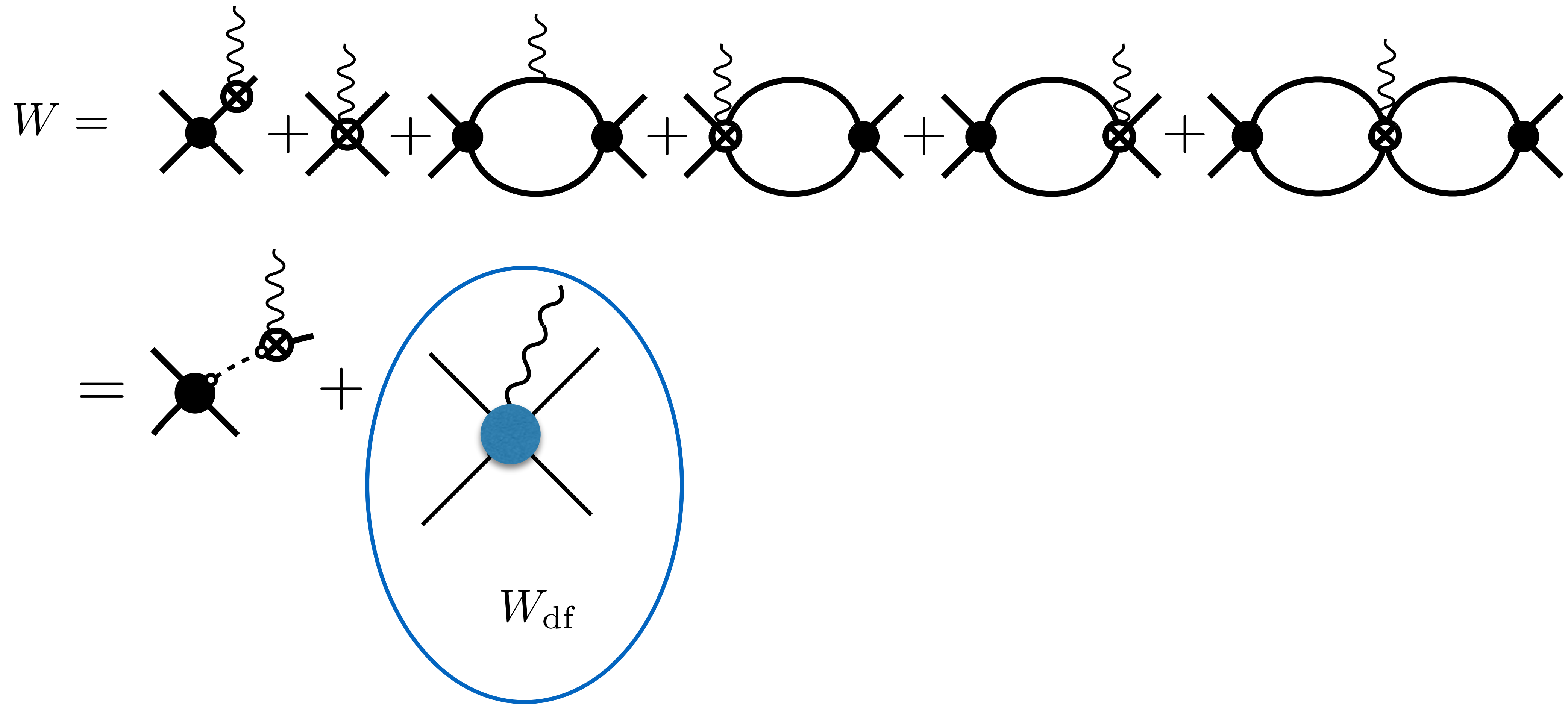
Fully dressed propagator $\quad i\Delta = \dfrac{i}{k^2 - m^2 + i\epsilon} + \text{smooth}$

 $$= iM \, \frac{i}{k^2 - m^2 + i\epsilon} \, i\omega_{\mu_1\cdots\mu_n} + \text{“smooth”}$$





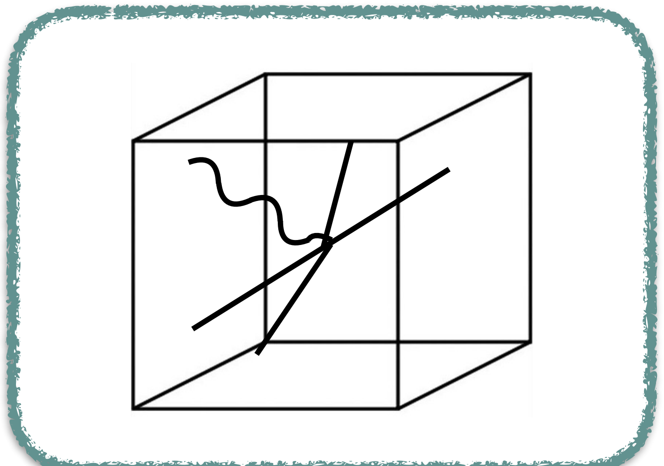
Two-to-two scattering **with current**

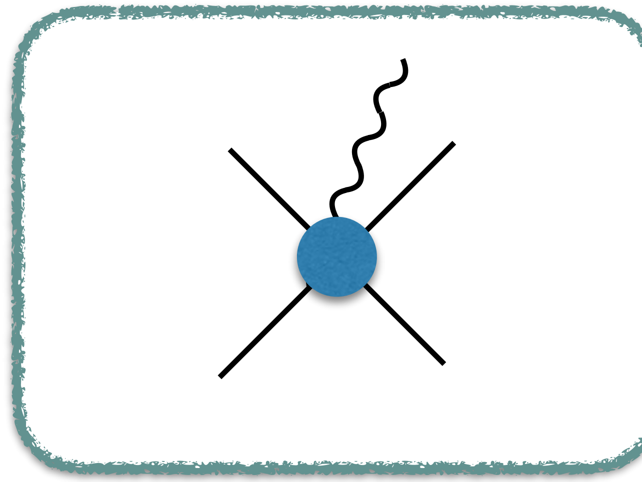


- Will be related to the FV matrix element is not the full scattering amplitude
- We can recover the full scattering amplitude adding back diagrams with kinematic singularities

$2 + \mathcal{J} \to 2$

FV matrix elements to infinite volume electroweak amplitudes





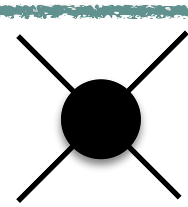
$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

- Briceño & Hansen (2016)
- AB, Briceño, Hansen, Ortega (2018)

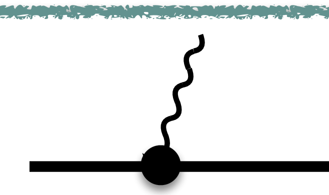
$2 + \mathcal{J} \to 2$

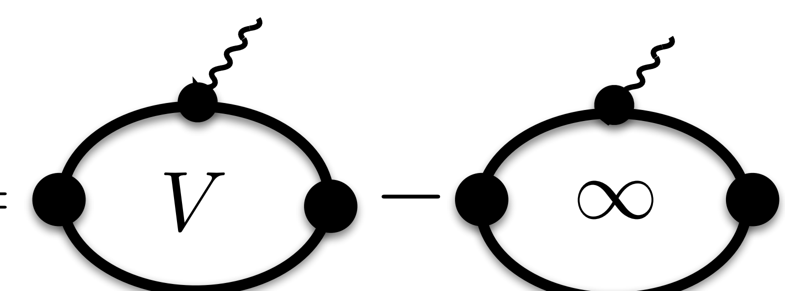
$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L, L) W_{L,\mathrm{df}} R(E_L, L) W_{L,\mathrm{df}}\right]$$

$$W_{L,\mathrm{df}} = W_{\mathrm{df}} + M G(L, w) M$$

$M =$ 

2→ 2 scatt. Amplitude
Calculated at FV energies

$w =$ 

$G(L, w) =$ 

$$= \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int d\mathbf{k}\right](\cdots)$$

Crucial ingredient that we studied

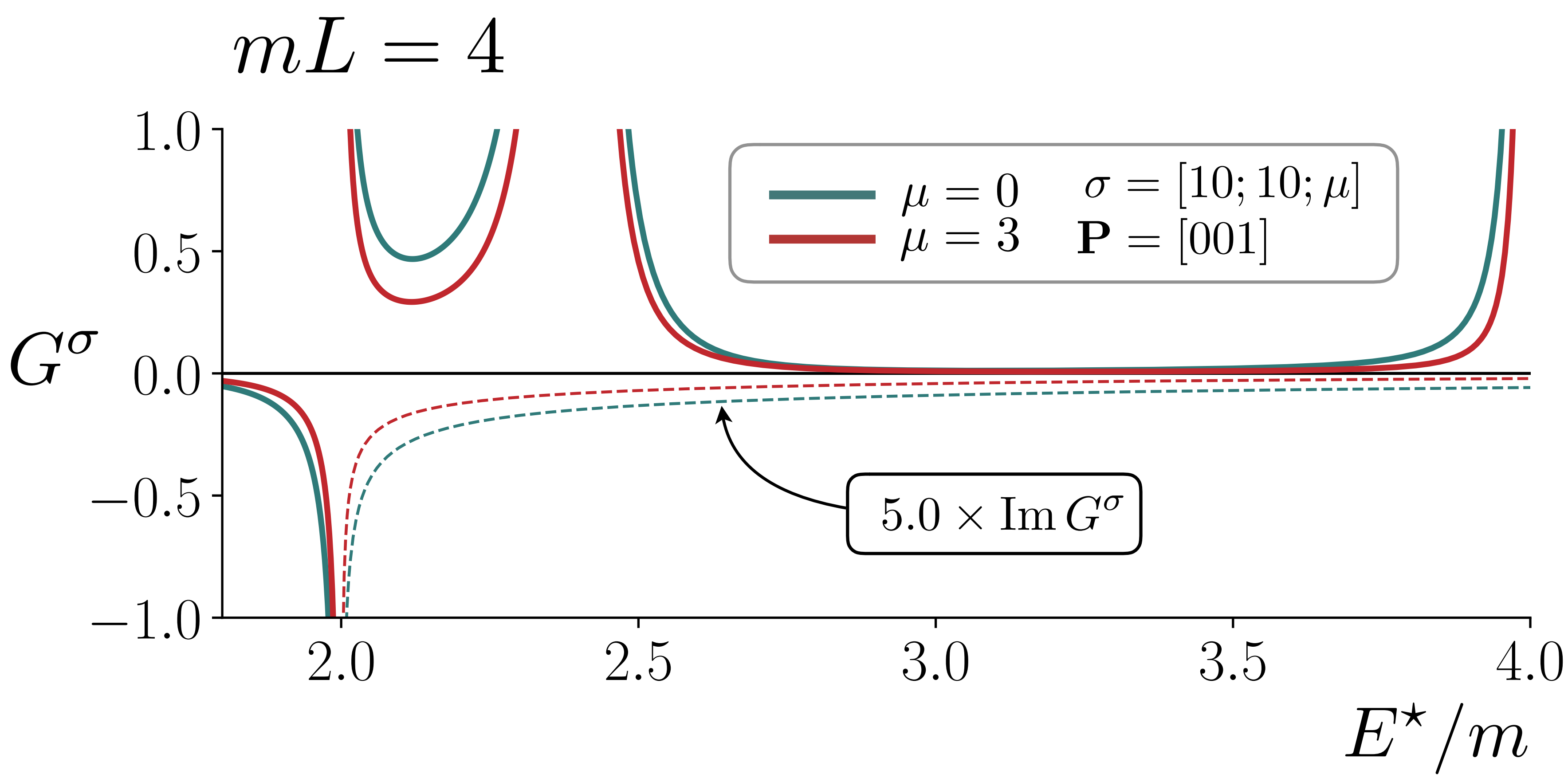
New kinematic function

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

- The sum is “straightforward”
- The integral is highly not trivial (spectator particle goes on-shell)
    - integrand singularities are two surfaces in three-dimension
    - standard principal value techniques in one dimension fail
    - techniques from other fields are not suitable
    - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D smooth integral

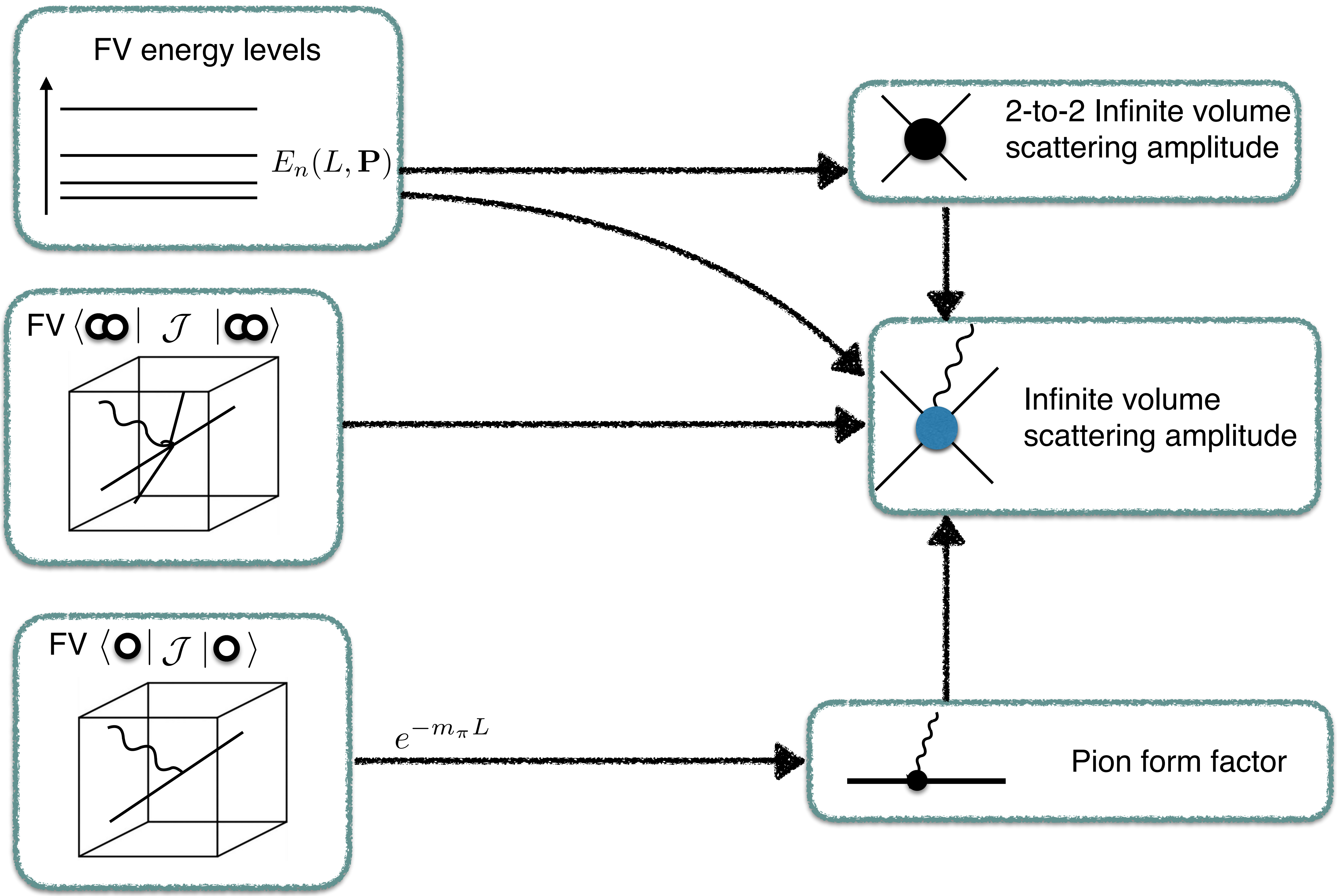
AB, R. A. Briceño, M. T. Hansen, F. Ortega (2018)

For equal initial and final momenta it has been shown that the above problem has an analytical solution



AB, R. A. Briceño, M. T. Hansen, F. Ortega (2018)

Summary


FV energy levels
$E_n(L, \mathbf{P})$
2-to-2 Infinite volume scattering amplitude
FV $\langle$ $|$ $\mathcal{J}$ $|$ $\rangle$
Infinite volume scattering amplitude
FV $\langle$ $|$ $\mathcal{J}$ $|$ $\rangle$
$e^{-m_\pi L}$
Pion form factor

- ✔ General, relativistic, EFT independent, finite volume formalism to treat processes relevant for many electroweak processes (nuclear and not only) derived

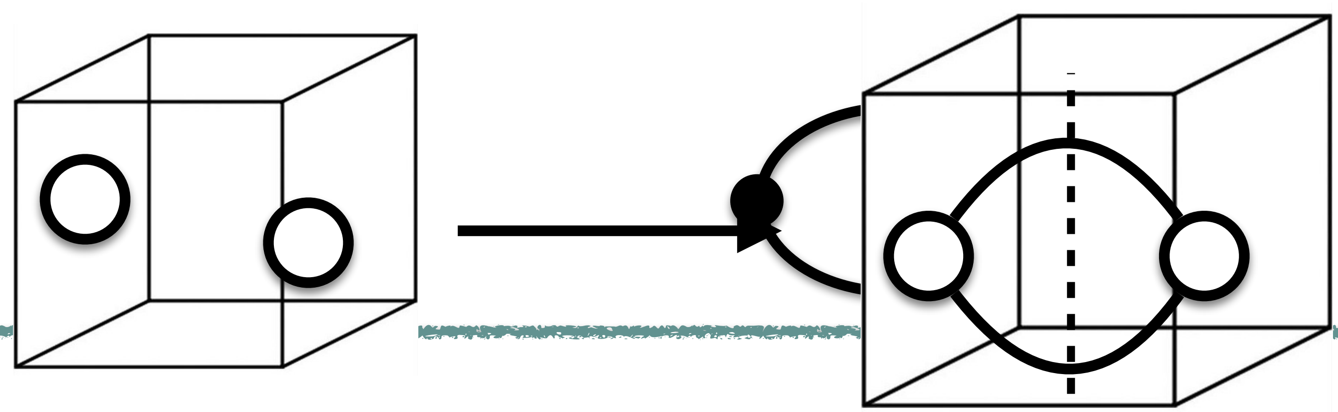
- ✔ A crucial ingredient the new kinematic function is under control

- ○ Think to apply all of this in a simple case

Thank you!

Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
  - On-shell intermediate states give singularities



Asymptotic states not in a FV

- Formalis
  - Brice
  - Rusetsky et al. (2012) - EFT dependent, NR
  - Briceño and Davoudi (2012) - EFT dependent, NR

    Many works.. not cited here
  - Luscher formalism (1986, 1991)

Finite volume correlator

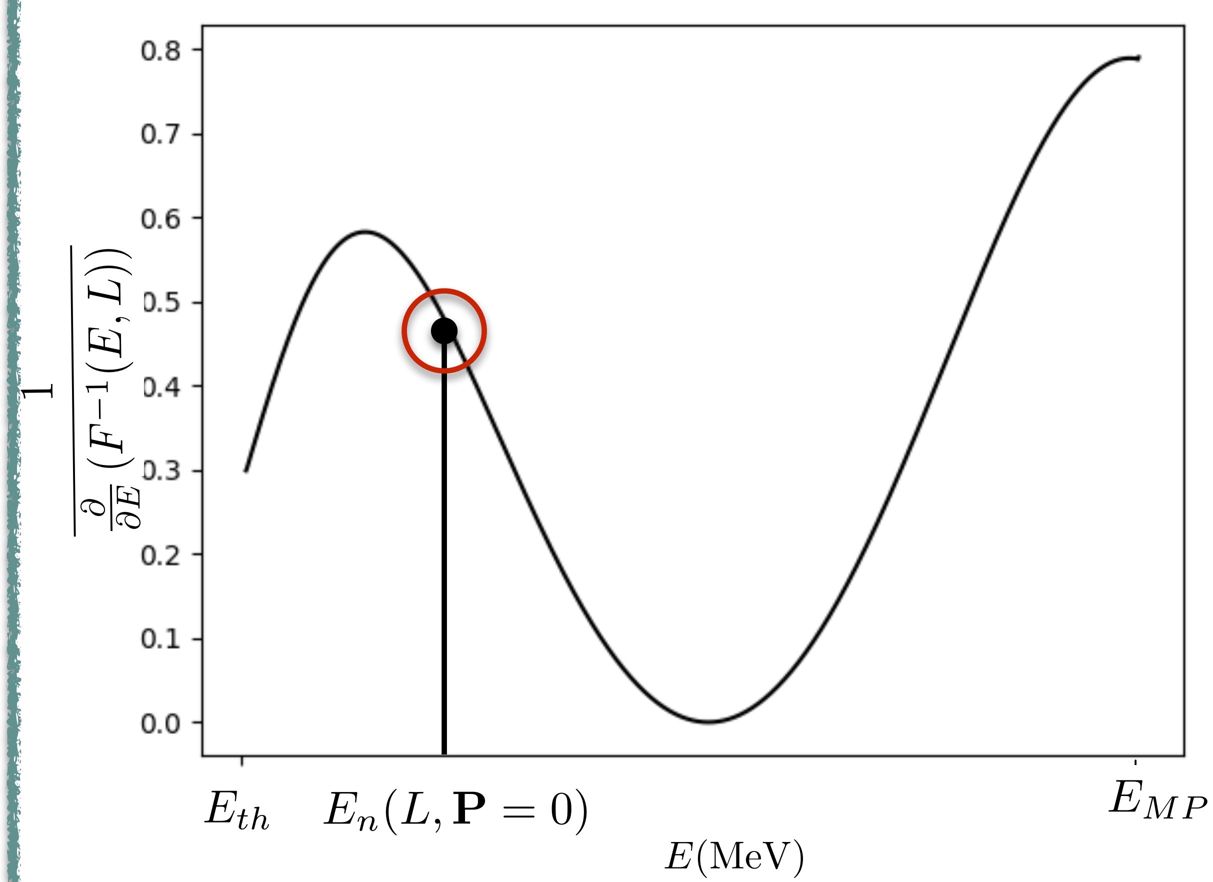
Rearranging

Kinematic functions I

$$R(E_L, L) = \left. \frac{1}{\frac{\partial}{\partial E}\left(F^{-1}(E,L)\right) + M(E))} \right|_{E=E_L}$$

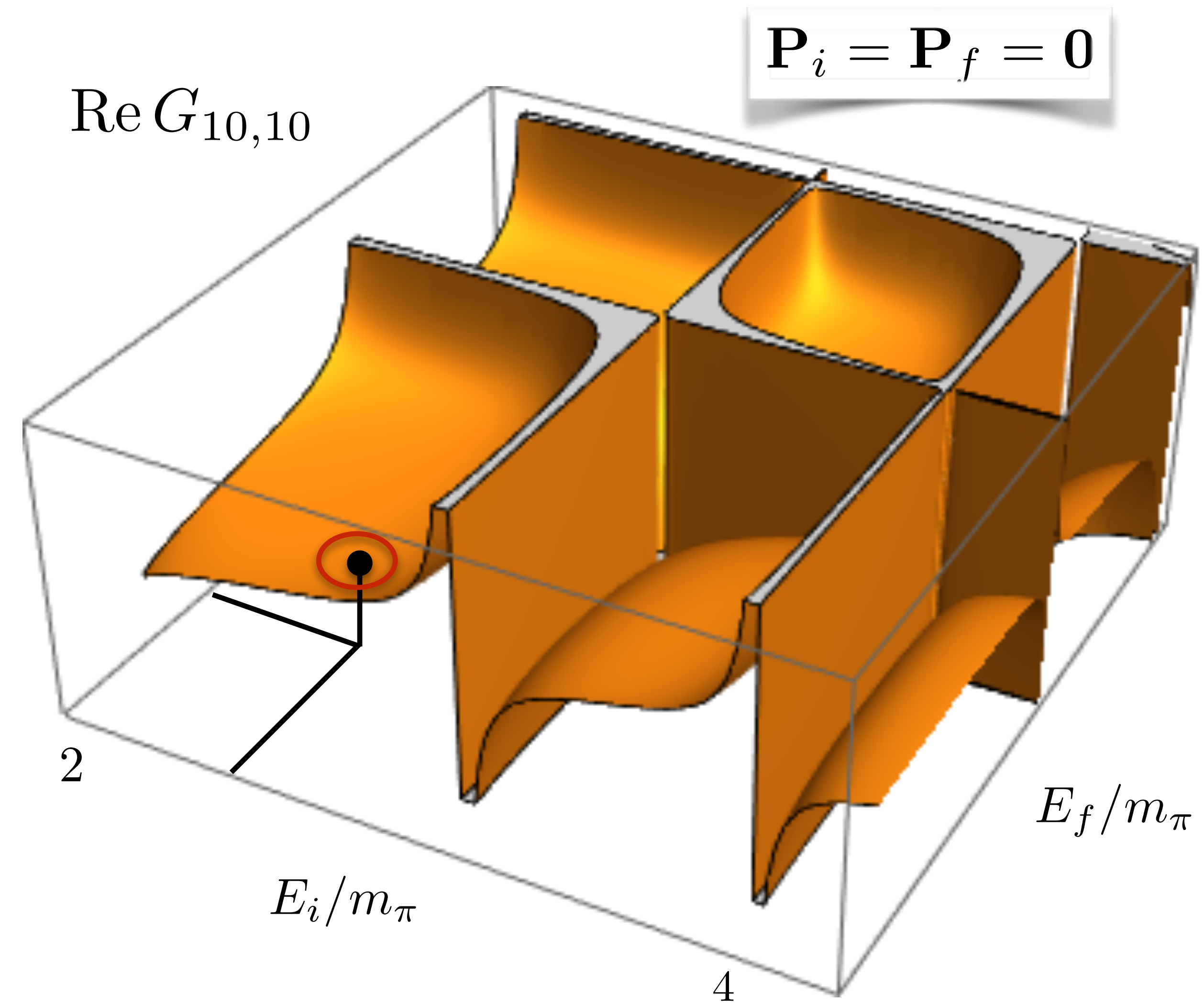
$G_{l_i m_i, l_f m_f}$

$m_\pi L = 4$



E_{th} = threshold energy

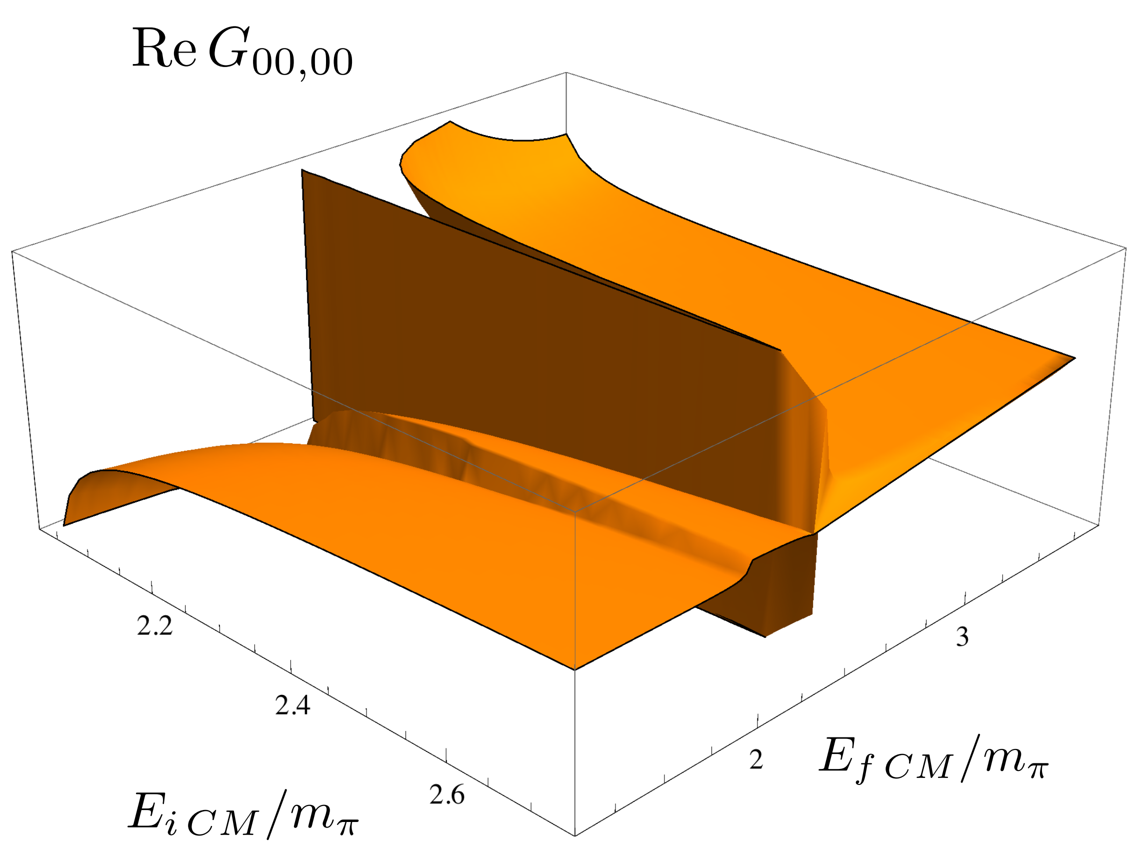
E_{MP} = multiparticle states energy

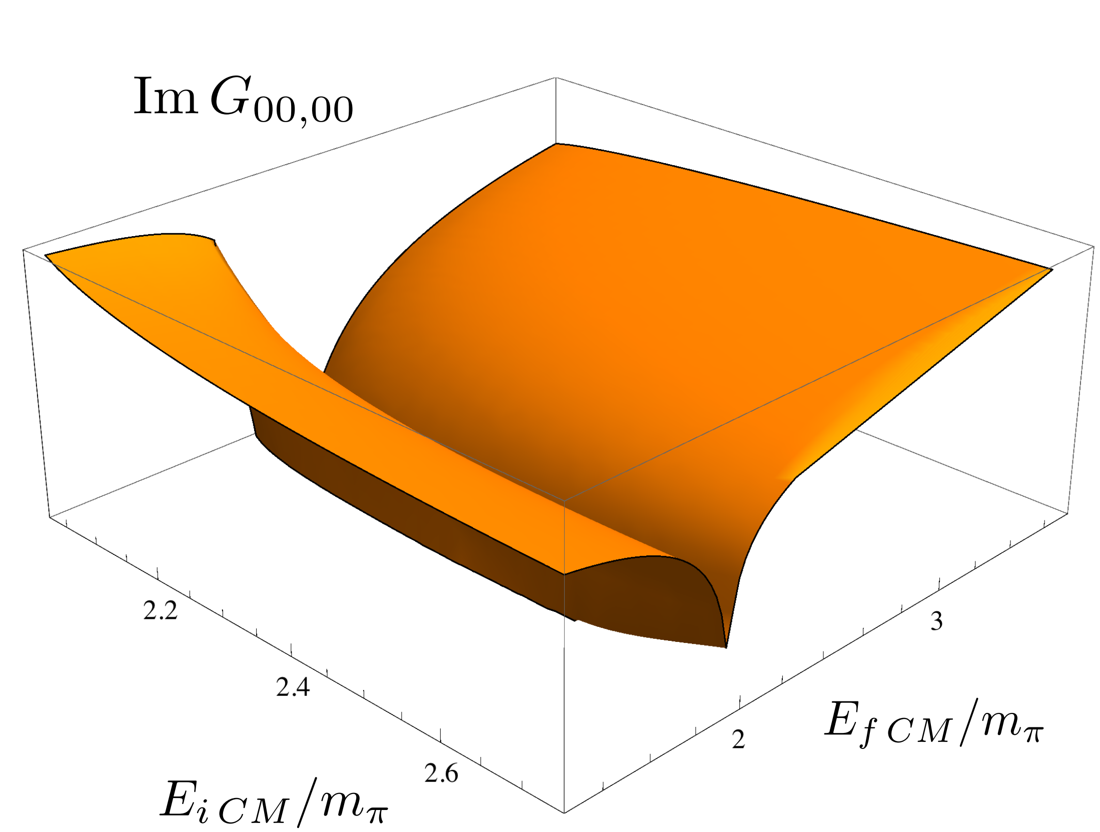


Singularities at free particle energies

Kinematic functions II

$$\mathbf{P}_i = \mathbf{0}\,, \quad \mathbf{P}_f = [0,0,2\pi/L]$$
$$m_\pi\, L = 4$$



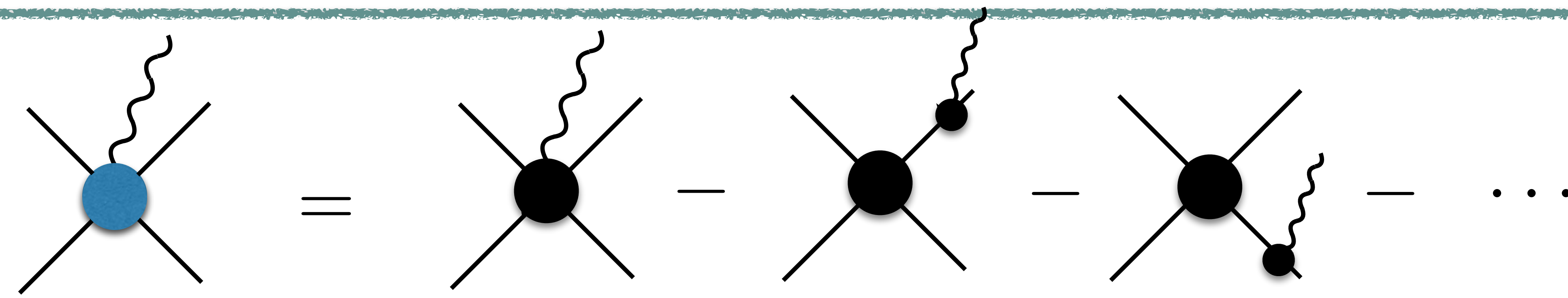


More plots in

AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
In preparation

A detail

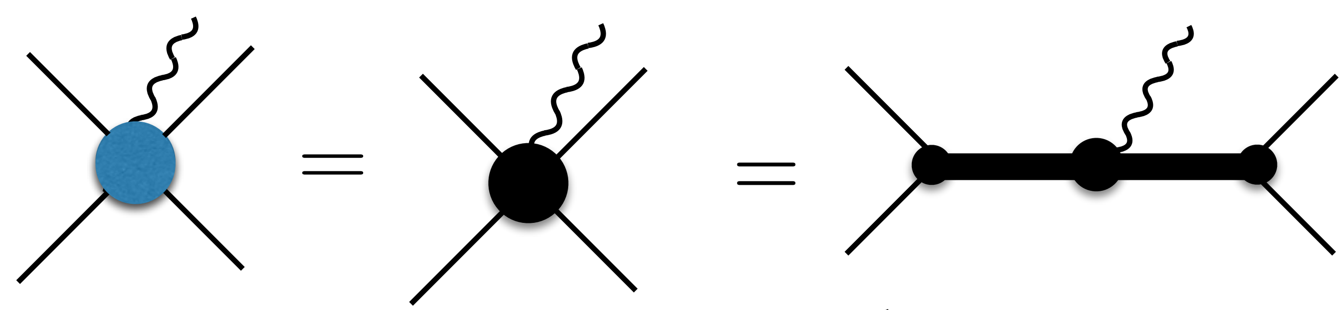
$W_{\mathrm{df}} =$



Kinematic poles cancel

But dynamical poles associated with resonances can still appear

$\lim_{E_i^{\mathrm{cm}}, E_f^{\mathrm{cm}} \to E_R}$



In the resonance region

Explain that. In order to do this bridge we need to know not only LECs from QCD but also form factors
Not only single nucleon for factors but also understand multi hadron…

Explain that. In order to do this bridge we need to know not only LECs from QCD but also form factor
Not only single nucleon for factors but also understand multi hadron…

Goals

Add pictures and make it pretty

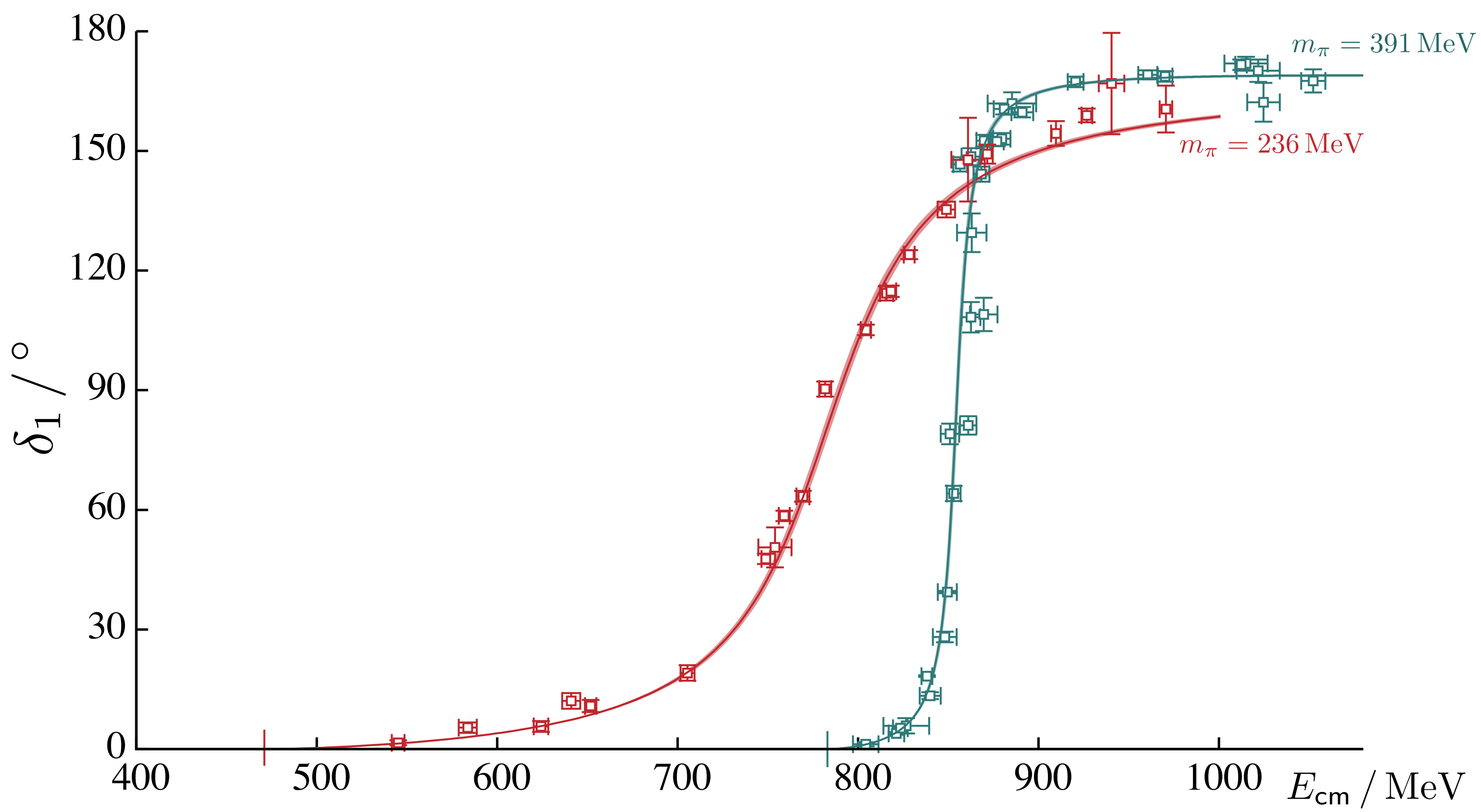
- Developing a framework for studying :
  - structure of composite states (structure -> Form factors)
    - structure of resonances
    - structure of the deuteron
  - Weak processes involving few-hadron systems
    - parity violation
    - p-p fusion, neutrino-nucleus

Add some extra explanation

Sketch of the derivation

Expand

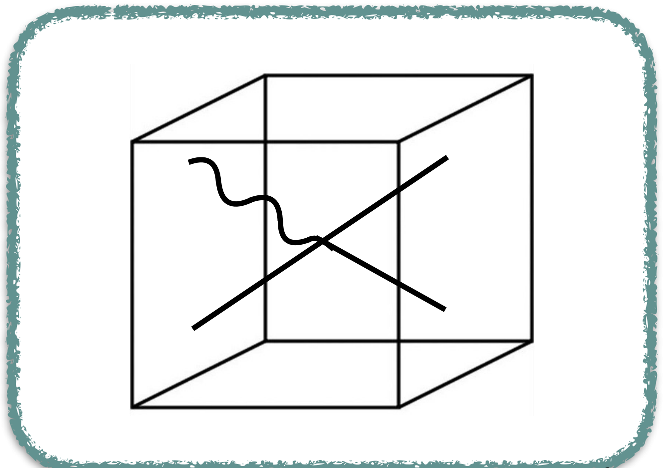
2→2

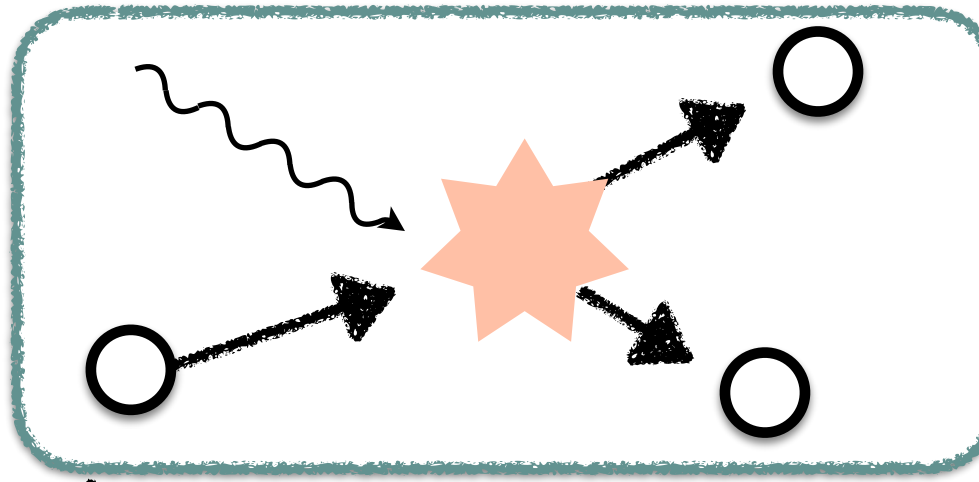


- Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$1 + \mathcal{J} \to 2$

FV matrix elements to infinite volume electroweak amplitudes

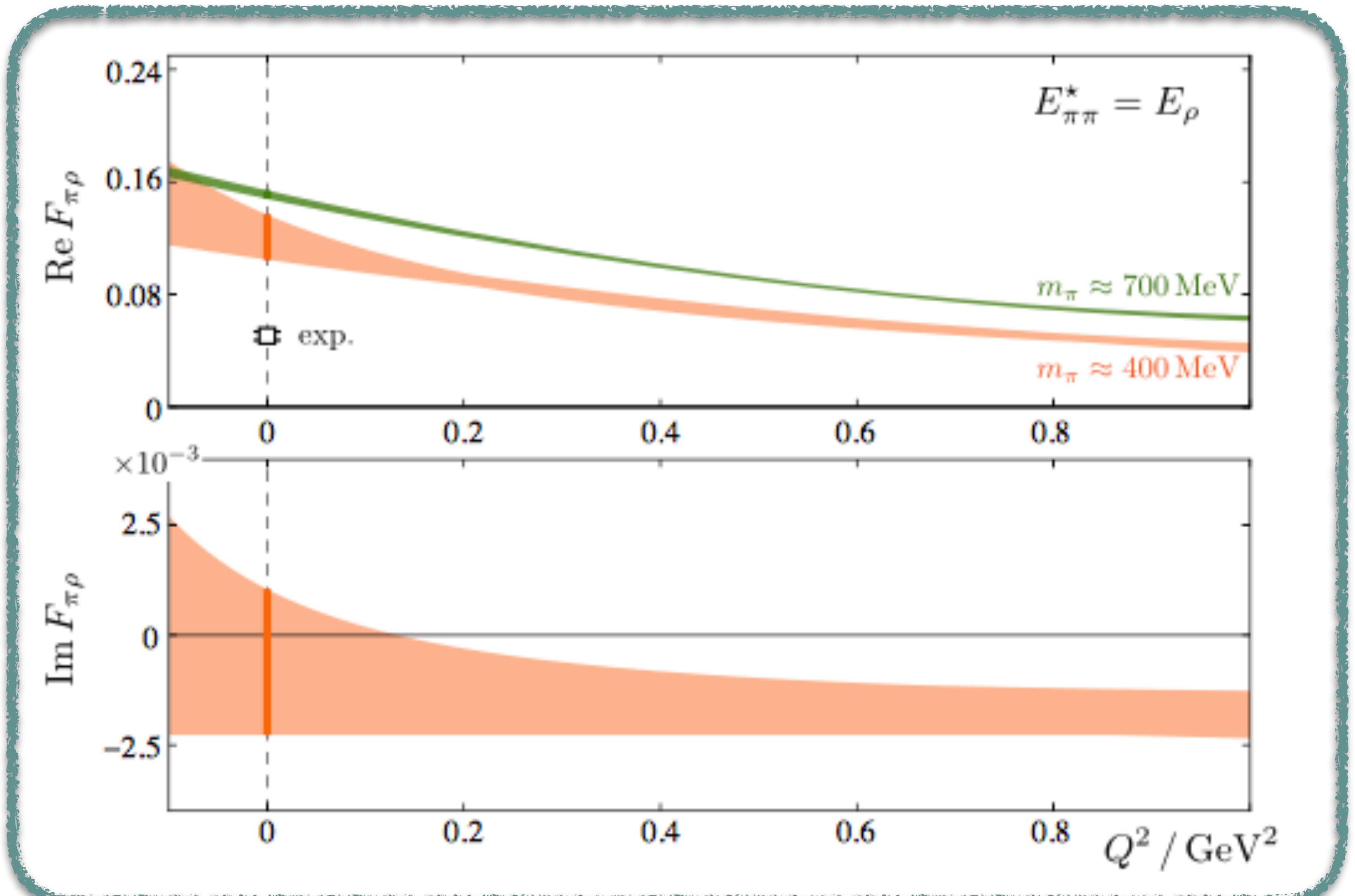




$$|\langle 2|\mathcal{J}|1\rangle|_L = \frac{1}{L^3}\sqrt{\langle 1|\mathcal{J}|2\rangle_\infty R(E_L, L)\langle 2|\mathcal{J}|1\rangle_\infty}$$

- Lellouch & Lüscher (2000) [K-to-$\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [moving K-to-$\pi\pi$]
- Hansen & Sharpe (2012) [D-to-$\pi\pi$/KK]
- Briceño, Hansen Walker-Loud /Briceño & Hansen(2014-2015)[general 1-to-2]

$1 + \mathcal{J} \rightarrow 2$



Briceño, Dudek, Edwards, Shultz, Thomas and Wilson PRL (2016)

Report some examples calculations in few body systems

BF 1 slide

Some open questions

Expand here with pics

Goals

- Developing a framework for studying :
  - structure of composite states
    - structure of resonances
    - structure of the deuteron
  - Weak processes involving few-hadron systems
    - parity violation
    - p-p fusion, neutrino-nucleus

Goals

- Developing a framework for studying :
  - structure of composite states
    - structure of resonances → Focus here
    - structure of the deuteron
  - Weak processes involving few-hadron systems
    - parity violation
    - p-p fusion, neutrino-nucleus

Why do we care about the structure of this objects?
get the charge radius etc etc, maybe mention nuclear physics stuff
Add stuff in this slide…..

Goals

- Developing a framework for studying :
  - structure of composite states
    - structure of resonances
    - structure of the deuteron
  - Weak processes involving few-hadron systems
    - parity violation, p-p fusion
    - neutrino-nucleus

Focus here

LQCD inputs for
low energy nuclear physics

Maybe expand here general goals nice but give specific examples

Goals

- Developing a framework for studying :
  - structure of composite states
    - structure of resonances
    - structure of the deuteron
  - Weak processes involving few-hadron systems
    - parity violation, p-p fusion
    - neutrino-nucleus

Focus here

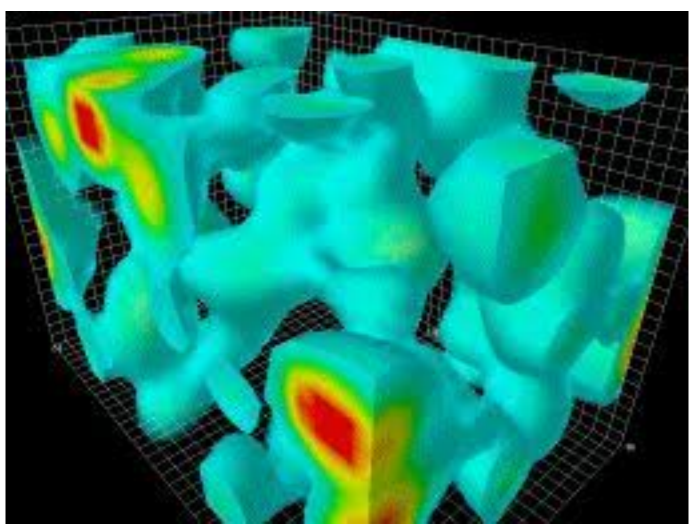
LQCD inputs for
low energy nuclear physics

LQCD inputs for neutrino-nucleus scattering

Nuclear electroweak interactions?

Atomic nuclei are a complex quantum-many body systems of strongly interacting nucleons

Hadronic matrix elements difficult because of QCD



? ?

Nuclear Decay Half-Lives for Nuclei A = 3 - . . . .

Nuclide (β^- Decay)	Half-Life	Date	Nuclide (β^+ Decay)	Half-Life	Date
^{3}H	12.323 ± 0.020 yr	September, 2015	^{7}Be	53.22 ± 0.06 days	August, 2001
^{6}He	$806.89^{+0.25}_{-0.22}$ ms	August, 2015	^{8}B	770.3 ± 0.4 ms	July, 2015
^{8}He	119.0 ± 1.6 ms	September, 2015	^{9}C	126.5 ± 1.0 ms	July, 2015
^{8}Li	838.79 ± 0.36 ms	August, 2015	^{10}C	19.3015 ± 0.0017 sec	May, 2016
^{9}Li	177.7 ± 0.6 ms	August, 2015	^{11}C	20.3401 ± 0.0070 min	March, 2018
^{10}Be	$(1.51 \pm 0.06) \times 10^6$ yr	August, 2015	^{12}N	11.000 ± 0.016 ms	August, 2015
^{11}Li	8.74 ± 0.15 ms	August, 2015	^{13}N	9.967 ± 0.005 min	July, 2015

$+ \cdots$

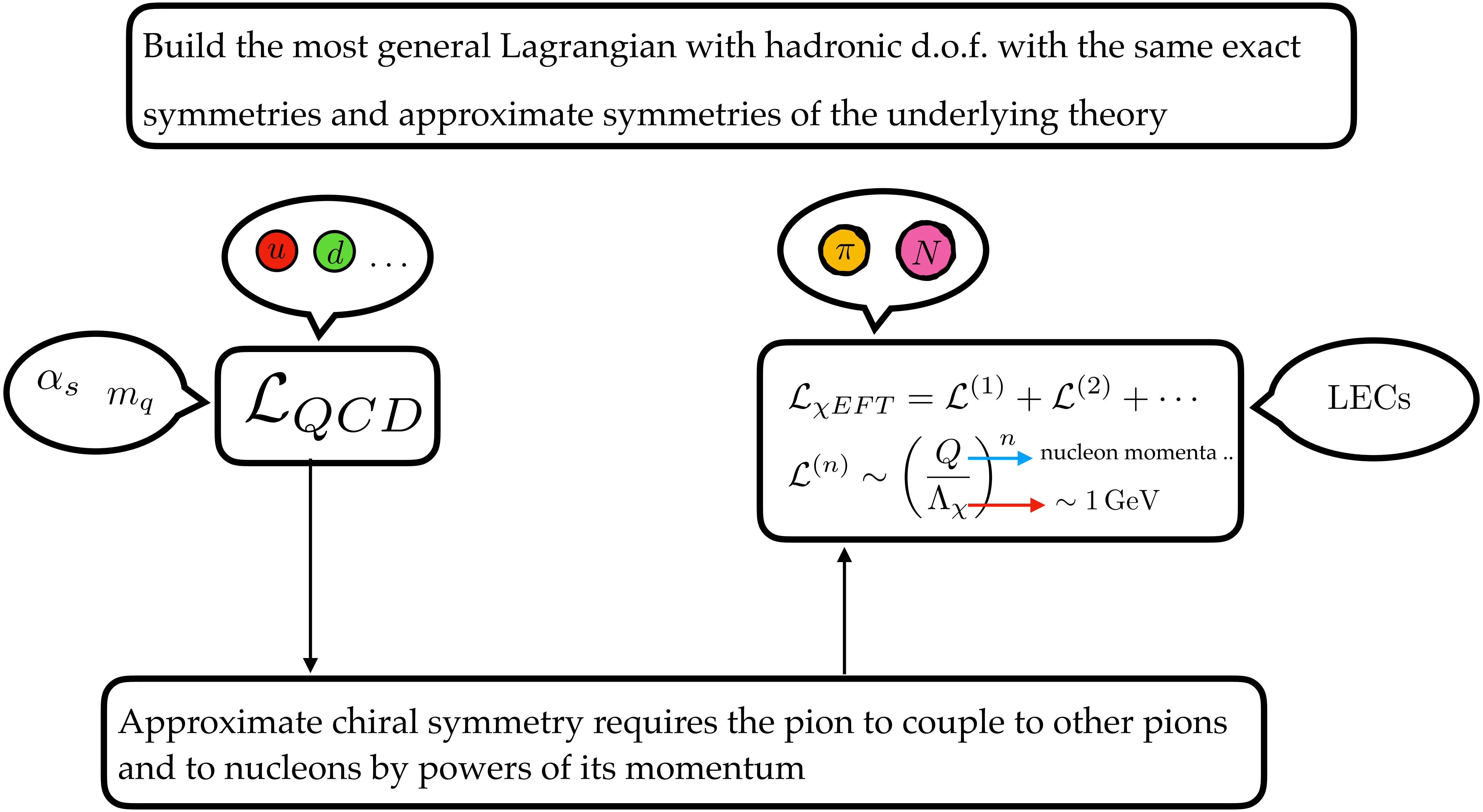
http://www.tunl.duke.edu/nucldata/HalfLife.shtml

Lattice QCD (non perturbative method)

Effective field theories (expansion in kinematic variables)

χ EFT

- General slide on workflow



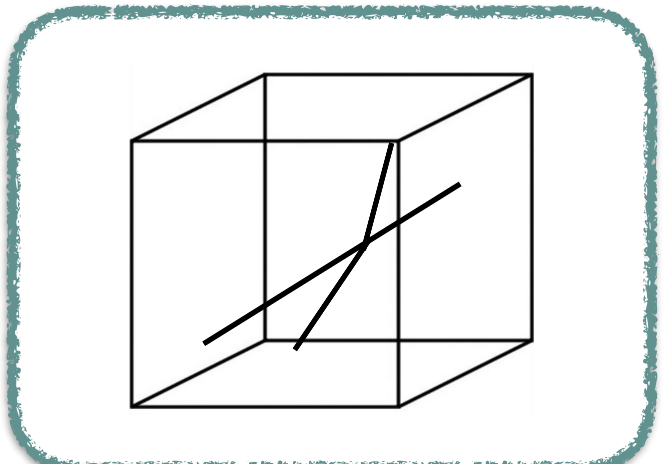
- S. Weinberg (1968-1979)

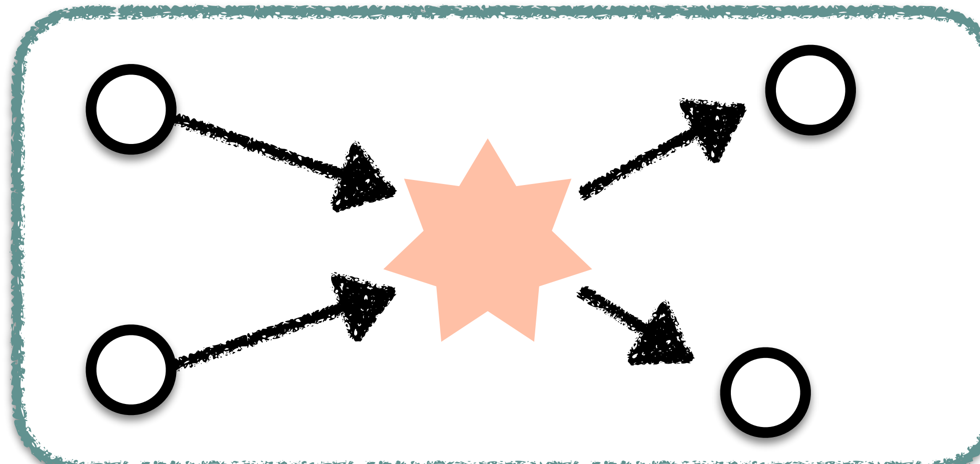
- Slide on LQCD generalities (only one is enough)

- Slide on (why we need) a FV formalism to deal with stuff

$2 \to 2$

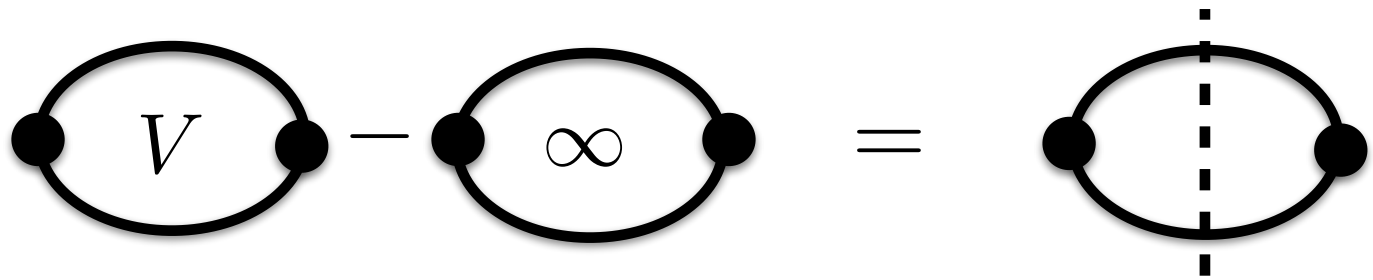
- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds



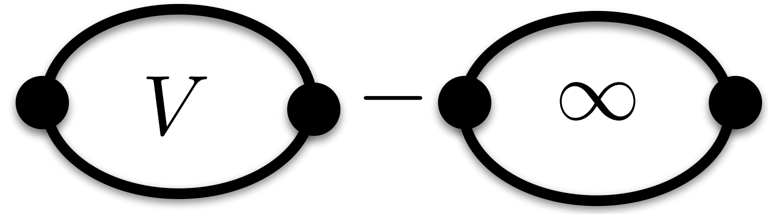


$$\det\left[F^{-1}(E_L, L) + M(E_L)\right] = 0$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]

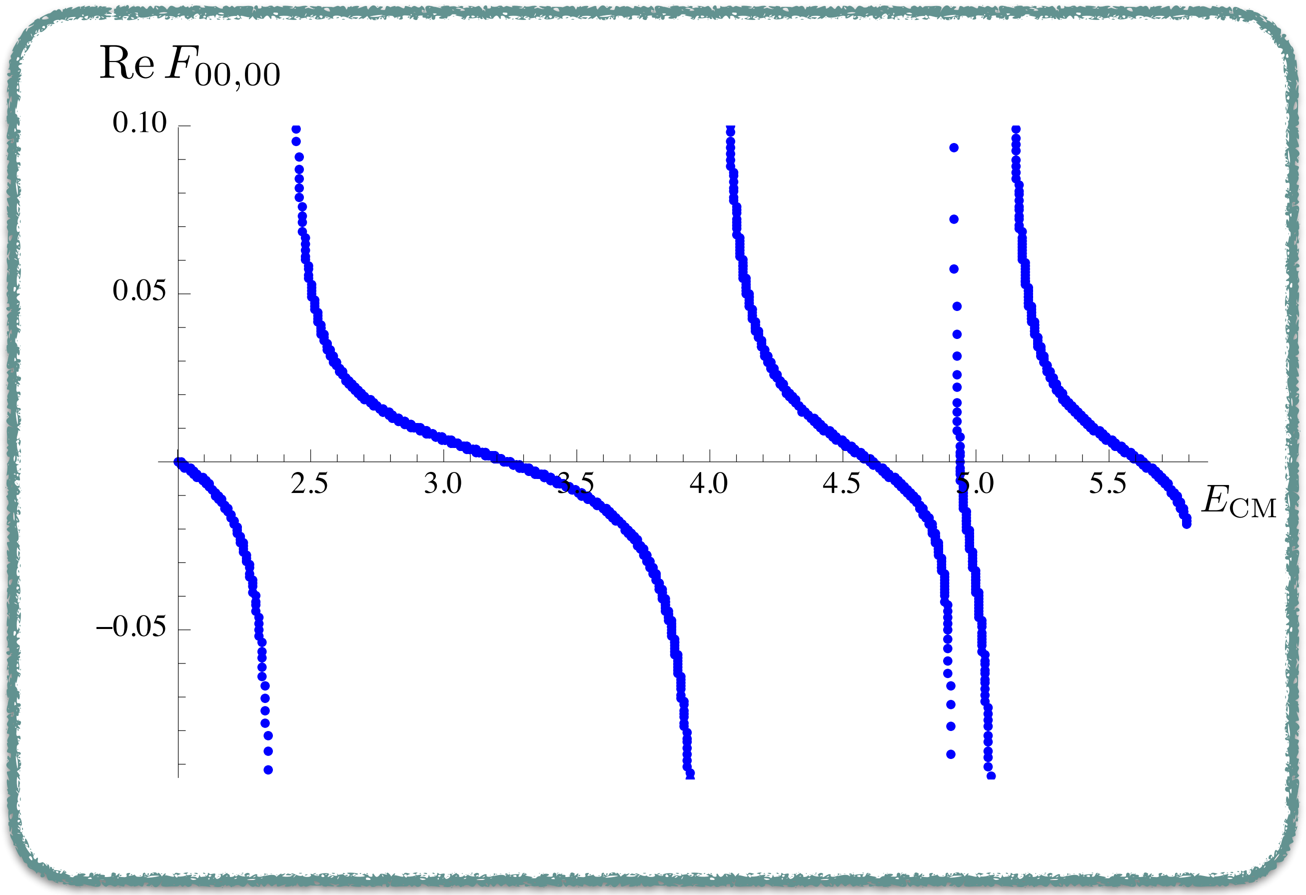

V
∞

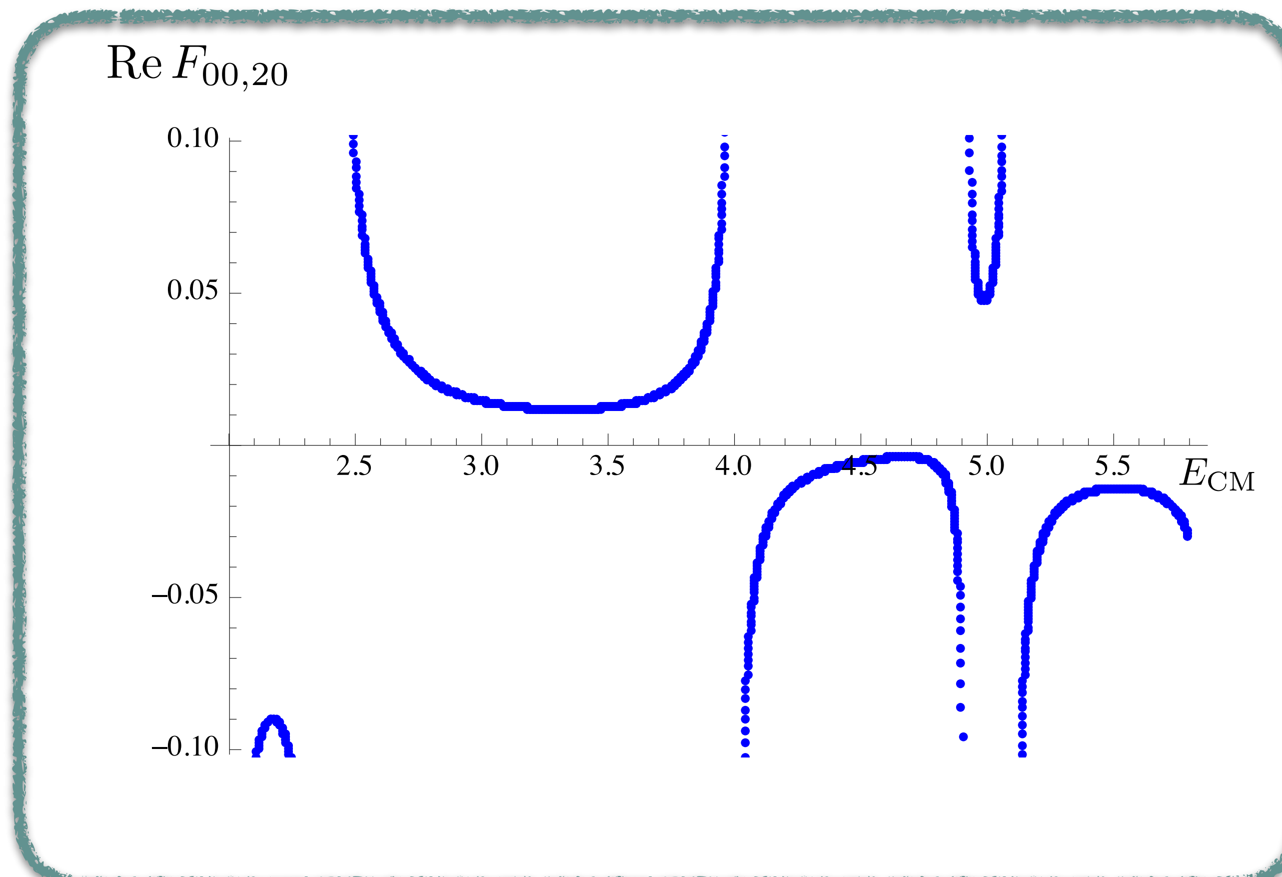
$$F_{l\,m,l'm'}(E_L, L) =$$



$$= \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int d\mathbf{k}\right](\cdots)$$

$\mathbf{P} = [0, 0, 2\pi/L]\,, \quad m_\pi L = 4$



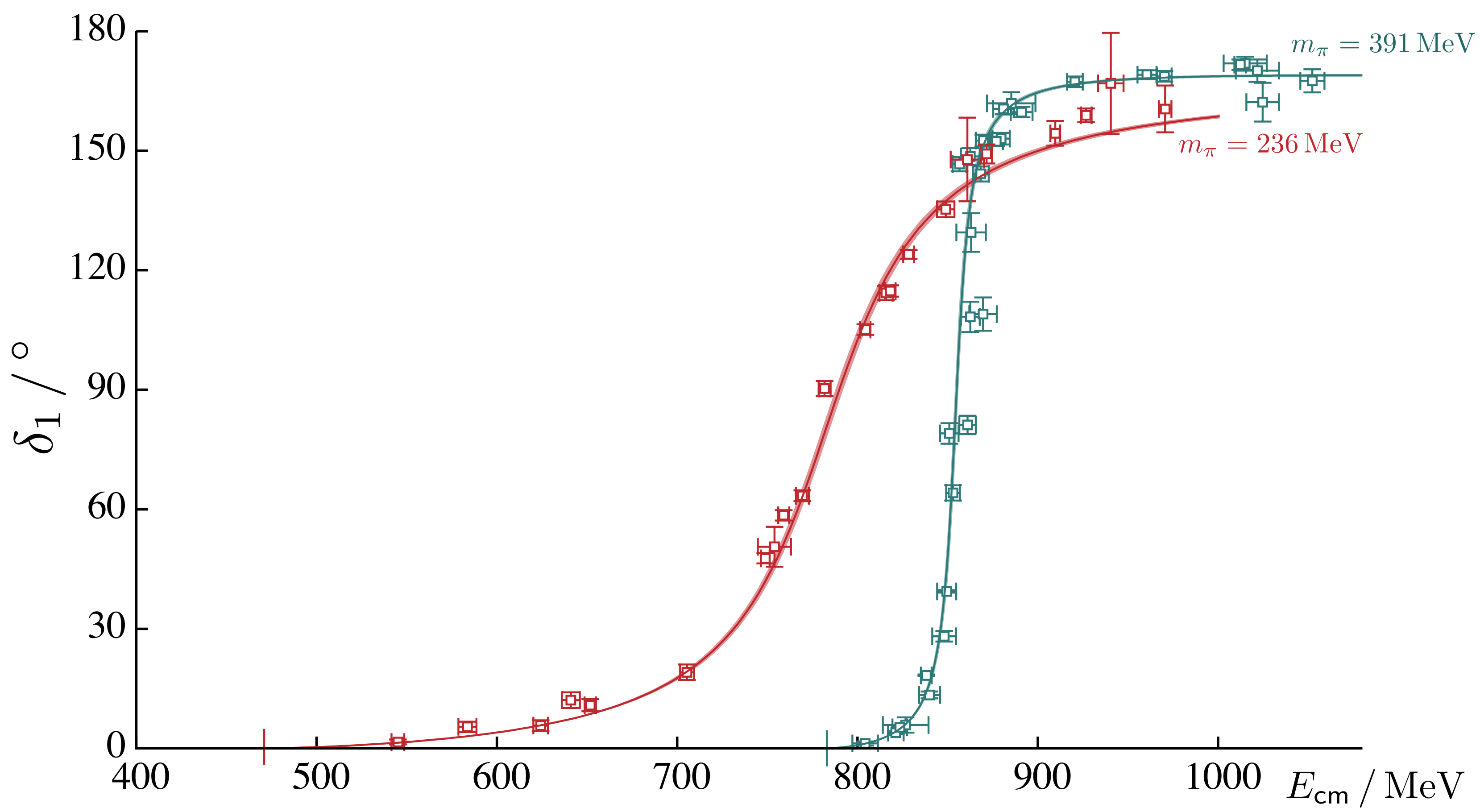


Gotta explain why the only thing we care about is this bubble (series can be resumed)

- Sketch the derivation…

- Sketch the derivation…

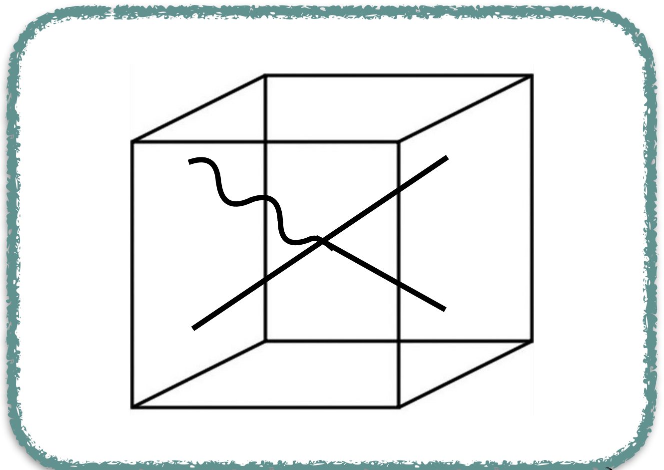
2→2

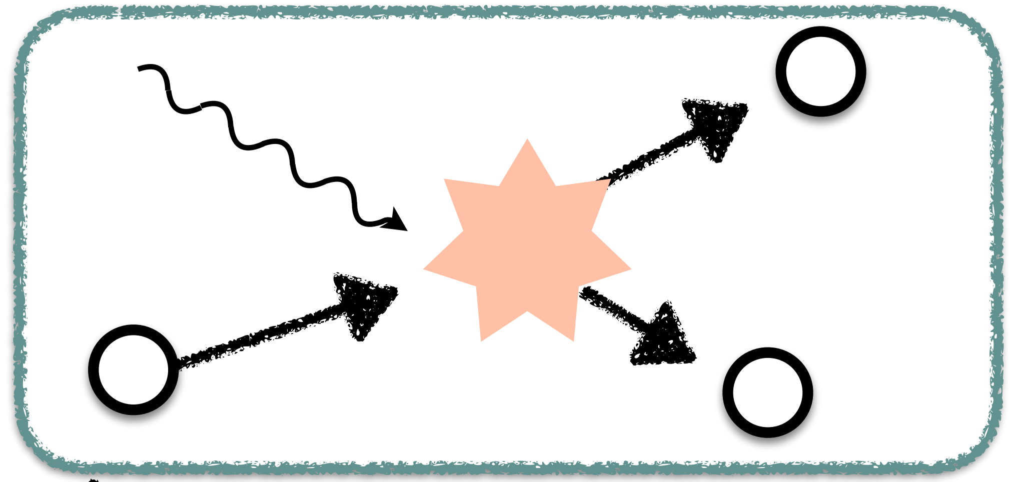


- Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$1 + \mathcal{J} \rightarrow 2$

FV matrix elements to infinite volume electroweak amplitudes





$$|\langle 2|\mathcal{J}|1\rangle|_L = \frac{1}{L^3}\sqrt{\langle 1|\mathcal{J}|2\rangle_\infty R(E_L, L)\langle 2|\mathcal{J}|1\rangle_\infty}$$

- Lellouch & Lüscher (2000) [K-to-$\pi\pi$ at rest]
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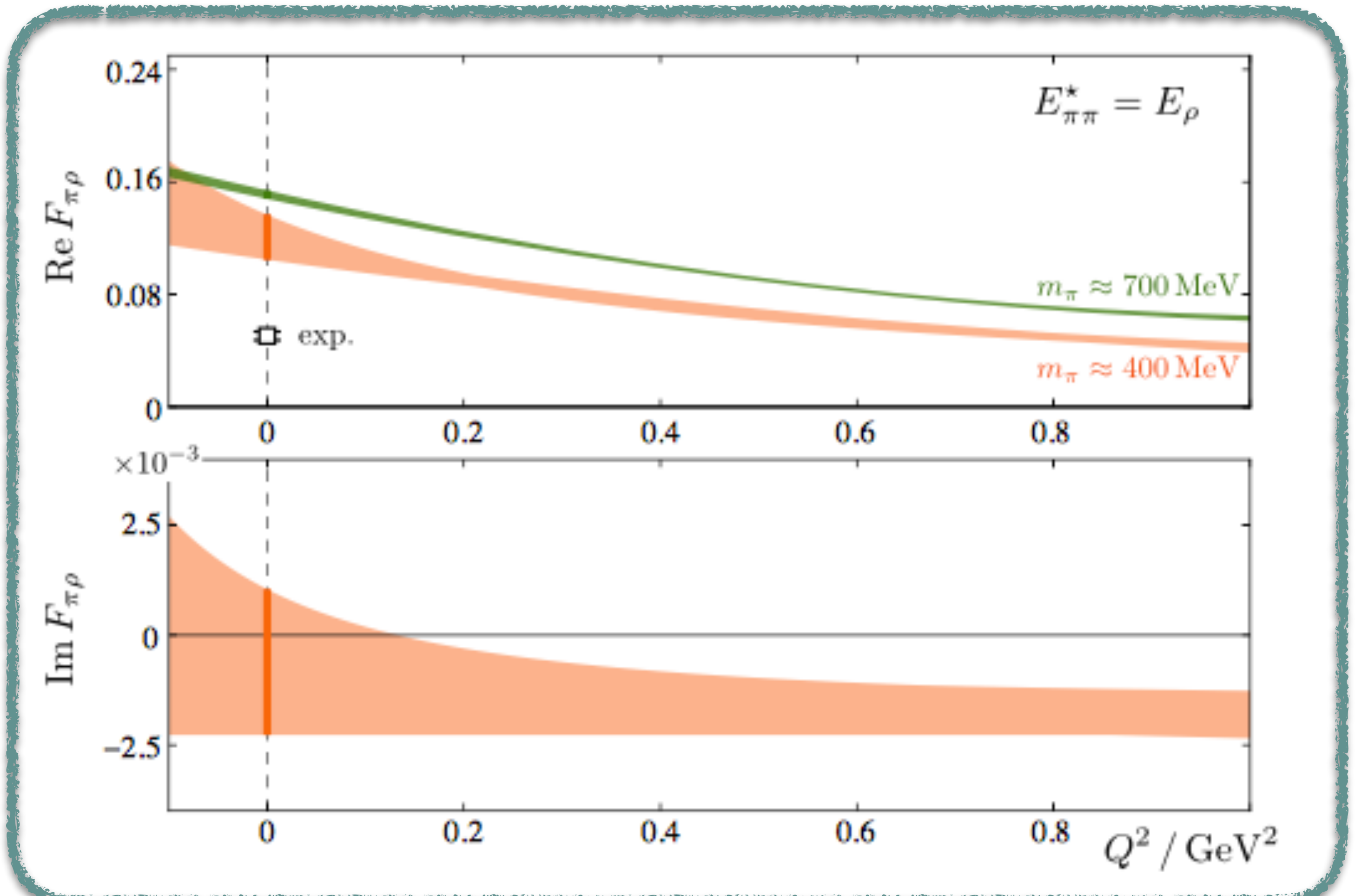
- Hansen & Sharpe (2012) [D-to-$\pi\pi$/KK]
- Briceño, Hansen Walker-Loud /Briceño & Hansen(2014-2015)[general 1-to-2]

Explain ingredients
what’s R?

Explain finite volume matrix elements

$1+\mathcal{J}\rightarrow 2$

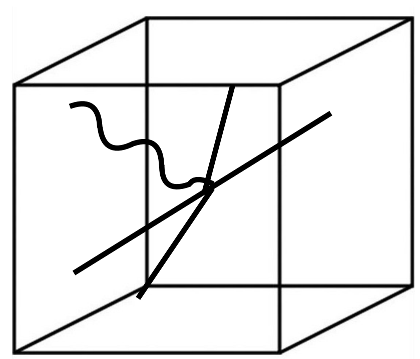


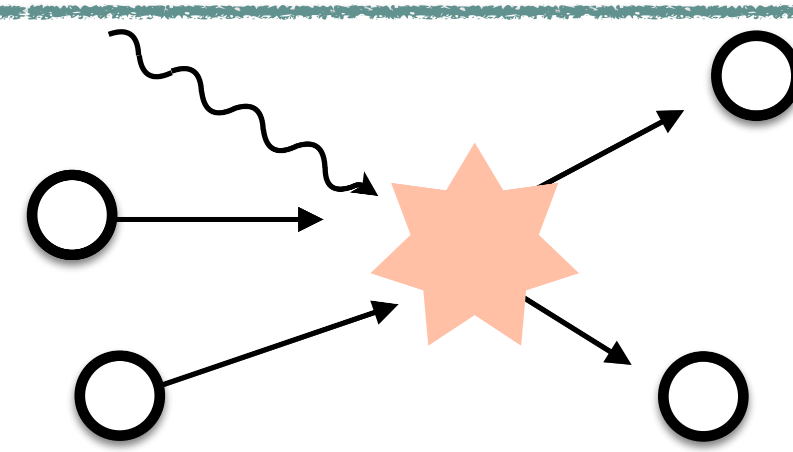
Understand carefully transition form factors

Briceño, Dudek, Edwards, Shultz, Thomas and Wilson PRL (2016)

$2+\mathcal{J} \to 2$

FV matrix elements to infinite volume electroweak amplitudes





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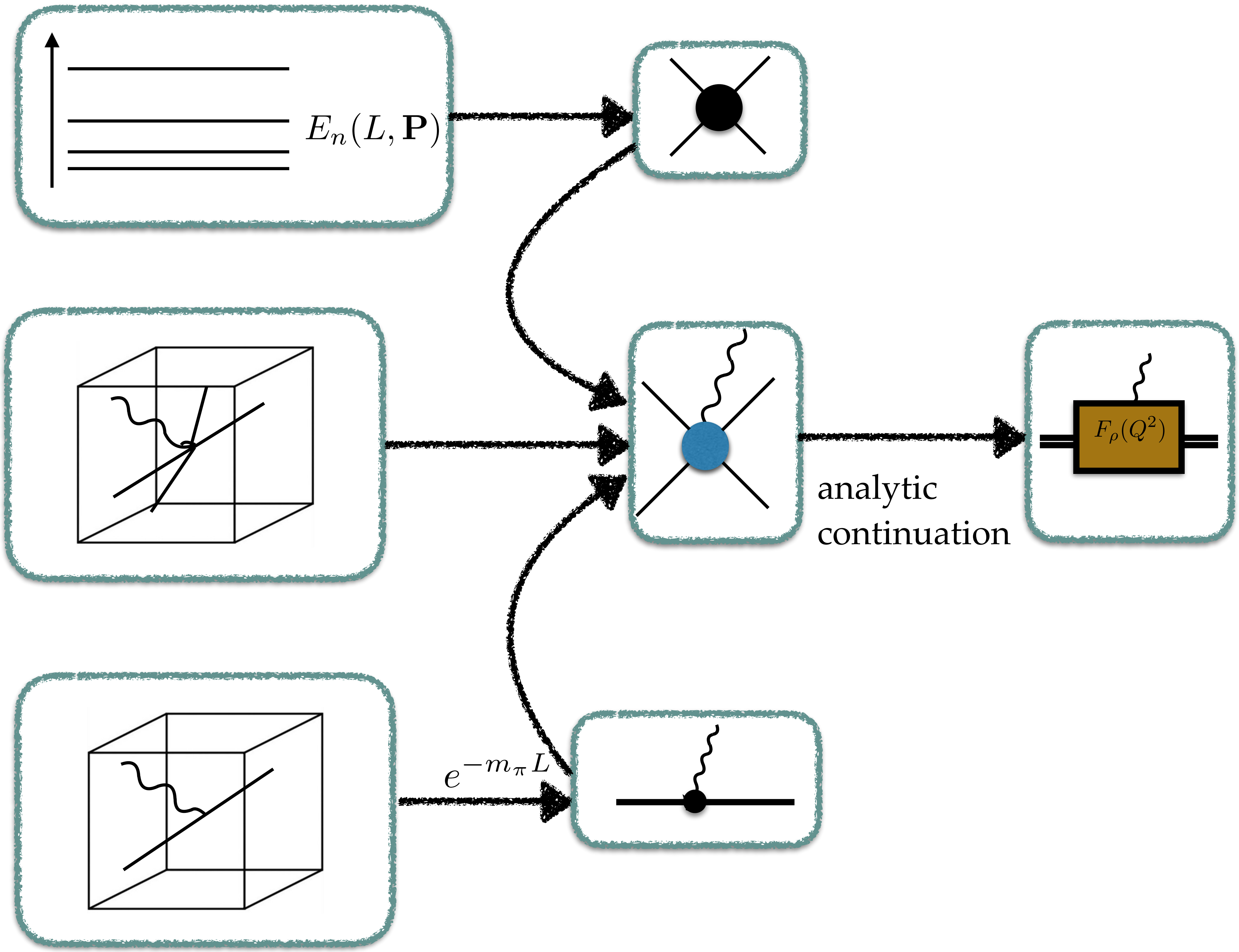
- Briceño & Hansen (2016)

Workflow

Explain that this stuff is already extrapolated at zero lattice spacing

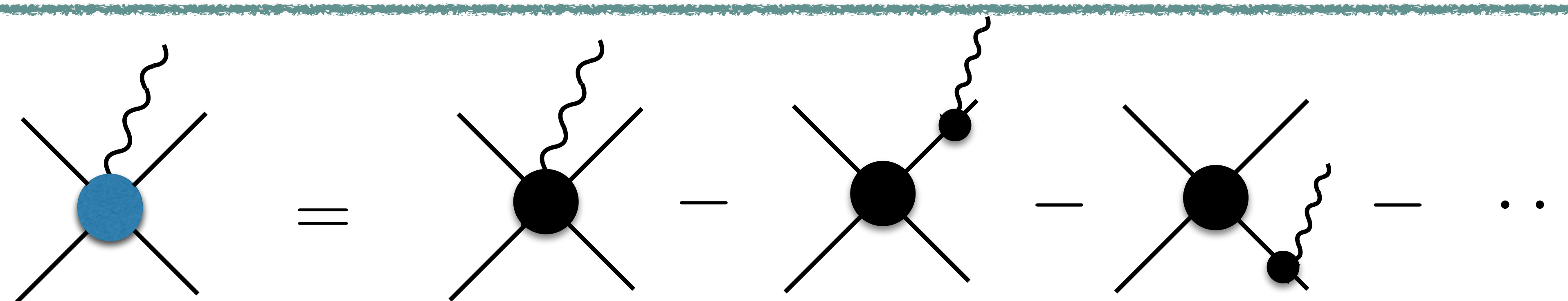


$2+\mathcal{J}\to 2$

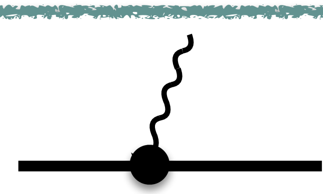
$$\langle 2|\mathcal{J}|2\rangle|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[R(E_L,L)W_{L,\mathrm{df}}R(E_L,L)W_{L,\mathrm{df}}\right]$$

$$W_{L,\mathrm{df}} = W_{\mathrm{df}} + MG(L,w)M$$

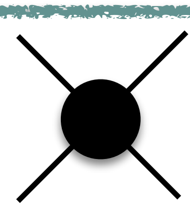
$W_{\mathrm{df}} =$



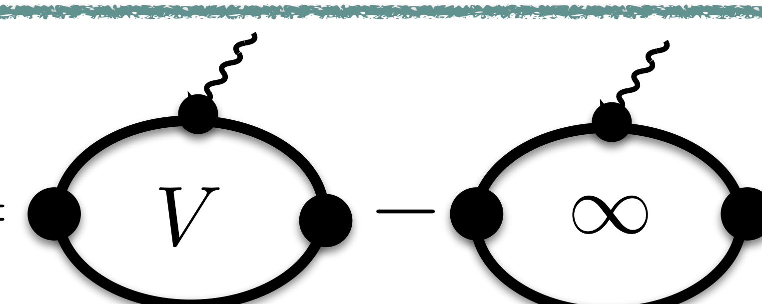
$w =$



$M =$



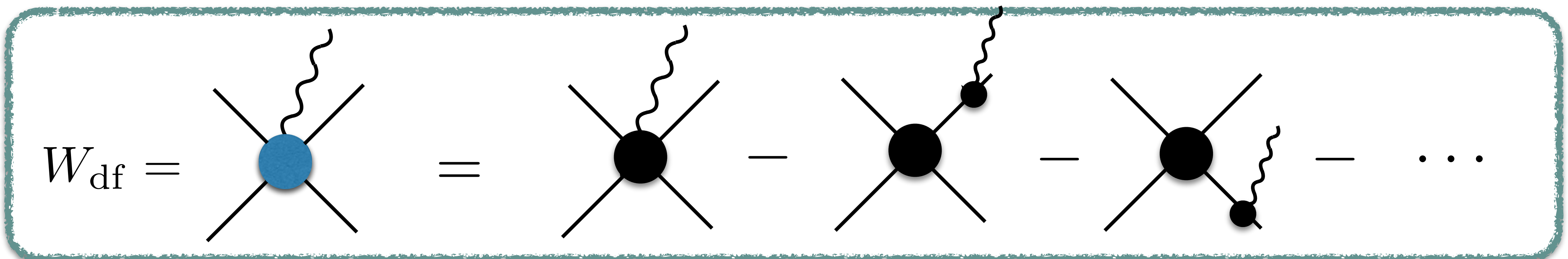
$G(L,w) =$



$$= \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int d\mathbf{k}\right](\cdots)$$

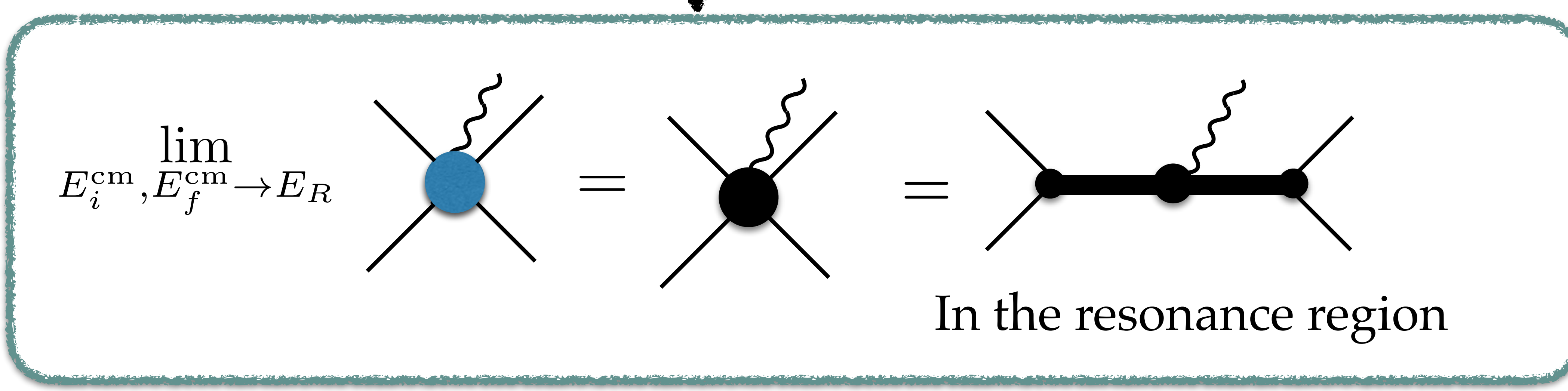
explain (or at least know) where these equations are coming from

A detail



Kinematic poles cancel

But dynamical poles associated with resonances can still appear



Outline

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

- The sum is “easy”
- The integral is highly not trivial (spectator particle goes on-shell)
  - integrand singularities are two surfaces in three-dimension
  - standard principal value techniques in one dimension fail
  - techniques from other fields are not suitable
  - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D smooth integral

AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)

Give details I

Discuss implementation

Give details II

Discuss implementation

Give details III

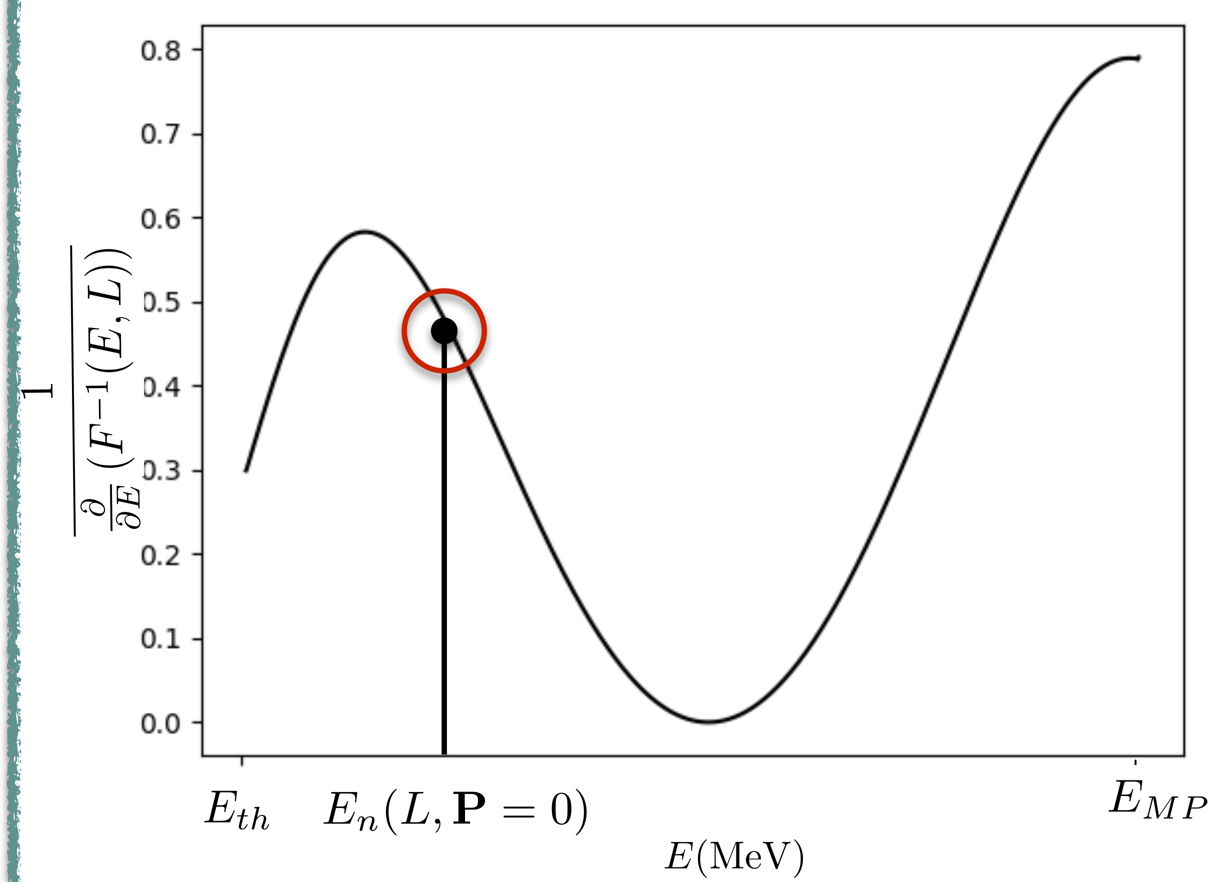
Discuss implementation

Kinematic functions I

$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E}\left(F^{-1}(E,L)\right) + M(E)}\bigg|_{E=E_L}$$

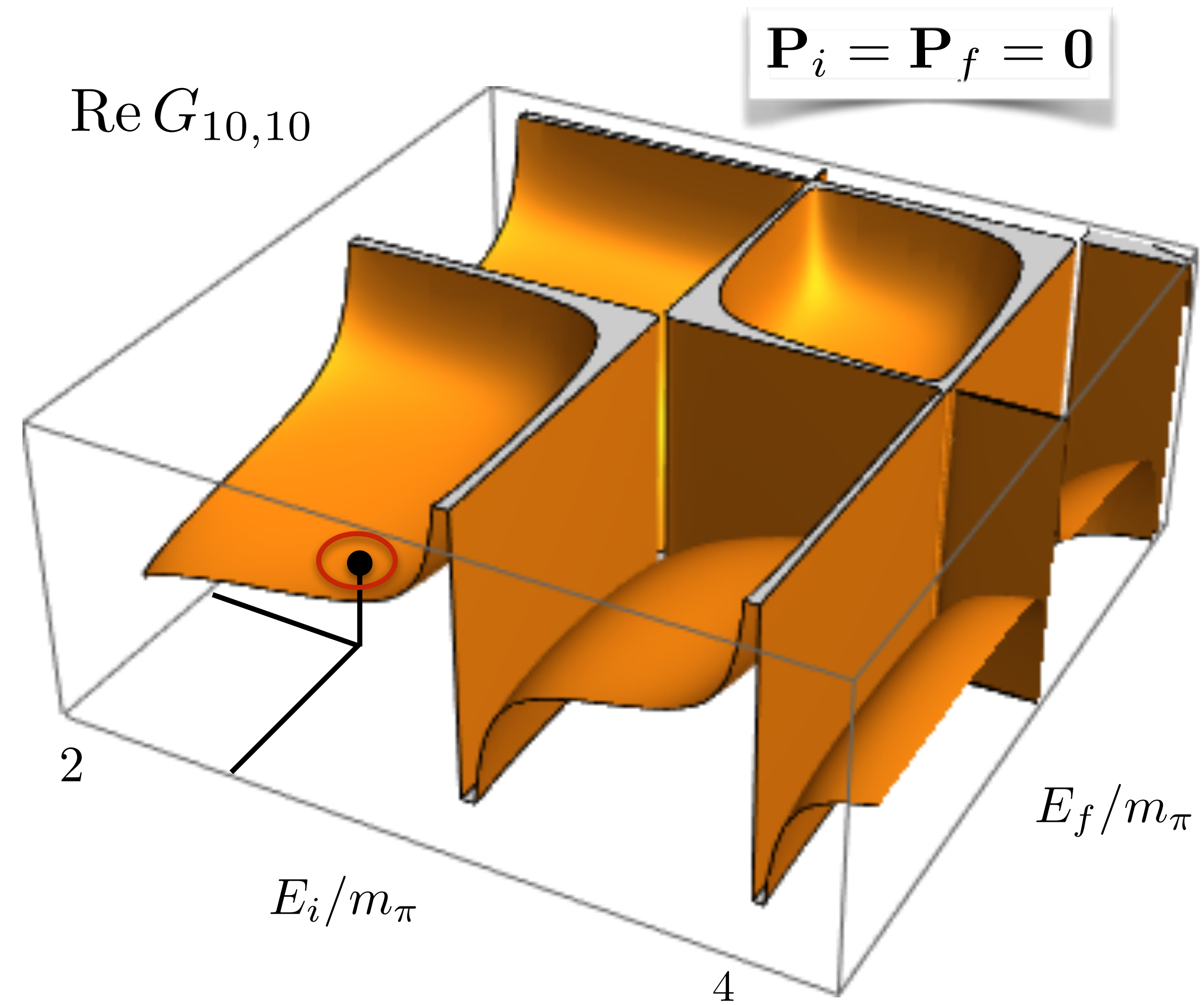
$G_{l_i m_i, l_f m_f}$

$m_\pi L = 4$



E_{th} = threshold energy

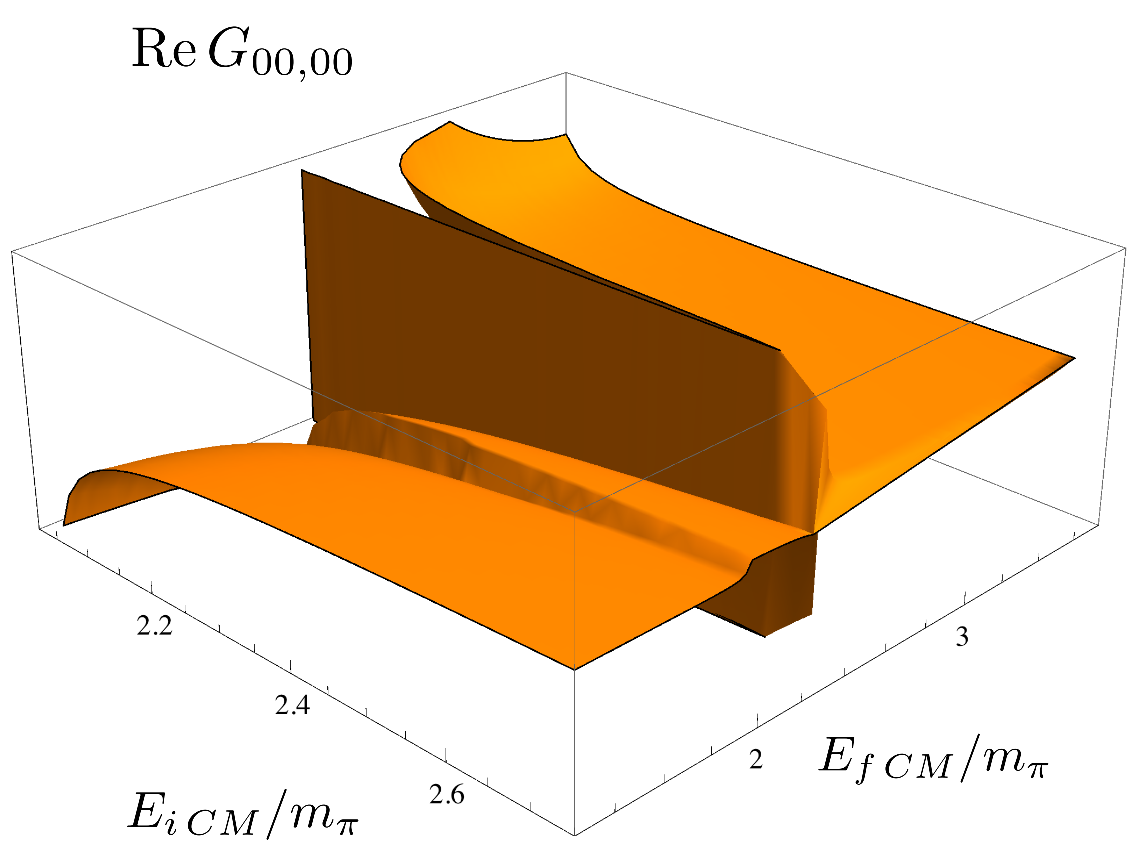
E_{MP} = multiparticle states energy

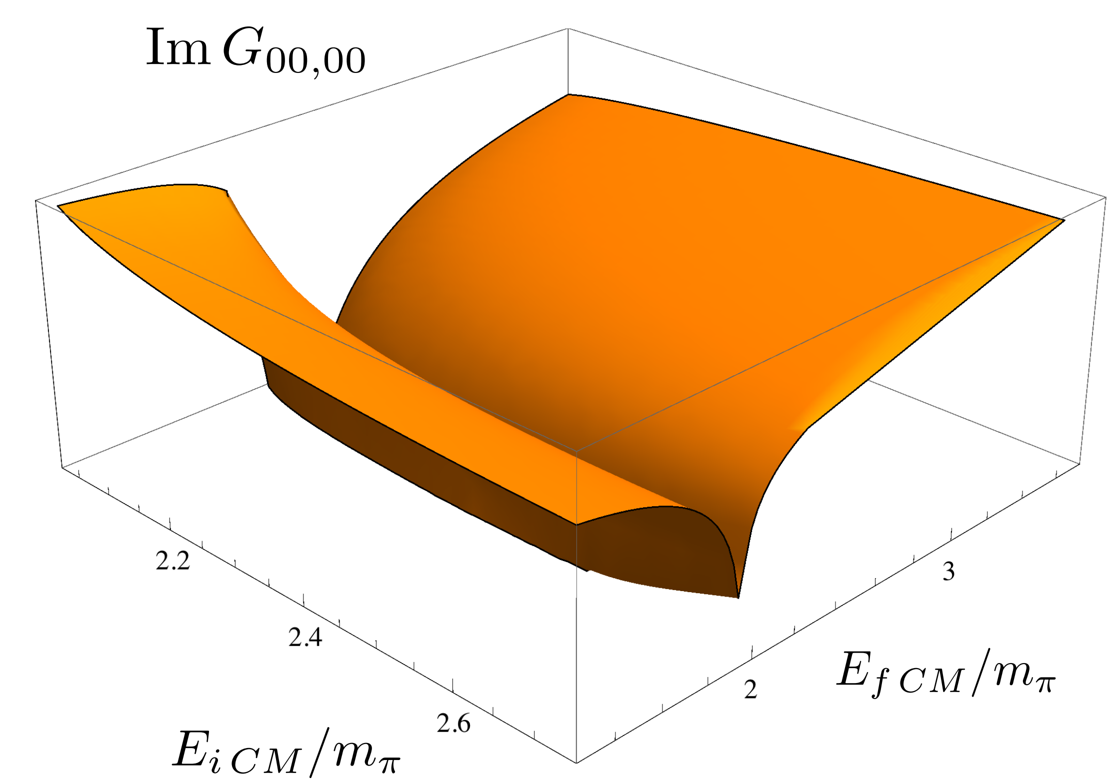


Singularities at free particle energies

Kinematic functions II

$$\mathbf{P}_i = \mathbf{0}\,, \quad \mathbf{P}_f = [0, 0, 2\pi/L]$$
$$m_\pi L = 4$$





More plots in

AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
In preparation

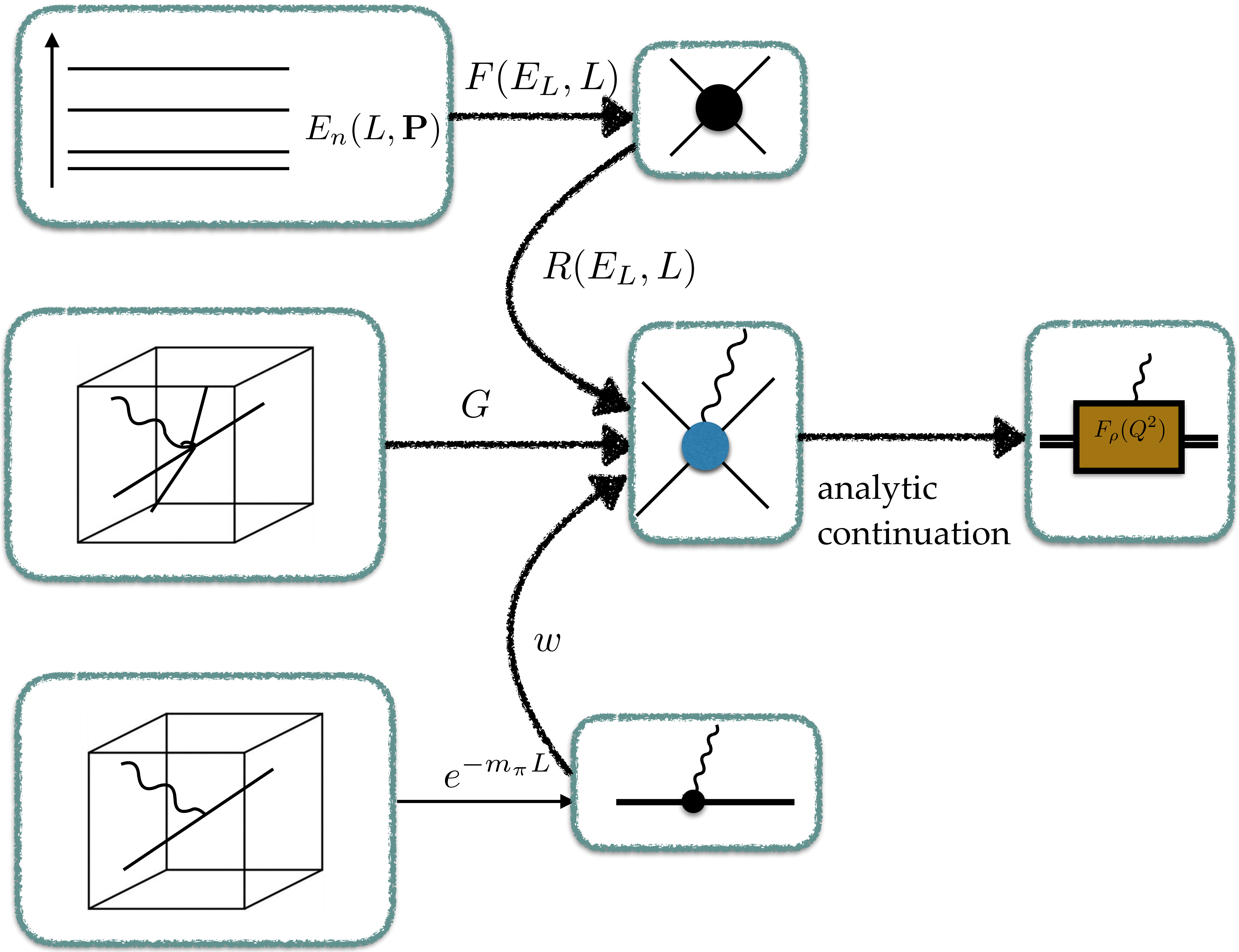
Discuss about opening challenges

Discuss about Lorentz decomposition (maybe..)

Discuss about other stuff for which it could be useful

Thank you!

Workflow



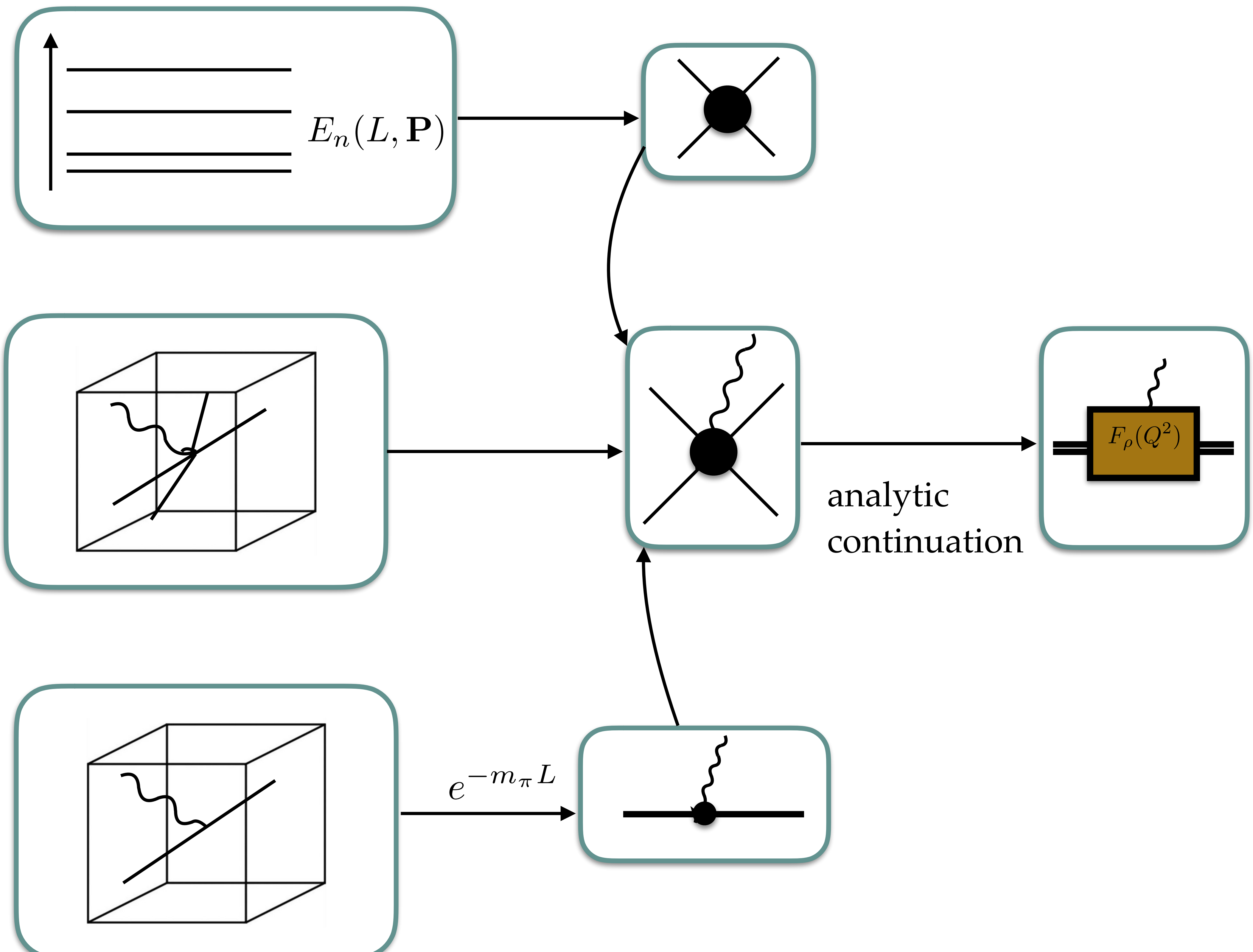
$E_n(L, \mathbf{P})$
$F(E_L, L)$
$R(E_L, L)$
G
$F_\rho(Q^2)$
analytic continuation
w
$e^{-m_\pi L}$

Some challenges

- A spin 1 particle between non-degenerate states has four form factors
- There is not a one-to-one mapping between matrix elements and amplitudes
    - Solved problem for spectrum analysis
- Analytical continuation of the amplitudes

Backup slides

Workflow 101



Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\rm FV} \to \langle 2|\mathcal{J}|2\rangle_\infty$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves…..

- From $\langle 2|\mathcal{J}|2\rangle_\infty$ how do we get the four form factors?
  - Analytic continuation