

2019

Finite Volume Matrix Elements of Two-Body States With One Current Insertion

Alessandro Baroni

Raúl Briceño
Old Dominion University, rbriceno@odu.edu

Maxwell Hansen

Felipe Ortega

Follow this and additional works at: https://digitalcommons.odu.edu/physics_fac_pubs



Part of the [Elementary Particles and Fields and String Theory Commons](#), and the [Quantum Physics Commons](#)

Original Publication Citation

Baroni, A., Briceño, R., Hansen, M., & Ortega, F. (2019). *Finite volume matrix elements of two-body states with one current insertion*. Lattice QCD, Santa Fe, New Mexico. <https://www.osti.gov/biblio/1561050>

This Conference Paper is brought to you for free and open access by the Physics at ODU Digital Commons. It has been accepted for inclusion in Physics Faculty Publications by an authorized administrator of ODU Digital Commons. For more information, please contact digitalcommons@odu.edu.

LA-UR-19-28892

Approved for public release; distribution is unlimited.

Title: Finite volume matrix elements of two-body states with one current insertion

Author(s): Baroni, Alessandro
Briceño, Raul
Hansen, Maxwell
Ortega, Felipe

Intended for: Lattice conference Santa Fe, 2019-08-26/2019-08-30 (Santa Fe, New Mexico, United States)

Issued: 2019-09-04

Disclaimer:

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA000001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

Finite volume matrix elements of two-body states with one current insertion

Alessandro Baroni
LANL



R. A. Briceño
JLab/ODU



M. T. Hansen
CERN



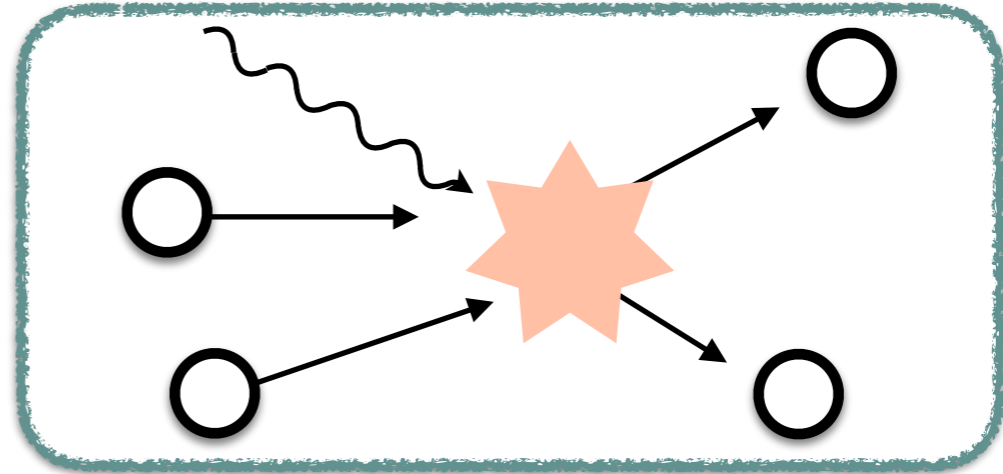
F. G. Ortega
William and Mary

Santa Fe 2019

Two-to-two scattering with current

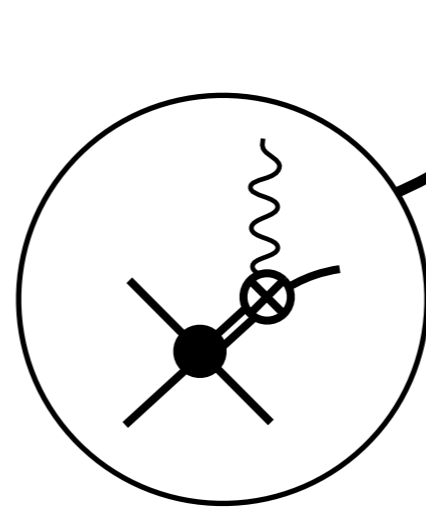
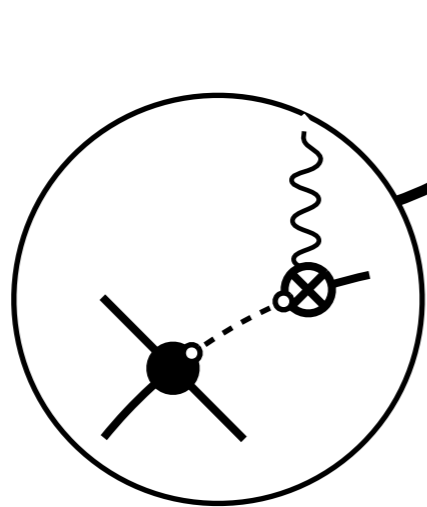
Kinematic divergences

$$i\mathcal{W}_{\mu_1 \dots \mu_n} = \text{[Diagram: vertex with wavy line]} + \dots$$



Fully dressed propagator $i\Delta = \frac{i}{k^2 - m^2 + i\epsilon} + \text{smooth}$

$$\text{[Diagram: vertex with wavy line]} = iM \frac{i}{k^2 - m^2 + i\epsilon} i\mathcal{W}_{\mu_1 \dots \mu_n} + \text{“smooth”}$$



Two-to-two scattering with current

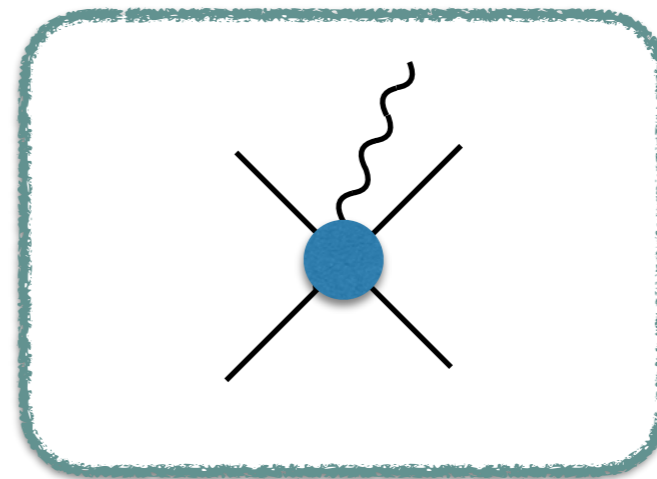
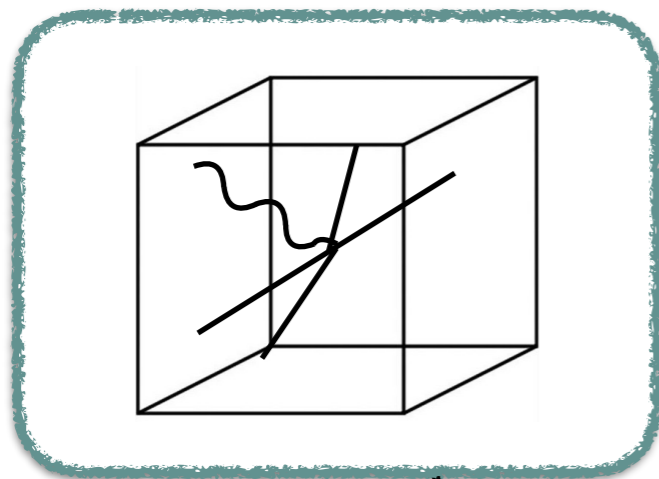
$$W = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]} + \text{[diagram 7]}$$
$$= \text{[diagram 8]} + \text{[diagram 9]}$$

The diagrams represent various Feynman diagrams for two-to-two scattering with a current. Diagram 1 is a contact term with a wavy line. Diagrams 2-7 are tree-level diagrams with one or two loops and a wavy line. Diagram 8 is a contact term with a dashed line. Diagram 9 is a loop diagram with a blue circle and labeled W_{df} .

- Will be related to the FV matrix element is not the full scattering amplitude
- We can recover the full scattering amplitude adding back diagrams with kinematic singularities

$$2 + \mathcal{J} \rightarrow 2$$

FV matrix elements to infinite volume electroweak amplitudes



$$\langle 2 | \mathcal{J} | 2 \rangle |^2_L = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L, \text{df}} R(E_L, L) W_{L, \text{df}}]$$

- Briceño & Hansen (2016)
- AB, Briceño, Hansen, Ortega (2018)

$$2 + \mathcal{J} \rightarrow 2$$

$$\langle 2 | \mathcal{J} | 2 \rangle \Big|_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + MG(L, \omega)M$$

$$M = \text{---} \bullet \text{---}$$

2 → 2 scatt. Amplitude
Calculated at FV energies

$$w = \text{---} \bullet \text{---}$$

Crucial ingredient that we studied

$$G(L, \omega) = \text{---} \bullet \text{---} \text{---} \bullet \text{---} - \text{---} \bullet \text{---} \text{---} \bullet \text{---} \infty$$

$$= \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots)$$

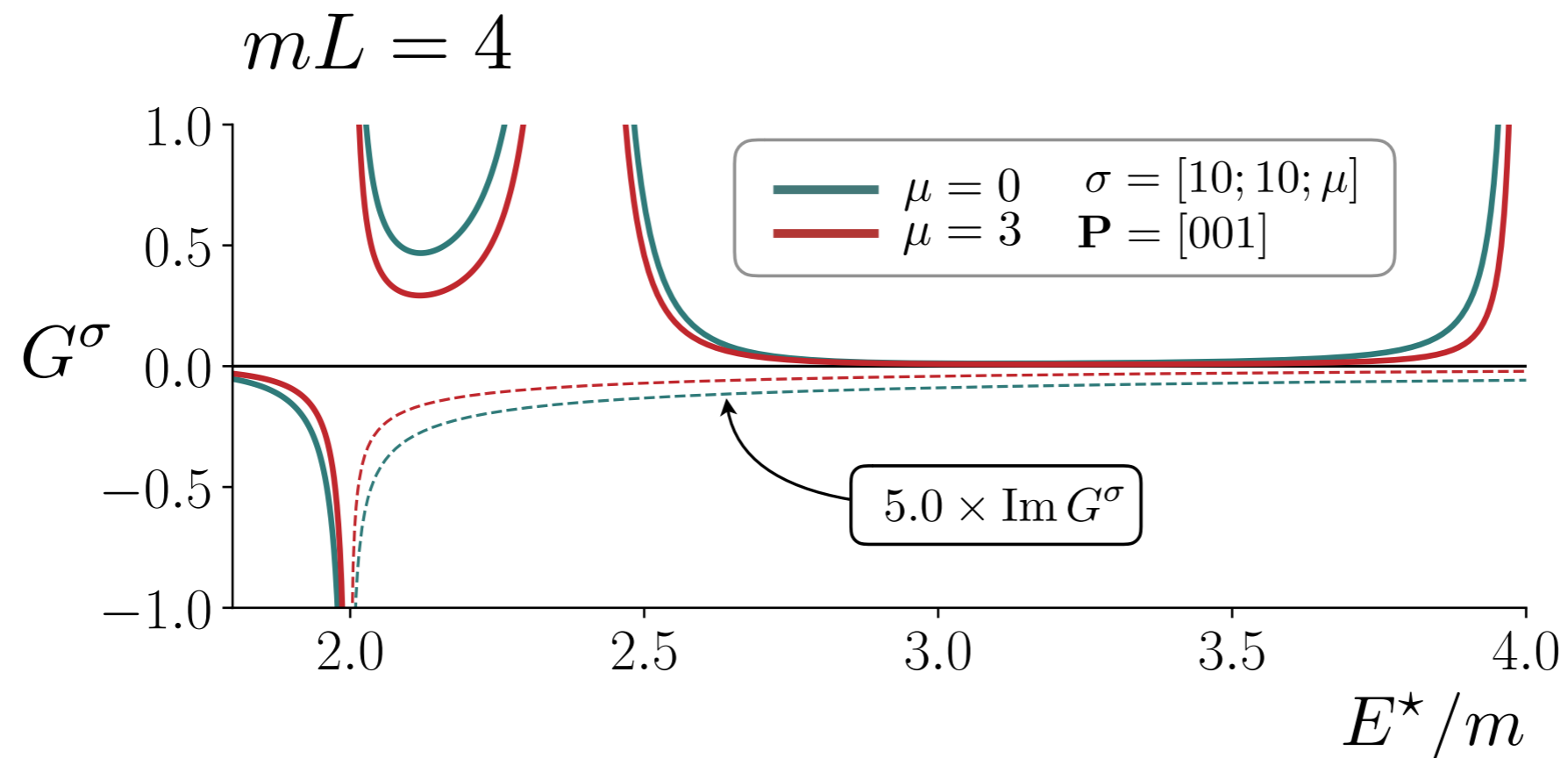
New kinematic function

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

- The sum is “straightforward”
- The integral is highly not trivial (spectator particle goes on-shell)
 - integrand singularities are two surfaces in three-dimension
 - standard principal value techniques in one dimension fail
 - techniques from other fields are not suitable
 - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D **smooth** integral

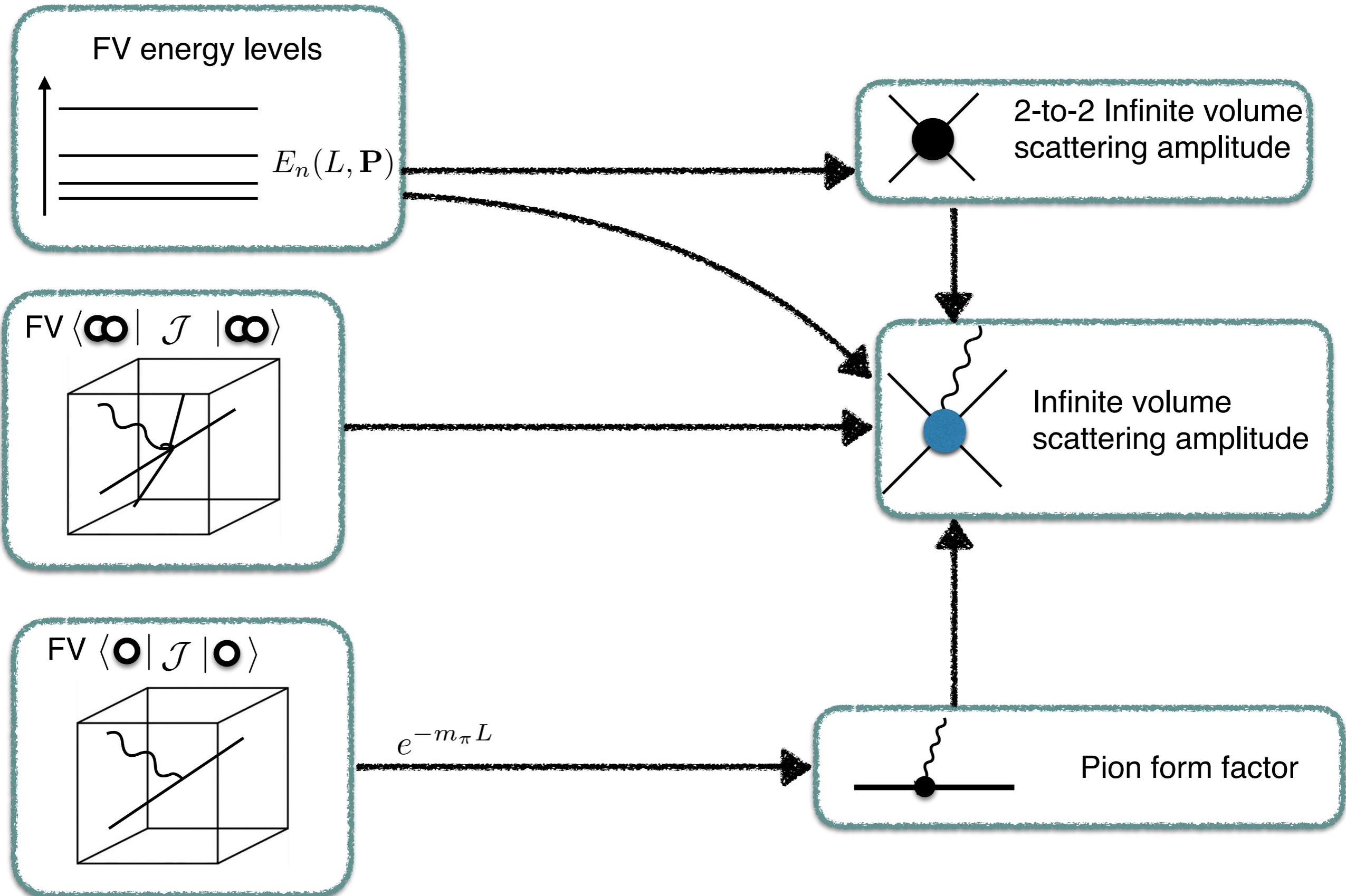
AB, R. A. Briceño, M. T. Hansen, F. Ortega (2018)

For equal initial and final momenta it has been shown that the above problem has an analytical solution



AB, R. A. Briceño, M. T. Hansen, F. Ortega (2018)

Summary



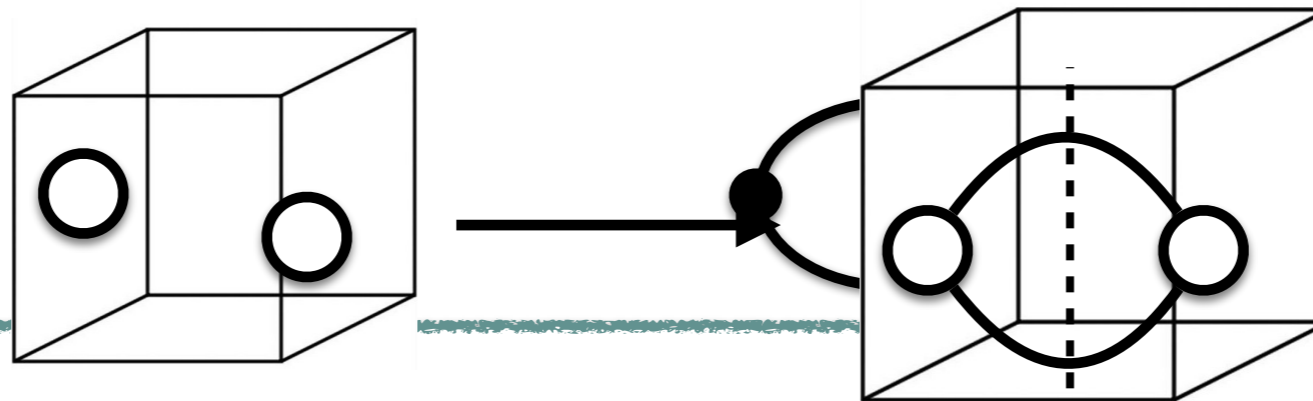


- General, relativistic, EFT independent, finite volume formalism to treat processes relevant for many electroweak processes (nuclear and not only) derived
- A crucial ingredient the new kinematic function is under control
- Think to apply all of this in a simple case

Thank you!

Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
 - On-shell intermediate states give singularities



Asymptotic states not in a FV

- Formalism

- Briceño
- Rusetsky et al. (2012) - EFT dependent, NR
- Briceño and Davoudi (2012) - EFT dependent, NR



Many works.. not cited here

- Luscher formalism (1986, 1991)

Finite volume correlator

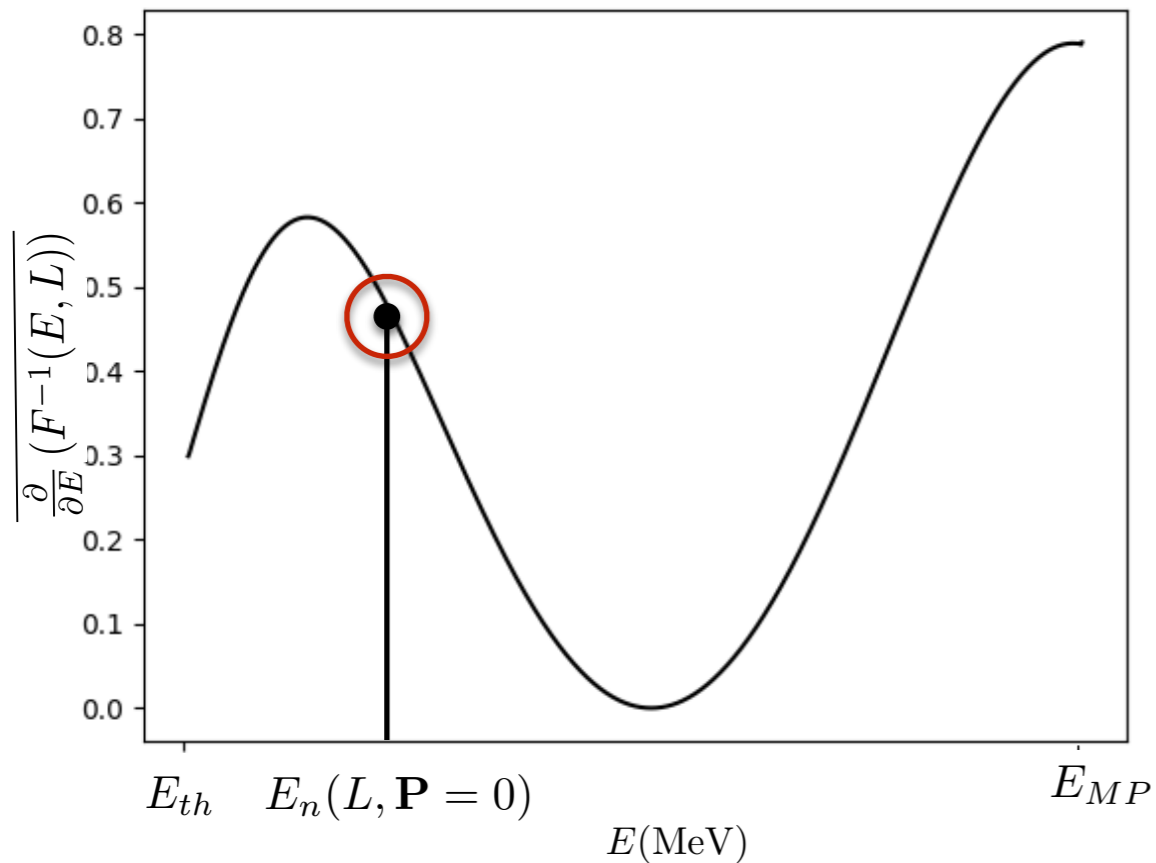
Rearranging

Kinematic functions I

$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E} (F^{-1}(E, L) + M(E))} \Big|_{E=E_L}$$

$$m_\pi L = 4$$

$$G_{l_i m_i, l_f m_f}$$

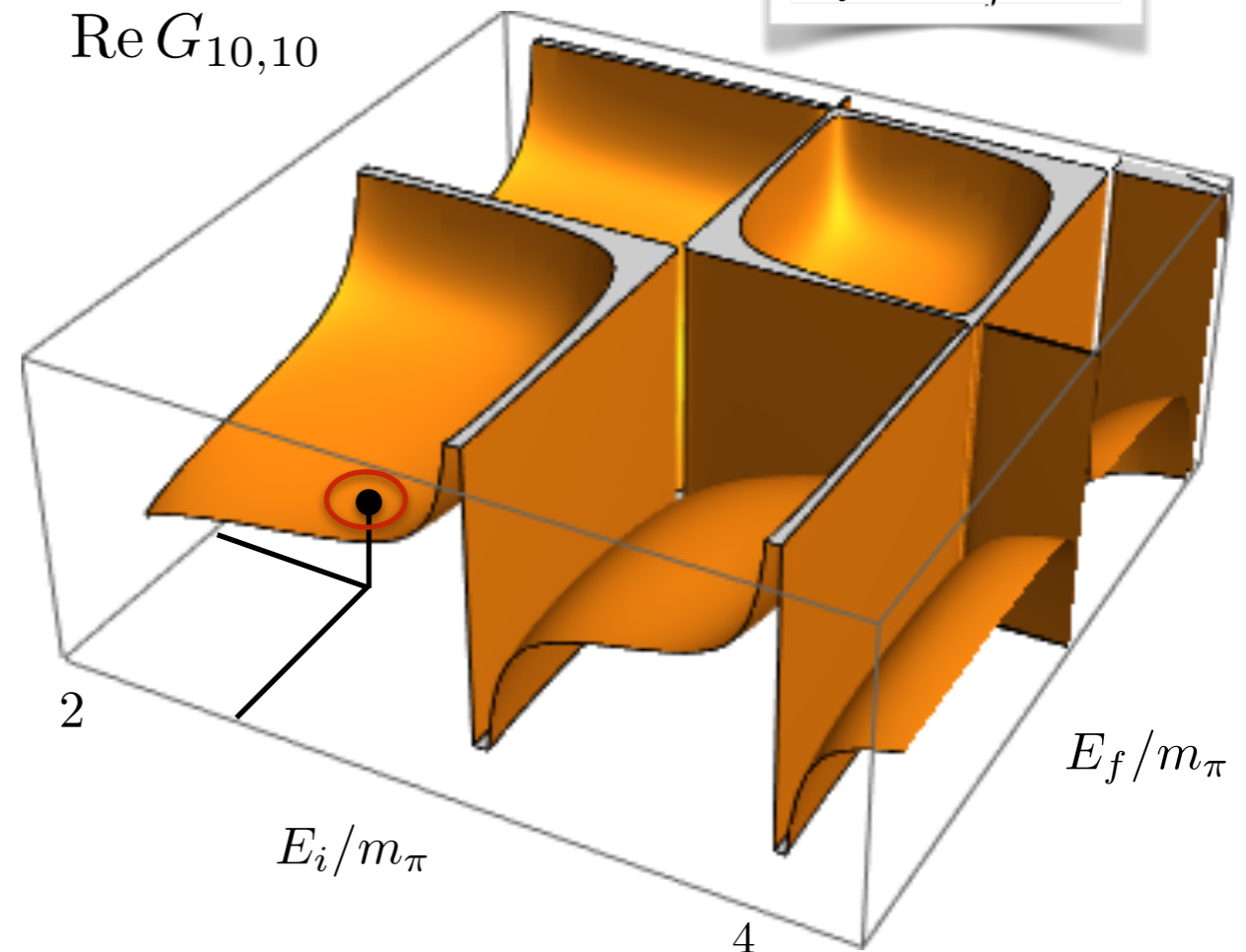


E_{th} = threshold energy

E_{MP} = multiparticle states energy

$\text{Re } G_{10,10}$

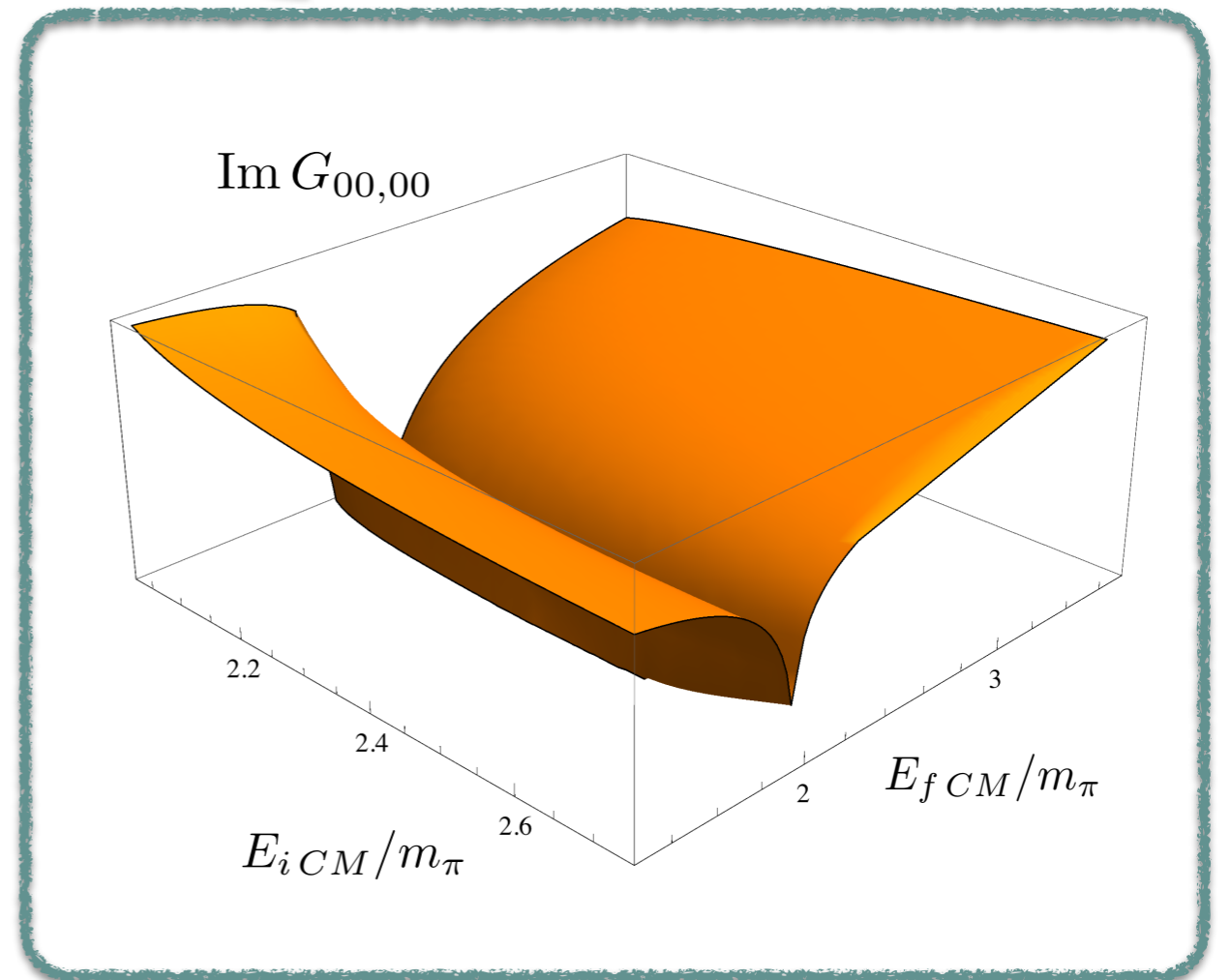
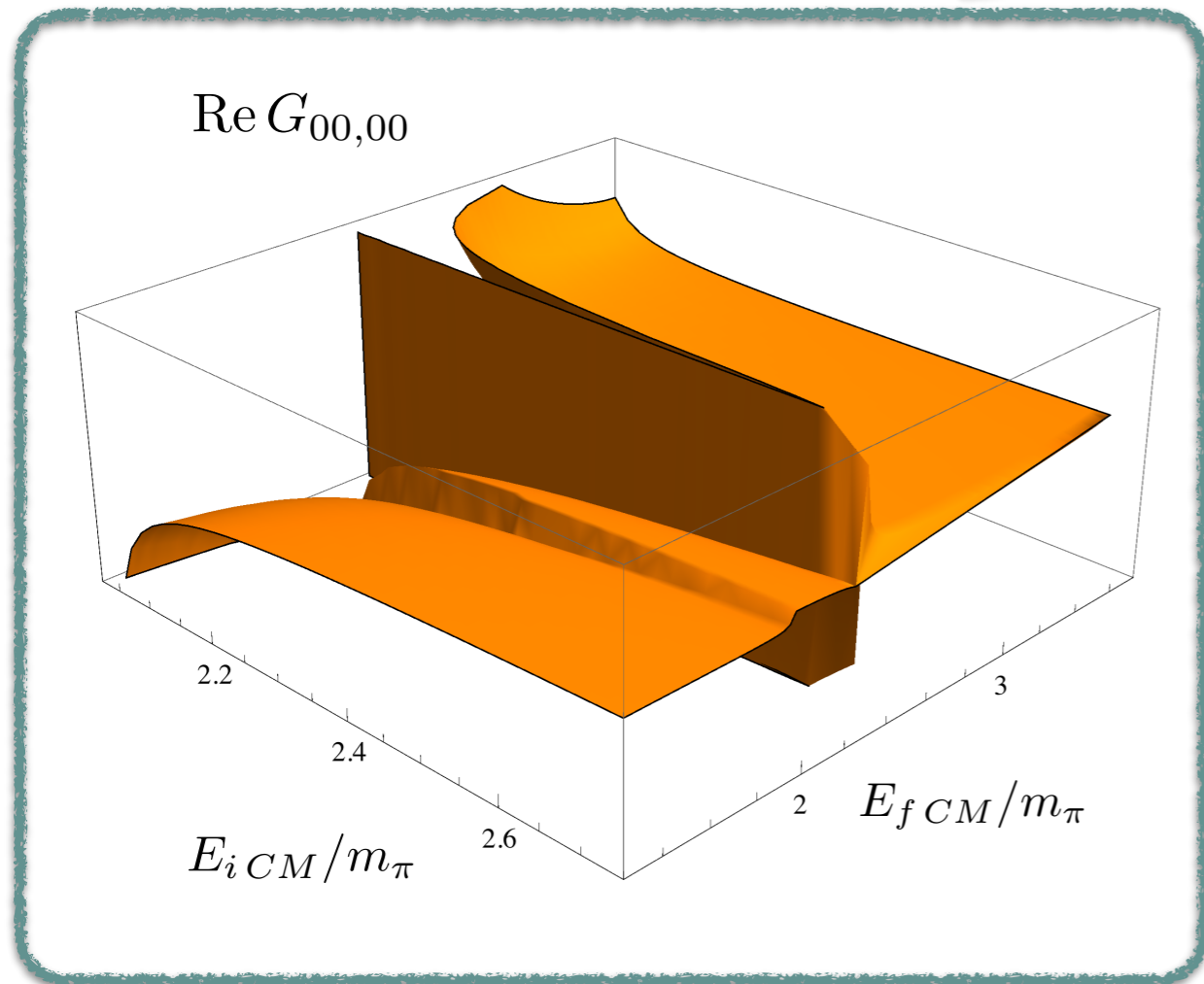
$$\mathbf{P}_i = \mathbf{P}_f = 0$$



Singularities at free particle energies

Kinematic functions II

$$\mathbf{P}_i = \mathbf{0}, \quad \mathbf{P}_f = [0, 0, 2\pi/L]$$
$$m_\pi L = 4$$



More plots in



AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
In preparation

A detail

$$W_{df} = \text{[Blue vertex]} = \text{[Black vertex]} - \text{[Black vertex with wavy line]} - \text{[Black vertex with wavy line]} - \dots$$

Kinematic poles cancel

But dynamical poles associated with resonances can still appear

$$\lim_{E_i^{cm}, E_f^{cm} \rightarrow E_R} \text{[Blue vertex]} = \text{[Black vertex]} = \text{[Resonance diagram]}$$

In the resonance region

Explain that. In order to do this bridge we need to know not only LECs from QCD but also form factors
Not only single nucleon for factors but also understand multi hadron...

Explain that. In order to do this bridge we need to know not only LECs from QCD but also form factors
Not only single nucleon for factors but also understand multi hadron...

Goals

Add pictures and make it pretty

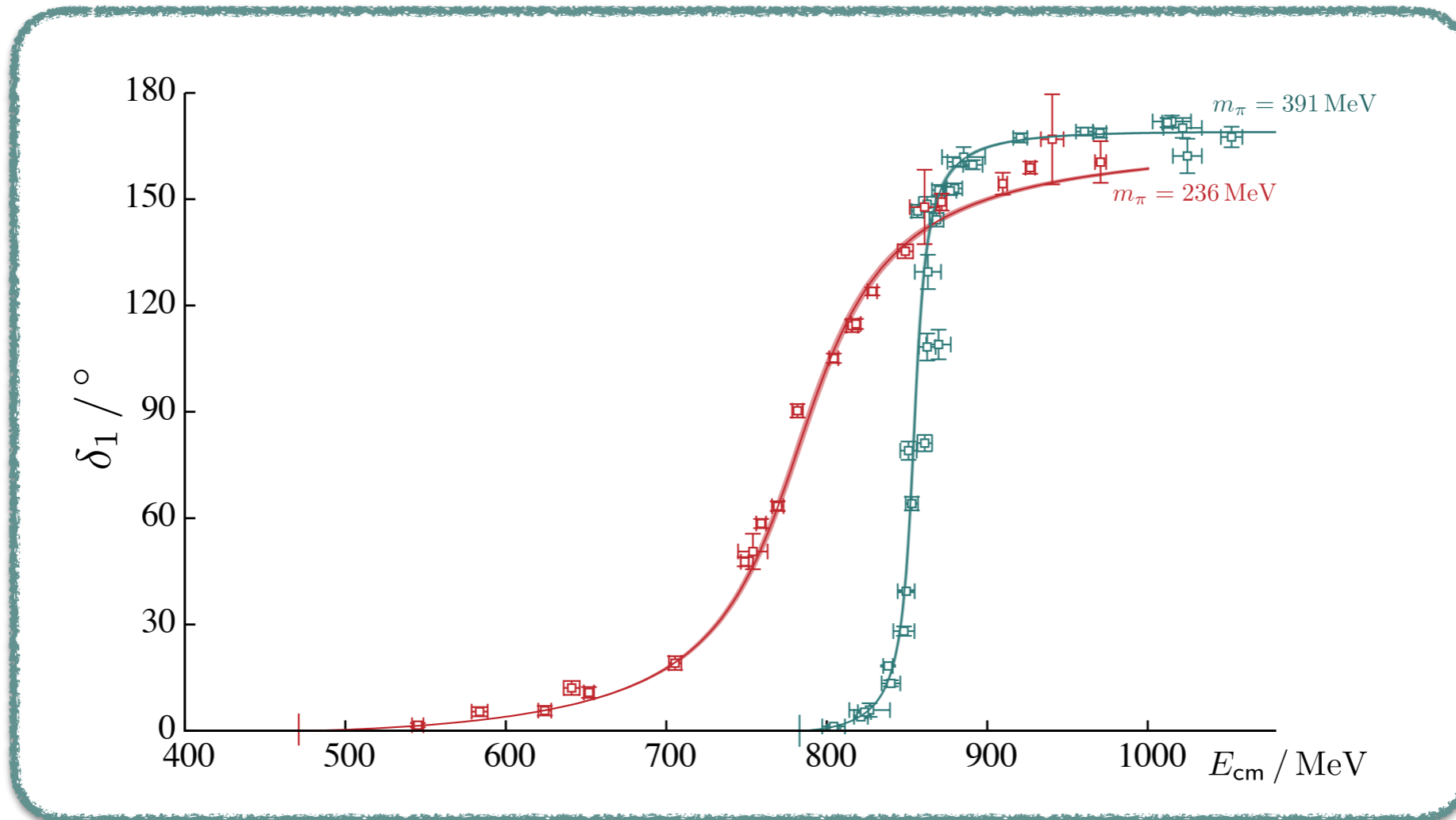
- Developing a framework for studying :
 - structure of composite states (structure -> Form factors)
 - structure of resonances
 - structure of the deuteron
 - Weak processes involving few-hadron systems
 - parity violation
 - p-p fusion, neutrino-nucleus

Add some extra explanation

Sketch of the derivation

Expand

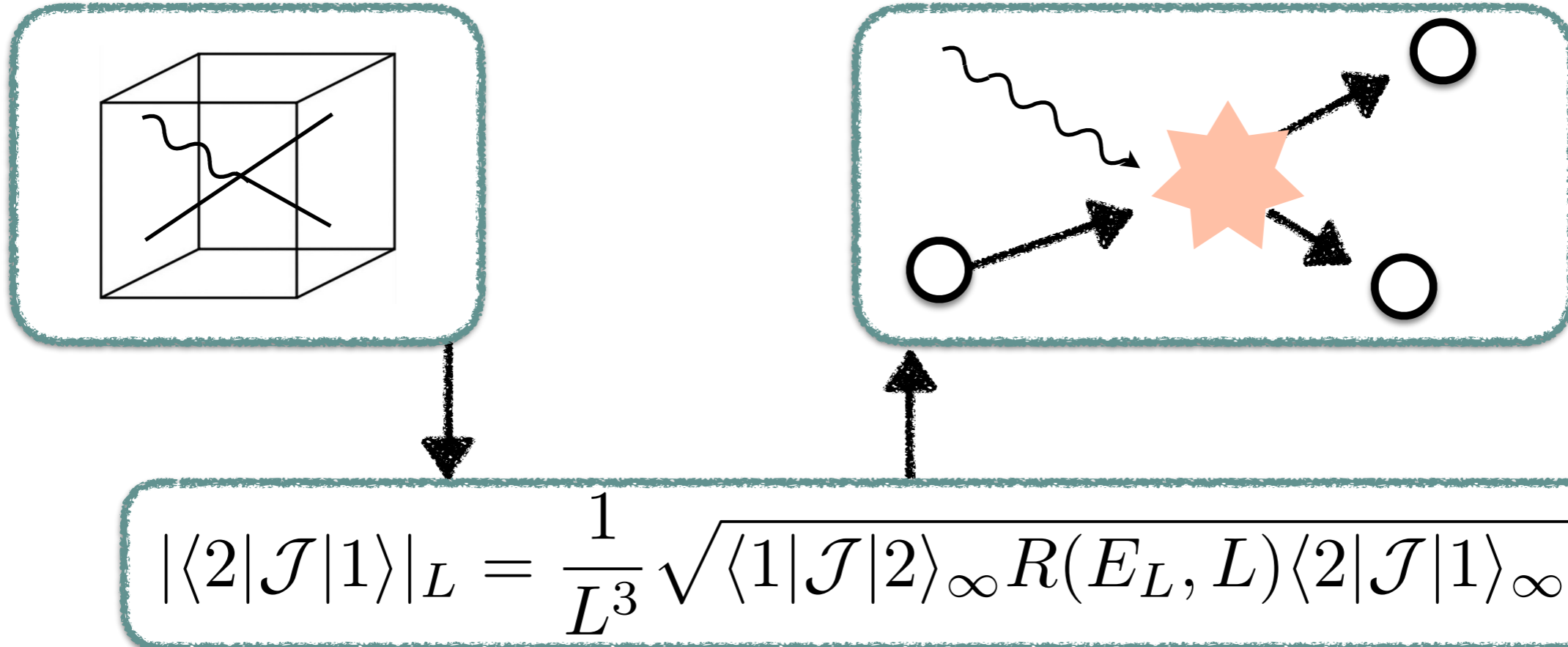
2 → 2



● Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

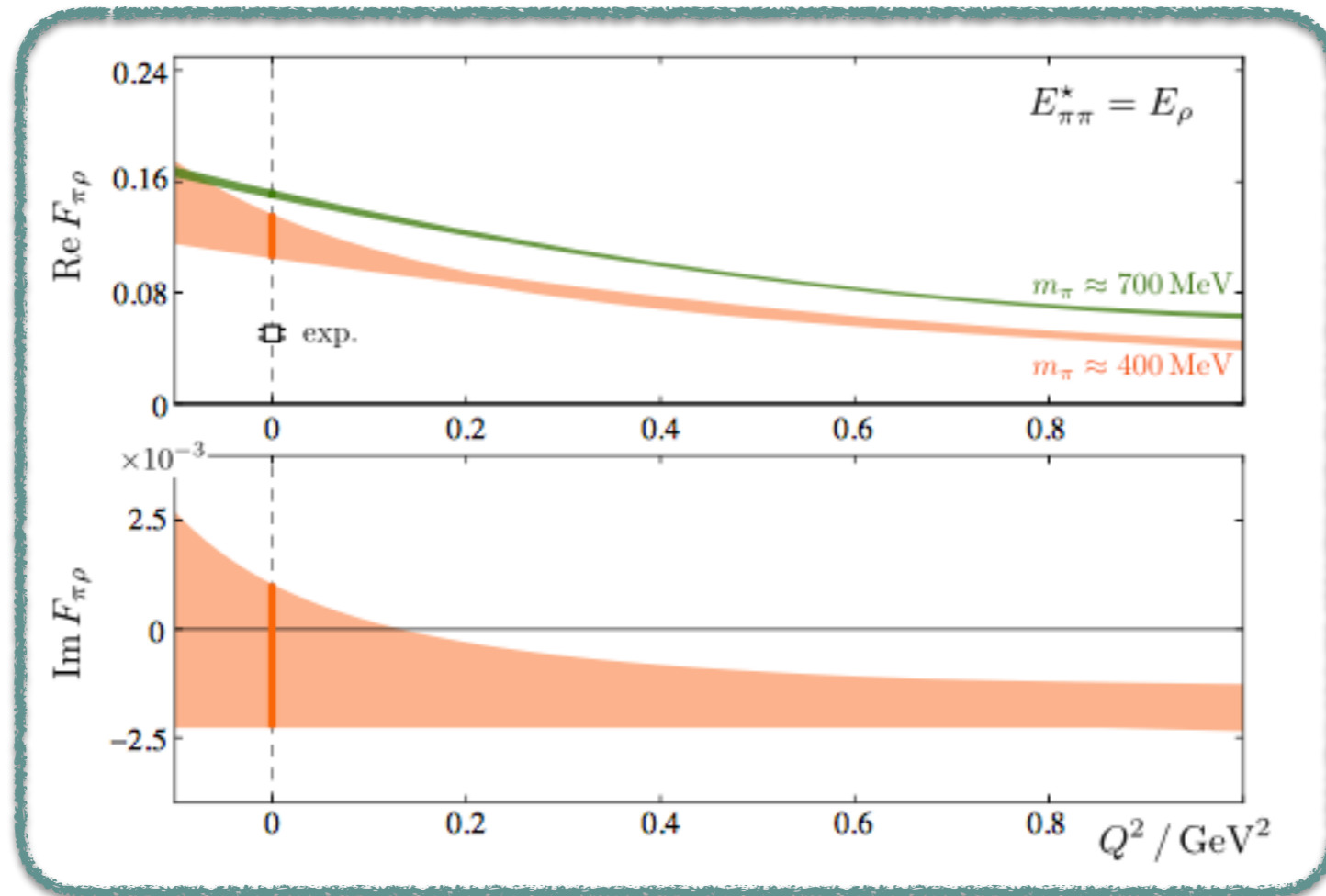
$1 + \mathcal{J} \rightarrow 2$

FV matrix elements to infinite volume electroweak amplitudes



- Lellouch & Lüscher (2000) [K-to- $\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to- $\pi\pi$]
- Hansen & Sharpe (2012) [D-to- $\pi\pi$ / KK]
- Briceño, Hansen Walker-Loud / Briceño & Hansen (2014-2015) [general 1-to-2]

$$1 + \mathcal{J} \rightarrow 2$$



Briceño, Dudek, Edwards, Shultz, Thomas and Wilson PRL (2016)

Report some examples calculations in few body systems

BF 1 slide


Some open questions

Expand here with pics

Goals

- Developing a framework for studying :
 - structure of composite states
 - structure of resonances
 - structure of the deuteron
 - Weak processes involving few-hadron systems
 - parity violation
 - p-p fusion, neutrino-nucleus

Goals

- Developing a framework for studying :
 - structure of composite states
 - structure of resonances
 - structure of the deuteron
 - Weak processes involving few-hadron systems
 - parity violation
 - p-p fusion, neutrino-nucleus
- 

Why do we care about the structure of these objects?
get the charge radius etc etc, maybe mention nuclear physics stuff
Add stuff in this slide.....

Goals

- Developing a framework for studying :
 - structure of composite states
 - structure of resonances
 - structure of the deuteron
 - Weak processes involving few-hadron systems
 - parity violation, p-p fusion
 - neutrino-nucleus

Focus here

LQCD inputs for
low energy nuclear physics

Maybe expand here general goals nice but give specific examples

Goals

- Developing a framework for studying :

- structure of composite states

- structure of resonances

- structure of the deuteron

Focus here

- Weak processes involving few-hadron systems

- parity violation, p-p fusion

- neutrino-nucleus

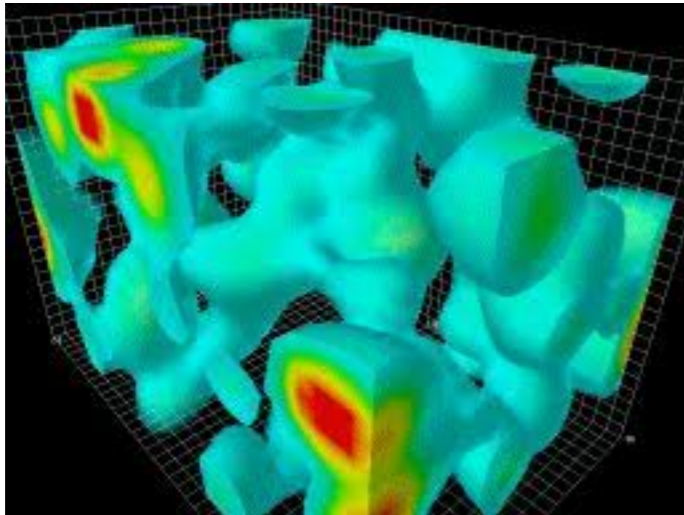
LQCD inputs for
low energy nuclear physics

LQCD inputs for neutrino-nucleus scattering

Nuclear electroweak interactions?

Atomic nuclei are a complex quantum-many body systems of **strongly** interacting nucleons

Hadronic matrix elements difficult because of QCD



Nuclear Decay Half-Lives for Nuclei A = 3 - . . .

Nuclide (β^- Decay)	Half-Life	Date	Nuclide (β^+ Decay)	Half-Life	Date
${}^3\text{H}$	12.323 ± 0.020 yr	September, 2015	${}^7\text{Be}$	53.22 ± 0.06 days	August, 2001
${}^6\text{He}$	$806.89^{+0.25}_{-0.22}$ ms	August, 2015	${}^8\text{B}$	770.3 ± 0.4 ms	July, 2015
${}^8\text{He}$	119.0 ± 1.6 ms	September, 2015	${}^9\text{C}$	126.5 ± 1.0 ms	July, 2015
${}^8\text{Li}$	838.79 ± 0.36 ms	August, 2015	${}^{10}\text{C}$	19.3015 ± 0.0017 sec	May, 2016
${}^9\text{Li}$	177.7 ± 0.6 ms	August, 2015	${}^{11}\text{C}$	20.3401 ± 0.0070 min	March, 2018
${}^{10}\text{Be}$	$(1.51 \pm 0.06) \times 10^6$ yr	August, 2015	${}^{12}\text{N}$	11.000 ± 0.016 ms	August, 2015
${}^{11}\text{Li}$	8.74 ± 0.15 ms	August, 2015	${}^{13}\text{N}$	9.967 ± 0.005 min	July, 2015

+ . . .

<http://www.tunl.duke.edu/nucldata/HalfLife.shtml>

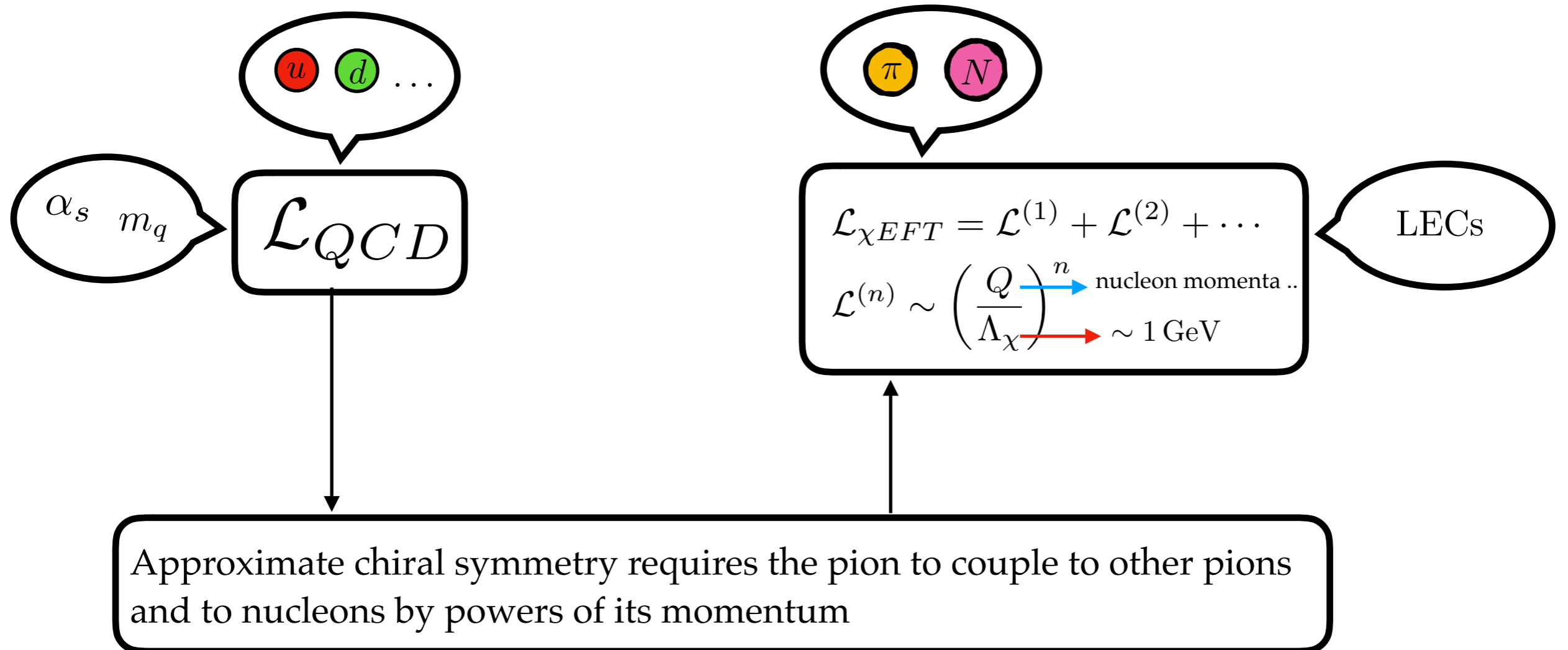
Lattice QCD (non perturbative method)

Effective field theories (expansion in kinematic variables)

χ EFT

- General slide on workflow

Build the most general Lagrangian with hadronic d.o.f. with the same exact symmetries and approximate symmetries of the underlying theory



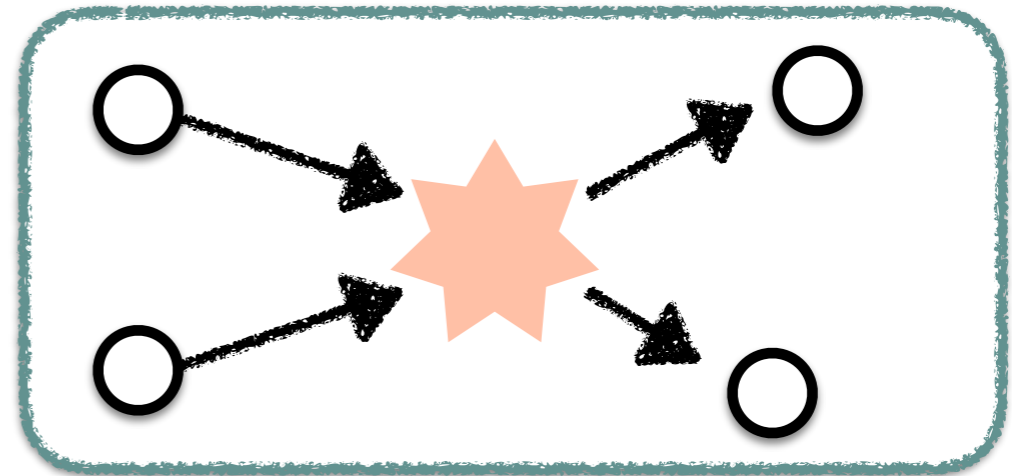
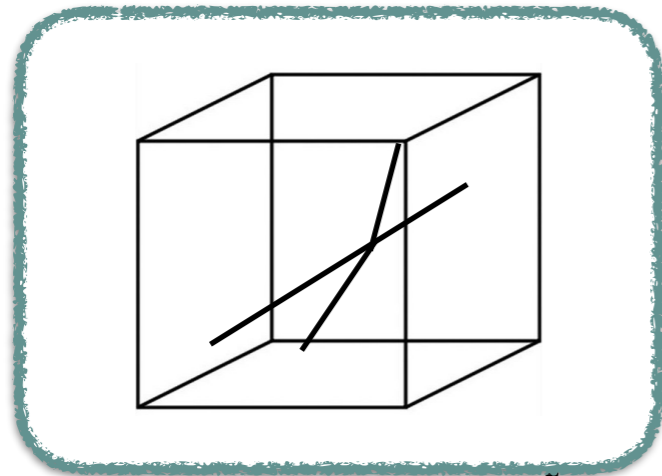
- S. Weinberg (1968-1979)

- Slide on LQCD generalities (only one is enough)

- Slide on (why we need) a FV formalism to deal with stuff

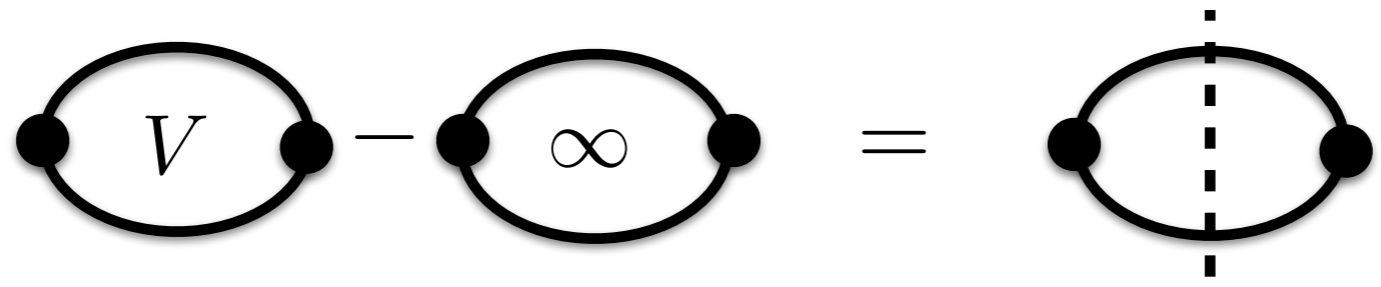
2→2

- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds



$$\det \left[F^{-1}(E_L, L) + M(E_L) \right] = 0$$

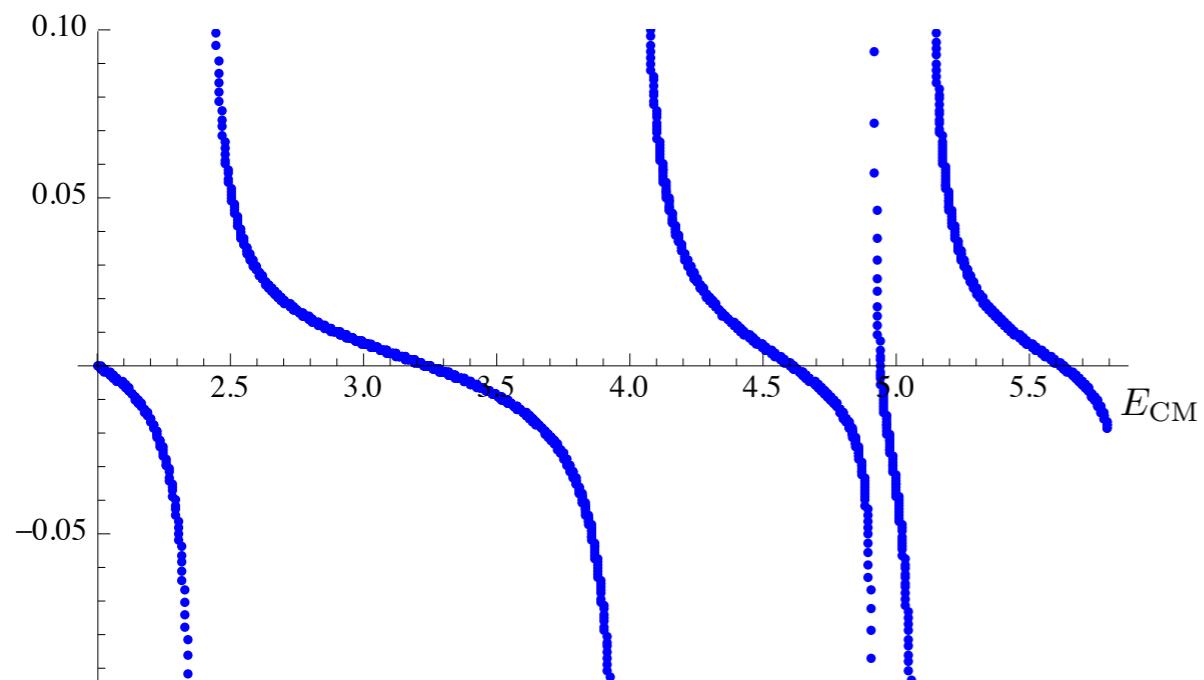
- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]



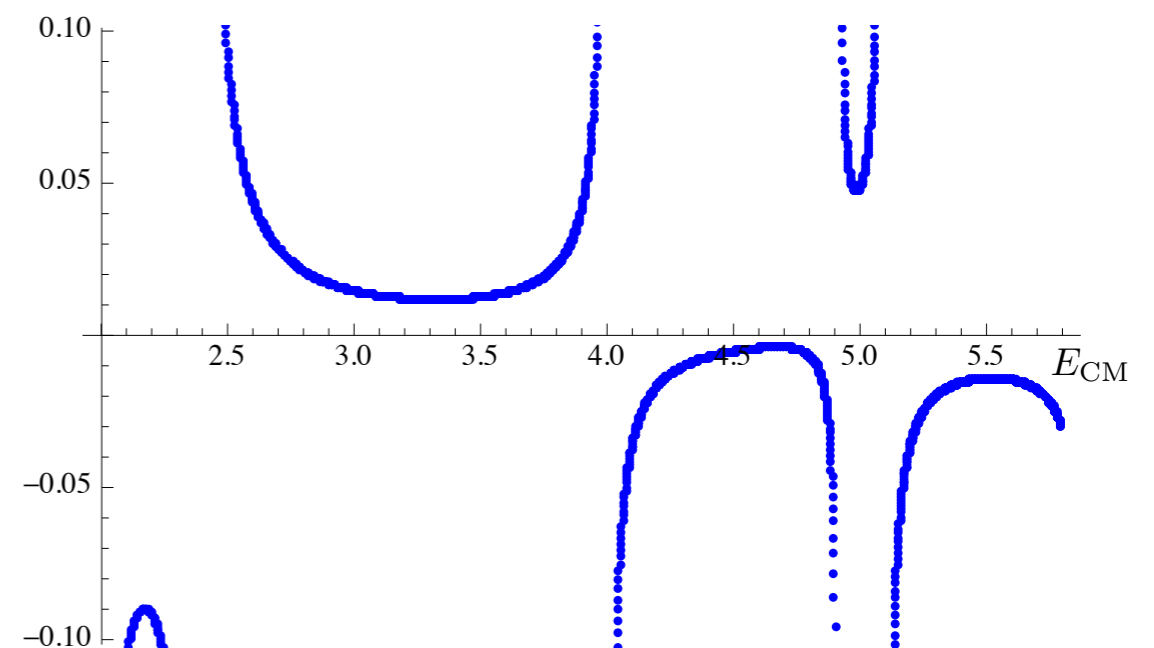
$$F_{l m, l' m'}(E_L, L) = \text{bubble}(V) - \text{bubble}(\infty) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots)$$

$$\mathbf{P} = [0, 0, 2\pi/L], \quad m_\pi L = 4$$

Re $F_{00,00}$



Re $F_{00,20}$

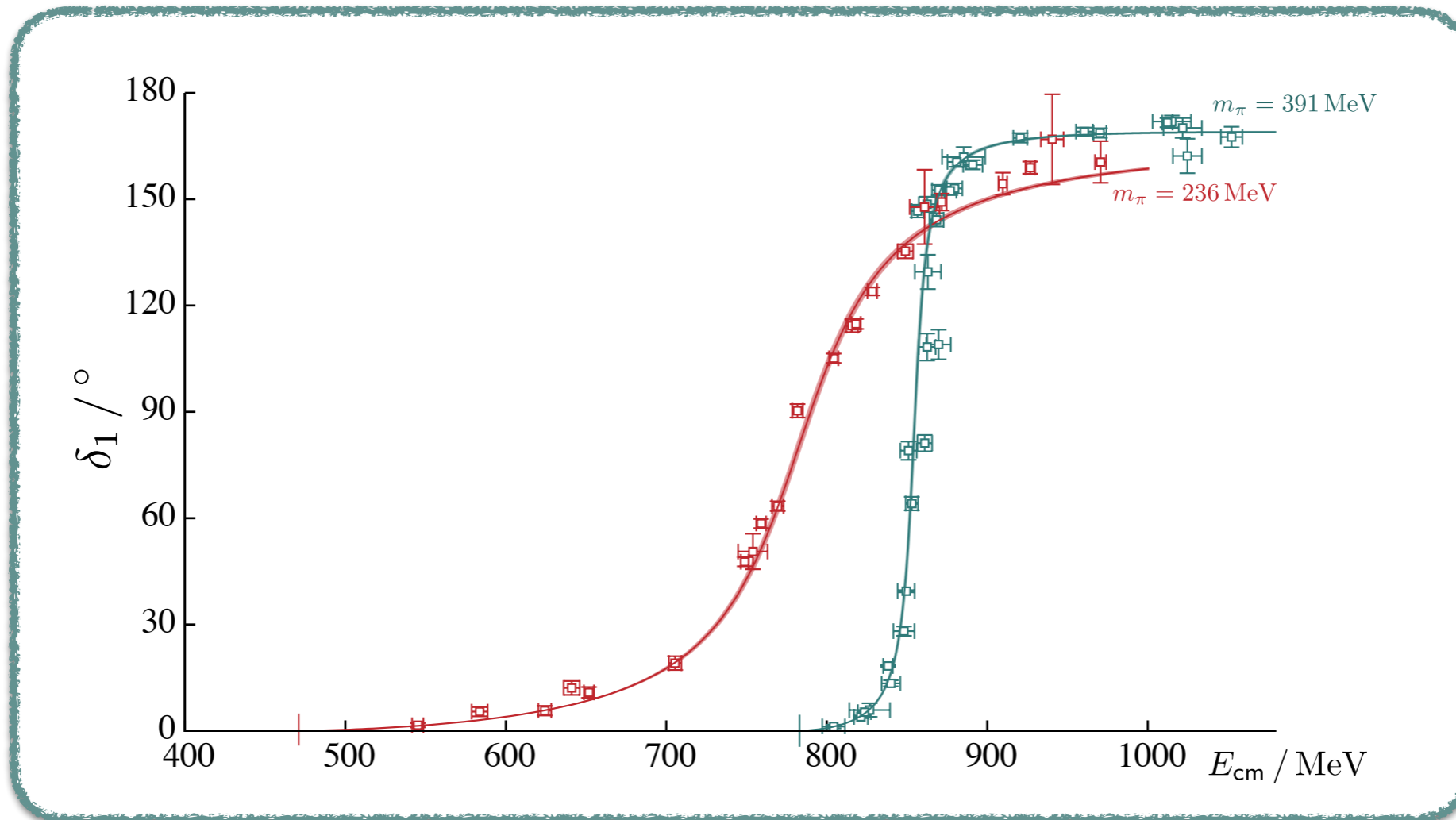


Gotta explain why the only thing we care about is this bubble (series can be resummed)

- Sketch the derivation...

- Sketch the derivation...

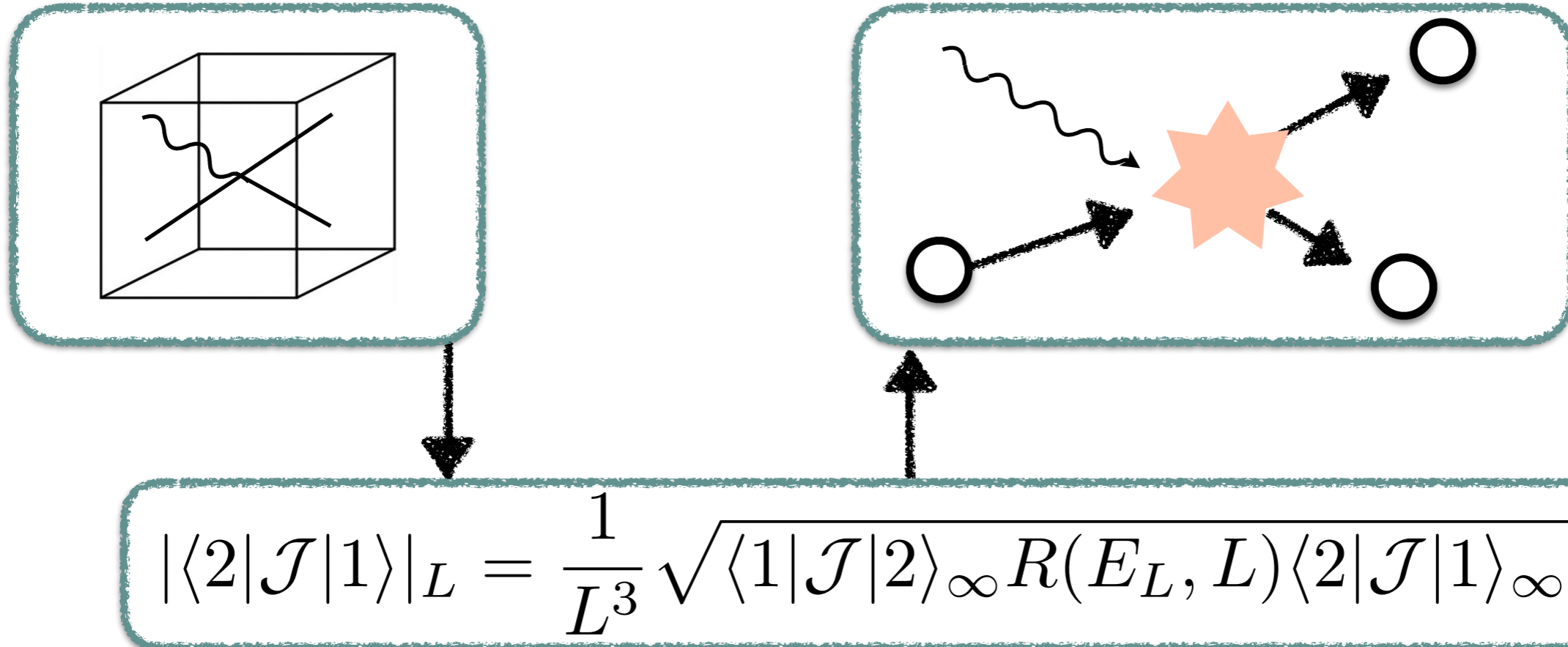
$2 \rightarrow 2$



● Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$$1 + \mathcal{J} \rightarrow 2$$

FV matrix elements to infinite volume electroweak amplitudes

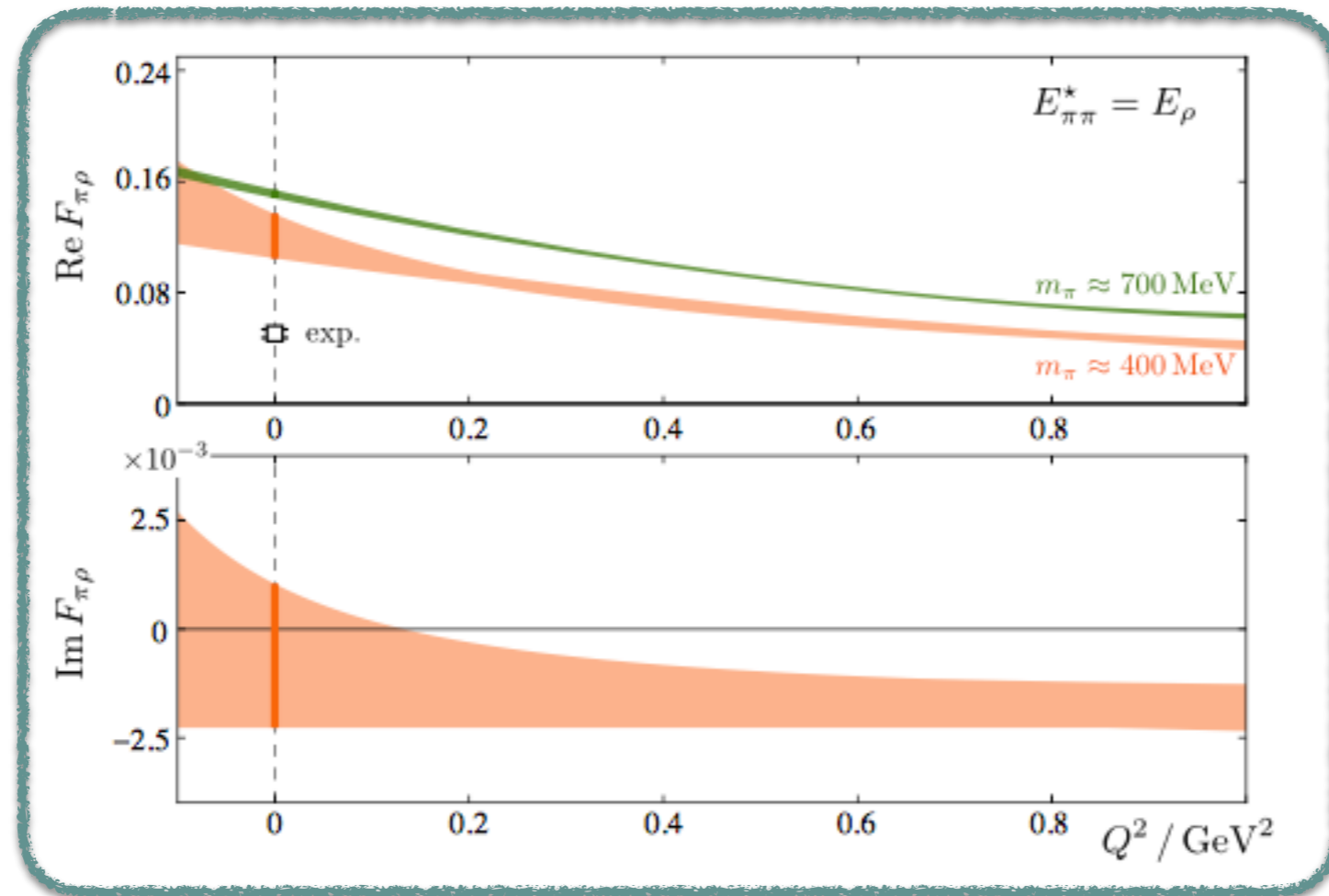


- Lellouch & Lüscher (2000) [K-to- $\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to- $\pi\pi$]
- Hansen & Sharpe (2012) [D-to- $\pi\pi$ / KK]
- Briceño, Hansen Walker-Loud / Briceño & Hansen (2014-2015) [general 1-to-2]

Explain ingredients
what's R?

Explain finite volume matrix elements

$$1 + \mathcal{J} \rightarrow 2$$

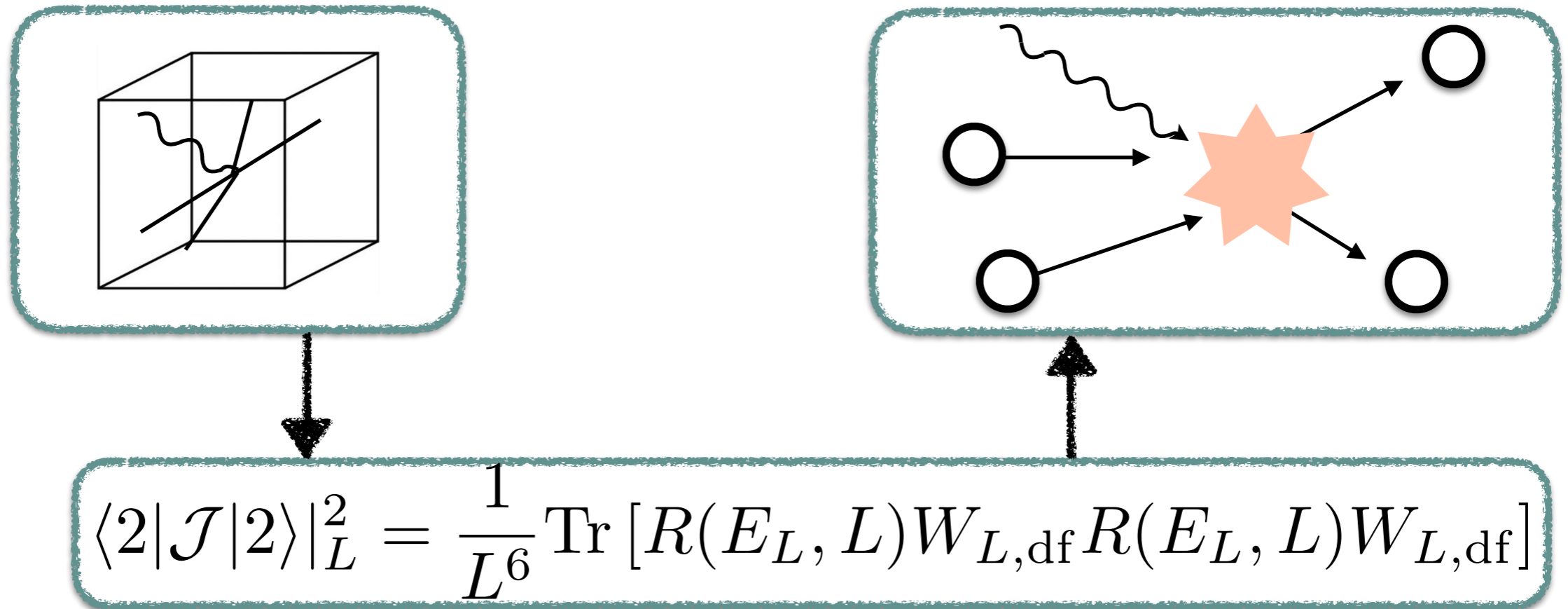


Understand carefully
transition form factors

Briceño, Dudek, Edwards, Shultz, Thomas and Wilson PRL (2016)

$$2 + \mathcal{J} \rightarrow 2$$

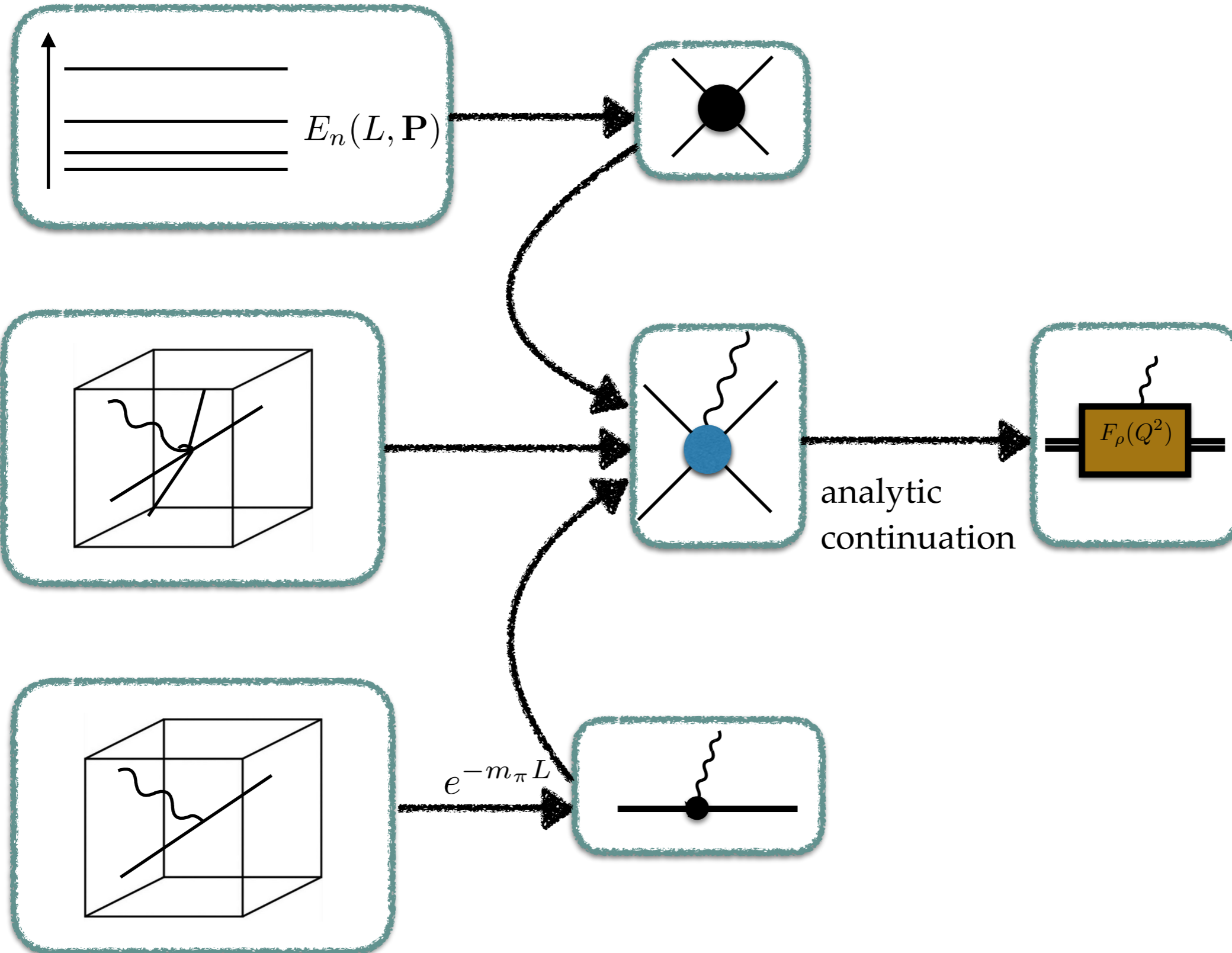
FV matrix elements to infinite volume electroweak amplitudes



● Briceño & Hansen (2016)

Workflow

Explain that this stuff is already extrapolated at zero lattice spacing



$2 + \mathcal{J} \rightarrow 2$

$$\langle 2 | \mathcal{J} | 2 \rangle_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + MG(L, w)M$$

$$W_{\text{df}} = \text{Diagram} = \text{Diagram} - \text{Diagram} - \text{Diagram} - \dots$$

The diagram shows the definition of W_{df} as a sum of diagrams. The first diagram is a blue circle with four external lines and a wavy line. This is equal to a series of diagrams: a black circle with four external lines and a wavy line, minus a black circle with four external lines, a wavy line, and a small black dot, minus a black circle with four external lines and a wavy line with a small black dot, minus an ellipsis.

$$w = \text{Diagram}$$

The diagram shows w as a horizontal line with a wavy line attached to a small black dot on the line.

$$M = \text{Diagram}$$

The diagram shows M as a black circle with four external lines.

$$G(L, w) = \text{Diagram} - \text{Diagram} = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots)$$

The diagram shows $G(L, w)$ as the difference between two diagrams. The first diagram is a loop with two external lines and a wavy line, labeled V . The second diagram is a loop with two external lines and a wavy line, labeled ∞ . Below this, the expression is given as $\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots)$.

explain (or at least know) where
these equations are coming from

A detail

$$W_{df} = \text{[Blue vertex]} = \text{[Black vertex]} - \text{[Black vertex with wavy line]} - \text{[Black vertex with wavy line]} - \dots$$

Kinematic poles cancel

But dynamical poles associated with resonances can still appear

$$\lim_{E_i^{cm}, E_f^{cm} \rightarrow E_R} \text{[Blue vertex]} = \text{[Black vertex]} = \text{[Resonance diagram]}$$

In the resonance region

Outline

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

- The sum is “easy”
- The integral is highly not trivial (spectator particle goes on-shell)
 - integrand singularities are two surfaces in three-dimension
 - standard principal value techniques in one dimension fail
 - techniques from other fields are not suitable
 - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D **smooth** integral

AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)

Give details I

Discuss implementation

Give details II

Discuss implementation

Give details III

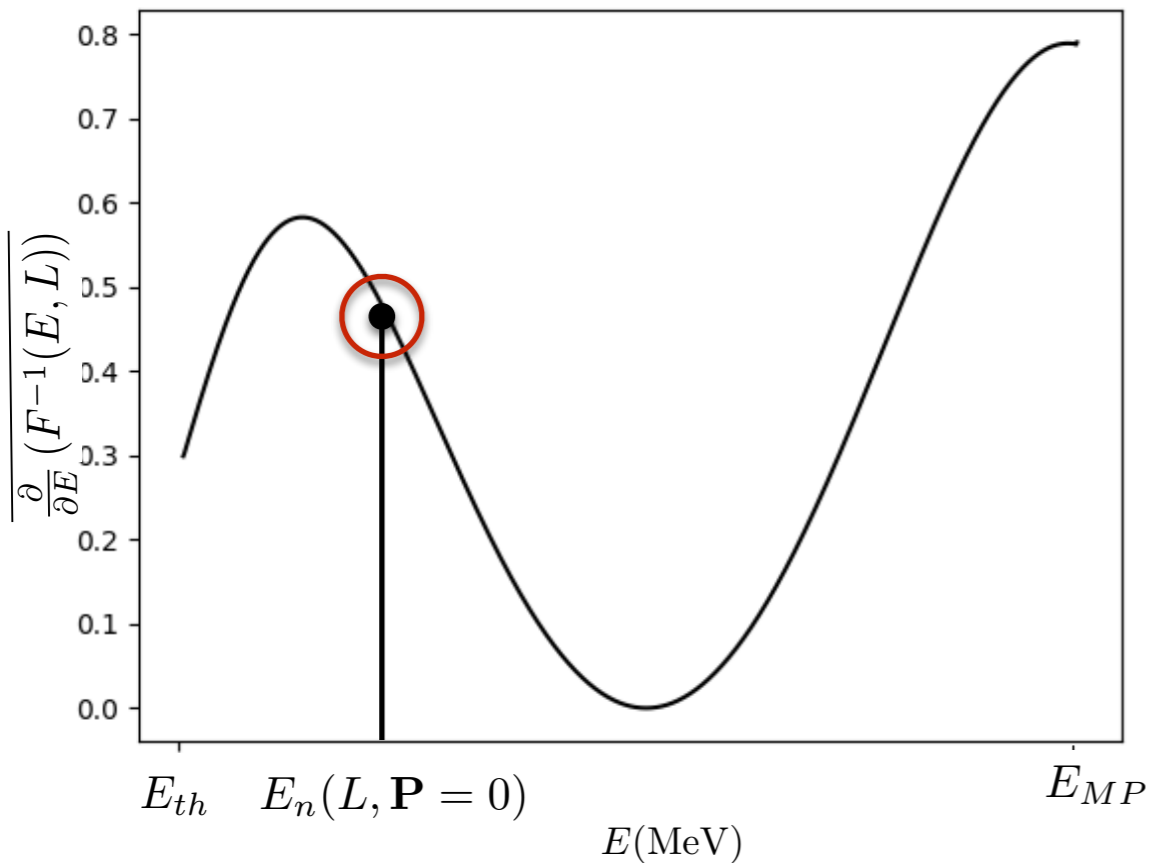
Discuss implementation

Kinematic functions I

$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E} (F^{-1}(E, L) + M(E))} \Big|_{E=E_L}$$

$$m_\pi L = 4$$

$$G_{l_i m_i, l_f m_f}$$

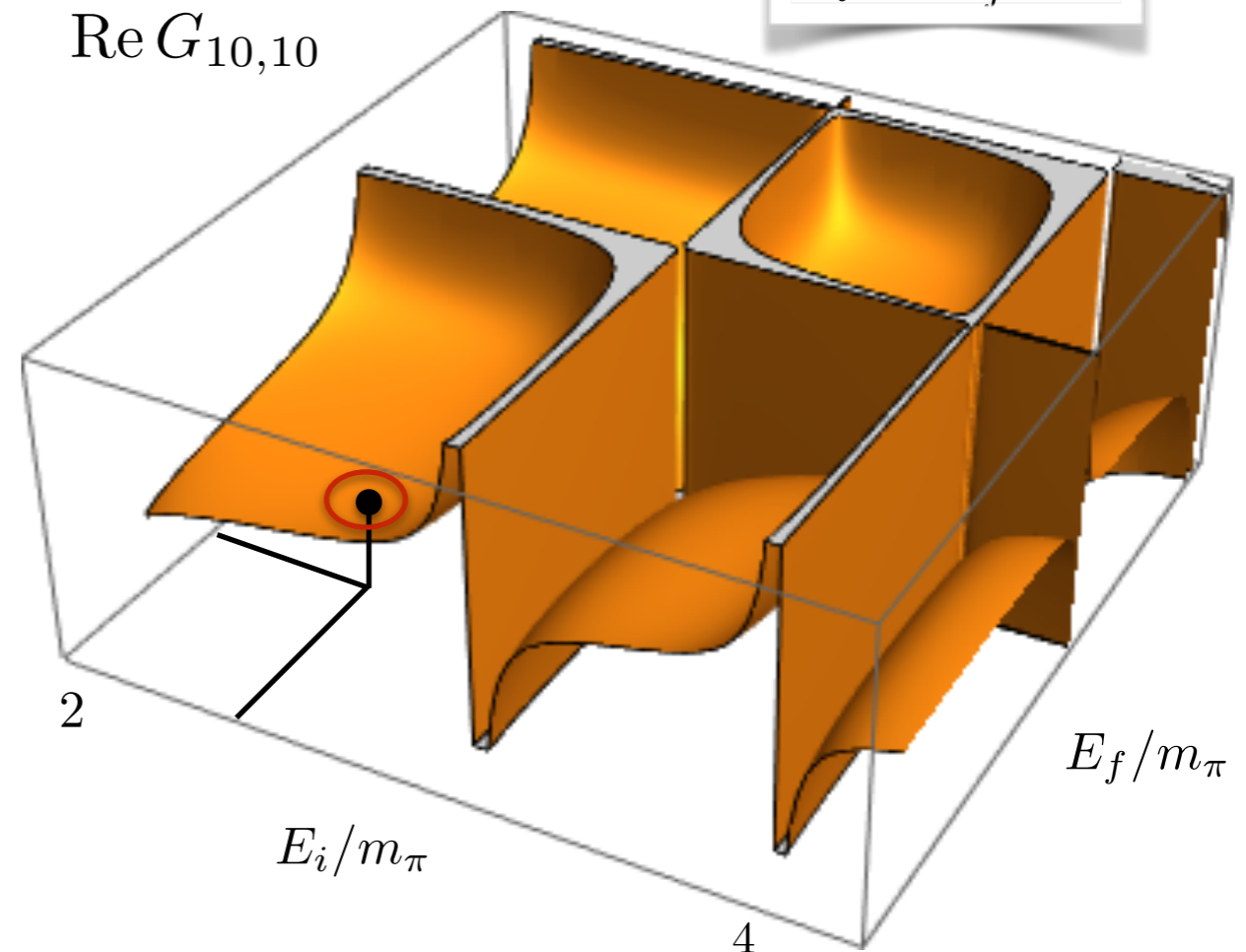


E_{th} = threshold energy

E_{MP} = multiparticle states energy

$\text{Re } G_{10,10}$

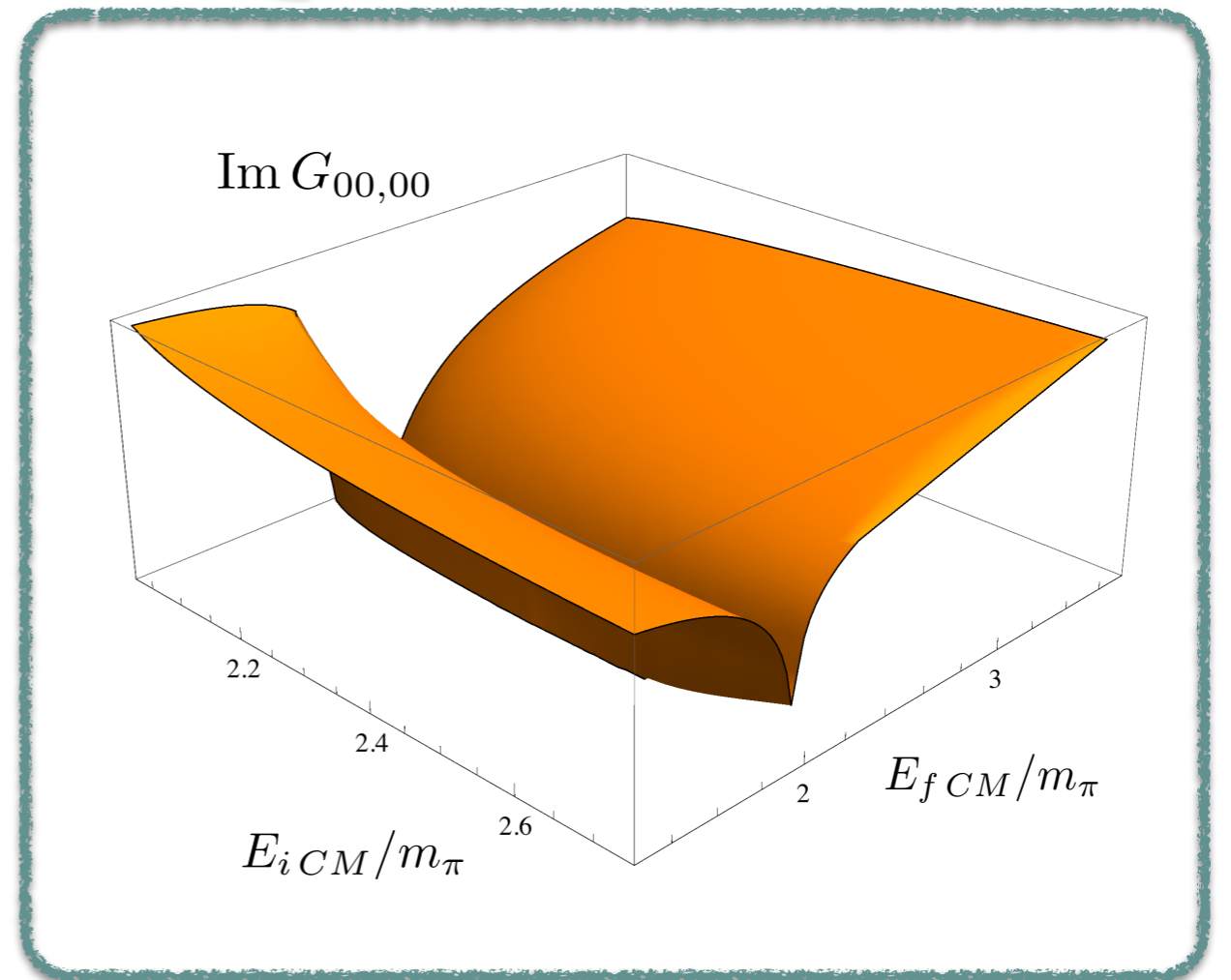
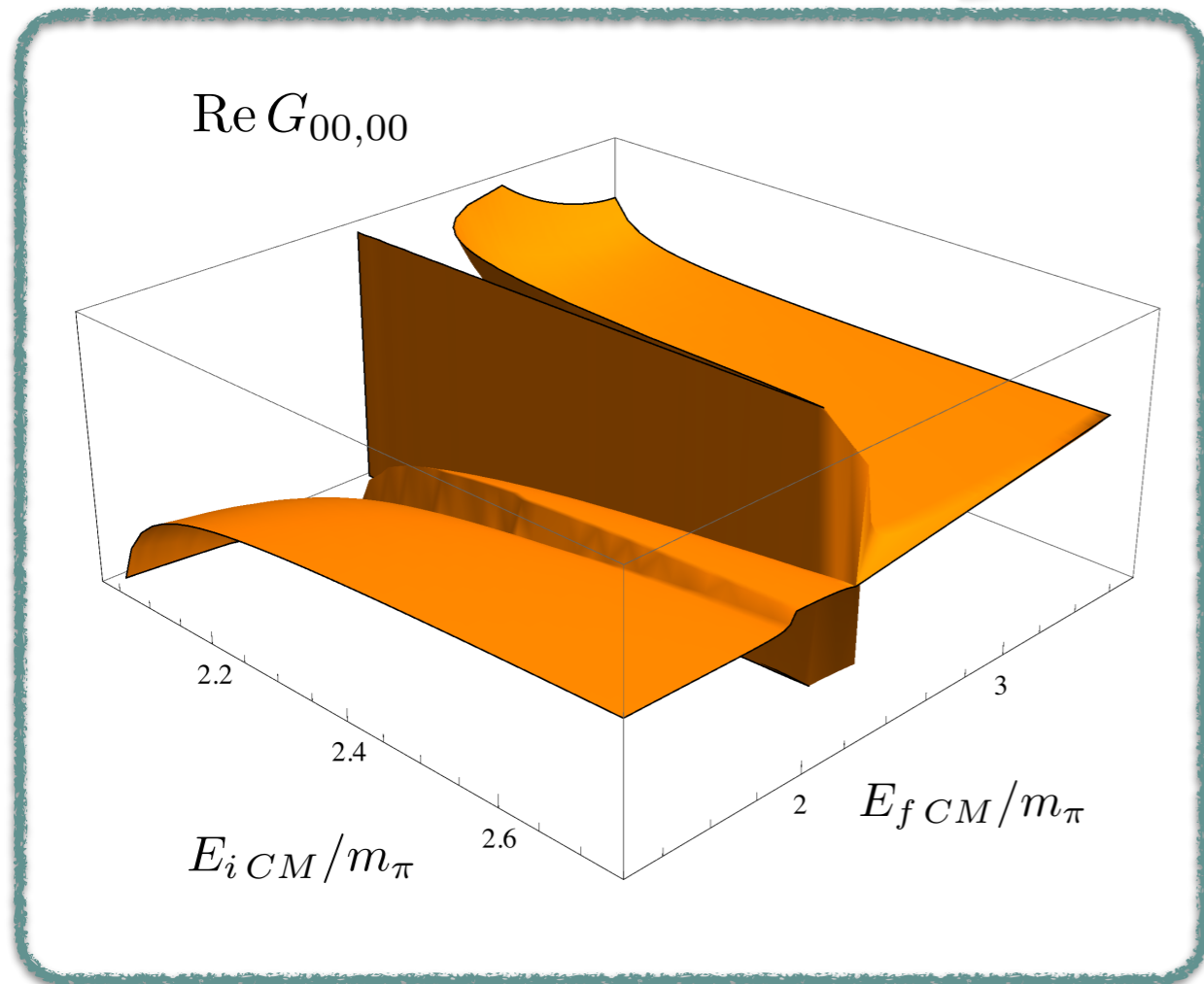
$$\mathbf{P}_i = \mathbf{P}_f = 0$$



Singularities at free particle energies

Kinematic functions II

$$\mathbf{P}_i = \mathbf{0}, \quad \mathbf{P}_f = [0, 0, 2\pi/L]$$
$$m_\pi L = 4$$



More plots in



AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
In preparation

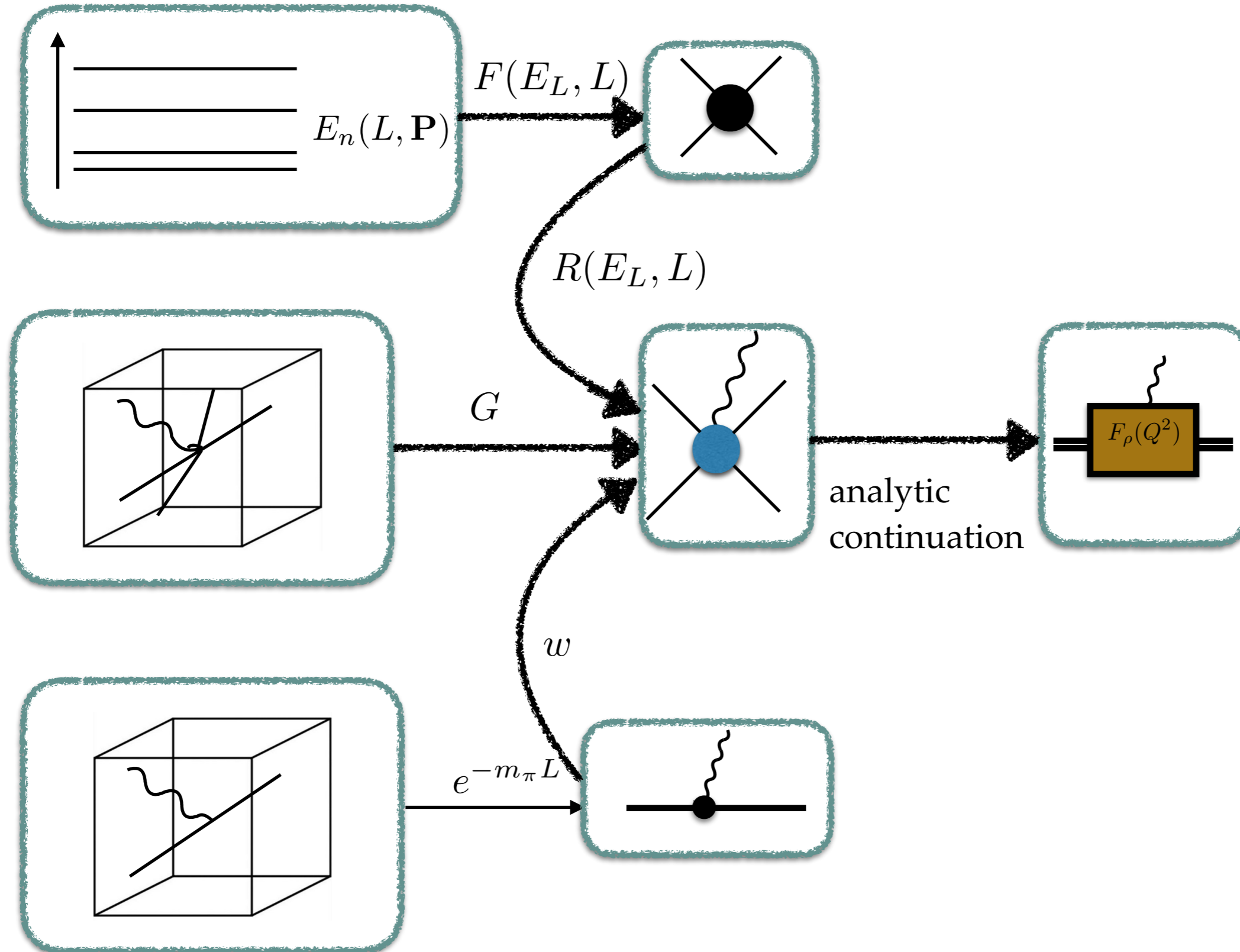
Discuss about opening
challenges

Discuss about Lorentz
decomposition (maybe..)

Discuss about other stuff for
which it could be useful

Thank you!

Workflow

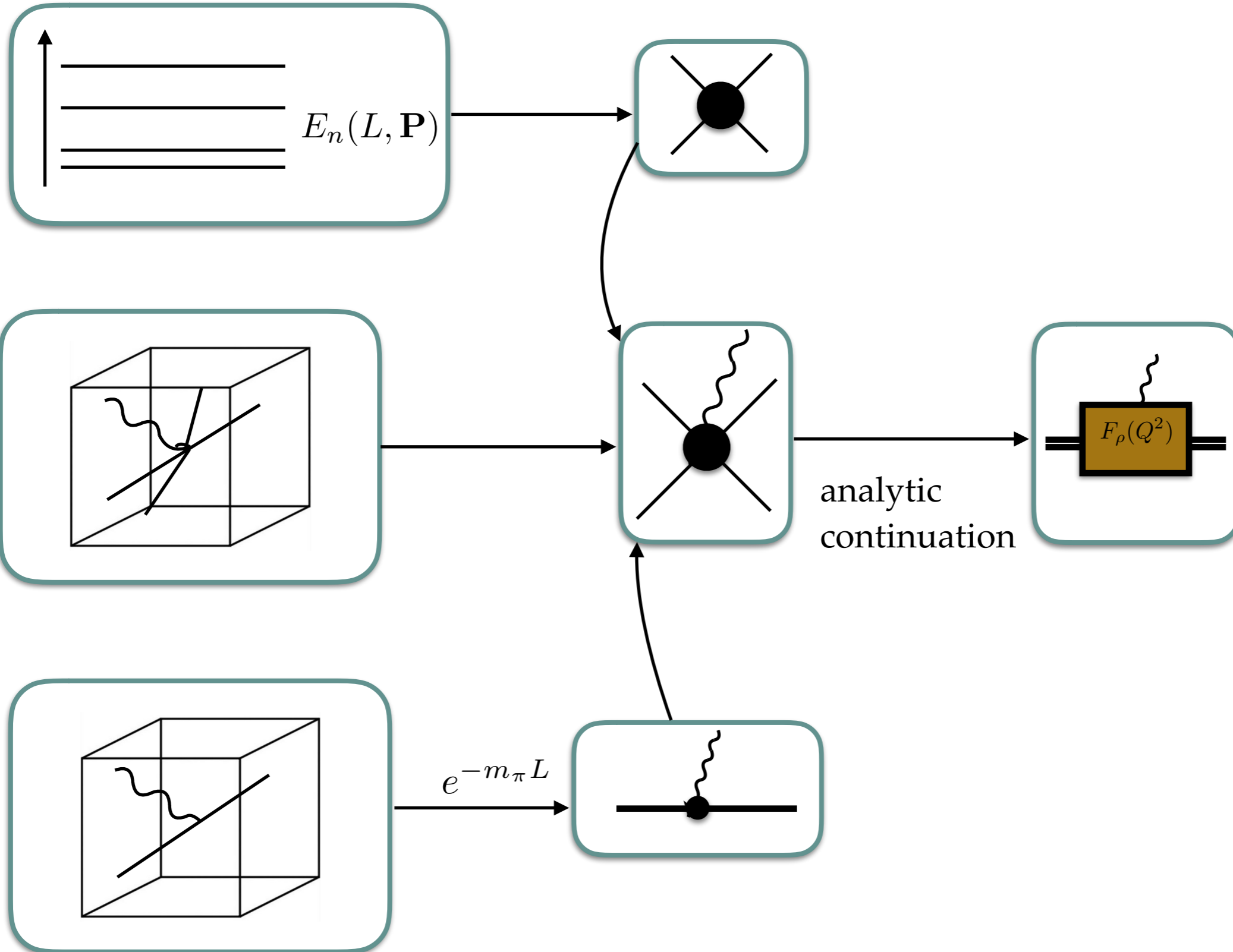


Some challenges

- A spin 1 particle between non-degenerate states has four form factors
- There is not a one-to-one mapping between matrix elements and amplitudes
 - Solved problem for spectrum analysis
- Analytical continuation of the amplitudes

Backup slides

Workflow 101



Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\text{FV}} \rightarrow \langle 2|\mathcal{J}|2\rangle_{\infty}$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves.....

- From $\langle 2|\mathcal{J}|2\rangle_{\infty}$ how do we get the four form factors?
 - **Analytic continuation**