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PHOTON IMPACT FACTOR AND k_T FACTORIZATION IN THE NEXT-TO-LEADING ORDER

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The photon impact factor for the BFKL pomeron is calculated in the next-to-leading order (NLO) approximation using the operator expansion in Wilson lines. The result is represented as a NLO k_T-factorization formula for the structure functions of small-x deep inelastic scattering.

Keywords: High energy; conformal invariance; wilson lines.

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1. Introduction

It is well known that the small-x behavior of structure functions of deep inelastic scattering is determined by the hard pomeron contribution. In the leading order the pomeron intercept is determined by the BFKL equation[1] and the pomeron residue (the $\gamma^*\gamma^*$-pomeron vertex) is given by the so-called impact factor. To find the small-x structure functions in the next-to-leading order, one needs to know both the pomeron intercept and the impact factor. The NLO pomeron intercept was found many years ago[2] but the analytic expression for the NLO impact factor is obtained for the first time in the present paper.

We calculate the NLO impact factor using the high-energy operator expansion of T-product of two vector currents in Wilson lines (see e.g the reviews [3, 4]). As a first step, we integrate over gluons with rapidities $Y > \eta$ and leave the integration over $Y < \eta$ for the later time. It is convenient to use the background field formalism: we integrate over gluons with $\alpha > \sigma = e^\eta$ and leave gluons with $\alpha < \sigma$ as a background field, to be integrated over later. Since the rapidities of the background gluons are very different from the rapidities of gluons in our Feynman diagrams, the background field can be taken in the form of a shock wave due to the Lorentz contraction. To derive the expression of a quark (or gluon) propagator in this shock-wave background we represent the propagator as a path integral over various trajectories, each of them weighed with the gauge factor $\mathrm{Pexp}(ig\int dx_\mu A^\mu)$ ordered along the propagation path. Now, since the shock wave is very thin, quarks (or gluons) do not have time to deviate in transverse direction so their trajectory inside the shock wave can be approximated by a segment of the straight line. Moreover, since there is no external field outside the shock wave, the integral over the segment of straight line can be formally extended to $\pm\infty$ limits yielding the Wilson-line gauge factor

$$U^\eta_x = \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} du p_1^\mu A^\sigma_\mu(up_1 + x_\perp)\Big],$$
$$A^\eta_\mu(x) = \int d^4k\theta(e^\eta - |\alpha_k|)e^{ik\cdot x}A_\mu(k), \tag{1}$$

where the Sudakov variable α_k is defined as usual, $k = \alpha_k p_1 + \beta_k p_2 + k_\perp$. We define the light-like vectors p_1 and p_2 such that $q = p_1 - x_B p_2$ and $p = p_2 + \frac{m_N^2}{s}p_1$ where q is the virtual photon momentum, p is the momentum of the target particle, and $x_B = Q^2/s \ll 1$ is the Bjorken variable (at large energies $s \simeq 2p\cdot q$). The structure of the propagator in a shock-wave background looks as follows:

$\big[$Free propagation from initial point x to the point of intersection with the shock wave $z\big]$
$\times$ $\big[$Interaction with the shock wave described by the Wilson-line operator $U_z\big]$
$\times$ $\big[$Free propagation from point of interaction z to the final point $y\big]$.

The explicit form of quark propagator in a shock-wave background can be taken from Ref. [5]

$$\langle T\{\hat\psi(x)\bar{\hat\psi}(y)\}\rangle_A \stackrel{x_*>0>y_*}{=} -\int d^4z\delta(z_*)\frac{(\not x - \not z)}{2\pi^2(x-z)^4}\not p_2 U_z\frac{(\not z - \not y)}{2\pi^2(x-z)^4}. \tag{2}$$

As usual, we label operators by hats and $\langle\hat{\mathcal{O}}\rangle_A$ means the vacuum average of the operator $\hat{\mathcal{O}}$ in the presence of an external field A. Hereafter use the notations $x_* = p_2^\mu x_\mu = \frac{\sqrt{s}}{2}x^+$, $x_\bullet = p_1^\mu x_\mu = \frac{\sqrt{s}}{2}x^-$ (and our metric is $(1,-1,-1,-1)$). Note that the Regge limit in the coordinate space can be achieved by rescaling

$$x \to \rho x_*\frac{2}{s}p_1 + x_\bullet\frac{2}{s\rho}p_2 + x_\perp, \quad y \to \rho y_*\frac{2}{s}p_1 + y_\bullet\frac{2}{s\rho}p_2 + y_\perp \tag{3}$$

with $\rho\to\infty$, see the discussion in Refs. [6, 7].

The result of the integration over gluons with rapidities $Y > \eta$ gives the impact factor — the amplitude of the transition of virtual photon in two-Wilson-lines operators (sometimes called "color dipole"). The LO impact factor is a product of two propagators (2),

$$\langle T\{\bar{\hat\psi}(x)\gamma^\mu\hat\psi(x)\bar{\hat\psi}(y)\gamma^\nu\hat\psi(y)\}\rangle_A = \frac{s^2}{2^9\pi^6x_*^2y_*^2}\int d^2z_{1\perp}d^2z_{2\perp}\frac{\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}}{(\kappa\cdot\zeta_1)^3(\kappa\cdot\zeta_2)^3}$$
$$\times\frac{\partial^2}{\partial x^\mu\partial y^\nu}\big[2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \kappa^2(\zeta_1\cdot\zeta_2)\big] + O(\alpha_s) \tag{4}$$

Here we introduced the conformal vectors[8, 9]

$$\kappa = \kappa_x - \kappa_y, \quad \kappa_x = \frac{\sqrt{s}}{2x_*}(\frac{p_1}{s} - x^2p_2 + x_\perp), \quad \zeta_i = \big(\frac{p_1}{s} + z_{i\perp}^2p_2 + z_{i\perp}\big) \tag{5}$$

and the notation $\mathcal{R} \equiv \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$. The above equation is explicitly Möbius invariant. In addition, it is easy to check that $\frac{\partial}{\partial x_\mu}$(r.h.s)=0.

Our goal is the NLO contribution to the r.h.s. of Eq. (4), but first let us briefly discuss the three remaining steps of the high-energy OPE. The evolution equation for color dipoles has the form[5, 10]

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\} = \frac{\alpha_s}{2\pi^2}\int d^2z_3\frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\} \times \mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}] + \mathrm{NLOcontribution} \tag{6}$$

(To save space, hereafter z_i stand for $z_{i\perp}$ so $z_{12}^2 \equiv z_{12\perp}^2$ etc.) The explicit form of the NLO contributions can be found in Refs. [11, 12, 4] while the agrument of the coupling constant in the above equation (following from the NLO calculations) is discussed in Refs. [13, 14].

It is worth noting that we performed the OPE program outlined above for scattering of scalar "particles" in $\mathcal{N}=4$ SYM and obtained the explicit expression for the four-point correlator of scalar operators at high energies in the next-to-leading order.[7] In QCD the analytic solution of the evolution equation for color dipoles with running coupling constant is not known at present. This prevents us from getting the explicit NLO amplitude as in $\mathcal{N}=4$ case. We can, however, perform the first two steps in our OPE program discussed in the Introduction: calculate the coefficient function (impact factor) and find the evolution equation for color dipoles. The next two steps, solution of the evolution equation (6) with appropriate initial conditions and the eventual comparison with experimental DIS data are discussed in many papers (see e.g. Ref. [15]). It is worth noting that, contrary to the evolution equation, the NLO correction to the impact factor has nothing to do with running of the coupling constant - it starts at the NNLO level. Thus, the argument of the coupling constant at the NLO level is determined solely by the evolution equation for color dipoles. For numerical estimates involving the impact factor one can take $\alpha_s(|x-y|)$ as the first approximation since the characteristic transverse distances in the impact factor are $\sim |x-y|$.

2. Calculation of the NLO Impact Factor

Now we would like to repeat the steps of operator expansion discussed above to the NLO accuracy. A general form of the expansion of T-product of the electromagnetic currents in color dipoles looks as follows:

$$(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\mathrm{LO}}_{\mu\nu}(z_1,z_2)\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\big[1+\frac{\alpha_s}{\pi}\big] + \int d^2z_3 I^{\mathrm{NLO}}_{\mu\nu}(z_1,z_2,z_3;\eta)[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\Big\} \tag{7}$$

Unfortunately, in terms of Wilson-line approach there is no direct way to get the NLO impact factor for the BFKL pomeron. One needs first to find the coefficient in front of the four-Wilson-line operator (which we will also call the NLO impact factor) and then linearize it.

The structure of the NLO contribution is clear from the topology of diagrams in the shock-wave background, see Fig. 1 below. Also, the term $\sim 1 + \frac{\alpha_s}{\pi}$ can be restored from the requirement that at $U = 1$ (no shock wave) one should get the perturbative series for the polarization operator $1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2)$.

In our notations

$$I^{\rm LO}_{\mu\nu}(z_1, z_2) = \frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\frac{\partial^2}{\partial x^\mu \partial y^\nu}\Big[(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\Big] \tag{8}$$

which corresponds to the well-known expression for the LO impact factor in the momentum space.

The NLO impact factor is given by the diagrams shown in Fig. 1. The calculation of these diagrams is performed in Ref. [16] (see also Ref.[12]) and the result is

$$\begin{aligned} I^{\rm NLO}_{\mu\nu}(z_1, z_2, z_3; \eta) &= I_1^{\mu\nu}(z_1, z_2, z_3; \eta) + I_2^{\mu\nu}(z_1, z_2, z_3), \\ I_1^{\mu\nu}(x, y; z_1, z_2, z_3; \eta) &= \frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}(z_1, z_2)\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\bigg[\int_0^\infty \frac{d\alpha}{\alpha} e^{i\alpha\frac{s}{4}\mathcal{Z}_3} - \int_0^\sigma \frac{d\alpha}{\alpha}\bigg] \\ &= -\frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\bigg[\ln\frac{\sigma s}{4}\mathcal{Z}_3 - \frac{i\pi}{2} + C\bigg] \end{aligned} \tag{9}$$

where C is the Euler constant and

$$\begin{aligned} (I_2)_{\mu\nu}(z_1, z_2, z_3) &= \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \\ &\times \bigg\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu \partial y^\nu}\bigg[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)} \\ &- \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\bigg] + \frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\times\frac{\partial^2}{\partial x^\mu \partial y^\nu}\bigg[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\bigg] + (\zeta_1 \leftrightarrow \zeta_2)\bigg\} \end{aligned} \tag{10}$$

(recall that $z_{ij\perp}^2 = 2(\zeta_i\cdot\zeta_j)$ and $\mathcal{Z}_i = \frac{4}{\sqrt{s}}(\kappa\cdot\zeta_i)$). The α integration in the NLO impact factor is cut from above by $\sigma = e^\eta$ in accordance with the definition of operators $\hat{U}^\eta$, see Eq. (1). Note that one should expect the NLO impact factor to be conformally invariant since it is determined by tree diagrams in Fig. 1. However, as discussed in Refs. 4, 7, 11, formally the light-like Wilson lines are conformally (Möbius) invariant but the longitudinal cutoff $\alpha < \sigma$ in Eq. (1) violates this property

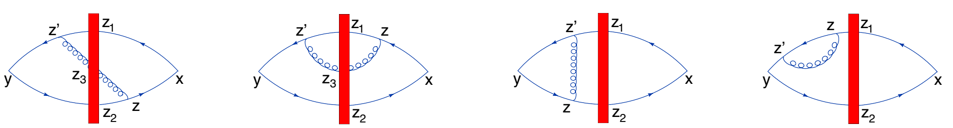

Fig. 1. Impact factor in the next-to-leading order.

so the term $\sim \ln\sigma\mathcal{Z}_3$ in the r.h.s. of Eq. (9) is not invariant. As was demonstrated in these papers, one can define a composite operator in the form

$$\begin{aligned}\big[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\big]_a &= \mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\} + \frac{\alpha_s}{2\pi^2}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_3}\} \\ &\quad\times \mathrm{tr}\{\hat{U}^\sigma_{z_3}\hat{U}^{\dagger\sigma}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{\sigma^2sz_{13}^2z_{23}^2} + O(\alpha_s^2), \qquad (11)\end{aligned}$$

where a is an arbitrary constant. It is convenient to choose the rapidity-dependent constant $a \to ae^{-2\eta}$ so that the $\big[\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}\big]_a^{\mathrm{conf}}$ does not depend on $\eta = \ln\sigma$ and all the rapidity dependence is encoded into a-dependence. Indeed, it is easy to see that $\frac{d}{d\eta}\big[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\big]_a^{\mathrm{conf}} = 0$ and $\frac{d}{da}\big[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\big]_a^{\mathrm{conf}}$ is determined by the NLO BK kernel which is a sum of the conformal part and the running-coupling part with our $O(\alpha_s^2)$ accuracy.[4,12]

Rewritten in terms of composite dipoles (11), the operator expansion (7) takes the form:

$$\begin{aligned}(x-y)^4T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} &= \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\mu\nu}_{\mathrm{LO}}(z_1,z_2)[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} \\ &\quad\times\big[1+\frac{\alpha_s}{\pi}\big] + \int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)} - 2C\Big)I^{\mu\nu}_{\mathrm{LO}} + I^{\mu\nu}_2\Big] \\ &\quad\times[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}. \qquad (12)\end{aligned}$$

Here the composite dipole $[\mathrm{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]_{a_0}$ is given by Eq. (11) with $a_0 = -\frac{4x_*y_*}{s(x-y)^2} + i\epsilon$ while $I^{\mu\nu}_{\mathrm{LO}}(z_1,z_2)$ and $I^{\mu\nu}_2(z_1,z_2,z_3)$ are given by Eqs. (8) and (10), respectively. The "new rapidity cutoff" a_0 is chosen in such a way that all the energy dependence is included in the matrix element(s) of Wilson-line operators so the impact factor should not depend on energy.

3. NLO Impact Factor for the BFKL Pomeron

For the studies of DIS with the linear NLO BFKL equation (up to two-gluon accuracy) we need the linearized version of Eq. (12). If we define

$$\hat{\mathcal{U}}_a(z_1,z_2) = 1 - \frac{1}{N_c}[\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a \qquad (13)$$

and consider the linearization

$$\frac{1}{N_c^2}\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}\mathrm{tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - \frac{1}{N_c}\mathrm{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} \simeq \hat{\mathcal{U}}(z_1,z_2) - \hat{\mathcal{U}}(z_1,z_3) - \hat{\mathcal{U}}(z_2,z_3)$$

one of the integrals over z_i in the r.h.s. of Eq. (12) can be performed. The result is

$$\begin{aligned}&\frac{1}{N_c}(x-y)^4T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} \\ &\quad= \frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1dz_2}{z_{12}^4}\hat{\mathcal{U}}_{a_0}(z_1,z_2)\big[\mathcal{I}^{\mathrm{LO}}_{\alpha\beta}\big(1+\frac{\alpha_s}{\pi}\big) + \mathcal{I}^{\mathrm{NLO}}_{\alpha\beta}\big], \qquad (14)\end{aligned}$$

where

$$\mathcal{I}^{\alpha\beta}_{\rm LO}(x,y;z_1,z_2)=\mathcal{R}^2\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)-\zeta_1^\alpha\zeta_2^\beta-\zeta_2^\alpha\zeta_1^\beta}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \tag{15}$$

(see Eq. (8)) and

$$\begin{aligned}
\mathcal{I}^{\alpha\beta}_{\rm NLO}(x,y;z_1,z_2)=&\frac{\alpha_sN_c}{4\pi^7}\mathcal{R}^2\Big\{\frac{\zeta_1^\alpha\zeta_2^\beta+\zeta_1\leftrightarrow\zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[4{\rm Li}_2(1-\mathcal{R})-\frac{2\pi^2}{3}+\frac{2\ln\mathcal{R}}{1-\mathcal{R}}\\
&+\frac{\ln\mathcal{R}}{\mathcal{R}}-4\ln\mathcal{R}+\frac{1}{2\mathcal{R}}-2+2\big(\ln\frac{1}{\mathcal{R}}+\frac{1}{\mathcal{R}}-2\big)\big(\ln\frac{1}{\mathcal{R}}+2C\big)-4C-\frac{2C}{\mathcal{R}}\Big]\\
&+\Big(\frac{\zeta_1^\alpha\zeta_1^\beta}{(\kappa\cdot\zeta_1)^2}+\zeta_1\leftrightarrow\zeta_2\Big)\times\Big[\frac{\ln\mathcal{R}}{\mathcal{R}}-\frac{2C}{\mathcal{R}}+2\frac{\ln\mathcal{R}}{1-\mathcal{R}}-\frac{1}{2\mathcal{R}}\Big]\\
&+\Big[\frac{\zeta_1^\alpha\kappa^\beta+\zeta_1^\beta\kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2}+\zeta_1\leftrightarrow\zeta_2\Big]\Big[-2\frac{\ln\mathcal{R}}{1-\mathcal{R}}-\frac{\ln\mathcal{R}}{\mathcal{R}}+\ln\mathcal{R}-\frac{3}{2\mathcal{R}}+\frac{5}{2}+2C+\frac{2C}{\mathcal{R}}\Big]\\
&-\frac{2}{\kappa^2}\Big(g^{\alpha\beta}-2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big)+\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[6\ln\mathcal{R}+\frac{2\pi^2}{3}-4{\rm Li}_2(1-\mathcal{R})\\
&-2\big(\ln\frac{1}{\mathcal{R}}+\frac{1}{\mathcal{R}}+\frac{1}{2\mathcal{R}^2}-3\big)\big(\ln\frac{1}{\mathcal{R}}+2C\big)-\frac{2}{\mathcal{R}}+2+\frac{3}{2\mathcal{R}^2}\Big]\Big\},
\end{aligned} \tag{16}$$

where ${\rm Li}_2(z)$ is the dilogarithm. Here one easily recognizes five conformal tensor structures discussed in Ref. [19].

4. Photon Impact factor in the Mellin representation

In preparation for Fourier transformation we calculated the Mellin transform of the photon impact factor (16). We project the impact factor on the conformal eigenfunctions of the BFKL equation[20]

$$E_{\nu,n}(z_{10},z_{20})=\Big[\frac{\tilde{z}_{12}}{\tilde{z}_{10}\tilde{z}_{20}}\Big]^{\frac12+i\nu+\frac{n}{2}}\Big[\frac{\bar{z}_{12}}{\bar{z}_{10}\bar{z}_{20}}\Big]^{\frac12+i\nu-\frac{n}{2}} \tag{17}$$

(here $\tilde{z}=z_x+iz_y,\bar{z}=z_x-iz_y$, $z_{10}\equiv z_1-z_0$ etc.). Since electromagnetic currents are vectors, the only non-vanishing contribution comes from projection on the eigenfunctions with spin 0 and spin 2. In this paper we will present only spin-0 results. The spin-0 projection has the form[23] (throughout the paper we reserve the notation γ for $\frac12+i\nu$):

$$\begin{aligned}
&\big(1+\frac{\alpha_s}{\pi}\big)\mathcal{J}^{\rm LO}_{\alpha\beta}(x,y;z_0,\nu)+\mathcal{J}^{\rm NLO}_{\alpha\beta}(x,y;z_0,\nu)\\
&\quad=\int\frac{d^2z_1d^2z_2}{z_{12}^4}\frac{\partial\kappa^\lambda}{\partial x^\alpha}\frac{\partial\kappa^\rho}{\partial y^\beta}\Big[\big(1+\frac{\alpha_s}{\pi}\big)\mathcal{I}^{\rm LO}_{\lambda\rho}(x,y;z_1,z_2)\\
&\qquad+\mathcal{I}^{\rm NLO}_{\lambda\rho}(x,y;z_1,z_2)\Big]\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^\gamma
\end{aligned}$$

$$
\begin{aligned}
&= \frac{B(\bar\gamma,\bar\gamma)}{4\pi^4}\Gamma(\gamma+1)\Gamma(2-\gamma)\Big\{-\frac{\gamma\bar\gamma}{3}(2S_1+S_2)_{\mu\nu}\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_1(\gamma)\Big]\\
&\quad -2S_{2\mu\nu}\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_2(\gamma)\Big]+2\gamma(S_2-S_3)_{\mu\nu}\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_3(\gamma)\Big]\\
&\quad +\frac{\bar\gamma\gamma^2}{(3-2\gamma)}\big(-\frac{1}{3}S_1-\frac{2}{3}S_2+S_3-2S_4\big)_{\mu\nu}\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_4(\gamma)\Big]\\
&\quad +(S_1+S_2)_{\mu\nu}(2+\bar\gamma\gamma)\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_5(\gamma)\Big]\Big\}\Big(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\Big)^{\gamma}, \qquad (18)
\end{aligned}
$$

where $\gamma \equiv \frac{1}{2}+i\nu$, $\bar\gamma \equiv 1-\gamma = \frac{1}{2}-i\nu$ and

$$
\begin{aligned}
&F_1(\gamma) = F(\gamma)+\frac{\chi_\gamma}{\gamma\bar\gamma}, F_2(\gamma) = F(\gamma)-1+\frac{1}{2\gamma\bar\gamma}+\chi_\gamma, F_3(\gamma) = F(\gamma)+\frac{\chi_\gamma}{2},\\
&F_4(\gamma) = F(\gamma)-\frac{6}{\gamma\bar\gamma}+\frac{3}{\gamma^2\bar\gamma^2}-\frac{2\chi_\gamma}{\gamma\bar\gamma}, F_5(\gamma) = F(\gamma)+\frac{3\bar\gamma\gamma\chi_\gamma+1-2\bar\gamma\gamma}{\gamma\bar\gamma(2+\bar\gamma\gamma)},\\
&F(\gamma) = \frac{2\pi^2}{3}+1-\frac{2\pi^2}{\sin^2\pi\gamma}-2C\chi_\gamma+\frac{\chi_\gamma-2}{\bar\gamma\gamma} \qquad (19)
\end{aligned}
$$

and

$$
\begin{aligned}
&S_1^{\mu\nu} \equiv \frac{\partial^2 \ln\kappa^2}{\partial x_\mu \partial y_\nu}, S_2^{\mu\nu} \equiv \frac{\partial \ln\kappa^2}{\partial x_\mu}\frac{\partial \ln\kappa^2}{\partial y_\nu}, \qquad (20)\\
&S_3^{\mu\nu} \equiv \frac{\partial \ln\kappa^2}{\partial x_\mu}\frac{\partial \ln\kappa\cdot\zeta_0}{\partial y_\nu}+\frac{\partial \ln\kappa\cdot\zeta_0}{\partial x_\mu}\frac{\partial \ln\kappa^2}{\partial y_\nu}, S_4^{\mu\nu} \equiv \frac{\partial \ln\kappa\cdot\zeta_0}{\partial x_\mu}\frac{\partial \ln\kappa\cdot\zeta_0}{\partial y_\nu}.
\end{aligned}
$$

The contribution of spin 2 in the t-channel has the form

$$
\begin{aligned}
&\big(1+\frac{\alpha_s}{\pi}\big)\mathcal{J}^{\rm LO}_{2,\alpha\beta}(x,y;z_0,\nu)+\mathcal{J}^{\rm NLO}_{2,\alpha\beta}(x,y;z_0,\nu)\\
&\quad = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\Big[\big(1+\frac{\alpha_s}{\pi}\big)\mathcal{I}^{\rm LO}_{\alpha\beta}(x,y;z_1,z_2)+\mathcal{I}^{\rm NLO}_{\alpha\beta}(x,y;z_1,z_2)\Big]\\
&\qquad \times\Big(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\Big)^{\gamma}\frac{\tilde z_{12}}{\tilde z_{10}\tilde z_{20}}\frac{\bar z_{10}\bar z_{20}}{\bar z_{12}}\\
&\quad = -\frac{1}{2\pi^4(x-y)^2}B(2-\gamma,2-\gamma)\Gamma(\gamma+2)\Gamma(3-\gamma)\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_6(\gamma)\Big]S_5^{\mu\nu}, \qquad (21)
\end{aligned}
$$

where

$$
F_6(\gamma) = F(\gamma)+\frac{2C}{\bar\gamma\gamma}-\frac{2}{\bar\gamma\gamma}+\frac{2}{\bar\gamma^2\gamma^2}+3\frac{1+\chi_\gamma-\frac{1}{\gamma\bar\gamma}}{2+\bar\gamma\gamma}-\frac{\chi_\gamma}{\bar\gamma\gamma} \qquad (22)
$$

and

$$
\begin{aligned}
S_5^{\mu\nu} \equiv &\Big[g^{\mu 1}-ig^{\mu 2}-2(x-z_0)^\mu\frac{\partial}{\partial\tilde z_0}\ln\kappa\cdot\zeta_0+\frac{4p_2^\mu}{\sqrt{s}}\frac{(\kappa_x\cdot\zeta_0)(\kappa_y\cdot\zeta_0)}{(\kappa\cdot\zeta_0)}\frac{\partial}{\partial\tilde z_0}\ln\frac{\kappa_x\cdot\zeta_0}{\kappa_y\cdot\zeta_0}\Big]\\
&\times\Big[g^{\nu 1}-ig^{\nu 2}-2(y-z_0)^\nu\frac{\partial}{\partial\tilde z_0}\ln\kappa\cdot\zeta_0+\frac{4p_2^\nu}{\sqrt{s}}\frac{(\kappa_x\cdot\zeta_0)(\kappa_y\cdot\zeta_0)}{(\kappa\cdot\zeta_0)}\frac{\partial}{\partial\tilde z_0}\ln\frac{\kappa_x\cdot\zeta_0}{\kappa_y\cdot\zeta_0}\Big]
\end{aligned}
$$

Using the decomposition of the product of the transverse δ-functions in conformal 3-point functions (17)

$$\delta^{(2)}(z_1-z_3)\delta^{(2)}(z_2-z_4)=\sum_{n=-\infty}^{\infty}\int\frac{d\nu}{\pi^4}\frac{\nu^2+\frac{n^2}{4}}{z_{12}^2z_{34}^2}\times\int d^2z_0E_{\nu,n}(z_{10},z_{20})E^*_{\nu,n}(z_{30},z_{40}) \tag{23}$$

we obtain

$$\hat{\mathcal{U}}(z_1,z_2)=\int\frac{d\nu}{\pi^2}\int d^2z_0\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{\gamma}\Big\{\nu^2\hat{\mathcal{U}}(z_0,\nu)+(\nu^2+1)\Big[\frac{\tilde{z}_{12}\bar{z}_{10}\bar{z}_{20}}{\tilde{z}_{10}\tilde{z}_{20}\bar{z}_{12}}\hat{\bar{\mathcal{U}}}^{(2)}(z_0,\nu)+\frac{\bar{z}_{12}\tilde{z}_{10}\tilde{z}_{20}}{\bar{z}_{10}\bar{z}_{20}\tilde{z}_{12}}\hat{\tilde{\mathcal{U}}}^{(2)}(z_0,\nu)\Big]\Big\}, \tag{24}$$

where

$$\begin{aligned}\hat{\mathcal{U}}_a(\nu,z_0)&\equiv\int\frac{d^2z_1d^2z_2}{\pi^2z_{12}^4}\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{\bar{\gamma}}\hat{\mathcal{U}}_{a_0}(z_1,z_2)\\ \hat{\bar{\mathcal{U}}}^{(2)}_a(\nu,z_0)&\equiv\int\frac{d^2z_1d^2z_2}{\pi^2z_{12}^4}\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{-\gamma}\frac{\bar{z}_{12}^2}{\bar{z}_{10}^2\bar{z}_{20}^2}\hat{\mathcal{U}}_a(z_1,z_2)\\ \hat{\tilde{\mathcal{U}}}^{(2)}_a(\nu,z_0)&\equiv\int\frac{d^2z_1d^2z_2}{\pi^2z_{12}^4}\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{-\gamma}\frac{\tilde{z}_{12}^2}{\tilde{z}_{10}^2\tilde{z}_{20}^2}\hat{\mathcal{U}}_a(z_1,z_2)\end{aligned} \tag{25}$$

is a composite dipole (11) in the Mellin representation.

Substituting the decomposition (23)) in Eq. (14) we get the high-energy OPE in the form

$$\begin{aligned}&\frac{1}{N_c}(x-y)^4T\{\hat{j}^\mu(x)\hat{j}^\nu(y)\}\\ &=\int\frac{d\nu}{\pi^2}\int d^2z_0\Big\{\nu^2\Big[\Big(1+\frac{\alpha_s}{\pi}\Big)\mathcal{J}^{\rm LO}_{\alpha\beta}(x,y;z_0,\nu)+\mathcal{J}^{\rm NLO}_{\alpha\beta}(x,y;z_0,\nu)\Big]\hat{\mathcal{U}}_{a_0}(\nu,z_0)\\ &\quad+(\nu^2+1)\Big[\Big\{\Big(1+\frac{\alpha_s}{\pi}\Big)\mathcal{J}^{\rm LO}_{2,\alpha\beta}(x,y;z_0,\nu)+\mathcal{J}^{\rm NLO}_{2,\alpha\beta}(x,y;z_0,\nu)\Big\}\hat{\bar{\mathcal{U}}}_{a_0}(\nu,z_0)+\text{c.c.}\Big]\Big\}.\end{aligned} \tag{26}$$

This equation (26) and its Fourier transform (30) are the main results of this paper.

5. Photon Impact Factor in the Momentum Space

In general, the rapidity evolution of color dipoles is non-linear but in this paper we assume that we can linearize it to the dipole form of the BFKL equation, like in the case of scattering of two virtual photons. Moreover, we will consider only the forward case which corresponds to deep inelastic scattering. In this case, one may write down the high-energy OPE in the form of k_T-factorization formula

$$\int d^4x e^{iqx}\langle p|T\{\hat{j}_\mu(x)\hat{j}_\nu(0)\}|p\rangle = \frac{s}{2}\int d^2k_\perp I_{\mu\nu}(q,k_\perp)\langle\langle p|\hat{\mathcal{U}}(k_\perp)|p\rangle\rangle \qquad (27)$$

where

$$\langle\hat{\mathcal{U}}(k_\perp)\rangle = \int dx_\perp e^{-i(k,x)_\perp}\langle\hat{\mathcal{U}}(x_\perp,0)\rangle,$$

$q = p_1 + \frac{q^2}{s}p_2$ and $p = p_2 + \frac{m^2}{s}p_1$ is the target's momentum. The reduced matrix element $\langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle$ is defined as

$$\begin{aligned}\langle p|\hat{\mathcal{U}}(k)|p+\beta p_2\rangle &= 2\pi\delta(\beta)\langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle\\ \langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle &= \int d^2z e^{-i(k,z)_\perp}\langle\langle p|\hat{\mathcal{U}}(z,0)|p\rangle\rangle\end{aligned} \qquad (28)$$

where the factor $2\pi\delta(\beta)$ reflects the fact that the forward matrix element of the operator $U_xU_y^\dagger$ contains an unrestricted integration along the p_1. Our goal in this Section is to find the impact factor $I_{\mu\nu}(q,k_\perp)$ in the next-to-leading order.

Since our "energy scale" $a_0 = -\kappa^{-2}$ for color dipoles depends on x and y, to perform the Fourier transformation of the OPE (26) one should express $\hat{\mathcal{U}}_{a_0}$ in terms of $\hat{\mathcal{U}}_{a_m}$ with a_m independent of coordinates x and y. A suitable choice is $a_m = 1/x_B$. With this choice, the impact factor does not scale with s and all the energy dependence is included in martix elements of color dipoles. This is similar to the choice $\mu^2 = Q^2$ for the DGLAP evolution: the coefficient functions in front of the light-ray operators will not depend on Q^2 (except for $\alpha_s(Q^2)$ of course) and all the Q^2 dependence is shifted to parton densities. The leading-order evolution of a color dipole $\hat{\mathcal{U}}_a$ is given by

$$\begin{aligned}\hat{\mathcal{U}}_{a_0}(\nu,z_0) &= \hat{\mathcal{U}}_{a_m}(\nu,z_0)\big(a_0x_B\big)^{\frac{\omega(\nu)}{2}},\\ \hat{\mathcal{U}}^{(2)}_{a_0}(\nu,z_0) &= \hat{\mathcal{U}}^{(2)}_{a_m}(\nu,z_0)\big(a_0x_B\big)^{\frac{\omega(2,\nu)}{2}},\end{aligned} \qquad (29)$$

so the Fourier transform of Eq. (26) yields[23]

$$\begin{aligned}&\frac{1}{N_c}\int d^4xd^4y\delta(y_\bullet)e^{iq\cdot(x-y)}T\{\bar{\psi}\gamma_\mu\psi(x)\bar{\psi}\gamma_\nu\psi(y)\}\\ &\quad= \int\frac{d\nu}{\pi^3}\int d^2z_0\bigg\{\frac{\Gamma\big(\bar{\gamma}+\frac{\omega(\nu)}{2}\big)\Gamma^2\big(2-\gamma+\frac{\omega(\nu)}{2}\big)}{\Gamma(4-2\gamma+\omega(\nu))\Gamma\big(2+\gamma+\frac{\omega(\nu)}{2}\big)}\frac{2\gamma-1}{2\gamma+1}\Gamma(2-\gamma)\\ &\quad\times\Big[(\gamma\bar{\gamma}+2)P_1^{\mu\nu}\quad\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\Phi_1(\nu)\Big)\\ &\quad+(3\gamma\bar{\gamma}+2)P_2^{\mu\nu}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\Phi_2(\nu)\Big)\Big]\hat{\mathcal{U}}_{a_m}(z_0,\nu)\end{aligned}$$

$$- \frac{\bar\gamma\Gamma(3-\gamma)\Gamma\big(\bar\gamma+\frac{\omega(2,\nu)}{2}\big)\Gamma^2\big(2-\gamma+\frac{\omega(2,\nu)}{2}\big)}{2\Gamma(4-2\gamma+\omega(2,\nu))\Gamma\big(2+\gamma+\frac{\omega(2,\nu)}{2}\big)}$$
$$\times \Big[\hat{\tilde{\mathcal{U}}}^{(2)}_{a_m}(\nu,z_0)\bar P^{\mu\nu} + \hat{\tilde{\mathcal{U}}}^{(2)}_{a_m}(\nu,z_0)\tilde P^{\mu\nu}\Big]$$
$$\times \left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_6(\nu)\right)\bigg\}\frac{\Gamma^2(\bar\gamma)}{\Gamma(2\bar\gamma)}\frac{(Q^2)^{\gamma-1}\Gamma(2+\gamma)}{4^{\gamma+1}}, \tag{30}$$

where

$$P_1^{\mu\nu} = g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad P_2^{\mu\nu} = \frac{1}{q^2}\Big(q^\mu - \frac{p_2^\mu q^2}{q\cdot p_2}\Big)\Big(q^\nu - \frac{p_2^\nu q^2}{q\cdot p_2}\Big)$$
$$\bar P^{\mu\nu} = \big(g^{\mu 1} - ig^{\mu 2}\big)\big(g^{\nu 1} - ig^{\nu 2}\big), \quad \tilde P^{\mu\nu} = \big(g^{\mu 1} + ig^{\mu 2}\big)\big(g^{\nu 1} + ig^{\nu 2}\big) \tag{31}$$

and

$$\Phi_1(\nu) = F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} - \frac{25}{18(2-\gamma)} + \frac{1}{2\bar\gamma} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2}$$
$$\Phi_2(\nu) = F(\gamma) + \frac{1}{2\bar\gamma\gamma} - \frac{7}{2(2+3\bar\gamma\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar\gamma\gamma}. \tag{32}$$

The last step is to rewrite Eq. (30) in the k_T-factorized form (27). After some algebra we get

$$I^{\mu\nu}(q,k_\perp) = \frac{N_c}{128}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\Big(\frac{k_\perp^2}{Q^2}\Big)^{\frac12 - i\nu}\bigg\{\Big(\frac94+\nu^2\Big)$$
$$\times\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_1(\nu)\Big]P_1^{\mu\nu} + \Big(\frac{11}{4}+3\nu^2\Big)\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_2(\nu)\Big]P_2^{\mu\nu}$$
$$+\Big(\frac18+\frac{\nu^2}{2}\Big)\Big[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}\mathcal{F}_3(\nu)\Big]P_3^{\mu\nu}\bigg\}, \tag{33}$$

where (as usual, $\gamma \equiv \frac12 + i\nu$)

$$\mathcal{F}_{1(2)}(\nu) = \Phi_{1(2)}(\nu) + \chi_\gamma\Psi(\nu), \mathcal{F}_3(\nu) = F_6(\nu) + \Big(\chi_\gamma - \frac{1}{\bar\gamma\gamma}\Big)\Psi(\nu),$$
$$\Psi(\nu) \equiv \psi(\bar\gamma) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma) \tag{34}$$

and

$$P_3^{\mu\nu} = \frac{1}{k_\perp^2}\big[\tilde k^2\bar P^{\mu\nu} + \bar k^2\tilde P^{\mu\nu}\big] = 2\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - g_\perp^{\mu\nu}.$$

The structures P_1 and P_2 correspond to unpolarized structure functions $F_1(x_B)$ and $F_2(x_B)$. The third term vanishes for nucleon structure function but contributes to polarized structure functions of a vector meson (or photon).

6. Conclusions

Let us present again the k_T factorization formula for DIS in the next-to-leading order:

$$\int d^4x e^{iqx}\langle p|T\{\hat{j}_\mu(x)\hat{j}_\nu(0)\}|p\rangle = \frac{s}{2}\int \frac{d^2k_\perp}{k_\perp^2} I_{\mu\nu}(q,k_\perp)\mathcal{V}_{a_m=x_B}(k_\perp) \tag{35}$$

where $I_{\mu\nu}(q,k_\perp)$ is given by Eq. (33) and the evolution equation for $\mathcal{V}_a(k_\perp)$ has the form[23]

$$\begin{aligned}
2a\frac{d}{da}\mathcal{V}_a(k) = {} & \frac{\alpha_s N_c}{\pi^2}\int d^2k'\Big\{\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2} - \frac{(k,k')\mathcal{V}_a(k)}{k'^2(k-k')^2}\Big] \\
& \times\Big(1+\frac{\alpha_s b}{4\pi}\Big[\ln\frac{\mu^2}{k^2}+\frac{N_c}{b}\big(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\big)\Big]\Big) \\
& -\frac{b\alpha_s}{4\pi}\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2}\ln\frac{(k-k')^2}{k'^2} - \frac{k^2\mathcal{V}_a(k)}{k'^2(k-k')^2}\ln\frac{(k-k')^2}{k^2}\Big] \\
& +\frac{\alpha_s N_c}{4\pi}\Big[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2}+F(k,k')+\Phi(k,k')\Big]\mathcal{V}_a(k')\Big\} + 3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{V}_a(k).
\end{aligned} \tag{36}$$

The analytic NLO photon impact factor in momentum space for the pomeron contribution (33) and the NLO k_{T} factorization formula (35) for the deep inelastic scattering are the main results of this paper. Previously, the impact factor was known only as a combination of analytic and numerical results.[24] It should be noted, however, that our final NLO result (33) is defined as a coefficient function in front of a composite operator (11) rather than in front of a usual dipole (with the rapidity cutoff) which corresponds to the impact factor defined in Ref. [24] papers.

It would be also instructive to compare our result (12) for the coefficient in front of the four-Wilson-line operator (relevant for the structure functions of DIS off a large nucleus) to similar result for the NLO impact factor obtained recently in Ref. [25] using the dipole model. However, as we already mentioned our final NLO result (33]) is defined as a coefficient function in front of composite operator (11) defined with a counterterm which restores the conformal invariance in $\mathcal{N}=4$ amplitudes and in our case leads to the conformal impact factor (since the impact factor is given by tree diagrams it should be conformally invariant even in QCD). As a consequence, the impact factor depends on a new parameter a (an analog of the factorization scale μ in usual OPE) which we chose in such a way that all the energy dependence is shifted in to the matrix element, leaving the impact factor energy-scale invariant. To compare with the result of Ref. [25] representing the coefficient function of a usual dipole (without counterterm subtraction) we should trace one step back and look at the impact factor $I^{\mathrm{NLO}}_{\mu\nu}(z_1,z_2,z_3;\eta)$ given by (9). One should then perform Fourier transformation to momentum space with respect to the positions x and y of the two electromagnetic currents in formula (7) and compare it to the result (58) from Ref. [25] integrated over z_1 (and z_2 when appropriate). Hopefully, after these integrations the two results will coincide.

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