2013

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Original Publication Citation

Balitsky, I. & Chiilli, G.A. (2013) Photon impact factor and k_T factorization for DIS in the next-to-leading order. *Physical Review D*, *87*(1), 1-15, Article 014013. https://doi.org/10.1103/PhysRevD.87.014013

PHYSICAL REVIEW D **87,** 014013 (2013)

Photon impact factor and k_T factorization for DIS in the next-to-leading order

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(Received 19 July 2012; published 15 January 2013)

The photon impact factor for the Balitsky-Fadin-Kuraev-Lipatov pomeron is calculated in the next-to-leading order approximation using the operator expansion in Wilson lines. The result is represented as a next-to-leading order k_T-factorization formula for the structure functions of small-x deep inelastic scattering.

DOI: 10.1103/PhysRevD.87.014013 PACS numbers: 12.38.Bx, 12.38.Cy

I. INTRODUCTION

It is well known that the small-x behavior of structure functions of deep inelastic scattering (DIS) is determined by the hard pomeron contribution. In the leading order, the pomeron intercept is determined by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [1] and the pomeron residue (the $\gamma^*\gamma^*$-pomeron vertex) is given by the so-called impact factor. To find the small-x structure functions in the next-to-leading order (NLO), one needs to know both the pomeron intercept and the impact factor. The NLO pomeron intercept was found many years ago [2] but the analytic expression for the NLO impact factor is obtained for the first time in the present paper.

We calculate the NLO impact factor using the high-energy operator expansion of the T product of two vector currents in Wilson lines (see, e.g., the reviews [3,4]). Let us recall the general logic of an operator expansion. In order to find a certain asymptotical behavior of an amplitude by operator product expansion (OPE) one should

(i) identify the relevant operators and factorize an amplitude into a product of coefficient functions and matrix elements of these operators,
(ii) find the evolution equations of the operators with respect to the factorization scale,
(iii) solve these evolution equations,
(iv) convolute the solution with the initial conditions for the evolution and get the amplitude.

Since we are interested in the small-x asymptotics of DIS it is natural to factorize in rapidity: we introduce the rapidity divide η which separates the "fast" gluons from the "slow" ones.

As a first step, we integrate over gluons with rapidities $Y > \eta$ and leave the integration over $Y < \eta$ for the later time; see Fig. 1.

It is convenient to use the background field formalism: we integrate over gluons with $\alpha > \sigma = e^{\eta}$ and leave gluons with $\alpha < \sigma$ as a background field, to be integrated over later. Since the rapidities of the background gluons are very different from the rapidities of gluons in our Feynman diagrams, the background field can be taken in the form of a shock wave due to the Lorentz contraction. To derive the expression of a quark (or gluon) propagator in this shock-wave background, we represent the propagator as a path integral over various trajectories, each of them weighed with the gauge factor $\mathrm{P}\exp(ig\int dx_\mu A^\mu)$ ordered along the propagation path. Now, since the shock wave is very thin, quarks (or gluons) do not have time to deviate in a transverse direction so their trajectory inside the shock wave can be approximated by a segment of the straight line. Moreover, since there is no external field outside the shock

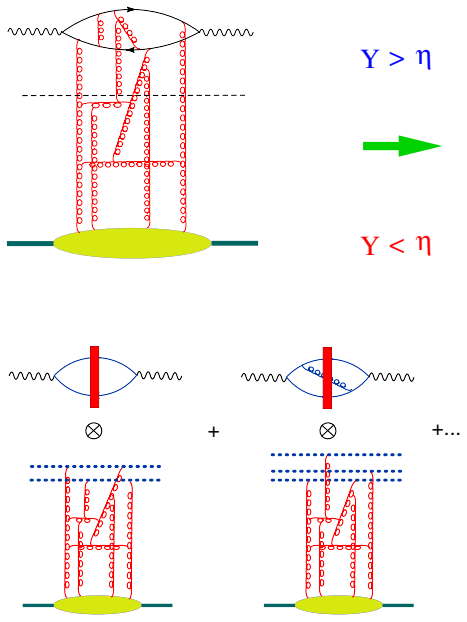

FIG. 1 (color online). Rapidity factorization. The impact factors with $Y > \eta$ are given by diagrams in the shock-wave background. Wilson-line operators with $Y < \eta$ are denoted by dotted lines.

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wave, the integral over the segment of the straight line can be formally extended to $\pm\infty$ limits yielding the Wilson-line gauge factor

$$U_x^\eta = \mathrm{P}\exp\left[ig\int_{-\infty}^{\infty} du p_1^\mu A_\mu^\sigma(up_1 + x_\perp)\right],$$
$$A_\mu^\eta(x) = \int d^4k\theta(e^\eta - |\alpha_k|)e^{ik\cdot x}A_\mu(k), \tag{1}$$

where the Sudakov variable α_k is defined as usual, $k = \alpha_k p_1 + \beta_k p_2 + k_\perp$. We define the lightlike vectors p_1 and p_2 such that $q = p_1 - x_B p_2$ and $p = p_2 + \frac{m_N^2}{s}p_1$, where q is the virtual photon momentum, p is the momentum of the target particle, and $x_B = Q^2/s \ll 1$ is the Bjorken variable (at large energies $s \simeq 2p\cdot q$). The structure of the propagator in a shock-wave background looks as follows (see Fig. 2):

[Free propagation from initial point x to the point of intersection with the shock wave z]
$\times$ [Interaction with the shock wave described by the Wilson-line operator U_z]
$\times$ [Free propagation from point of interaction z to the final point y].

The explicit form of a quark propagator in a shock-wave background can be taken from Ref. [5]:

$$\langle T\{\hat\psi(x)\bar{\hat\psi}(y)\}\rangle_A \overset{x_*>0>y_*}{=} -\int d^4z\delta(z_*)\frac{(\not x - \not z)}{2\pi^2(x-z)^4}\not p_2 \times U_z\frac{(\not z - \not y)}{2\pi^2(x-z)^4}. \tag{2}$$

As usual, we label operators by hats and $\langle\hat{\mathcal{O}}\rangle_A$ means the vacuum average of the operator $\hat{\mathcal{O}}$ in the presence of an external field A. Hereafter, we use the notations $x_* = p_2^\mu x_\mu = \frac{\sqrt{s}}{2}x^+$, $x_\bullet = p_1^\mu x_\mu = \frac{\sqrt{s}}{2}x^-$ [and our metric is $(1,-1,-1,-1)$]. Note that the Regge limit in the coordinate space can be achieved by rescaling

$$x \to \rho x_*\frac{2}{s}p_1 + x_\bullet\frac{2}{s\rho}p_2 + x_\perp,$$
$$y \to \rho y_*\frac{2}{s}p_1 + y_\bullet\frac{2}{s\rho}p_2 + y_\perp, \tag{3}$$

with $\rho \to \infty$; see the discussion in Refs. [6,7].

The result of the integration over gluons with rapidities $Y > \eta$ gives the impact factor—the amplitude of the transition of virtual photon in two-Wilson-lines operators (sometimes called "color dipole"). The LO impact factor is a product of two propagators (2) (see Fig. 3)

$$\langle T\{\bar{\hat\psi}(x)\gamma^\mu\hat\psi(x)\bar{\hat\psi}(y)\gamma^\nu\hat\psi(y)\}\rangle_A = \frac{s^2}{2^9\pi^6x_*^2y_*^2}\int d^2z_{1\perp}d^2z_{2\perp}\frac{\mathrm{tr}\{U_{z_1}U_{z_2}^\dagger\}}{(\kappa\cdot\zeta_1)^3(\kappa\cdot\zeta_2)^3}\frac{\partial^2}{\partial x^\mu\partial y^\nu} \times[2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \kappa^2(\zeta_1\cdot\zeta_2)] + O(\alpha_s). \tag{4}$$

FIG. 2 (color online). Propagator in a shock-wave background.

Here we introduced the conformal vectors [8,9]:

$$\kappa = \kappa_x - \kappa_y,$$
$$\kappa_x = \frac{\sqrt{s}}{2x_*}\left(\frac{p_1}{s} - x^2p_2 + x_\perp\right),$$
$$\zeta_i = \left(\frac{p_1}{s} + z_{i\perp}^2p_2 + z_{i\perp}\right), \tag{5}$$

and the notation $\mathcal{R} \equiv \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}$. The above equation is explicitly Möbius invariant. In addition, it is easy to check that $\frac{\partial}{\partial x_\mu}(\mathrm{r.h.s}) = 0$.

Our goal is the NLO contribution to the right-hand side of Eq. (4), but first let us briefly discuss the three remaining steps of the high-energy OPE. The evolution equation for color dipoles has the form [5,10]

$$\frac{d}{d\eta}\mathrm{tr}\{\hat U_{z_1}^\eta\hat U_{z_2}^{\dagger\eta}\} = \frac{\alpha_s}{2\pi^2}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2} \times[\mathrm{tr}\{\hat U_{z_1}^\eta\hat U_{z_3}^{\dagger\eta}\}\mathrm{tr}\{\hat U_{z_3}^\eta\hat U_{z_2}^{\dagger\eta}\} - N_c\mathrm{tr}\{\hat U_{z_1}^\eta\hat U_{z_2}^{\dagger\eta}\}] + \text{NLO contribution}. \tag{6}$$

(To save space, hereafter z_i stands for $z_{i\perp}$ so $z_{12}^2 \equiv z_{12\perp}^2$ etc.) The explicit form of the NLO contributions can be found in Refs. [4,11,12] while the argument of the coupling constant in the above equation (following from the NLO calculations) is discussed in Refs. [13,14].

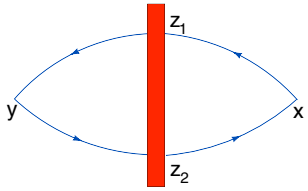

FIG. 3 (color online). Impact factor in the leading order. Solid lines represent quarks.

It is worth noting that we performed the OPE program outlined above for the scattering of scalar "particles" in $\mathcal{N}=4$ super Yang-Mills and obtained the explicit expression for the four-point correlator of scalar operators at high energies in the next-to-leading order [7]. In QCD, the analytic solution of the evolution equation for color dipoles with a running-coupling constant is not known at present. This prevents us from getting the explicit NLO amplitude as in the $\mathcal{N}=4$ case. We can, however, perform the first two steps in our OPE program discussed in the Introduction: calculate the coefficient function (impact factor) and find the evolution equation for color dipoles. The next two steps, solution of the evolution equation (6) with appropriate initial conditions and the eventual comparison with experimental DIS data are discussed in many papers (see, e.g., [15]). It is worth noting that, contrary to the evolution equation, the NLO correction to the impact factor has nothing to do with the running of the coupling constant—it starts at the next-to-next-to-leading order level. Thus, the argument of the coupling constant at the NLO level is determined solely by the evolution equation for color dipoles. For numerical estimates involving the impact factor, one can take $\alpha_s(|x-y|)$ as the first approximation since the characteristic transverse distances in the impact factor are $\sim|x-y|$.

The paper is organized as follows: in Secs. II and III we calculate the NLO impact factor in the coordinate representation (the results of these sections were published previously in a Brief Report [16]). The Mellin representation of the impact factor is presented in Sec. IV and V contains the impact factor in the momentum representation for the forward case corresponding to deep inelastic scattering. Finally, we present the NLO BFKL kernel and discuss the k_T factorization for DIS in Sec. VI.

II. CALCULATION OF THE NLO IMPACT FACTOR

Now we would like to repeat the steps of operator expansion discussed above to the NLO accuracy. A general form of the expansion of the T product of the electromagnetic currents in color dipoles looks as follows:

$$
\begin{aligned}
&(x-y)^4 T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\\
&\quad=\int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\rm LO}_{\mu\nu}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big]{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\\
&\qquad+\int d^2z_3 I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3;\eta)\\
&\qquad\times[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\}-N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\Big\}.
\end{aligned}
\tag{7}
$$

Unfortunately, in terms of the Wilson-line approach, there is no direct way to get the NLO impact factor for the BFKL pomeron. One needs first to find the coefficient in front of the four-Wilson-line operator (which we will also call the NLO impact factor) and then linearize it.

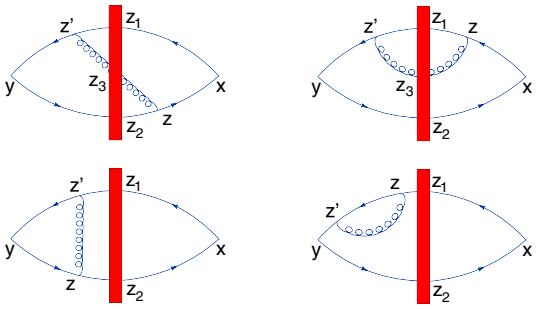

FIG. 4 (color online). Impact factor in the next-to-leading order.

The structure of the NLO contribution is clear from the topology of diagrams in the shock-wave background; see Fig. 4 below. Also, the term $\sim 1+\frac{\alpha_s}{\pi}$ can be restored from the requirement that at $U=1$ (no shock wave) one should get the perturbative series for the polarization operator $1+\frac{\alpha_s}{\pi}+O(\alpha_s^2)$.

In our notations

$$
\begin{aligned}
I^{\rm LO}_{\mu\nu}(z_1,z_2)=\frac{\mathcal{R}^2}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[&(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)\\
&-\frac{1}{2}\kappa^2(\zeta_1\cdot\zeta_2)\Big],
\end{aligned}
\tag{8}
$$

which corresponds to the well-known expression for the LO impact factor in the momentum space.

The NLO impact factor is given by the diagrams shown in Fig. 4. The calculation of these diagrams is similar to the calculation of the NLO impact factor for scalar currents in the $\mathcal{N}=4$ SYM carried out in our previous paper [12]. The gluon propagator in the shock-wave background at $x_*>0>y_*$ in the lightlike gauge $p_2^\mu A_\mu=0$ is given by [17,18]

$$
\begin{aligned}
&\langle T\{\hat{A}^a_\mu(x)\hat{A}^b_\nu(y)\}\rangle\\
&\overset{x_*>0>y_*}{=}-\frac{i}{2}\int d^4z\delta(z_*)\frac{x_*g^\perp_{\mu\xi}-p_{2\mu}(x-z)^\perp_\xi}{\pi^2[(x-z)^2+i\epsilon]^2}\\
&\qquad\times U^{ab}_{z_\perp}\frac{1}{\partial^{(z)}_*}\frac{y_*\delta^{\perp\xi}_\nu-p_{2\nu}(y-z)^\xi_\perp}{\pi^2[(z-y)^2+i\epsilon]^2},
\end{aligned}
\tag{9}
$$

where $\frac{1}{\partial_*}$ can be either $\frac{1}{\partial_*+i\epsilon}$ or $\frac{1}{\partial_*-i\epsilon}$ which leads to the same result. [This is obvious for the leading order and correct in NLO after subtraction of the leading-order contribution, see Eq. (16) below.]

The diagrams in Fig. 4 can be calculated using the conformal integral

$$\int d^4z \frac{\not{x}-\not{z}}{(x-z)^4}\gamma_\mu \frac{\not{z}-\not{y}}{(z-y)^4}\frac{z_\nu}{z^4} - \mu \leftrightarrow \nu$$
$$= \frac{\pi^2}{x^2y^2(x-y)^2}\Big[-\not{x}\gamma_\mu\not{y}\Big(\frac{x_\nu}{x^2}+\frac{y_\nu}{y^2}\Big) + \frac{1}{2}(\not{x}\gamma_\mu\gamma_\nu - \gamma_\mu\gamma_\nu\not{y}) + 2x_\mu y_\nu \frac{\not{x}-\not{y}}{(x-y)^2}\Big] - \mu \leftrightarrow \nu, \tag{10}$$

which gives the three-point $\psi\bar{\psi}F_{\mu\nu}$ Green function in the leading order in g. Using Eqs. (2), (9), and (10), performing integrals over $z_\bullet$'s, and taking the traces one gets after some algebra, the NLO contribution of the diagrams in Fig. 4 is in the form

$$I^{\text{Fig.4}}_{\mu\nu}(z_1,z_2,z_3) = \tilde{I}^{\mu\nu}_1(z_1,z_2,z_3) + \tilde{I}^{\mu\nu}_2(z_1,z_2,z_3), \tag{11}$$

where

$$\tilde{I}^{\mu\nu}_1(z_1,z_2,z_3) = \frac{\alpha_s}{4\pi^2} I^{\text{LO}}_{\mu\nu}(z_1,z_2)\frac{(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)} \times \int_0^\infty \frac{d\alpha}{\alpha} e^{i\alpha\frac{s}{4}\sigma Z_3} \tag{12}$$

and

$$(I_2)_{\mu\nu}(z_1,z_2,z_3) = \frac{\alpha_s}{16\pi^8}\frac{\mathcal{R}^2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big\{\frac{(\kappa\cdot\zeta_2)}{(\kappa\cdot\zeta_3)}\frac{\partial^2}{\partial x^\mu \partial y^\nu}\Big[-\frac{(\kappa\cdot\zeta_1)^2}{(\zeta_1\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)} + \frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)(\zeta_1\cdot\zeta_2)}{(\zeta_1\cdot\zeta_3)(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_2)}{(\zeta_2\cdot\zeta_3)}\Big] + \frac{(\kappa\cdot\zeta_2)^2}{(\kappa\cdot\zeta_3)^2}\frac{\partial^2}{\partial x^\mu\partial y^\nu}\Big[\frac{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_3)}{(\zeta_2\cdot\zeta_3)} - \frac{\kappa^2(\zeta_1\cdot\zeta_3)}{2(\zeta_2\cdot\zeta_3)}\Big] + (\zeta_1\leftrightarrow\zeta_2)\Big\} \tag{13}$$

[recall that $z^2_{ij\perp} = 2(\zeta_i\cdot\zeta_j)$ and $\mathcal{Z}_i = \frac{4}{\sqrt{s}}(\kappa\cdot\zeta_i)$]. We obtained this expression at $x_* > 0 > y_*$, but from the conformal structure of the result it is clear that this expression holds true at $x_* < 0 < y_*$ as well.

The integral over α in the right-hand side of Eq. (12) diverges. This divergence reflects the fact that the contributions of the diagrams in Fig. 4 are not exactly the NLO impact factor since we must subtract the matrix element of the leading-order contribution. Indeed, the NLO impact factor is a coefficient function defined according to Eq. (7). To find the NLO impact factor, we consider the operator equation (7) in the shock-wave background [in the leading order $\langle\hat{U}_{z_3}\rangle_A = U_{z_3}$]:

$$\langle T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\rangle_A - \int\frac{d^2z_1d^2z_2}{z_{12}^4} I^{\text{LO}}_{\mu\nu}(x,y;z_1,z_2)\langle\text{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\rangle_A = \int\frac{d^2z_1d^2z_2}{z_{12}^4}d^2z_3 I^{\text{NLO}}_{\mu\nu}(x,y;z_1,z_2,z_3;\eta) \times[\text{tr}\{U_{z_1}U^\dagger_{z_3}\}\text{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\text{tr}\{U_{z_1}U^\dagger_{z_2}\}]. \tag{14}$$

The NLO matrix element $\langle T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\rangle_A$ is given by Eq. (11) while the subtracted term is

$$\frac{\alpha_s}{2\pi^2}\int\frac{d^2z_1d^2z_2}{z_{12}^4}I^{\text{LO}}_{\mu\nu}(z_1,z_2)\int_0^\sigma\frac{d\alpha}{\alpha}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2} \times[\text{tr}\{U_{z_1}U^\dagger_{z_3}\}\text{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\text{tr}\{U_{z_1}U^\dagger_{z_2}\}], \tag{15}$$

as follows from Eq. (6). The α integration is cut from above by $\sigma = e^\eta$ in accordance with the definition of operators $\hat{U}^\eta$; see Eq. (1). Subtracting (15) from Eq. (11) we get

$$I^{\text{NLO}}_{\mu\nu}(z_1,z_2,z_3;\eta) = I^{\mu\nu}_1(z_1,z_2,z_3;\eta) + I^{\mu\nu}_2(z_1,z_2,z_3),$$
$$I^{\mu\nu}_1(x,y;z_1,z_2,z_3;\eta) = \frac{\alpha_s}{2\pi^2}I^{\text{LO}}_{\mu\nu}(z_1,z_2)\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[\int_0^\infty\frac{d\alpha}{\alpha}e^{i\alpha\frac{s}{4}Z_3} - \int_0^\sigma\frac{d\alpha}{\alpha}\Big] = -\frac{\alpha_s}{2\pi^2}I^{\text{LO}}_{\mu\nu}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[\ln\frac{\sigma s}{4}Z_3 - \frac{i\pi}{2} + C\Big], \tag{16}$$

where C is the Euler constant. Note that one should expect the NLO impact factor to be conformally invariant since it is determined by tree diagrams in Fig. 4. However, as discussed in Refs. [4,7,11], formally, the lightlike Wilson lines are conformally (Möbius) invariant but the longitudinal cutoff $\alpha < \sigma$ in Eq. (1) violates this property so the term $\sim\ln\sigma\mathcal{Z}_3$ in the right-hand side of Eq. (16) is not invariant. As was demonstrated in these papers, one can define a composite operator in the form

$$[\text{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a = \text{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\} + \frac{\alpha_s}{2\pi^2}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2} \times[\text{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_3}\}\text{tr}\{\hat{U}^\sigma_{z_3}\hat{U}^{\dagger\sigma}_{z_2}\} - N_c\text{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]\ln\frac{4az_{12}^2}{\sigma^2sz_{13}^2z_{23}^2} + O(\alpha_s^2), \tag{17}$$

where a is an arbitrary constant. It is convenient to choose the rapidity-dependent constant $a \to ae^{-2\eta}$ so that the $[\text{tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]^{\text{conf}}_a$ does not depend on $\eta = \ln\sigma$ and all the rapidity dependence is encoded into a dependence. Indeed, it is easy to see that $\frac{d}{d\eta}[\text{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]^{\text{conf}}_a = 0$ and $\frac{d}{da}\times[\text{tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]^{\text{conf}}_a$ is determined by the NLO Balitsky-Kovchegov kernel which is a sum of the conformal part and the running-coupling part with our $O(\alpha_s^2)$ accuracy [4,12].

Rewritten in terms of composite dipoles (17), the operator expansion (7) takes the form

$$T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\rm LO}_{\mu\nu}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a + \int d^2z_3 I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3;a)[{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}{\rm tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a\Big\}. \quad (18)$$

We need to choose the "new rapidity cutoff" a in such a way that all the energy dependence is included in the matrix element(s) of Wilson-line operators so the impact factor should not depend on energy. A suitable choice of a is given by $a_0 = -\kappa^{-2} + i\epsilon = -\frac{4x_*y_*}{s(x-y)^2} + i\epsilon$ so we obtain

$$(x-y)^4T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big\{I^{\mu\nu}_{\rm LO}(z_1,z_2)\Big[1+\frac{\alpha_s}{\pi}\Big][{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} + \int d^2z_3\Big[\frac{\alpha_s}{4\pi^2}\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big(\ln\frac{\kappa^2(\zeta_1\cdot\zeta_3)(\zeta_1\cdot\zeta_3)}{2(\kappa\cdot\zeta_3)^2(\zeta_1\cdot\zeta_2)} - 2C\Big)I^{\mu\nu}_{\rm LO} + I_2^{\mu\nu}\Big][{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}{\rm tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - N_c{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0}\Big\}. \quad (19)$$

Here the composite dipole $[{\rm tr}\{\hat{U}^\sigma_{z_1}\hat{U}^{\dagger\sigma}_{z_2}\}]_{a_0}$ is given by Eq. (17) with $a_0 = -\frac{4x_*y_*}{s(x-y)^2} + i\epsilon$ while $I^{\mu\nu}_{\rm LO}(z_1,z_2)$ and $I_2^{\mu\nu}(z_1,z_2,z_3)$ are given by Eqs. (8) and (13), respectively.

III. NLO IMPACT FACTOR FOR THE BFKL POMERON

For the studies of DIS with the linear NLO BFKL equation (up to two-gluon accuracy) we need the linearized version of Eq. (19). If we define

$$\hat{\mathcal{U}}_a(z_1,z_2) = 1 - \frac{1}{N_c}[{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_a, \quad (20)$$

and consider the linearization

$$\frac{1}{N_c^2}\,{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_3}\}{\rm tr}\{\hat{U}_{z_3}\hat{U}^\dagger_{z_2}\} - \frac{1}{N_c}\,{\rm tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}]_{a_0} \simeq \hat{\mathcal{U}}(z_1,z_2) - \hat{\mathcal{U}}(z_1,z_3) - \hat{\mathcal{U}}(z_2,z_3),$$

one of the integrals over z_i in the right-hand side of Eq. (19) can be performed. The result is

$$\frac{1}{N_c}(x-y)^4T\{\bar{\hat{\psi}}(x)\gamma^\mu\hat{\psi}(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} = \frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1dz_2}{z_{12}^4}\hat{\mathcal{U}}_{a_0}(z_1,z_2)\Big[\mathcal{I}^{\rm LO}_{\alpha\beta}\Big(1+\frac{\alpha_s}{\pi}\Big) + \mathcal{I}^{\rm NLO}_{\alpha\beta}\Big], \quad (21)$$

where

$$\mathcal{I}^{\alpha\beta}_{\rm LO}(x,y;z_1,z_2) = \mathcal{R}^2\frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2) - \zeta_1^\alpha\zeta_2^\beta - \zeta_2^\alpha\zeta_1^\beta}{\pi^6(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)} \quad (22)$$

[see Eq. (8)] and

$$\begin{aligned}\mathcal{I}^{\alpha\beta}_{\rm NLO}(x,y;z_1,z_2) = \frac{\alpha_sN_c}{4\pi^7}\mathcal{R}^2\Big\{&\frac{\zeta_1^\alpha\zeta_2^\beta + \zeta_1\leftrightarrow\zeta_2}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[4{\rm Li}_2(1-\mathcal{R}) - \frac{2\pi^2}{3} + \frac{2\ln\mathcal{R}}{1-\mathcal{R}} + \frac{\ln\mathcal{R}}{\mathcal{R}} - 4\ln\mathcal{R} + \frac{1}{2\mathcal{R}} \\ &- 2 + 2\Big(\ln\frac{1}{\mathcal{R}} + \frac{1}{\mathcal{R}} - 2\Big)\Big(\ln\frac{1}{\mathcal{R}} + 2C\Big) - 4C - \frac{2C}{\mathcal{R}}\Big] \\ &+ \Big(\frac{\zeta_1^\alpha\zeta_1^\beta}{(\kappa\cdot\zeta_1)^2} + \zeta_1\leftrightarrow\zeta_2\Big)\Big[\frac{\ln\mathcal{R}}{\mathcal{R}} - \frac{2C}{\mathcal{R}} + 2\frac{\ln\mathcal{R}}{1-\mathcal{R}} - \frac{1}{2\mathcal{R}}\Big] \\ &+ \Big[\frac{\zeta_1^\alpha\kappa^\beta + \zeta_1^\beta\kappa^\alpha}{(\kappa\cdot\zeta_1)\kappa^2} + \zeta_1\leftrightarrow\zeta_2\Big]\Big[-2\frac{\ln\mathcal{R}}{1-\mathcal{R}} - \frac{\ln\mathcal{R}}{\mathcal{R}} + \ln\mathcal{R} - \frac{3}{2\mathcal{R}} + \frac{5}{2} + 2C + \frac{2C}{\mathcal{R}}\Big] \\ &- \frac{2}{\kappa^2}\Big(g^{\alpha\beta} - 2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\Big) + \frac{g^{\alpha\beta}(\zeta_1\cdot\zeta_2)}{(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2)}\Big[\frac{2\pi^2}{3} - 4{\rm Li}_2(1-\mathcal{R}) \\ &- 2\Big(\ln\frac{1}{\mathcal{R}} + \frac{1}{\mathcal{R}} + \frac{1}{2\mathcal{R}^2} - 3\Big)\Big(\ln\frac{1}{\mathcal{R}} + 2C\Big) + 6\ln\mathcal{R} - \frac{2}{\mathcal{R}} + 2 + \frac{3}{2\mathcal{R}^2}\Big]\Big\}, \end{aligned} \quad (23)$$

where ${\rm Li}_2(z)$ is the dilogarithm. Here one easily recognizes five conformal tensor structures discussed in Ref. [19].

While it is easy to see that

$$\frac{d}{dx_\mu}\frac{1}{(x-y)^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\mathcal{I}^{\rm LO}_{\alpha\beta}(x,y;z_i) = 0, \quad (24)$$

one should be careful when checking the electromagnetic gauge invariance in the next-to-leading order. The reason is that the composite dipole $\hat{\mathcal{U}}^{a_0}(z_1,z_2)$ depends on x via the rapidity cutoff $a_0 = -\frac{4x_*y_*}{s(x-y)^2}$, so from Eq. (21) we get

$$\frac{\partial}{\partial x_\mu}\frac{1}{(x-y)^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\int\frac{dz_1dz_2}{z_{12}^4}\hat{\mathcal{U}}_{a_0}(z_1,z_2)\mathcal{I}^{\rm NLO}_{\alpha\beta}(x,y;z_i)=\left(2\frac{(x-y)^\mu}{(x-y)^2}-\frac{p_2^\mu}{x_*}\right)\frac{1}{(x-y)^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}$$
$$\times\int\frac{dz_1dz_2}{z_{12}^4}\mathcal{I}^{\rm LO}_{\alpha\beta}(x,y;z_i)a\frac{d}{da}\hat{\mathcal{U}}_a(z_1,z_2)|_{a_0}. \tag{25}$$

Using the leading-order BFKL equation in the dipole form [linearization of Eq. (6)]

$$a\frac{d}{da}\hat{\mathcal{U}}_a(z_1,z_2)=\frac{\alpha_sN_c}{4\pi^2}\int d^2z_3\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\hat{\mathcal{U}}_a(z_1,z_3)+\hat{\mathcal{U}}_a(z_2,z_3)-\hat{\mathcal{U}}_a(z_1,z_2)], \tag{26}$$

we obtain the following consequence of gauge invariance:

$$\frac{\partial}{\partial x_\mu}\frac{1}{(x-y)^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\mathcal{I}^{\rm NLO}_{\alpha\beta}(x,y;z_i)=\frac{\alpha_s}{\pi^8}\frac{y_*}{x_*(x-y)^6}\mathcal{R}^3\Bigg[\left(\frac{1}{2\mathcal{R}}-3-\ln\mathcal{R}\right)\frac{\partial\ln\kappa^2}{\partial y^\nu}$$
$$+\left(\frac{\ln\mathcal{R}}{\mathcal{R}}+\frac{5}{2\mathcal{R}}-\frac{1}{2\mathcal{R}^2}\right)\frac{\partial}{\partial y^\nu}[\ln(\kappa\cdot\zeta_1)+\ln(\kappa\cdot\zeta_2)]\Bigg]. \tag{27}$$

We have verified that the expression (23) satisfies the above equation.

IV. PHOTON IMPACT FACTOR IN THE MELLIN REPRESENTATION

In preparation for the Fourier transformation, we calculated the Mellin transform of the photon impact factor (23). We project the impact factor on the conformal eigenfunctions of the BFKL equation [20]

$$E_{\nu,n}(z_{10},z_{20})=\left[\frac{\tilde{z}_{12}}{\tilde{z}_{10}\tilde{z}_{20}}\right]^{\frac{1}{2}+i\nu+\frac{n}{2}}\left[\frac{\bar{z}_{12}}{\bar{z}_{10}\bar{z}_{20}}\right]^{\frac{1}{2}+i\nu-\frac{n}{2}} \tag{28}$$

(here $\tilde{z}=z_x+iz_y$, $\bar{z}=z_x-iz_y$, $z_{10}\equiv z_1-z_0$ etc.). Since electromagnetic currents are vectors, the only nonvanishing contribution comes from a projection on the eigenfunctions with spin 0 and spin 2. The spin-0 projection has the form (throughout the paper we reserve the notation γ for $\frac{1}{2}+i\nu$)

$$\left(1+\frac{\alpha_s}{\pi}\right)\mathcal{J}^{\rm LO}_{\alpha\beta}(x,y;z_0,\nu)+\mathcal{J}^{\rm NLO}_{\alpha\beta}(x,y;z_0,\nu)$$
$$=\int\frac{d^2z_1d^2z_2}{z_{12}^4}\frac{\partial\kappa^\lambda}{\partial x^\alpha}\frac{\partial\kappa^\rho}{\partial y^\beta}\left[\left(1+\frac{\alpha_s}{\pi}\right)\mathcal{I}^{\rm LO}_{\lambda\rho}(x,y;z_1,z_2)+\mathcal{I}^{\rm NLO}_{\lambda\rho}(x,y;z_1,z_2)\right]\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^\gamma$$
$$=\frac{1}{4\pi^4}B(\bar\gamma,\bar\gamma)\Gamma(\gamma+1)\Gamma(2-\gamma)\left(\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\right)^\gamma\Bigg\{-\frac{\gamma\bar\gamma}{3}(2S_1+S_2)_{\mu\nu}\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_1(\gamma)\right]$$
$$-2S_{2\mu\nu}\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_2(\gamma)\right]+2\gamma(S_2-S_3)_{\mu\nu}\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_3(\gamma)\right]$$
$$+\frac{\bar\gamma\gamma^2}{(3-2\gamma)}\left(-\frac{1}{3}S_1-\frac{2}{3}S_2+S_3-2S_4\right)_{\mu\nu}\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_4(\gamma)\right]$$
$$+(S_1+S_2)_{\mu\nu}(2+\bar\gamma\gamma)\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_5(\gamma)\right]\Bigg\}, \tag{29}$$

where $\gamma\equiv\frac{1}{2}+i\nu$, $\bar\gamma\equiv1-\gamma=\frac{1}{2}-i\nu$ and

$$F_1(\gamma)=F(\gamma)+\frac{\chi_\gamma}{\gamma\bar\gamma},\qquad F_2(\gamma)=F(\gamma)-1+\frac{1}{2\gamma\bar\gamma}+\chi_\gamma,\qquad F_3(\gamma)=F(\gamma)+\frac{1}{2}\chi_\gamma,$$
$$F_4(\gamma)=F(\gamma)-\frac{6}{\gamma\bar\gamma}+\frac{3}{\gamma^2\bar\gamma^2}-\frac{2\chi_\gamma}{\gamma\bar\gamma},\qquad F_5(\gamma)=F(\gamma)+\frac{3\bar\gamma\gamma\chi_\gamma+1-2\bar\gamma\gamma}{\gamma\bar\gamma(2+\bar\gamma\gamma)}, \tag{30}$$
$$F(\gamma)=\frac{2\pi^2}{3}+1-\frac{2\pi^2}{\sin^2\pi\gamma}-2C\chi_\gamma+\frac{\chi_\gamma-2}{\bar\gamma\gamma},$$

and

$$S_1^{\mu\nu} \equiv \frac{\partial^2 \ln\kappa^2}{\partial x_\mu \partial y_\nu}, \qquad S_2^{\mu\nu} \equiv \frac{\partial \ln\kappa^2}{\partial x_\mu}\frac{\partial \ln\kappa^2}{\partial y_\nu},$$
$$S_3^{\mu\nu} \equiv \frac{\partial \ln\kappa^2}{\partial x_\mu}\frac{\partial \ln\kappa\cdot\zeta_0}{\partial y_\nu} + \frac{\partial \ln\kappa\cdot\zeta_0}{\partial x_\mu}\frac{\partial \ln\kappa^2}{\partial y_\nu}, \tag{31}$$
$$S_4^{\mu\nu} \equiv \frac{\partial \ln\kappa\cdot\zeta_0}{\partial x_\mu}\frac{\partial \ln\kappa\cdot\zeta_0}{\partial y_\nu}.$$

The contribution of spin 2 in the t channel has the form

$$\begin{aligned}
&\left(1+\frac{\alpha_s}{\pi}\right)\mathcal{J}^{\mathrm{LO}}_{2,\alpha\beta}(x,y;z_0,\nu) + \mathcal{J}^{\mathrm{NLO}}_{2,\alpha\beta}(x,y;z_0,\nu)\\
&\quad= \int\frac{d^2z_1 d^2z_2}{z_{12}^4}\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\Bigg[\left(1+\frac{\alpha_s}{\pi}\right)I^{\mathrm{LO}}_{\alpha\beta}(x,y;z_1,z_2)\\
&\qquad + I^{\mathrm{NLO}}_{\alpha\beta}(x,y;z_1,z_2)\Bigg]\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^\gamma\frac{\tilde z_{12}}{\tilde z_{10}\tilde z_{20}}\frac{\bar z_{10}\bar z_{20}}{\bar z_{12}}\\
&\quad= -\frac{1}{2\pi^4(x-y)^2}B(2-\gamma,2-\gamma)\Gamma(\gamma+2)\\
&\qquad\times\Gamma(3-\gamma)\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_s N_c}{2\pi}F_6(\gamma)\right]S_5^{\mu\nu},
\end{aligned}\tag{32}$$

where

$$F_6(\gamma) = F(\gamma) + \frac{2C}{\bar\gamma\gamma} - \frac{2}{\bar\gamma\gamma} + \frac{2}{\bar\gamma^2\gamma^2} + 3\frac{1+\chi_\gamma-\frac{1}{\gamma\bar\gamma}}{2+\bar\gamma\gamma} - \frac{\chi_\gamma}{\bar\gamma\gamma} \tag{33}$$

and

$$\begin{aligned}
S_5^{\mu\nu} \equiv &\left[g^{\mu 1} - ig^{\mu 2} - 2(x-z_0)^\mu\frac{\partial}{\partial\tilde z_0}\ln\kappa\cdot\zeta_0\right.\\
&\left.+\frac{4p_2^\mu}{\sqrt{s}}\frac{(\kappa_x\cdot\zeta_0)(\kappa_y\cdot\zeta_0)}{(\kappa\cdot\zeta_0)}\frac{\partial}{\partial\tilde z_0}\ln\frac{\kappa_x\cdot\zeta_0}{\kappa_y\cdot\zeta_0}\right]\\
&\times\left[g^{\nu 1} - ig^{\nu 2} - 2(y-z_0)^\nu\frac{\partial}{\partial\tilde z_0}\ln\kappa\cdot\zeta_0\right.\\
&\left.+\frac{4p_2^\nu}{\sqrt{s}}\frac{(\kappa_x\cdot\zeta_0)(\kappa_y\cdot\zeta_0)}{(\kappa\cdot\zeta_0)}\frac{\partial}{\partial\tilde z_0}\ln\frac{\kappa_x\cdot\zeta_0}{\kappa_y\cdot\zeta_0}\right].
\end{aligned}\tag{34}$$

Using the decomposition of the product of the transverse δ functions in conformal three-point functions (28)

$$\begin{aligned}
&\delta^{(2)}(z_1-z_3)\delta^{(2)}(z_2-z_4)\\
&\quad= \sum_{n=-\infty}^{\infty}\int\frac{d\nu}{\pi^4}\frac{\nu^2+\frac{n^2}{4}}{z_{12}^2 z_{34}^2}\int d^2z_0 E_{\nu,n}(z_{10},z_{20})E^*_{\nu,n}(z_{30},z_{40}),
\end{aligned}\tag{35}$$

we obtain

$$\begin{aligned}
\hat{\mathcal{U}}(z_1,z_2) = &\int\frac{d\nu}{\pi^2}\int d^2z_0\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^\gamma\left\{\nu^2\hat{\mathcal{U}}(z_0,\nu) + (\nu^2+1)\right.\\
&\times\left[\frac{\tilde z_{12}\bar z_{10}\bar z_{20}}{\bar z_{10}\tilde z_{20}\bar z_{12}}\hat{\tilde{\mathcal{U}}}^{(2)}(z_0,\nu)\right.\\
&\left.\left.+\frac{\bar z_{12}\tilde z_{10}\tilde z_{20}}{\bar z_{10}\bar z_{20}\tilde z_{12}}\hat{\tilde{\mathcal{U}}}^{(2)}(z_0,\nu)\right]\right\},
\end{aligned}\tag{36}$$

where

$$\begin{aligned}
\hat{\mathcal{U}}_a(\nu,z_0) &\equiv \int\frac{d^2z_1 d^2z_2}{\pi^2 z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^{\bar\gamma}\hat{\mathcal{U}}_{a_0}(z_1,z_2),\\
\hat{\tilde{\mathcal{U}}}^{(2)}_a(\nu,z_0) &\equiv \int\frac{d^2z_1 d^2z_2}{\pi^2 z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^{-\gamma}\frac{\bar z_{12}^2}{\bar z_{10}^2\bar z_{20}^2}\hat{\mathcal{U}}_a(z_1,z_2),\\
\hat{\tilde{\mathcal{U}}}^{(2)}_a(\nu,z_0) &\equiv \int\frac{d^2z_1 d^2z_2}{\pi^2 z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2 z_{20}^2}\right)^{-\gamma}\frac{\tilde z_{12}^2}{\tilde z_{10}^2\tilde z_{20}^2}\hat{\mathcal{U}}_a(z_1,z_2),
\end{aligned}\tag{37}$$

is a composite dipole (17) in the Mellin representation.

Substituting the decomposition (35) in Eq. (21) we get the high-energy OPE in the form

$$\begin{aligned}
&\frac{1}{N_c}(x-y)^4 T\{\hat j^\mu(x)\hat j^\nu(y)\}\\
&\quad= \int\frac{d\nu}{\pi^2}\int d^2z_0\left\{\nu^2\left[\left(1+\frac{\alpha_s}{\pi}\right)\mathcal{J}^{\mathrm{LO}}_{\alpha\beta}(x,y;z_0,\nu)\right.\right.\\
&\qquad\left.+\mathcal{J}^{\mathrm{NLO}}_{\alpha\beta}(x,y;z_0,\nu)\right]\hat{\mathcal{U}}_{a_0}(\nu,z_0)\\
&\qquad+(\nu^2+1)\left[\left\{\left(1+\frac{\alpha_s}{\pi}\right)\mathcal{J}^{\mathrm{LO}}_{2,\alpha\beta}(x,y;z_0,\nu)\right.\right.\\
&\qquad\left.\left.\left.+\mathcal{J}^{\mathrm{NLO}}_{2,\alpha\beta}(x,y;z_0,\nu)\right\}\hat{\tilde{\mathcal{U}}}_{a_0}(\nu,z_0)+\text{c.c.}\right]\right\}.
\end{aligned}\tag{38}$$

Equation (38) and its Fourier transform (46) are the main results of this paper.

At this point it is instructive to check again the photon gauge invariance $\frac{\partial}{\partial x^\mu}T\{\hat j^\mu(x)\hat j^\nu(y)\} = 0$. Since $a_0 = -\kappa^2 + i\epsilon$ we need to differentiate $\hat{\mathcal{U}}_{a_0}(\nu,z_0)$ too:

$$\frac{\partial}{\partial x^\mu}\hat{\mathcal{U}}_{a_0}(\nu,z_0) = a\frac{d}{da}\hat{\mathcal{U}}_a(\nu,z_0)|_{a=a_0}\frac{\partial}{\partial x^\mu}\ln a_0. \tag{39}$$

Let us start with a spin-2 contribution. It is easy to see that

$$\frac{\partial}{\partial x^\mu}\frac{S_5^{\mu\nu}}{(x-y)^6} = 0, \qquad S_5^{\mu\nu}\frac{\partial}{\partial x^\mu}\ln a_0 = 0,$$

and therefore the second term in the right-hand side of Eq. (38) is gauge invariant (recall that $a_0 = -\kappa^{-2} + i\epsilon$).

For the spin-0 part we need to use Eq. (39). Since $\hat{\mathcal{U}}^a(\nu,z_0)$ are the projections of color dipoles on the eigenfunctions (28) of the BFKL equation, the evolution equation (26) simplifies to

$$2a\frac{d}{da}\hat{\mathcal{U}}_a(\nu, z_0) = \omega(\nu)\hat{\mathcal{U}}^a(\nu, z_0), \tag{40}$$

where $\omega(\nu) = \frac{\alpha_s N_c}{\pi}\chi_\gamma$ is the BFKL pomeron intercept (as usual $\gamma = \frac{1}{2} + i\nu$). We obtain

$$\frac{\partial}{\partial x^\mu}\hat{\mathcal{U}}_{a_0}(\nu, z_0) = -\frac{\omega(\nu)}{2}\hat{\mathcal{U}}_{a_0}(\nu, z_0)\frac{\partial}{\partial x^\mu}\ln\kappa^2. \tag{41}$$

In the leading order, the derivative (41) does not contribute so the formula for gauge invariance is simply

$$\frac{\partial}{\partial x^\mu}\mathcal{J}^{\mathrm{LO}}_{\alpha\beta}(x, y; z_0, \nu) = 0.$$

It is easy to demonstrate that $\mathcal{J}^{\mathrm{LO}}_{\alpha\beta}$ in the right-hand side of Eq. (29) satisfies this requirement.

In the NLO, we need both $\mathcal{J}^{\mathrm{NLO}}_{\alpha\beta}$ and $\omega\mathcal{J}^{\mathrm{LO}}_{\alpha\beta}$ parts so the requirement for electromagnetic gauge invariance takes the form

$$\frac{\partial}{\partial x^\mu}\mathcal{J}^{\mathrm{NLO}}_{\alpha\beta}(x, y; z_0, \nu) = \frac{\omega(\nu)}{2}\mathcal{J}^{\mathrm{LO}}_{\alpha\beta}(x, y; z_0, \nu)\frac{\partial\ln\kappa^2}{\partial x^\mu}. \tag{42}$$

We have checked that the right-hand side of Eq. (29) satisfies this equation.

V. PHOTON IMPACT FACTOR IN THE MOMENTUM SPACE

In general, the rapidity evolution of color dipoles is nonlinear but in this paper we assume that we can linearize it to the dipole form of the BFKL equation, like in the case of scattering of two virtual photons. Moreover, we will consider only the forward case which corresponds to deep inelastic scattering. In this case, one may write down the high-energy OPE in the form of a k_T-factorization formula

$$\int d^4x e^{iqx}\langle p|T\{\hat{j}_\mu(x)\hat{j}_\nu(0)\}|p\rangle = \frac{s}{2}\int d^2k_\perp I_{\mu\nu}(q, k_\perp)\langle\langle p|\hat{\mathcal{U}}(k_\perp)|p\rangle\rangle, \tag{43}$$

where

$$\langle\hat{\mathcal{U}}(k_\perp)\rangle = \int dx_\perp e^{-i(k,x)_\perp}\langle\hat{\mathcal{U}}(x_\perp, 0)\rangle,$$

$q = p_1 + \frac{q^2}{s}p_2$ and $p = p_2 + \frac{m^2}{s}p_1$ is the target's momentum. The reduced matrix element $\langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle$ is defined as

$$\langle p|\hat{\mathcal{U}}(k)|p + \beta p_2\rangle = 2\pi\delta(\beta)\langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle,$$
$$\langle\langle p|\hat{\mathcal{U}}(k)|p\rangle\rangle = \int d^2z e^{-i(k,z)_\perp}\langle\langle p|\hat{\mathcal{U}}(z, 0)|p\rangle\rangle, \tag{44}$$

where the factor $2\pi\delta(\beta)$ reflects the fact that the forward matrix element of the operator $U_x U_y^\dagger$ contains an unrestricted integration along the p_1. Our goal in this section is to find the impact factor $I_{\mu\nu}(q, k_\perp)$ in the next-to-leading order.

Since our "energy scale" $a_0 = -\kappa^{-2}$ for color dipoles depends on x and y, to perform the Fourier transformation of the OPE (38) one should express $\hat{\mathcal{U}}_{a_0}$ in terms of $\hat{\mathcal{U}}_{a_m}$ with a_m independent of coordinates x and y. A suitable choice is $a_m = 1/x_B$. With this choice, the impact factor does not scale with s and all the energy dependence is included in matrix elements of color dipoles. This is similar to the choice $\mu^2 = Q^2$ for the DGLAP evolution: the coefficient functions in front of the light-ray operators will not depend on Q^2 [except for $\alpha_s(Q^2)$ of course] and all the Q^2 dependence is shifted to parton densities. The leading-order evolution of a color dipole $\hat{\mathcal{U}}_a$ is given by Eq. (40)

$$\hat{\mathcal{U}}_{a_0}(\nu, z_0) = \hat{\mathcal{U}}_{a_m}(\nu, z_0)(a_0x_B)^{\frac{\omega(\nu)}{2}},$$
$$\hat{\mathcal{U}}^{(2)}_{a_0}(\nu, z_0) = \hat{\mathcal{U}}^{(2)}_{a_m}(\nu, z_0)(a_0x_B)^{\frac{\omega(2,\nu)}{2}}, \tag{45}$$

so the Fourier transform of Eq. (38) yields

$$\begin{aligned}
&\frac{1}{N_c}\int d^4x d^4y\delta(y_\bullet)e^{iq\cdot(x-y)}T\{\bar{\psi}\gamma_\mu\psi(x)\bar{\psi}\gamma_\nu\psi(y)\}\\
&= \int\frac{d\nu}{\pi^3}\int d^2z_0\Big\{\frac{\Gamma(\bar{\gamma} + \frac{\omega(\nu)}{2})\Gamma^2(2 - \gamma + \frac{\omega(\nu)}{2})}{\Gamma(4 - 2\gamma + \omega(\nu))\Gamma(2 + \gamma + \frac{\omega(\nu)}{2})}\frac{2\gamma - 1}{2\gamma + 1}\\
&\times\Gamma(2 - \gamma)\Big[(\gamma\bar{\gamma} + 2)P_1^{\mu\nu}\Big(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s N_c}{2\pi}\Phi_1(\nu)\Big) + (3\gamma\bar{\gamma} + 2)P_2^{\mu\nu}\Big(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s N_c}{2\pi}\Phi_2(\nu)\Big)\Big]\hat{\mathcal{U}}_{a_m}(z_0, \nu)\\
&- \frac{\bar{\gamma}\Gamma(3 - \gamma)\Gamma(\bar{\gamma} + \frac{\omega(2,\nu)}{2})\Gamma^2(2 - \gamma + \frac{\omega(2,\nu)}{2})}{2\Gamma(4 - 2\gamma + \omega(2, \nu))\Gamma(2 + \gamma + \frac{\omega(2,\nu)}{2})}[\hat{\tilde{\mathcal{U}}}^{(2)}_{a_m}(\nu, z_0)\bar{P}^{\mu\nu} + \hat{\tilde{\mathcal{U}}}^{(2)}_{a_m}(\nu, z_0)\tilde{P}^{\mu\nu}]\\
&\times\Big(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s N_c}{2\pi}F_6(\nu)\Big)\Big\}\frac{\Gamma^2(\bar{\gamma})}{\Gamma(2\bar{\gamma})}\frac{(Q^2)^{\gamma-1}\Gamma(2 + \gamma)}{4^{\gamma+1}},
\end{aligned} \tag{46}$$

where

$$P_1^{\mu\nu} = g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2},$$
$$P_2^{\mu\nu} = \frac{1}{q^2}\left(q^\mu - \frac{p_2^\mu q^2}{q\cdot p_2}\right)\left(q^\nu - \frac{p_2^\nu q^2}{q\cdot p_2}\right),$$
$$\bar{P}^{\mu\nu} = (g^{\mu 1} - ig^{\mu 2})(g^{\nu 1} - ig^{\nu 2}),$$
$$\tilde{P}^{\mu\nu} = (g^{\mu 1} + ig^{\mu 2})(g^{\nu 1} + ig^{\nu 2}), \tag{47}$$

and

$$\Phi_1(\nu) = F(\gamma) + \frac{3\chi_\gamma}{2+\bar\gamma\gamma} - \frac{25}{18(2-\gamma)} + \frac{1}{2\bar\gamma} - \frac{1}{2\gamma} - \frac{7}{18(1+\gamma)} + \frac{10}{3(1+\gamma)^2},$$
$$\Phi_2(\nu) = F(\gamma) + \frac{1}{2\bar\gamma\gamma} - \frac{7}{2(2+3\bar\gamma\gamma)} + \frac{\chi_\gamma}{1+\gamma} + \frac{\chi_\gamma(1+3\gamma)}{2+3\bar\gamma\gamma}. \tag{48}$$

The last step is to rewrite Eq. (46) in the k_T-factorized form (43). Using

$$\langle\hat{\mathcal{U}}(z_0,\nu)\rangle = \frac{\Gamma(-\bar\gamma)\Gamma(1-2\gamma)\Gamma^2(\gamma)}{4^{\bar\gamma}\Gamma(2-\gamma)\Gamma^2(\bar\gamma)\Gamma(2\gamma)}\int\frac{d^2k}{4\pi^2}k^{2\bar\gamma}\langle\hat{\mathcal{U}}(k)\rangle,$$
$$\langle\hat{\tilde{\mathcal{U}}}^{(2)}(z_0,\nu)\rangle = -\frac{\Gamma(\gamma)\Gamma(3-2\gamma)\Gamma^2(1+\gamma)}{4^{\bar\gamma}\Gamma(3-\gamma)\Gamma^2(2-\gamma)\Gamma(1+2\gamma)}\int\frac{d^2k}{4\pi^2}\frac{\bar k}{k}k^{2\bar\gamma}\langle\hat{\tilde{\mathcal{U}}}^{(2)}(k)\rangle, \tag{49}$$

we get

$$\int d^4x dz_* e^{iqx}\langle p|T\left\{\hat j_\mu\left(x+\frac{2}{s}z_*p_1\right)\hat j_\nu\left(\frac{2}{s}z_*p_1\right)\right\}|p\rangle$$
$$= N_c\frac{s}{2}\int\frac{d\nu}{32\pi^3}\int\frac{d^2k}{4\pi^2}\left(\frac{k^2}{Q^2}\right)^{\bar\gamma}\frac{\Gamma^3(\bar\gamma)\Gamma^3(\gamma)}{\Gamma(2\bar\gamma)\Gamma(2\gamma)}\frac{\langle p|\hat{\mathcal{U}}(k)|p\rangle}{3+4\bar\gamma\gamma}\left\{\frac{\Gamma(\bar\gamma+\frac{\omega(\nu)}{2})\Gamma^2(2-\gamma+\frac{\omega(\nu)}{2})\Gamma(4-2\gamma)\Gamma(2+\gamma)}{\Gamma(\bar\gamma)\Gamma^2(2-\gamma)\Gamma(4-2\gamma+\omega(\nu))\Gamma(2+\gamma+\frac{\omega(\nu)}{2})}\right.$$
$$\times\left[(\gamma\bar\gamma+2)P_1^{\mu\nu}\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\Phi_1(\nu)\right)+(3\gamma\bar\gamma+2)P_2^{\mu\nu}\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\Phi_2(\nu)\right)\right]$$
$$\left.+\frac{\Gamma(\bar\gamma+\frac{\omega(2,\nu)}{2})\Gamma^2(2-\gamma+\frac{\omega(2,\nu)}{2})\Gamma(4-2\gamma)\Gamma(2+\gamma)}{\Gamma(\bar\gamma)\Gamma^2(2-\gamma)\Gamma(4-2\gamma+\omega(2,\nu))\Gamma(2+\gamma+\frac{\omega(2,\nu)}{2})}\frac{\bar\gamma\gamma}{2}P_3^{\mu\nu}\left(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}F_6(\nu)\right)\right\}, \tag{50}$$

where

$$P_3^{\mu\nu} = \frac{1}{k_\perp^2}[\tilde k^2\bar P^{\mu\nu} + \bar k^2\tilde P^{\mu\nu}] = 2\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - g_\perp^{\mu\nu}.$$

There is a subtle point in the Fourier transform of Eq. (46): the contribution of infinite z_0 doubles the result from finite z_0 as shown in the Appendix.

Note that $\omega(\nu) = \frac{\alpha_sN_c}{\pi}\chi_\gamma$ and $\omega(2,\nu) = \frac{\alpha_sN_c}{\pi}(\chi_\gamma - \frac{1}{\bar\gamma\gamma})$ are of order α_s so one should expand the expressions like $\Gamma(4-2\gamma+\omega(\nu))$ in Eq. (46) up to the first order in $\omega(\nu)$. Using definitions (43) and (44) one obtains the impact factor in the form

$$I^{\mu\nu}(q,k_\perp) = \frac{N_c}{128}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\left(\frac{k_\perp^2}{Q^2}\right)^{\frac12-i\nu}$$
$$\times\left\{\left(\frac94+\nu^2\right)\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_1(\nu)\right]P_1^{\mu\nu}\right.$$
$$+\left(\frac{11}{4}+3\nu^2\right)\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_2(\nu)\right]P_2^{\mu\nu}$$
$$\left.+\left(\frac18+\frac{\nu^2}{2}\right)\left[1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_3(\nu)\right]P_3^{\mu\nu}\right\}, \tag{51}$$

where (as usual, $\gamma \equiv \frac12 + i\nu$)

$$\mathcal{F}_{1(2)}(\nu) = \Phi_{1(2)}(\nu) + \chi_\gamma\Psi(\nu),$$
$$\mathcal{F}_3(\nu) = F_6(\nu) + \left(\chi_\gamma - \frac{1}{\bar\gamma\gamma}\right)\Psi(\nu),$$
$$\Psi(\nu) \equiv \psi(\bar\gamma) + 2\psi(2-\gamma) - 2\psi(4-2\gamma) - \psi(2+\gamma). \tag{52}$$

The structures P_1 and P_2 correspond to unpolarized structure functions $F_1(x_B)$ and $F_2(x_B)$. The third term vanishes for nucleon structure function but contributes to the polarized structure functions of a vector meson (or photon).

It is instructive to compare Eq. (51) with the well-known double-integral representation of the leading-order impact factor (see, e.g., Refs. [3,21]):

$$I^{\mu\nu}(q,k) = \frac{N_c}{32}\int_0^1 du dv\frac{k_\perp^2}{Q^2\bar u u + k_\perp^2\bar v v}\left\{(1-2\bar u u - 2\bar v v + 4\bar u u\bar v v)P_1^{\mu\nu} + (1-2\bar u u - 2\bar v v + 12\bar u u\bar v v)P_2^{\mu\nu} + \frac{2\bar u u\bar v v}{k_\perp^2}(\tilde P^{\mu\nu}\bar k^2 + \bar P^{\mu\nu}\tilde k^2)\right\}. \tag{53}$$

(Note that the definition of $I^{\mu\nu}(q,k)$ differs in sign and N_c from that of Ref. [3].) It is easy to see that Eq. (53) is equal to LO terms in the right-hand side of Eq. (51).

VI. NLO BFKL FOR COLOR DIPOLES

For completeness, in this section we present the (linearized) evolution equation for composite color dipoles and discuss how it is related to the usual NLO BFKL approach [2]. The evolution equation for forward matrix elements of color dipoles $\mathcal{U}(z_{12}) \equiv \langle \hat{\mathcal{U}}(z_1, z_2)\rangle$ reads [11,12]

$$
\begin{aligned}
&2a\frac{d}{da}\mathcal{U}_a(z)\\
&\quad=\frac{\alpha_s N_c}{2\pi^2}\int d^2z'\frac{z^2}{z'^2(z-z')^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\Big(\ln\frac{z^2\mu^2}{4}+2C\Big)\\
&\qquad-b\frac{(z-z')^2-z'^2}{z^2}\ln\frac{(z-z')^2}{z'^2}+\Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c\\
&\qquad-\frac{10n_f}{9}\Big]\Big\}[\mathcal{U}_a(z')+\mathcal{U}_a(z-z')-\mathcal{U}_a(z)]\\
&\qquad+\frac{\alpha_s^2N_c^2}{4\pi^3}\int d^2z'\frac{z^2}{z'^2}\Big[-\frac{1}{(z-z')^2}\ln^2\frac{z^2}{z'^2}+F(z,z')\\
&\qquad+\Phi(z,z')\Big]\mathcal{U}_a(z')+3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{U}_a(z),
\end{aligned}
\tag{54}
$$

where

$$
\begin{aligned}
F(z,z')&=\Big(1+\frac{n_f}{N_c^3}\Big)\frac{3(z,z')^2-2z^2z'^2}{16z^2z'^2}\Big(\frac{2}{z^2}+\frac{2}{z'^2}+\frac{z^2-z'^2}{z^2z'^2}\\
&\quad\times\ln\frac{z^2}{z'^2}\Big)-\Big[3+\Big(1+\frac{n_f}{N_c^3}\Big)\Big(1-\frac{(z^2+z'^2)^2}{8z^2z'^2}\\
&\quad+\frac{3z^4+3z'^4-2z^2z'^2}{16z^4z'^4}(z,z')^2\Big)\Big]\\
&\quad\times\int_0^\infty\frac{dt}{z^2+t^2z'^2}\ln\frac{1+t}{|1-t|},
\end{aligned}
\tag{55}
$$

and

$$
\begin{aligned}
\Phi(z,z')&=\frac{(z^2-z'^2)}{(z-z')^2(z+z')^2}\Big[\ln\frac{z^2}{z'^2}\ln\frac{z^2z'^2(z-z')^4}{(z^2+z'^2)^4}\\
&\quad+2\mathrm{Li}_2\Big(-\frac{z'^2}{z^2}\Big)-2\mathrm{Li}_2\Big(-\frac{z^2}{z'^2}\Big)\Big]\\
&\quad-\Big(1-\frac{(z^2-z'^2)^2}{(z-z')^2(z+z')^2}\Big)\Big[\int_0^1-\int_1^\infty\Big]\\
&\quad\times\frac{du}{(z-z'u)^2}\ln\frac{u^2z'^2}{z^2}.
\end{aligned}
\tag{56}
$$

Note that the kernel is a sum of the "running-coupling" part proportional to $b=\frac{11}{3}N_c-\frac{2}{3}n_f$ and the conformal part; see the discussion in Ref. [4]. Here $\alpha_s\equiv\alpha_s(\mu)$ and μ is the normalization point in the $\overline{\mathrm{MS}}$ scheme.

With the k_T factorization in view, let us rewrite the evolution equation (54) in terms of

$$
\mathcal{V}_{a_m}(z_\perp)\equiv-\partial_\perp^2\langle\langle\hat{\mathcal{U}}_{a_m}(z_\perp,0)\rangle\rangle, \tag{57}
$$

proportional to the dipole unintegrated gluon distribution $\mathcal{D}(x_B,z_\perp,\mu)$,

$$
\mathcal{V}_{x_B}(z_\perp,\mu)=\frac{4\pi^2x_B}{N_c}\alpha_s(\mu)\mathcal{D}(x_B,z_\perp,\mu), \tag{58}
$$

where

$$
\begin{aligned}
&\mathcal{D}(x_B,z_\perp,\mu)\\
&\quad\equiv\Big(\frac{2}{s}\Big)^2\int\frac{dz_*}{\pi x_B}\Big\langle p|\mathrm{Tr}\Big\{\Big[\infty p_1+z_\perp,\frac{2}{s}z_*p_1+z_\perp\Big]\\
&\qquad\times\hat{F}_{\bullet\xi}\Big(\frac{2}{s}z_*p_1+z_\perp\Big)\Big[\frac{2}{s}x_*p_1+z_\perp,-\infty p_1+z_\perp\Big]\\
&\qquad\times[-\infty p_1,0]\hat{F}_\bullet{}^\xi(0)[0,\infty p_1]\Big\}|p\Big\rangle^{a_m=x_B}.
\end{aligned}
\tag{59}
$$

Hereafter we use the notation

$$
[x,y]\equiv\mathrm{P}\exp\Big\{ig\int_0^1du(x-y)^\mu A_\mu(ux+\bar{u}y)\Big\}
$$

for the gauge link connecting points x and y. The color dipole is renorm-invariant so $\mathcal{D}$ depends on μ to compensate $g^2(\mu)$ dependence.

The Fourier transform

$$
\mathcal{D}(x_B,k_\perp)=\int dz_\perp e^{i(k,z)\perp}\mathcal{D}(x_B,z_\perp)
$$

is called the dipole gluon TMD (transverse momentum dependent distribution). Note, however, that the dipole gluon TMD defined above differs from the definition

$$
\begin{aligned}
&\tilde{\mathcal{D}}(x_B,z_\perp,\mu,\eta)\\
&\quad=\Big(\frac{2}{s}\Big)^2\int\frac{dz_*}{\pi x_B}e^{-ix_Bz_*}\Big\langle p|\mathrm{Tr}\Big\{\Big[\infty p_1+z_\perp,\frac{2}{s}z_*p_1+z_\perp\Big]\\
&\qquad\times\hat{F}_{\bullet\xi}\Big(\frac{2}{s}z_*p_1+z_\perp\Big)\Big[\frac{2}{s}x_*p_1+z_\perp,-\infty p_1+z_\perp\Big]\\
&\qquad\times[-\infty p_1,0]\hat{F}_\bullet{}^\xi(0)[0,\infty p_1]\Big\}|p\Big\rangle^\eta,
\end{aligned}
\tag{60}
$$

which reduces to the usual parton density at $z_\perp=0$. It should be emphasized that Eq. (60) is a more complex operator than (59). The difference is especially clear in the case of $\mathcal{N}=4$ theory: the dipole gluon TMD (59) is UV finite while Eq. (60) is UV divergent so it needs additional UV counterterms; see the discussion in [22]. [These UV divergent terms are directly proportional to x_B so they vanish for the definition (59).] Also, the role of parameter x_B is different in the two definitions: in Eq. (59) it is defined as a rapidity cutoff a_m while in Eq. (60) the rapidity cutoff η should be imposed separately from x_B.

Differentiating Eq. (54) two times with respect to z we obtain the NLO BFKL evolution for dipole gluon TMD (59) in the form

$$2a\frac{d}{da}\mathcal{V}_a(z)=\frac{\alpha_s N_c}{\pi^2}\int d^2z'\Big\{\Big(1+\frac{\alpha_s b}{4\pi}\Big[\ln\frac{z^2\mu^2}{4}+2C+\frac{67N_c}{9b}-\frac{\pi^2N_c}{3b}-\frac{10n_f}{9b}\Big]\Big)\Big[\frac{\mathcal{V}_a(z')}{(z-z')^2}-\frac{(zz')\mathcal{V}_a(z)}{z'^2(z-z')^2}\Big]$$
$$+\frac{\alpha_s b}{2\pi}\frac{\mathcal{V}_a(z')-\mathcal{V}_a(z)}{(z-z')^2}\ln\frac{(z-z')^2}{z'^2}+\frac{\alpha_s N_c}{4\pi}\Big[-\frac{\ln^2(z^2/z'^2)}{(z-z')^2}+F(z,z')+\Phi(z,z')\Big]\mathcal{V}_a(z')\Big\}$$
$$+3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{V}_a(z). \tag{61}$$

Next we need to perform the Fourier transformation of Eq. (61). It can be demonstrated that

$$\int\frac{d^2qd^2q'}{4\pi^2}e^{i(q,z)-i(q',z')}\Big[-\frac{\ln^2(q^2/q'^2)}{(q-q')^2}+F(q,q')+\Phi(q,q')\Big]=-\frac{\ln^2(z^2/z'^2)}{(z-z')^2}+F(z,z')+\Phi(z,z') \tag{62}$$

so the conformal part of the kernel looks the same in coordinate and momentum representations.

Performing also the Fourier transformation of the running-coupling part, one obtains the momentum-representation kernel in the form

$$2a\frac{d}{da}\mathcal{V}_a(k)=\frac{\alpha_s N_c}{\pi^2}\int d^2k'\Big\{\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2}-\frac{(k,k')\mathcal{V}_a(k)}{k'^2(k-k')^2}\Big]\Big(1+\frac{\alpha_s b}{4\pi}\Big[\ln\frac{\mu^2}{k^2}+\frac{N_c}{b}\Big(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\Big)\Big]\Big)$$
$$-\frac{b\alpha_s}{4\pi}\Big[\frac{\mathcal{V}_a(k')}{(k-k')^2}\ln\frac{(k-k')^2}{k'^2}-\frac{k^2\mathcal{V}_a(k)}{k'^2(k-k')^2}\ln\frac{(k-k')^2}{k^2}\Big]$$
$$+\frac{\alpha_s N_c}{4\pi}\Big[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2}+F(k,k')+\Phi(k,k')\Big]\mathcal{V}_a(k')\Big\}+3\frac{\alpha_s^2N_c^2}{2\pi^2}\zeta(3)\mathcal{V}_a(k), \tag{63}$$

where $\mathcal{V}(k)\equiv\int dz e^{-i(k,z)_\perp}\mathcal{V}(z)$.

In terms of Mellin projections ($\gamma\equiv\frac{1}{2}+i\nu$ as usual)

$$\mathcal{V}(k)=\sum_{n=0}^{\infty}\int\frac{d\nu}{2\pi^2}\mathcal{V}(n,\nu)(k^2)^{\gamma-1}(\tilde{k}/\bar{k})^{n/2}$$
$$\mathcal{V}(n,\nu)=\int d^2k(k^2)^{-\gamma}(\bar{k}/\tilde{k})^{n/2}\mathcal{V}(k)$$
$$=4\pi i^n\frac{\Gamma(\bar{\gamma}+\frac{n}{2})}{\Gamma(\gamma+\frac{n}{2})}\int d^2z(z^2)^{\gamma-1}\Big(\frac{\bar{z}}{\tilde{z}}\Big)^{n/2}\mathcal{V}(z), \tag{64}$$

the kernel (63) takes the form

$$2a\frac{d}{da}\mathcal{V}_a(n,\nu)$$
$$=\frac{\alpha_s N_c}{\pi}\Big\{\chi(n,\gamma)+\frac{\alpha_s N_c}{4\pi}\Big[\frac{b}{N_c}\chi(n,\gamma)\Big(\ln\mu^2+\frac{d}{d\gamma}$$
$$-\frac{\chi(n,\gamma)}{2}+\frac{\chi'(n,\gamma)}{2\chi(n,\gamma)}\Big)+f(n,\gamma)\Big]\Big\}\mathcal{V}_a(n,\nu). \tag{65}$$

Here

$$f(n,\gamma)=\Big[\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\Big]\chi(n,\gamma)-\chi''(n,\gamma)$$
$$+F(n,\gamma)-2\Phi(n,\gamma)-2\Phi(n,1-\gamma)+6\zeta(3), \tag{66}$$

where

$$F(n,\gamma)=\Big\{-\Big[3+\Big(1+\frac{n_f}{N_c^3}\Big)\frac{2+3\gamma\bar{\gamma}}{(3-2\gamma)(1+2\gamma)}\Big]\delta_{0n}$$
$$+\Big(1+\frac{n_f}{N_c^3}\Big)\frac{\gamma\bar{\gamma}}{2(3-2\gamma)(1+2\gamma)}\delta_{2n}\Big\}$$
$$\times\frac{\pi^2\cos\pi\gamma}{(1-2\gamma)\sin^2\pi\gamma}$$

and

$$\Phi(n,\gamma)=\int_0^1\frac{dt}{1+t}t^{\gamma-1+\frac{n}{2}}\Big\{\frac{\pi^2}{12}-\frac{1}{2}\psi'\Big(\frac{n+1}{2}\Big)-\mathrm{Li}_2(t)$$
$$-\mathrm{Li}_2(-t)-\Big[\psi(n+1)-\psi(1)+\ln(1+t)$$
$$+\sum_{k=1}^{\infty}\frac{(-t)^k}{k+n}\Big]\ln t-\sum_{k=1}^{\infty}\frac{t^k}{(k+n)^2}[1-(-1)^k]\Big\}. \tag{67}$$

To compare to NLO BFKL from Ref. [2], one should rewrite the above equation in terms of

$$\mathcal{L}(k)=\frac{1}{g^2(k)}\mathcal{V}(k) \tag{68}$$

since two gluons in the dipole $\mathcal{U}(k)$ come with an extra g^2 factor. Equation (65) turns into

$$2a\frac{d}{da}\mathcal{L}_a(k) = \frac{\alpha_s(k^2)N_c}{\pi^2}\int d^2k'\Bigg\{\Bigg[\frac{\mathcal{L}_a(k')}{(k-k')^2} - \frac{(k,k')\mathcal{L}_a(k)}{k'^2(k-k')^2}\Bigg]\Bigg[1+\frac{\alpha_s N_c}{4\pi}\Big(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\Big)\Bigg] - \frac{b\alpha_s}{4\pi}\Bigg[\frac{\mathcal{L}_a(k')}{(k-k')^2} - \frac{k^2\mathcal{L}_a(k)}{k'^2(k-k')^2}\Bigg]\ln\frac{(k-k')^2}{k^2} + \frac{\alpha_s N_c}{4\pi}\Bigg[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2} + F(k,k') + \Phi(k,k')\Bigg]\mathcal{L}_a(k')\Bigg\} + 3\frac{\alpha_s^2 N_c^2}{2\pi^2}\zeta(3)\mathcal{L}_a(k), \tag{69}$$

with the eigenvalues

$$\frac{\alpha_s(k^2)N_c}{\pi^2}\int d^2k'\Bigg\{\Bigg[\frac{(k'^2/k^2)^{-\bar\gamma}e^{in\phi}}{(k-k')^2} - \frac{(k,k')}{k'^2(k-k')^2}\Bigg]\Bigg[1+\frac{\alpha_s N_c}{4\pi}\Big(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10n_f}{9N_c}\Big)\Bigg] - \frac{b\alpha_s}{4\pi}\Bigg[\frac{(k'^2/k^2)^{-\bar\gamma}e^{in\phi}}{(k-k')^2} - \frac{k^2}{k'^2(k-k')^2}\Bigg]\ln\frac{(k-k')^2}{k^2} + \frac{\alpha_s N_c}{4\pi}\Bigg[-\frac{\ln^2(k^2/k'^2)}{(k-k')^2} + F(k,k') + \Phi(k,k')\Bigg](k'^2/k^2)^{-\bar\gamma}e^{in\phi}\Bigg\} + 3\frac{\alpha_s^2 N_c^2}{2\pi^2}\zeta(3)$$
$$= \frac{\alpha_s(k^2)N_c}{\pi}\Bigg\{\chi(n,\gamma) + \frac{\alpha_s N_c}{4\pi}\Bigg[-\frac{b}{2N_c}[\chi^2(n,\gamma)+\chi'(n,\gamma)] + f(n,\gamma)\Bigg]\Bigg\}, \tag{70}$$

which coincide with the eigenvalues of the kernel [2] of the partial wave of the forward Reggeized gluon scattering amplitude

$$\omega G_\omega(q,q') = \delta^{(2)}(q-q') + \int d^2p K(q,p)G_\omega(p,q'), \tag{71}$$

$$\int d^2p\Big(\frac{p^2}{q^2}\Big)^{\gamma-1}e^{in\phi}K(q,p) = \frac{\alpha_s(q)}{\pi}N_c\Bigg[\chi(n,\gamma) + \frac{\alpha_s N_c}{4\pi}\Bigg(f(n,\gamma) - \frac{b}{2N_c}[\chi'(n,\gamma)+\chi^2(n,\gamma)]\Bigg)\Bigg]. \tag{72}$$

This is somewhat surprising since the evolution of the composite (in $\mathcal{N}=4$ SYM—conformal) dipole with respect to a gives the evolution of a forward Reggeized gluon scattering amplitude with respect to rapidity η (of which ω is the Mellin transform). To illustrate the transition between the two evolutions let us consider the calculation of the dipole evolution directly from the NLO BFKL for Reggeized gluons.

The impact factor $\Phi_A(q)$ for the color dipole $\mathcal{U}(x,y)$ is proportional to $\alpha_s(q)(e^{iqx}-e^{iqy})(e^{-iqx}-e^{-iqy})$ so one obtains the cross section of the scattering of color dipole in the form

$$\mathcal{U}^\eta(x) = \int\frac{d^2q}{q^2}\frac{d^2q'}{q'^2}\frac{\alpha_s(q)}{4\pi^2}(e^{iqx}-1)(e^{-iqx}-1)\Phi_B(q') \times\int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\Big(\frac{se^\eta}{qq'}\Big)^\omega G_\omega(q,q'). \tag{73}$$

$\Phi_B(q')$ is the target impact factor. To get the evolution equation with respect to rapidity one should change the energy scale to q'^2.

$$\mathcal{U}^\eta(x) = \int\frac{d^2q}{q^2}\frac{d^2q'}{q'^2}\frac{\alpha_s(q)}{4\pi^2}(e^{iqx}-1)(e^{-iqx}-1)\Phi_B(q') \times\int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\Big(\frac{se^\eta}{q'^2}\Big)^\omega\tilde G_\omega(q,q'), \tag{74}$$

where $\tilde G_\omega(q,q')$ is the modified kernel with eigenvalues shifted by $2\chi(n,\nu)\chi'(n,\nu)\frac{\alpha_s^2N_c^2}{4\pi^2}$; see Ref. [2]. The corresponding equation for $\mathcal{V}^\eta(n,\nu)$ takes the form [11]

$$\frac{d}{d\eta}\mathcal{V}^\eta(n,\nu) = \frac{\alpha_s N_c}{\pi}\Bigg\{\chi(n,\gamma) + \frac{\alpha_s N_c}{4\pi}\Bigg[\frac{b}{N_c}\chi(n,\gamma) \times\Big(\ln\mu^2 + \frac{d}{d\gamma} - \frac{\chi(n,\gamma)}{2} + \frac{\chi'(n,\gamma)}{2\chi(n,\gamma)}\Big) + f(n,\gamma) + 2\chi(n,\nu)\chi'(n,\nu)\Bigg]\Bigg\}\mathcal{V}^\eta(n,\nu). \tag{75}$$

Let us demonstrate that it agrees with Eq. (65). The Mellin projection of composite dipole can be obtained from Eq. (17):

$$\mathcal{V}^a(n,\nu) = \left(1 + \frac{\alpha_s N_c}{4\pi}\left[\chi^2(n,\gamma) + 3\chi'(n,\gamma) + 2\chi(n,\gamma)\left(\ln a - 2\eta + 2C - \frac{d}{d\gamma}\right)\right]\right)$$
$$+ \left(\frac{\alpha_s N_c}{4\pi}\right)^2\left\{2\chi^2(n,\gamma)\left(\ln a - 2\eta - \frac{d}{d\gamma}\right)^2 + 2(\ln a - 2\eta)\left[\frac{b}{N_c}\chi(n,\gamma)\left(\ln\mu^2 + \frac{d}{d\gamma} - \frac{\chi(n,\gamma)}{2} + \frac{\chi'(n,\gamma)}{2\chi(n,\gamma)}\right)\right.\right.$$
$$\left.\left. + f(n,\gamma) + 3\chi(n,\gamma)\chi'(n,\gamma) + \chi^3(n,\gamma)\right] + X(n,\nu)\right\}\mathcal{V}^\eta(n,\nu). \quad (76)$$

The first-order term can be derived from Eq. (17) while the term $\sim\alpha_s^2$ can be restored from the condition $\frac{d}{d\eta}\mathcal{V}^a(n,\nu) = 0$ up to an unknown function $X(n,\nu)$ which requires a next-to-next-to-leading order calculation (and does not contribute to the NLO evolution). Now one can see that the derivative with respect to a gives Eq. (65). Thus, the transition between the evolution of the composite dipole $\mathcal{V}^a(n,\nu)$ with respect to a and the rapidity evolution of the dipole $\mathcal{V}^\eta(n,\nu)$ correspond to the shift in eigenvalues on the function $2\chi(n,\nu)\chi'(n,\nu)\frac{\alpha_s^2 N_c^2}{4\pi^2}$—the same transition that describes the shift of eigenvalues when going from energy scale qq' to q'^2 in formula (74).

VII. CONCLUSIONS

Let us present again the k_T-factorization formula for DIS in the next-to-leading order:

$$\int d^4x e^{iqx}\langle p|T\{\hat{j}_\mu(x)\hat{j}_\nu(0)\}|p\rangle$$
$$= \frac{s}{2}\int\frac{d^2k_\perp}{k_\perp^2} I_{\mu\nu}(q,k_\perp)\mathcal{V}_{a_m=x_B}(k_\perp), \quad (77)$$

where $I_{\mu\nu}(q,k_\perp)$ is given by Eq. (51) and the evolution equation for $\mathcal{V}_a(k_\perp)$ by Eq. (63) [or Eq. (65) in the Mellin representation].

The analytic NLO photon impact factor in momentum space for the pomeron contribution (51) and the NLO k_T-factorization formula (77) for the deep inelastic scattering are the main results of this paper.

Since the composite dipole (68) obeys the same equation as the forward scattering amplitude of two Reggeized gluons (71) the impact factor (51) may be obtained as a NLO amplitude of emission of two Reggeized gluons by the virtual photon. There were several attempts in the literature to obtain this amplitude [23], but at present such an impact factor is known only as a combination of analytical and numerical results [24]. Indeed, in Ref. [25] the $\gamma^* - \gamma^*$ cross section has been calculated using only the LO impact factor and the LO and NLO BFKL amplitude for two Reggeized gluons. The authors explain in the paper that the NLO impact factor known at present in Ref. [24] is difficult to handle for a numerical calculation since it is not in a full analytic form. On the other hand, the result of this paper, provided that one knows the solution of the NLO BFKL with the running-coupling constant, allows us to compute the full NLO total cross section for the $\gamma^* - \gamma^*$ scattering process.

An attempt to calculate the NLO impact factor in an analytic form using an approach based on the analytic properties of the amplitude can be found in Ref. [26].

In the past few years, there has been some activity on the calculation of the NLO impact factor of other processes as well: in Ref. [27] the calculation of the NLO impact factor for Mueller-Navelet jets has been performed, while the impact factor for the virtual photon to light vector meson transition has been performed in Ref. [28].

It would be also instructive to compare our result (19) for the coefficient in front of the four-Wilson-line operator (relevant for the structure functions of DIS off a large nucleus) to similar results for the NLO impact factor obtained recently in Ref. [29] using the dipole model. However, as we already mentioned, our final NLO result (51) is defined as a coefficient function in front of a composite operator (17) defined with a counterterm which restores the conformal invariance in $\mathcal{N} = 4$ amplitudes and in our case leads to the conformal impact factor (since the impact factor is given by tree diagrams it should be conformally invariant even in QCD). As a consequence, the impact factor depends on a new parameter a (an analog of the factorization scale μ in the usual OPE) which we chose in such a way that all the energy dependence is shifted in to the matrix element, leaving the impact factor energy scale invariant. To compare with the result of Ref. [29] representing the coefficient function of a usual dipole (without the counterterm subtraction), we should trace one step back and look at the impact factor $I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3;\eta)$ given by (16). One should then perform Fourier transformation to momentum space with respect to the positions x and y of the two electromagnetic currents in formula (7) and compare it to the result (58) from Ref. [29] integrated over z_1 (and cz_2 when appropriate). Hopefully, after these integrations the two results will coincide.

ACKNOWLEDGMENTS

This work was supported by Contract No. DE-AC05-06OR23177 under which the Jefferson Science Associates, LLC operate the Thomas Jefferson National Accelerator Facility, and by Grant No. DE-AC02-05CH11231.

APPENDIX

There is a subtle point in the Fourier transformation of Eq. (38) in the forward case (cf. Ref. [20]). To illustrate it, consider the simplest term in the right-hand side of Eq. (23)

$$-\frac{\partial\kappa^\alpha}{\partial x^\mu}\frac{\partial\kappa^\beta}{\partial y^\nu}\frac{2}{\kappa^2}\left(g^{\alpha\beta}-2\frac{\kappa^\alpha\kappa^\beta}{\kappa^2}\right)\mathcal{R}^2 = \frac{2\mathcal{R}^2}{(x-y)^2}\left(g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right). \tag{A1}$$

The corresponding contribution to the T product of currents (21) is proportional to

$$\int\frac{dz_1dz_2}{z_{12}^4}\hat{\mathcal{U}}(z_1,z_2)\frac{\mathcal{R}^2}{(x-y)^6}\left[g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right]. \tag{A2}$$

Consider now the Fourier transform of this equation for the case of forward scattering

$$\int d^4xd^4y\delta(y_\bullet)e^{iq\cdot(x-y)}\int\frac{dz_1dz_2}{z_{12}^4}\frac{\mathcal{R}^2}{(x-y)^6} \times\left[g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right]\mathcal{U}(z_{12}), \tag{A3}$$

where $\mathcal{U}(z_{12})=\langle\hat{\mathcal{U}}(z_1,z_2)\rangle$. We have calculated such integrals by using the representation of the type

$$\mathcal{R}^2=\int\frac{d\nu}{\pi^2}\nu^2r(\nu)\int d^2z_0\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^\gamma\left[\frac{\kappa^2}{(2\kappa\cdot\zeta_0)^2}\right]^{\bar\gamma}$$

based on the decomposition (35) of transverse δ functions. [For this example $r(\nu)=B(\gamma,\gamma)\Gamma(1+\gamma)\Gamma(2-\gamma)$.] The integral over z_1 and z_2 in Eq. (A3) is of the form

$$\int\frac{dz_1dz_2}{z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\frac12+i\nu}f(z_{12}^2) = \int\frac{d\mu}{2\pi}\int\frac{dz_1dz_2}{z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\frac12+i\nu}(z_{12}^2)^{\frac12-i\mu}f(\mu), \tag{A4}$$

where $f(\mu)=\int dz_{12}^2(z_{12}^2)^{-\frac32+i\mu}f(z_{12}^2)$. To calculate the integral in the right-hand side this equation, we take the orthogonality condition for conformal eigenfunctions [20]

$$\int\frac{d^2z_1d^2z_2}{z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\frac12+i\nu}\left(\frac{z_{12}^2}{z_1^2z_2^2}\right)^{\frac12-i\mu} = \left[\delta(\nu-\mu)\delta^{(2)}(z_0)+\delta(\nu+\mu)(z_0^2)^{-1+2i\mu} \times\frac{2i\mu B(\frac12+i\mu)}{\pi B(\frac12-i\mu)}\right]\frac{\pi^4}{2\nu^2} \tag{A5}$$

and perform the inversion $z_i\to\frac{z_i}{z^2}$. We obtain

$$\int\frac{d^2z_1d^2z_2}{z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\frac12+i\nu}(z_{12}^2)^{\frac12-i\mu} = \frac{\pi^4}{2\nu^2}\left[\delta(\nu-\mu)(z_0^2)^{-1-2i\nu}\delta^{(2)}\left(\frac{1}{z_0}\right) + \delta(\nu+\mu)\frac{2i\mu B(\frac12+i\mu)}{\pi B(\frac12-i\mu)}\right] \tag{A6}$$

so Eq. (A4) turns into

$$\int\frac{dz_1dz_2}{z_{12}^4}\left(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\right)^{\frac12+i\nu}f(z_{12}^2) = \frac{\pi^3}{4\nu^2}(z_0^2)^{-1-2i\nu}f(\nu)\delta^{(2)}\left(\frac{1}{z_0}\right)-\frac{i\pi^2B(\frac12-i\nu)}{2\nu B(\frac12+i\nu)}f(-\nu). \tag{A7}$$

Substituting this equation with $f(z_{12}^2)=\mathcal{U}(z_{12})=z_{12}^2\mathcal{V}(z_{12})$ into Eq. (A3) we get

$$\frac{\pi}{4}\int d^4xd^4y\delta(y_\bullet)\frac{e^{iq\cdot(x-y)}}{(x-y)^6}\left[g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right] \times r(\nu)\left\{\left[\frac{(x-y)^2x_*y_*}{(x_*-y_*)^2}\right]^{\bar\gamma}\mathcal{V}(\nu) + \frac{B(\bar\gamma)}{B(\gamma)}\left[\frac{(x-y)^2x_*y_*}{(x_*-y_*)^2}\right]^{\gamma}\mathcal{V}(-\nu)\right\}, \tag{A8}$$

where $V(\nu)\equiv\frac{1}{\pi}\int d^2z(z^2)^{-\frac12+i\nu}\mathcal{V}(z)$. Here the first term comes from $z_0=\infty$ while the second comes from finite z_0. It is easy to see that the two terms coincide after a change of the integration variable $\nu\leftrightarrow-\nu$ so effectively the contribution of the integral over finite z_0 is doubled:

$$\int d^4xd^4y\delta(y_\bullet)e^{iq\cdot(x-y)}\int\frac{dz_1dz_2}{z_{12}^4}\frac{\mathcal{R}^2}{(x-y)^6} \times\left[g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right]\mathcal{U}(z_{12}) = \frac{\pi}{2}\int d^4xd^4y\delta(y_\bullet)\left[g_{\mu\nu}-2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right] \times r(\nu)\frac{B(\bar\gamma)}{B(\gamma)}\frac{e^{iq\cdot(x-y)}}{(x-y)^6}\left[\frac{(x-y)^2x_*y_*}{(x_*-y_*)^2}\right]^{\gamma}V(-\nu). \tag{A9}$$

Since we have not used the explicit form of the Lorentz structure in μ and ν indices, it is clear that the doubling effect is general for any contribution to the forward Fourier transform of Eq. (46) (see also the discussion of zero transfer momentum limit in Ref. [20]).

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