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COMMENTARY

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Key Points:

- We review the conclusion from Sydorenko and Rankin (2017) on the existence of the ionospheric feedback instability in Earth's ionosphere
- We show that this conclusion contradicts to numerous theoretical and observational studies of this phenomena
- We present arguments supporting the existence of IFI in the Earth's magnetosphere-ionosphere system

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On the Existence of Ionospheric Feedback Instability in the Earth's Magnetosphere-Ionosphere System

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Abstract The ionospheric feedback instability (IFI) has been considered one of the main generation mechanisms for large-amplitude ultralow frequency waves and small-scale field-aligned currents in the auroral and subauroral regions for more than 40 years. Sydorenko and Rankin (2017, <https://doi.org/10.1002/2017GL073415>) have recently challenged the very existence of the IFI for any realistic geophysical conditions in the Earth's ionosphere-magnetosphere system. Because this conclusion contradicts numerous theoretical, numerical, and experimental works successfully used IFI to explain and predict results from observations for more than four decades, it deserves special attention. We show that this conclusion is mainly based on the specific ionospheric density profile and boundary conditions used in two runs of simulations presented in Sydorenko and Rankin (2017), and the generalization of this result is not justified. The effect of the collisions between ionospheric ions and neutrals on the development of the instability has been well studied since 1981, and these studies demonstrate that it does not prevent the development of the instability. Furthermore, excellent agreement of the theoretical and numerical results with observations verify without doubt the IFI existence and significance in the Earth's magnetosphere-ionosphere system.

1. Introduction

The basic idea of IFI is that the field-aligned current (FAC) in the ultralow-frequency Alfvén waves interacting with the ionosphere changes the ionospheric density (conductivity) by precipitating or removing electrons into the *E* layer, and these variations in the conductivity *feedback* on the structure and amplitude of the incident wave and the corresponding current. When the large-scale background electric field exists in the ionosphere, the variations in density will change the wave reflection coefficient and the Joule dissipation of the background electric field in that particular location, which in turn, generates some additional FAC, contributing to the reflected current. When the Alfvén waves are trapped in some resonator cavity in the magnetosphere, the ionospheric feedback can work in a constructive way and generate large-amplitude waves and density disturbances in the *E* region.

The IFI was introduced by Atkinson (1970) and extensively studied after that analytically, numerically, and experimentally in the global magnetospheric resonator, formed by the entire magnetic flux tube with both boundaries in the ionosphere, and the ionospheric Alfvén resonator (IAR), formed by the ionospheric *E* region and a strong gradient in the Alfvén velocity at the altitude 0.5–1.0 R_E .

Because IFI operates in the ultralow frequency (ULF) range, it considers the conducting bottom of the ionosphere as a narrow layer where the density and the electric field are relatively uniform, because the size of the conducting portion of the ionosphere is always much less than the parallel wavelength. In that case, the simplest mathematical model of the ionospheric feedback mechanism can be given by two equations connecting the perpendicular electric field, \mathbf{E}_\perp , and the plasma density, n , in the ionosphere with the density of the FAC, j_\parallel . One is the Poisson equation, derived by integrating the current continuity equation, $\nabla \cdot \mathbf{j} = \mathbf{0}$, over the effective thickness of the ionospheric *E* region ($h \approx 10$ –12 km):

$$\nabla \cdot (\Sigma_p \mathbf{E}_\perp) = \pm j_\parallel, \quad (1)$$

and another is the ionospheric density continuity equation:

$$\frac{\partial n}{\partial t} = \frac{j_{\parallel}}{eh} - \alpha(n^2 - n_0^2). \quad (2)$$

Here $\Sigma_p = M_p n h e$ is the ion Pedersen mobility; e is the elementary charge; the “+” sign should be used in the Northern Hemisphere, and “-” sign should be used in the Southern Hemisphere; α is the coefficient of recombination; and the term αn^2 on the right-hand side of (2) represents losses due to the recombination, and the term αn_0^2 represents the unperturbed source of the ionospheric plasma, which provide an equilibrium state of the ionosphere.

The boundary condition given by equations (1) and (2) works as follows: On every time step, the magnetospheric part of the model provides j_{\parallel} on the magnetosphere-ionosphere interface, and equation (1) is solved to find the corresponding density, n . After that, new Σ_p is calculated, and equation (2) is used to find E_{\perp} in the ionosphere. This E_{\perp} is used in the magnetospheric part of the model to calculate j_{\parallel} on the next time step.

The model can be advanced further by including the Hall conductivity, additional ionization of the ionosphere by the energetic electrons precipitated in the upward current channel and the effects of the neutral wind. None of these effects changes the essence of the feedback mechanism.

This particular model of IFI has been extensively studied in a number of papers, (e.g., Lysak, 1991; Lysak & Song, 2002; Miura & Sato, 1980; Pokhotelov et al., 2000, 2001; Russell et al., 2013; Streltsov & Lotko, 2004, 2005; Trakhtengertz & Feldstein, 1981, 1991; Watanabe et al., 1993), which established three favorable conditions for the IFI development: (1) the large-scale electric field in the ionosphere, (2) the low ionospheric density in the E region, and (3) the *matching impedance* between the ionosphere and the magnetosphere (which means $\Sigma_p \approx \Sigma_A \equiv 1/\mu_0 v_A$, where v_A is the value of the Alfvén speed above the ionosphere).

Each of these conditions has a clear physical meaning. The large-scale electric field in the ionosphere provides the free energy for the IFI development driving the plasma convection flow that causes overreflection of Alfvén waves. The low ionospheric density (a) provides a low conductance of the E region, which allows the electric field generated by the dynamo processes in the equatorial magnetosphere to penetrate into the ionosphere and (b) reduces the effects of the recombination, which saturate the instability. The matching impedance condition promotes efficient exchange of ULF energy between the ionosphere and the magnetosphere. (It may be pointed out here that the matching impedance condition eliminates large-amplitude waves in the global magnetospheric resonator driven by the driver in the magnetosphere because, in this case, the energy leaks from the resonator through the ionosphere. But the same condition promotes the development of large-amplitude waves in the same resonator when the driver is in the ionosphere.)

Thus, theoretical and numerical studies predict that IFI can develop in the nighttime, during winter season, or in the close vicinity of bright auroral arcs, where the return/downward currents deplete the ionospheric density by removing electrons from the E region and induces there a strong electric field. These predictions are in a good quantitative agreement with the observations of ULF waves conducted with ground magnetometers, sounding rockets, and satellites.

2. Discussion of SR17

The Sydorenko and Rankin (2017; hereafter, SR17) paper presents results from two runs of simulations of 2-D the magnetohydrodynamic model. The model includes perpendicular motion of the ions in the ionosphere and effects of the collisions between ions and neutrals. The simulations are performed for one particular profile of the ionospheric density shown in Figure 1a. In one case, the ionosphere is treated as a narrow conducting slab where collisions occur uniformly through the thickness of the slab. In another, the ionosphere is considered as a distributed medium with the inhomogeneous profile of the collision frequency over a height of 50 km. This inhomogeneity causes shear in the perpendicular velocity of the ions and, as a result, localized disturbances in the ionospheric plasma density may be *smoothed out* with altitude.

Based on the results from these two simulation runs, where the instability had been observed in the first run of simulation and had not been observed in the second run, SR17 concludes that “... the ionospheric feedback instability (IFI) does not develop in an E layer plasma with densities resolved in altitude” and suggests that “... the instability cannot occur in Earth’s ionosphere because ion-neutral collision frequencies always have a significant variation with altitude through the E layer.”

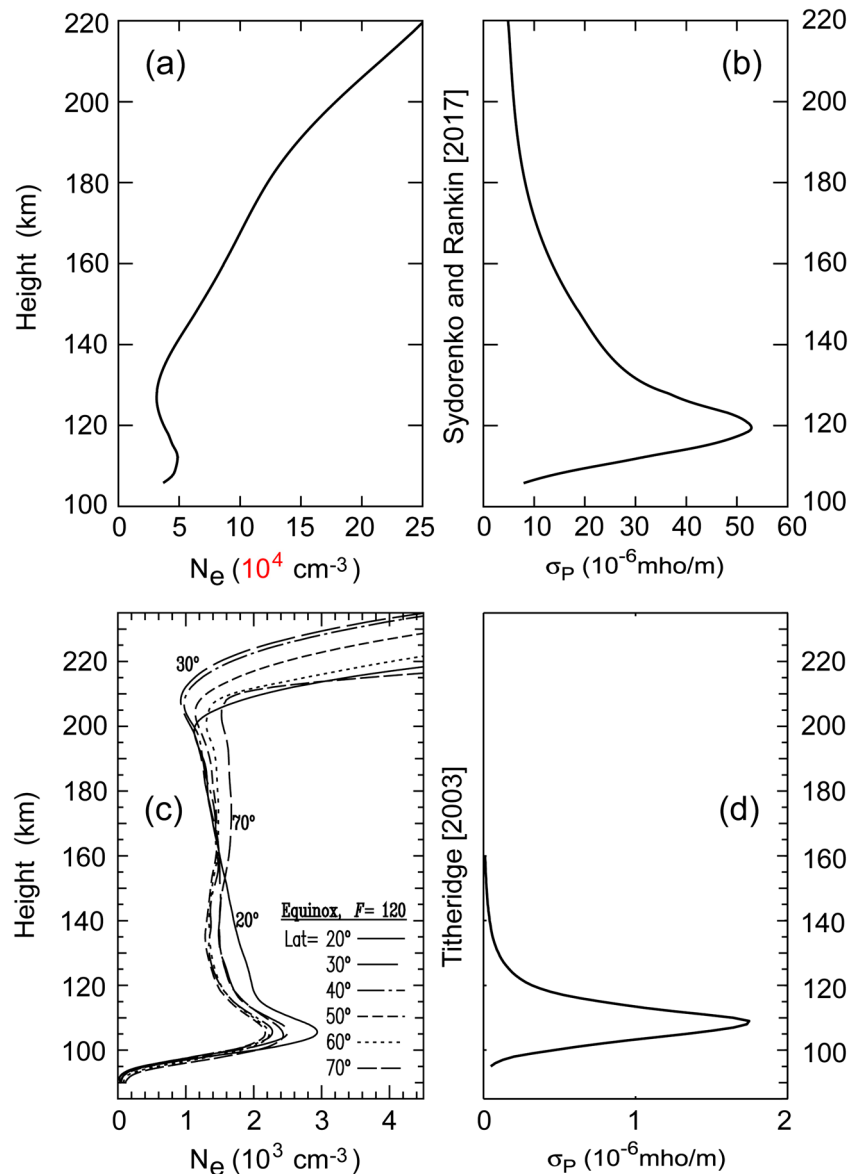


Figure 1. Density (a) and the Pedersen conductivity (b) used in SR17. Density (c) and the Pedersen conductivity (d) in the night time ionosphere according to Titheridge (2003).

That conclusion is not justified by the materials presented in the paper, and there are several reasons why. First of all, the effect of altitude-dependent ion-neutral collision frequency in the ionosphere on the development of IFI has been rigorously studied by Trakhtengertz and Feldstein (1981, 1984, 1991; hereafter, TF81, TF84, and TF91, respectively). These papers are not mentioned in SR17, but they demonstrate that the shear in the ion velocity due to the collisions with neutrals is extremely important for the development of the instability. In particular, TF91 stressed that “The collisions between charged and neutral particles result in the deceleration of magnetospheric convection at the altitude of the ionospheric dynamo region where the ion-neutral collision frequency exceeds the ion gyrofrequency. As a result, an inhomogeneous altitude profile of the convection velocity is formed, thereby leading, as is known from hydrodynamics, to the development of various instabilities ...”

TF81 and TF91 explain in detail how the collisions change the growth rate and the threshold for the instability. The instability criterion for the IAR frequency $f \approx 0.3\text{--}1.0$ Hz and the perpendicular wavelength $\lambda_{\perp} \geq 1$ km can be found after averaging equation (2) in TF91 over the effective height of the E region assuming the exponential dependence of the ion-neutral collision frequency on the altitude. For example, in the

low-density case, for $\lambda_{\perp} = 1$ km and $f = 0.5$ Hz the threshold value of the electric field in the ionosphere can be estimated from equation (7) in TF91:

$$E_{\text{th}} = E_0 \frac{\Omega_i}{\nu_i} \text{ [mV/m]}. \quad (3)$$

Here Ω_i is the ion gyrofrequency; $\nu_i > \Omega_i$ is the ion-neutral collision frequency at the bottom of the E layer; and E_0 is a numerical coefficient, which is equal to 25 in equation (7) in TF91. In fact, the value of E_0 should be somewhat greater because TF91 assumes that the E layer density is constant while integrating over the layer height. For the profile in Figure 1c, it means underestimating of E_0 by a factor of 2. At any rate, E_{th} is in the range of a few tens of millivolts per meter, and these values are quite reasonable for the magnitudes of the electric field observed in the high-latitude ionosphere during magnetically active time, particularly in the vicinity of discrete auroral arcs.

Also, TF81 and TF91 provided the boundary condition in the ionosphere (e.g., equations –(A11) in TF91), which describes IFI when the inhomogeneous altitude profile of the ion-neutral collision frequency is taken into account. It includes the tensor of the height-integrated ionospheric conductivity, $\hat{\Sigma}$, which depends on the collision and wave frequencies, the background electric field, and the perpendicular wave numbers. TF91's boundary conditions significantly differ from the boundary conditions used in the SR17 simulation. The boundary conditions given by equation (8) in SR17 postulates that the azimuthal component of the wave magnetic field is equal to 0 in the ionosphere. That means that no magnetic variations penetrate beneath the E layer and thus cannot be detected on the ground, which contradicts numerous observations of ULF waves associated with IAR by the magnetometers on the ground. Also, SR17 used different boundary conditions in the simulation with the height-integrated ionosphere (where the IFI is observed) and in the simulation with the distributed ionosphere (where the IFI is not observed). The different boundary conditions make these two models completely different from each other, and it is not appropriate to make any conclusions about occurrence of the IFI by comparing the results from these simulations.

The important conclusion from the analysis presented in TF81, TF84, and TF91 is that the shear in the perpendicular ion velocity due to the variation of the collision frequency with altitude changes the growth rate and the threshold of the ionospheric feedback instability but does not prevent it from developing. Obviously, the effect of the inhomogeneity of the collision frequency with altitude strongly depends on the density profile in the ionosphere, and the more the conductivity is localized in the narrow region, the less important this effect is.

To clarify this point, we compare the density and conductivity profiles used in the SR17 paper with the density and conductivity profiles observed in the nighttime ionosphere. The SR17 profiles are shown in Figures 1a and 1b and the nighttime density and conductivity profiles from Titheridge (2003) are shown in Figures 1c and 1d. The Pedersen conductivity profile shown in Figure 1d is calculated with the plasma density profile averaged between the profiles at 60° and 70° shown in Figure 1c, and the neutral atmosphere model from Mass Spectrometer and Incoherent Scatter radar model at 65° GLat. Figure 1 shows that the magnitude of the density used by SR17 is more 20 times larger than in the nighttime ionosphere, and also, the conductivity profile is more than 2 times wider.

Therefore, Figure 1 demonstrates that the parameters of the ionospheric density used in the SR17 paper are more typical for the daytime ionosphere with high density, when the IFI is not normally observed. According to Miura and Sato (1980) and Streltsov and Lotko (2005), it is reasonable to expect that IFI will not be developed for the density model used in SR17 paper, even when the ionosphere is treated as a height-integrated slab due to the recombination. But, despite its profound importance for IFI dynamics, the recombination was not included in the SR17 model.

Figure 1 also demonstrates that during the nighttime, the density in the ionosphere is low, and the Pedersen conductivity indeed is concentrated in the narrow slab with ≈ 10 -km effective height. In this case, effects of velocity shear are not important for the development of IFI, and the active ionospheric response on the magnetospheric FAC in ULF frequency range can be adequately described with equations (1) and (2).

3. Discussion and Conclusion

In general, the high-latitude ionosphere is a very dynamic medium with a complex structure, particularly during magnetically active times and in the vicinity of discrete auroral arcs. Besides, it is hard to measure the low

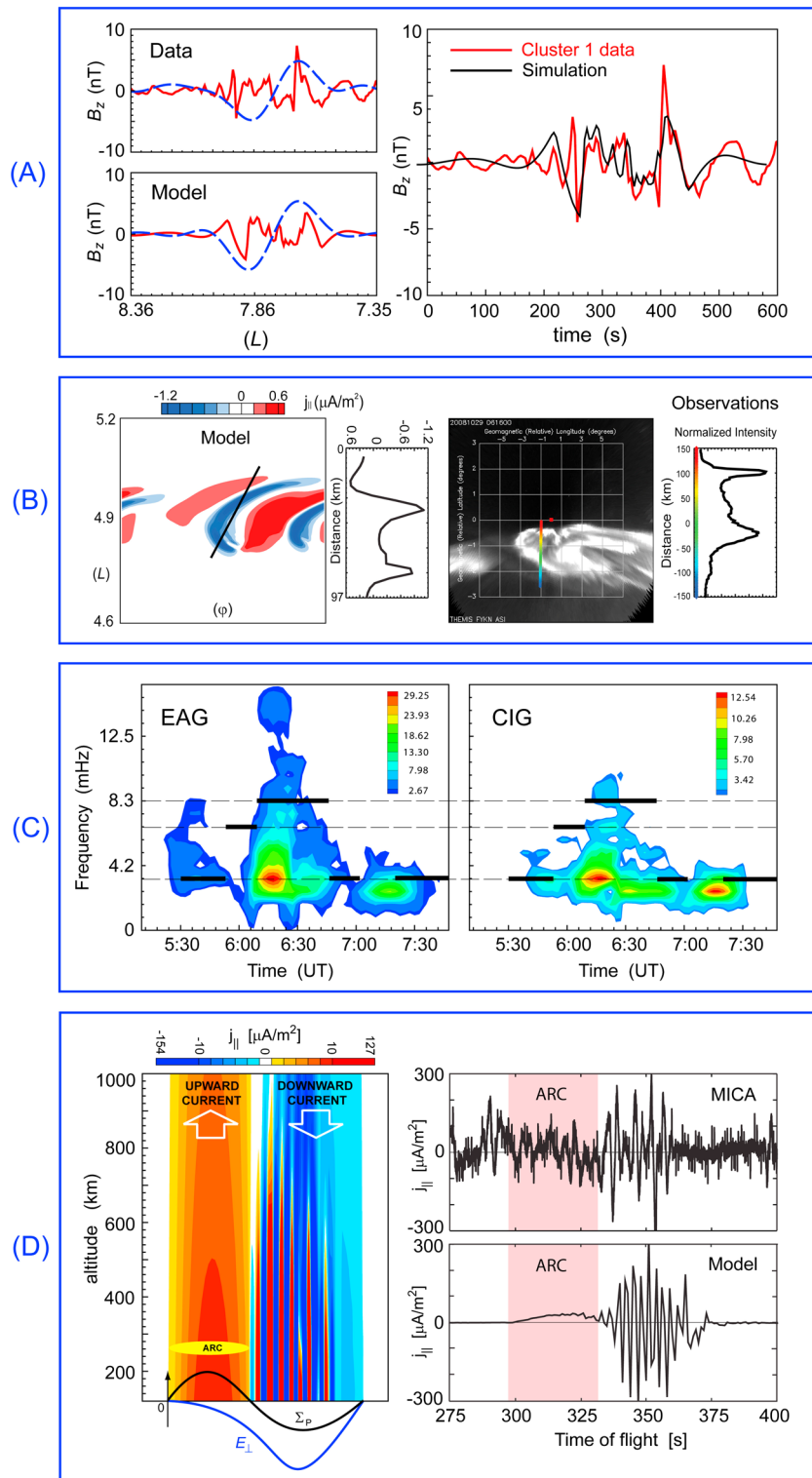


Figure 2. (A) Simulations of the data measured by the Cluster satellites (from Streltsov and Karlsson ; 2008). (B) Simulations of 3D dynamics of auroral arcs (from Jia and Streltsov ; 2014). (C) Prediction of the frequencies of ULF waves used in active experiments at High frequency Active Auroral Research Program to initiate a substorm (from Streltsov et al. 2011). (D) Prediction of the structure and location of small-scale ULF waves observed in the vicinity of discrete arcs by the MICA sounding rocket flight (from; Tulegenov & Streltsov, 2017).

density in the ionosphere using ground-based sensors (e.g., ionosondes operate at $f > 1$ MHz and measure the electron plasma density $> 1.24 \times 10^4 \text{ cm}^{-3}$). Therefore, the most reliable way to verify a complex, nonlinear, time-dependent, multidimensional numerical model and its basic assumptions is to compare the model results with the observations and to use the model to predict future observations. Some examples of successful explanations and predictions of ULF waves and currents obtained by the model given by equations (1) and (2) are

- the structure and amplitude of electromagnetic measurements from the Defense Meteorological Satellite Program satellites in the subauroral ionosphere (Streltsov & Mishin, 2003);
- the measurements of ULF waves by the Cluster satellites in the magnetosphere at the radial distance $5 R_E$ (Figure 2a; Streltsov & Karlsson, 2008);
- the dynamics of divergent electric fields in the downward current channels (Streltsov & Lotko, 2004);
- generation, spatial structure, and temporal dynamics of discrete auroral arcs (Figure 2b; Jia & Streltsov, 2014; Holzer & Sato, 1973; Miura & Sato, 1980; Sato & Holzer, 1973; Sato, 1978; Streltsov et al., 2012; Watanabe et al., 1993);
- the predicted frequencies of ULF waves excited in the ionospheric heating experiments conducted at High frequency Active Auroral Research Program (Figure 3c; Streltsov et al., 2011);
- the waves and currents observed by the Auroral Current and Electrodynamics Structure sounding rocket in the auroral zone (Cohen et al., 2013);
- the predicted conditions and locations of the strong ULF waves generated by the IFI inside the IAR near the discrete arcs. A special, dedicated sounding rocket experiment, Magnetosphere-Ionosphere Coupling in the Alfvén resonator, was conducted to verify that prediction in 2012. Results from the Magnetosphere-Ionosphere Coupling in the Alfvén resonator experiment and numerical modeling are in a good quantitative agreement (Figure 2d; Tulegenov & Streltsov, 2017).

Another argument supporting modeling of the ionosphere as a narrow conducting slab during the nighttime conditions came from the active experiments dealing with the generation of ULF waves by heating the ionospheric *D* and *E* regions with powerful high-frequency transmitters. These experiments are based on the assumption that heating produces a localized disturbance in the conductance, which generates waves/FACs when there is a large-scale electric field in the ionosphere (this is the so-called Getmantsev effect; Getmantsev et al., 1977). However, the SR17 paper concludes that any localized disturbance of conductivity always will be smoothed out by the shear in the ion drift motion, and if that conclusion were correct, then these experiments would not produce any positive results. Contrary to that, the active experiments involving generation of ULF waves have been successfully conducted for many years at all major heating facilities in the world: European Incoherent Scatter (Norway), SURA (Russia), and High frequency Active Auroral Research Program (USA), (e.g., Cohen et al., 2011, Papadopoulos et al., 2003).

In summary, more than 40 years of theoretical and experimental studies confirmed by the relevant observations provide a solid basis to conclude that contrary to the statement from SR17, the IFI is a well-established geophysical process existing in the Earth's magnetosphere-ionosphere system, which can be successfully modeled during the nighttime condition by considering the bottom of the ionosphere as a narrow conducting slab.

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